Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/46-1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [664]. This is test number [46].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (664)	0.00 (0)
Mathematica	99.70 (662)	0.30 (2)
Fricas	80.57 (535)	19.43 (129)
Maple	74.70 (496)	25.30 (168)
Giac	65.66 (436)	34.34 (228)
Mupad	54.22 (360)	45.78 (304)
Maxima	45.48 (302)	54.52 (362)
Sympy	38.25 (254)	61.75 (410)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

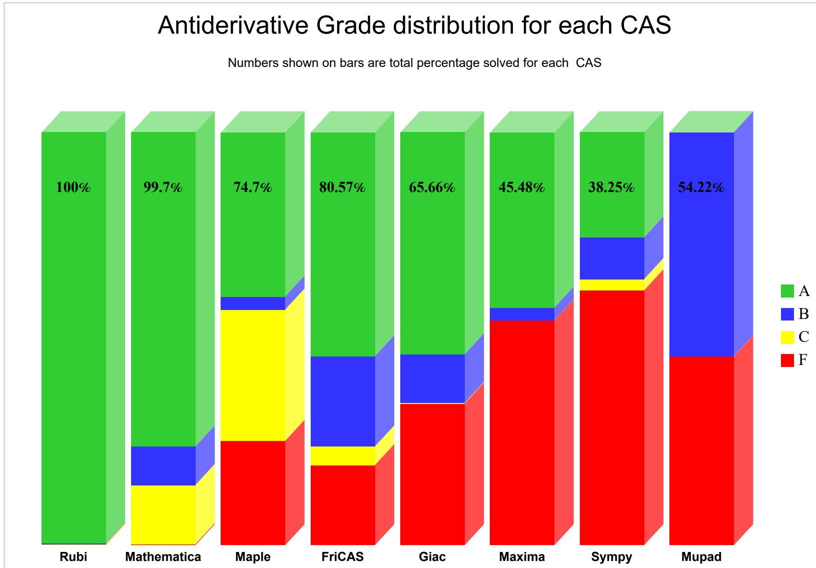
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

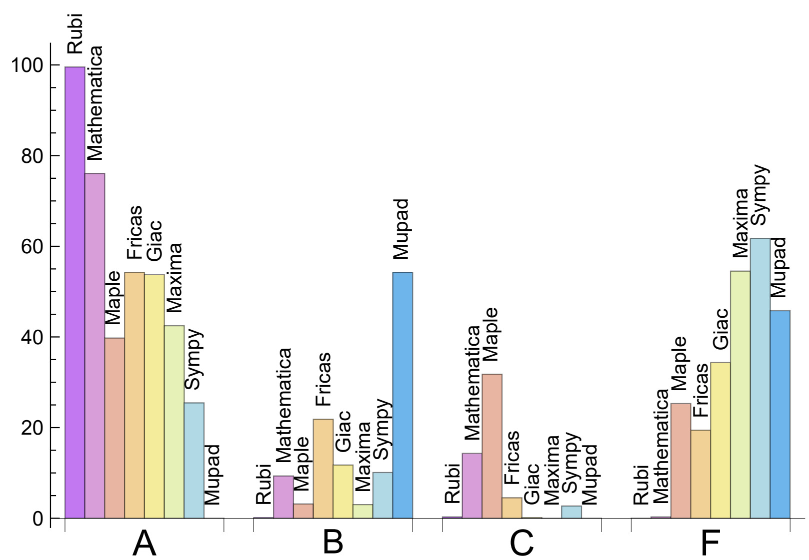
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.548	0.151	0.301	0.000
Mathematica	76.054	9.337	14.307	0.301
Fricas	54.217	21.837	4.518	19.428
Giac	53.765	11.747	0.151	34.337
Maxima	42.470	3.012	0.000	54.518
Maple	39.759	3.163	31.777	25.301
Sympy	25.452	10.090	2.711	61.747
Mupad	0.000	54.217	0.000	45.783

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	129	66.67	10.85	22.48
Maple	168	100.00	0.00	0.00
Giac	228	96.05	1.32	2.63
Mupad	304	0.00	100.00	0.00
Maxima	362	77.07	0.55	22.38
Sympy	410	76.34	23.17	0.49

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.11
Maxima	0.23
Fricas	0.31
Giac	0.35
Mathematica	0.71
Maple	1.54
Sympy	4.33
Mupad	6.55

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	81.13	0.80	55.50	0.76
Maple	138.26	1.00	62.00	0.66
Mathematica	154.11	1.09	89.50	0.94
Rubi	157.95	1.00	137.00	1.00
Giac	316.85	1.83	96.00	0.83
Sympy	511.19	3.80	79.00	1.00
Fricas	691.01	3.14	131.00	1.26
Mupad	1652.88	7.04	119.00	0.99

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

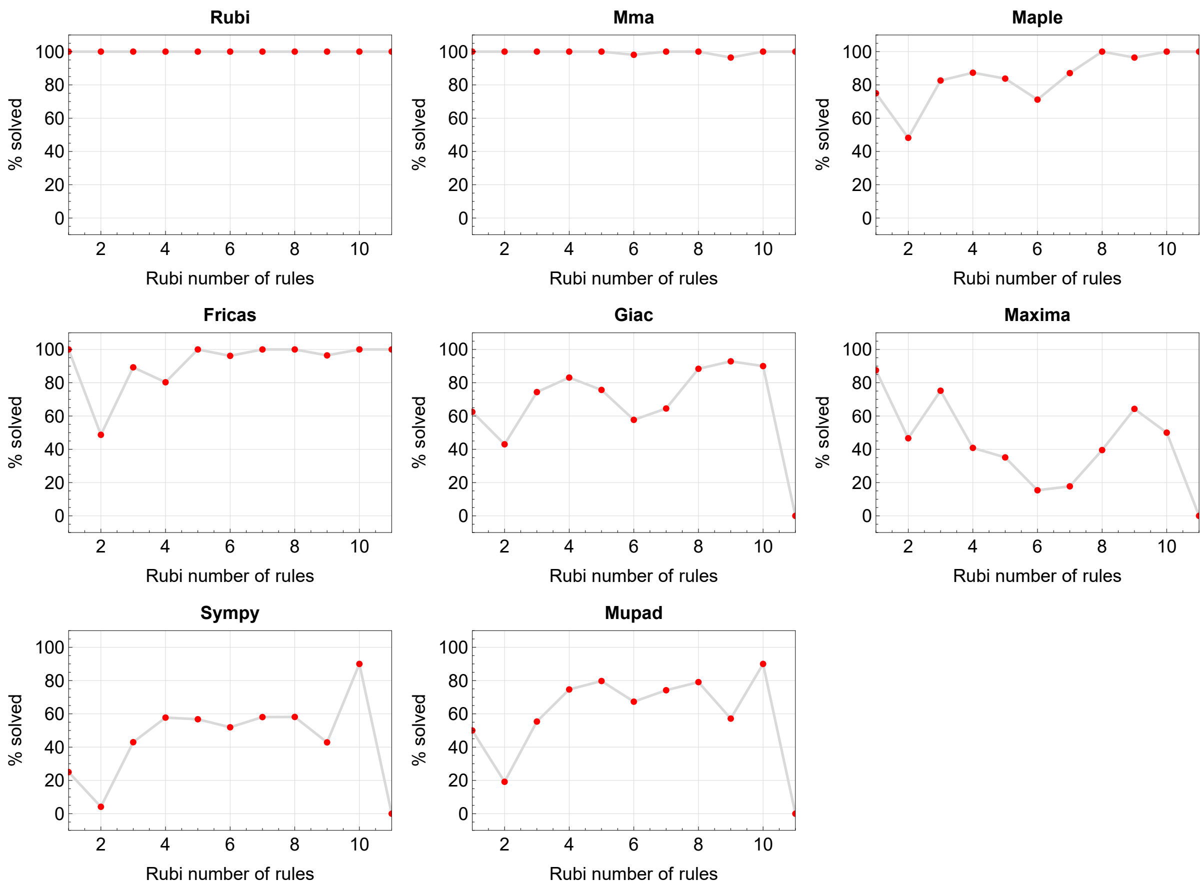


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

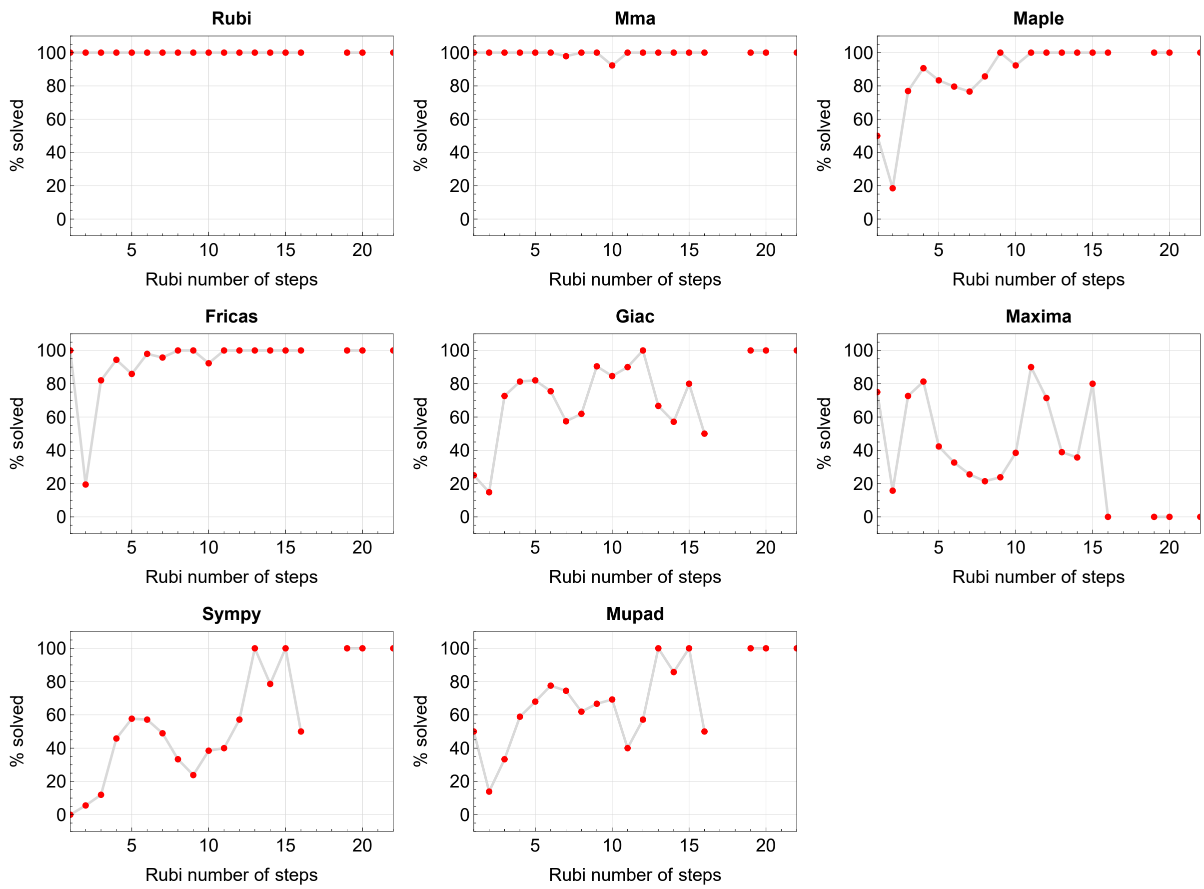


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

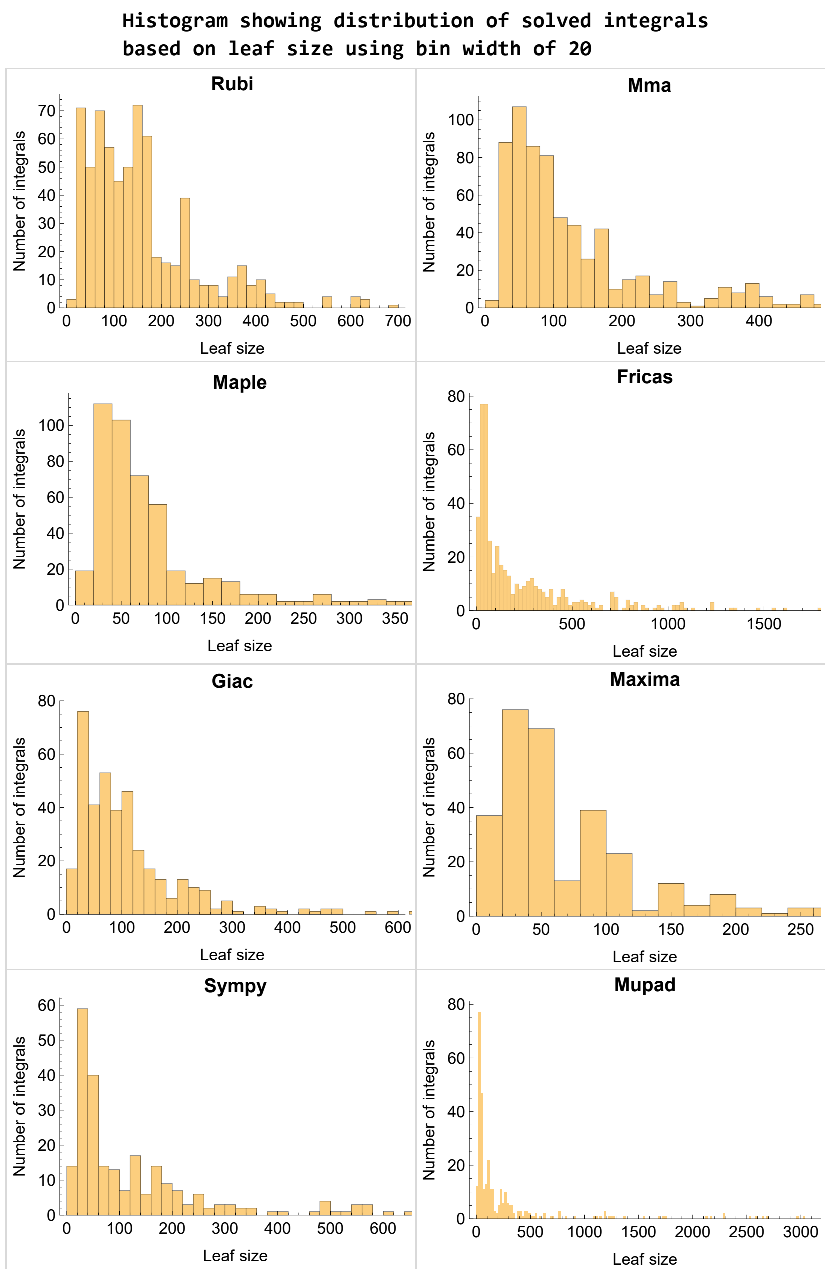


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

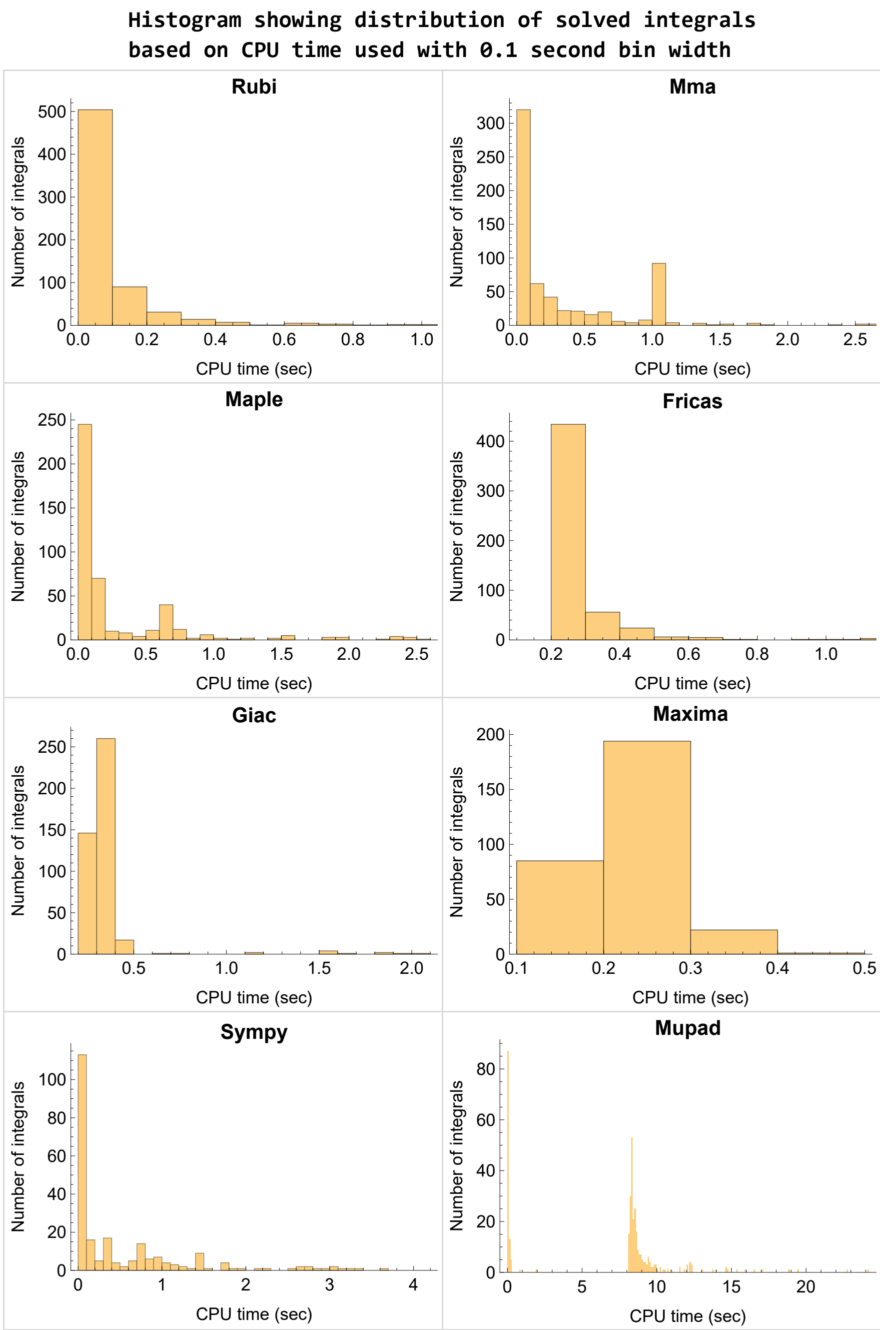


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

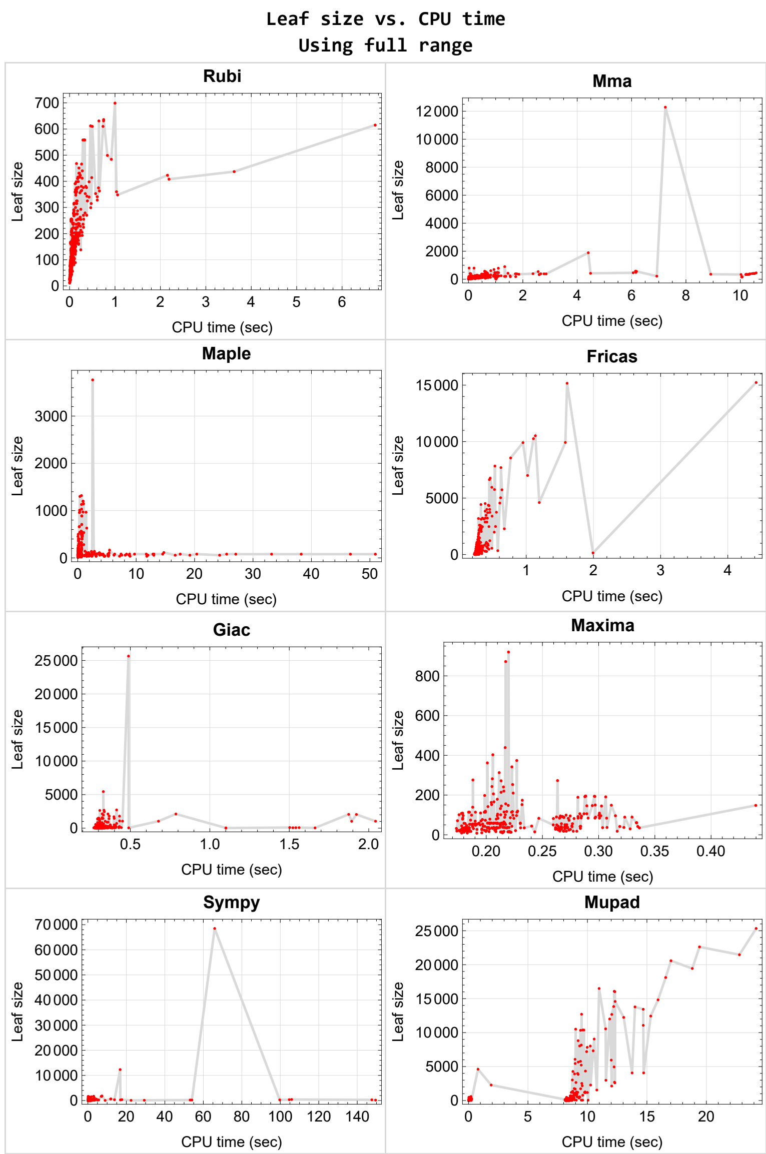


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {132, 250, 256, 309, 327, 347, 367, 385, 602}

Maple {6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 76, 79, 82, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 465, 515, 522, 548, 596, 597, 598}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

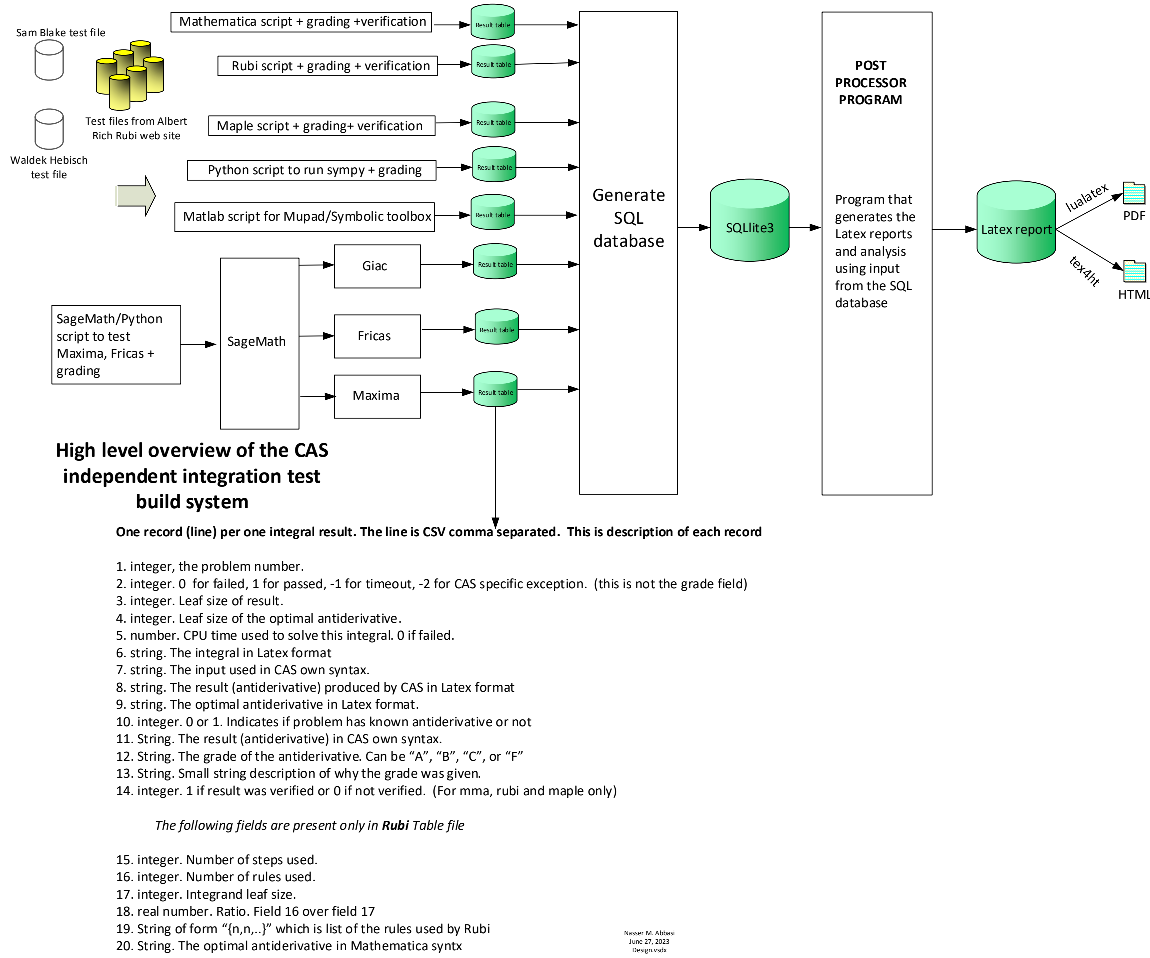
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664 }

B grade { 154 }

C grade { 176, 478 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 328, 330, 332, 337, 339, 342, 345, 348, 349, 350, 352, 355, 368, 369, 370, 371, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 565, 566, 567, 572, 579, 582, 583, 584, 585, 586, 587, 588, 593, 596, 597, 598, 599, 604, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663 }

B grade { 9, 12, 15, 30, 39, 61, 76, 100, 103, 106, 108, 111, 154, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 232, 233, 243, 244, 245, 246, 247, 252, 564, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 589, 590, 591, 592, 594, 595, 600, 601, 602, 603, 605, 607, 608, 609, 610, 611, 612, 664 }

C grade { 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 184, 250, 251, 257, 258, 309, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 333, 334, 335, 336, 338, 340, 341, 343, 344, 346, 347, 351, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409, 410, 411, 457, 458, 478, 500, 501, 502, 503, 504, 505, 546, 547, 559, 560, 561 }

F normal fail { 661, 662 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 35, 36, 37, 38, 40, 41, 43, 44, 46, 47, 49, 50, 51, 53, 54, 56, 57, 59, 60, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 81, 83, 84, 86, 87, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 171, 174, 177, 180, 183, 237, 238, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 317, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 352, 354, 355, 356, 368, 370, 372, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 533, 540, 544, 572, 579 }

B grade { 154, 248, 433, 434, 438, 549, 550, 551, 552, 553, 554, 555, 565, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade { 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 39, 42, 45, 48, 52, 55, 58, 61, 64, 67, 76, 79, 82, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 281, 282, 283, 284, 285, 286, 287, 288, 289, 311, 313, 315, 319, 320, 321, 322, 323, 324, 325, 326, 338, 340, 341, 343, 344, 346, 351, 353, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 396, 397, 398, 399, 400, 401, 402, 403, 404, 456, 457, 458, 465, 500, 501, 504, 505, 515, 522, 548, 556, 557, 558, 559, 560, 561, 596, 597, 598, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 240, 241, 242, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 294, 296, 298, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 352, 354, 355, 356, 368, 370, 374, 376, 378, 379, 380, 381, 382, 384, 386, 388, 392, 394, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 460, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 478, 479, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 565, 572, 579, 586, 616, 617, 639, 641, 642 }

B grade { 82, 143, 144, 145, 146, 147, 148, 149, 150, 154, 239, 248, 293, 295, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 369, 371, 372, 373, 375, 377, 383, 387, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 454, 456, 457, 458, 470, 471, 552, 556, 557, 558, 559, 560, 561, 593, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade { 176, 281, 282, 283, 284, 285, 286, 287, 288, 289, 338, 340, 341, 343, 344, 346, 351, 353, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 500, 505 }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 476, 477, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 599, 600, 601, 606, 661, 662 }

F(-1) timedout fail { 459, 461, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492 }

F(-2) exception fail { 472, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 602, 603, 604, 605 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 348, 350, 352, 354, 356, 368, 370, 372, 374, 376, 386, 387, 388, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 548, 596, 597, 598 }

B grade { 9, 21, 42, 45, 48, 79, 82, 85, 88, 154, 389, 391, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade { }

F normal fail { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 472, 476, 477, 478, 481, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

F(-1) timedout fail { 390, 572 }

F(-2) exception fail { 138, 139, 140, 141, 142, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 239, 240, 241, 242, 310, 312, 314, 316, 318, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439 }

Giac

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 188, 189, 204, 205, 222, 237, 238, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 453, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 496, 500, 501, 502, 515, 516, 517, 518, 523, 524, 525, 552, 614, 616, 619, 622, 624, 628, 637, 639, 641, 663, 664 }

B grade { 45, 61, 82, 118, 119, 125, 154, 170, 172, 173, 175, 178, 179, 181, 223, 248, 249, 311, 313, 315, 317, 349, 355, 389, 391, 454, 456, 473, 474, 522, 558, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 620, 621, 623, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 638, 640, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

C grade { 176 }

F normal fail { 1, 2, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 347, 367, 385, 457, 458, 472, 476, 477, 478, 493, 494, 495, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-1) timedout fail { 449, 450, 479 }

F(-2) exception fail { 182, 184, 451, 452, 459, 643 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 125, 126, 127, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 204, 205, 222, 223, 224, 225, 236, 237, 238, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 455, 456, 457, 458, 460, 465, 468, 469, 470, 471, 473, 474, 475, 478, 485, 496, 552, 565, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 449, 450, 453, 454, 459, 461, 462, 463, 464, 466, 467, 472, 476, 477, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

F(-2) exception fail { }

Sympy

A grade { 5, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 388, 390, 392, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 440, 441, 442, 443, 444, 445, 446, 447, 448, 456, 457, 460, 461, 462, 463, 464, 465, 466, 493, 613, 615, 618, 620, 638, 640, 643, 645 }

B grade { 138, 139, 140, 141, 248, 249, 310, 312, 314, 316, 387, 389, 391, 393, 395, 412, 413, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 544, 596, 597, 598, 607, 608, 609, 610, 611, 612, 614, 616, 617, 621, 622, 624, 631, 633, 639, 641, 642, 646, 647, 649, 655, 657, 663, 664 }

C grade { 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 338, 340, 341, 343, 344, 346 }

F normal fail { 1, 2, 3, 4, 9, 10, 11, 12, 13, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 252, 253, 254, 255, 271, 290, 327, 347, 367, 385, 449, 450, 451, 452, 453, 454, 455, 459, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 500, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 546, 547, 556, 557, 558, 559, 560, 561, 562, 563, 564, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 602, 603, 604, 605, 661, 662 }

F(-1) timedout fail { 6, 7, 8, 14, 15, 16, 17, 18, 19, 20, 21, 22, 142, 150, 250, 251, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 309, 318, 319, 320, 321, 324, 325, 326, 419, 420, 421, 428, 429, 430, 438, 439, 458, 480, 489, 492, 508, 509, 513, 514, 543, 545, 548, 549, 550, 551, 552, 553, 554, 555, 565, 566, 567, 600, 601, 606, 619, 623, 625, 626, 627, 628, 629, 630, 632, 634, 635, 636, 637, 644, 648, 650, 651, 652, 653, 654, 656, 658, 659, 660 }

F(-2) exception fail { 498, 499 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	46	53	0	0	40
N.S.	1	1.00	0.67	0.75	0.88	1.02	0.00	0.00	0.77
time (sec)	N/A	0.028	0.049	0.773	0.213	0.259	0.000	0.000	8.335

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	0	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16
time (sec)	N/A	0.003	0.007	0.634	0.213	0.269	0.000	0.000	8.181

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	17	21	0	14	21
N.S.	1	1.00	1.00	0.96	0.74	0.91	0.00	0.61	0.91
time (sec)	N/A	0.003	0.010	0.582	0.221	0.270	0.000	0.316	8.144

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	46	41	38	54	0	52	51
N.S.	1	1.00	0.60	0.53	0.49	0.70	0.00	0.68	0.66
time (sec)	N/A	0.039	0.255	0.585	0.215	0.282	0.000	0.334	8.277

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	36	37	46	48	38	51
N.S.	1	1.00	1.00	0.75	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.016	0.019	0.419	0.307	0.273	0.071	0.301	0.054

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	39	31	83	13	0	23	59
N.S.	1	1.00	0.49	0.39	1.05	0.16	0.00	0.29	0.75
time (sec)	N/A	0.016	1.020	0.187	0.225	0.285	0.000	0.299	8.222

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	0	29	0
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.015	1.007	4.595	0.222	0.283	0.000	0.298	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	0	29	0
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.015	1.011	3.115	0.222	0.274	0.000	0.305	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	88	23	52	13	0	22	33
N.S.	1	1.00	2.44	0.64	1.44	0.36	0.00	0.61	0.92
time (sec)	N/A	0.018	0.327	0.093	0.217	0.255	0.000	0.305	8.399

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	0	29	0
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.00	0.37	0.00
time (sec)	N/A	0.012	1.007	1.931	0.215	0.269	0.000	0.314	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	10	10	0	20	0
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.00	0.27	0.00
time (sec)	N/A	0.009	0.006	1.556	0.215	0.257	0.000	0.291	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	75	454	26	96	11	0	28	109
N.S.	1	1.00	6.05	0.35	1.28	0.15	0.00	0.37	1.45
time (sec)	N/A	0.014	0.692	0.062	0.210	0.256	0.000	0.304	8.371

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	38	36	14	14	0	29	0
N.S.	1	1.00	0.49	0.47	0.18	0.18	0.00	0.38	0.00
time (sec)	N/A	0.015	1.010	2.390	0.215	0.254	0.000	0.315	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	37	34	15	15	0	26	0
N.S.	1	1.00	0.50	0.46	0.20	0.20	0.00	0.35	0.00
time (sec)	N/A	0.014	1.007	3.159	0.220	0.255	0.000	0.300	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	75	178	28	99	17	0	43	112
N.S.	1	1.00	2.37	0.37	1.32	0.23	0.00	0.57	1.49
time (sec)	N/A	0.015	0.220	0.074	0.224	0.260	0.000	0.289	8.356

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	13	13	0	30	33
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.00	0.39	0.43
time (sec)	N/A	0.015	1.012	4.524	0.214	0.253	0.000	0.293	8.230

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	35	15	15	0	31	35
N.S.	1	1.00	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.015	1.009	5.245	0.220	0.258	0.000	0.320	8.169

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	37	22	86	13	0	30	33
N.S.	1	1.00	0.47	0.28	1.09	0.16	0.00	0.38	0.42
time (sec)	N/A	0.014	1.011	0.056	0.221	0.256	0.000	0.319	8.173

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	35	15	15	0	31	35
N.S.	1	1.00	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.015	1.008	7.162	0.221	0.285	0.000	0.297	8.172

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	35	15	15	0	31	35
N.S.	1	1.00	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.015	1.010	8.955	0.219	0.248	0.000	0.296	8.271

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	39	24	117	15	0	31	35
N.S.	1	1.00	0.49	0.30	1.48	0.19	0.00	0.39	0.44
time (sec)	N/A	0.016	1.013	0.066	0.218	0.240	0.000	0.306	8.136

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	35	15	15	0	31	35
N.S.	1	1.00	0.49	0.44	0.19	0.19	0.00	0.39	0.44
time (sec)	N/A	0.015	1.014	11.827	0.243	0.239	0.000	0.330	8.206

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.029	1.017	8.139	0.234	0.252	0.000	0.287	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	167	113	42	114	35	0	67	0
N.S.	1	1.40	0.95	0.35	0.96	0.29	0.00	0.56	0.00
time (sec)	N/A	0.038	0.515	0.118	0.227	0.252	0.000	0.311	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.028	1.015	6.202	0.218	0.264	0.000	0.288	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.027	1.019	5.336	0.218	0.254	0.000	0.324	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	78	113	31	83	35	0	45	46
N.S.	1	1.00	1.45	0.40	1.06	0.45	0.00	0.58	0.59
time (sec)	N/A	0.035	0.466	0.100	0.247	0.253	0.000	0.306	8.319

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.028	1.015	3.764	0.231	0.248	0.000	0.284	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.027	1.014	3.145	0.209	0.259	0.000	0.308	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	112	23	52	35	0	44	36
N.S.	1	1.00	3.11	0.64	1.44	0.97	0.00	1.22	1.00
time (sec)	N/A	0.020	0.709	0.087	0.212	0.249	0.000	0.308	8.485

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	0
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	0.00
time (sec)	N/A	0.024	1.014	1.858	0.205	0.274	0.000	0.312	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	59	56	32	32	0	64	0
N.S.	1	1.00	0.36	0.35	0.20	0.20	0.00	0.40	0.00
time (sec)	N/A	0.023	1.012	1.524	0.206	0.261	0.000	0.302	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	60	54	152	32	0	65	0
N.S.	1	1.00	0.38	0.34	0.95	0.20	0.00	0.41	0.00
time (sec)	N/A	0.030	1.016	0.083	0.205	0.270	0.000	0.311	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	67	0
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.027	1.014	2.433	0.220	0.266	0.000	0.329	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	65	0
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.40	0.00
time (sec)	N/A	0.026	1.016	3.132	0.202	0.261	0.000	0.312	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	62	59	156	38	0	85	0
N.S.	1	1.00	0.39	0.37	0.97	0.24	0.00	0.53	0.00
time (sec)	N/A	0.032	1.015	0.109	0.217	0.262	0.000	0.286	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	69	0
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.00
time (sec)	N/A	0.027	1.013	4.434	0.212	0.240	0.000	0.300	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	68	0
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.42	0.00
time (sec)	N/A	0.028	1.015	5.162	0.217	0.237	0.000	0.319	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	612	59	220	39	0	86	0
N.S.	1	1.00	3.78	0.36	1.36	0.24	0.00	0.53	0.00
time (sec)	N/A	0.034	0.820	0.091	0.216	0.238	0.000	0.287	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	70	0
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.00
time (sec)	N/A	0.026	1.012	7.541	0.219	0.258	0.000	0.343	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	61	58	37	37	0	67	0
N.S.	1	1.00	0.38	0.36	0.23	0.23	0.00	0.41	0.00
time (sec)	N/A	0.026	1.013	8.781	0.201	0.263	0.000	0.299	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	161	266	52	253	39	0	85	0
N.S.	1	1.00	1.65	0.32	1.57	0.24	0.00	0.53	0.00
time (sec)	N/A	0.030	0.309	0.089	0.223	0.263	0.000	0.408	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	61	57	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.92
time (sec)	N/A	0.026	1.012	12.025	0.216	0.258	0.000	0.318	8.227

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	57	37	37	0	69	151
N.S.	1	1.00	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.026	1.016	13.024	0.207	0.262	0.000	0.281	8.253

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	59	41	148	35	0	68	151
N.S.	1	1.00	1.44	1.00	3.61	0.85	0.00	1.66	3.68
time (sec)	N/A	0.012	1.011	0.087	0.220	0.269	0.000	0.291	8.223

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	57	37	37	0	69	151
N.S.	1	1.00	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.027	1.013	16.682	0.209	0.257	0.000	0.330	8.163

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	57	37	37	0	69	151
N.S.	1	1.00	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.027	1.014	19.193	0.219	0.255	0.000	0.329	8.118

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	84	61	44	179	37	0	69	151
N.S.	1	1.00	0.73	0.52	2.13	0.44	0.00	0.82	1.80
time (sec)	N/A	0.027	1.012	0.097	0.210	0.261	0.000	0.394	8.198

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	57	37	37	0	69	151
N.S.	1	1.00	0.37	0.34	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.028	1.014	24.284	0.201	0.276	0.000	0.285	8.278

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.041	1.023	14.543	0.206	0.281	0.000	0.292	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.039	1.022	13.043	0.209	0.269	0.000	0.292	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	135	53	145	57	0	105	0
N.S.	1	1.00	0.84	0.33	0.91	0.36	0.00	0.66	0.00
time (sec)	N/A	0.077	0.809	0.281	0.213	0.263	0.000	0.301	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.038	1.021	9.722	0.203	0.263	0.000	0.283	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.037	1.016	8.354	0.221	0.251	0.000	0.284	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	119	135	42	114	57	0	105	0
N.S.	1	1.00	1.13	0.35	0.96	0.48	0.00	0.88	0.00
time (sec)	N/A	0.062	0.751	0.172	0.223	0.241	0.000	0.293	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.041	1.018	6.322	0.210	0.253	0.000	0.322	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.038	1.016	5.226	0.203	0.243	0.000	0.284	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	78	135	31	83	57	0	67	0
N.S.	1	1.00	1.73	0.40	1.06	0.73	0.00	0.86	0.00
time (sec)	N/A	0.037	0.693	0.110	0.211	0.248	0.000	0.285	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	0
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.037	1.018	3.750	0.230	0.257	0.000	0.289	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	0
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.037	1.017	2.852	0.212	0.265	0.000	0.295	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	82	23	52	57	0	66	36
N.S.	1	1.00	2.28	0.64	1.44	1.58	0.00	1.83	1.00
time (sec)	N/A	0.020	1.017	0.105	0.215	0.285	0.000	0.321	8.214

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	0
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	0.00
time (sec)	N/A	0.036	1.019	1.913	0.212	0.285	0.000	0.293	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	81	78	53	53	0	101	0
N.S.	1	1.00	0.33	0.32	0.21	0.21	0.00	0.41	0.00
time (sec)	N/A	0.032	1.014	1.470	0.192	0.255	0.000	0.297	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	251	251	82	75	206	55	0	104	0
N.S.	1	1.00	0.33	0.30	0.82	0.22	0.00	0.41	0.00
time (sec)	N/A	0.048	1.019	0.082	0.206	0.274	0.000	0.282	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	105	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.041	1.018	2.378	0.202	0.289	0.000	0.306	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	103	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.41	0.00
time (sec)	N/A	0.038	1.017	2.814	0.217	0.294	0.000	0.292	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	252	252	85	81	214	61	0	124	0
N.S.	1	1.00	0.34	0.32	0.85	0.24	0.00	0.49	0.00
time (sec)	N/A	0.049	1.018	0.105	0.213	0.274	0.000	0.314	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	59	59	0	107	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.037	1.019	4.334	0.201	0.258	0.000	0.304	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	106	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.040	1.017	5.160	0.214	0.284	0.000	0.297	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	276	82	282	61	0	126	0
N.S.	1	1.00	1.10	0.33	1.12	0.24	0.00	0.50	0.00
time (sec)	N/A	0.047	0.804	0.122	0.205	0.289	0.000	0.299	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	107	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.039	1.018	7.413	0.203	0.286	0.000	0.290	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	105	0
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.039	1.016	8.876	0.210	0.262	0.000	0.285	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	279	82	313	61	0	127	0
N.S.	1	1.00	1.11	0.33	1.24	0.24	0.00	0.50	0.00
time (sec)	N/A	0.046	0.748	0.223	0.212	0.268	0.000	0.284	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	0
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	0.00
time (sec)	N/A	0.038	1.017	11.737	0.203	0.268	0.000	0.294	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	106	0
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	0.00
time (sec)	N/A	0.039	1.012	12.872	0.214	0.261	0.000	0.301	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	252	252	696	81	342	61	0	125	0
N.S.	1	1.00	2.76	0.32	1.36	0.24	0.00	0.50	0.00
time (sec)	N/A	0.046	0.938	0.326	0.223	0.277	0.000	0.290	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	0
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	0.00
time (sec)	N/A	0.038	1.016	17.542	0.203	0.275	0.000	0.287	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	105	0
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	0.00
time (sec)	N/A	0.039	1.015	20.374	0.205	0.272	0.000	0.315	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	251	251	268	72	374	61	0	123	0
N.S.	1	1.00	1.07	0.29	1.49	0.24	0.00	0.49	0.00
time (sec)	N/A	0.047	0.495	0.632	0.227	0.265	0.000	0.296	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.24	0.24	0.00	0.43	0.92
time (sec)	N/A	0.039	1.018	25.488	0.210	0.254	0.000	0.311	8.348

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	1.018	27.049	0.204	0.266	0.000	0.317	8.362

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	81	58	210	57	0	106	231
N.S.	1	1.00	1.98	1.41	5.12	1.39	0.00	2.59	5.63
time (sec)	N/A	0.014	1.016	1.203	0.224	0.276	0.000	0.295	8.367

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.038	1.018	33.178	0.214	0.262	0.000	0.298	8.346

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.038	1.015	38.250	0.230	0.272	0.000	0.300	8.448

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	84	84	83	68	241	59	0	107	231
N.S.	1	1.00	0.99	0.81	2.87	0.70	0.00	1.27	2.75
time (sec)	N/A	0.029	1.015	2.323	0.215	0.257	0.000	0.293	8.360

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	1.013	46.670	0.207	0.265	0.000	0.300	8.343

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	79	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	1.016	50.938	0.208	0.281	0.000	0.293	8.340

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	128	83	66	272	59	0	107	231
N.S.	1	1.00	0.65	0.52	2.12	0.46	0.00	0.84	1.80
time (sec)	N/A	0.039	1.025	4.142	0.213	0.264	0.000	0.296	8.352

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	240	240	131	77	109	123	0	146	0
N.S.	1	1.00	0.55	0.32	0.45	0.51	0.00	0.61	0.00
time (sec)	N/A	0.085	1.042	4.101	0.301	0.259	0.000	0.296	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	235	235	128	74	106	106	0	143	0
N.S.	1	1.00	0.54	0.31	0.45	0.45	0.00	0.61	0.00
time (sec)	N/A	0.080	1.026	3.208	0.283	0.257	0.000	0.301	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	51	22	15	13	0	22	33
N.S.	1	1.00	1.16	0.50	0.34	0.30	0.00	0.50	0.75
time (sec)	N/A	0.024	0.110	0.335	0.214	0.264	0.000	0.309	8.425

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	202	109	47	98	304	0	124	0
N.S.	1	1.00	0.54	0.23	0.49	1.50	0.00	0.61	0.00
time (sec)	N/A	0.059	1.025	2.265	0.300	0.278	0.000	0.303	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	202	202	109	47	98	299	0	122	0
N.S.	1	1.00	0.54	0.23	0.49	1.48	0.00	0.60	0.00
time (sec)	N/A	0.077	1.016	1.876	0.301	0.264	0.000	0.289	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	134	31	43	18	0	32	48
N.S.	1	1.00	1.68	0.39	0.54	0.22	0.00	0.40	0.60
time (sec)	N/A	0.023	0.171	0.447	0.223	0.245	0.000	0.305	8.571

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	238	238	133	93	106	103	0	131	0
N.S.	1	1.00	0.56	0.39	0.45	0.43	0.00	0.55	0.00
time (sec)	N/A	0.072	1.024	2.737	0.287	0.276	0.000	0.310	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	243	240	140	94	106	143	0	125	0
N.S.	1	0.99	0.58	0.39	0.44	0.59	0.00	0.51	0.00
time (sec)	N/A	0.072	1.025	3.569	0.289	0.275	0.000	0.304	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	125	122	175	44	73	33	0	50	75
N.S.	1	0.98	1.40	0.35	0.58	0.26	0.00	0.40	0.60
time (sec)	N/A	0.034	0.179	0.472	0.199	0.269	0.000	0.284	8.516

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	280	280	235	89	149	512	0	185	0
N.S.	1	1.00	0.84	0.32	0.53	1.83	0.00	0.66	0.00
time (sec)	N/A	0.088	1.050	3.856	0.311	0.277	0.000	0.312	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	276	276	235	87	146	503	0	176	0
N.S.	1	1.00	0.85	0.32	0.53	1.82	0.00	0.64	0.00
time (sec)	N/A	0.089	1.042	3.245	0.289	0.280	0.000	0.307	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	143	23	16	26	0	24	34
N.S.	1	1.00	3.76	0.61	0.42	0.68	0.00	0.63	0.89
time (sec)	N/A	0.020	0.351	0.059	0.209	0.256	0.000	0.285	8.237

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	277	237	90	147	514	0	176	0
N.S.	1	1.00	0.86	0.32	0.53	1.86	0.00	0.64	0.00
time (sec)	N/A	0.085	1.055	1.934	0.294	0.299	0.000	0.302	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	286	286	235	88	145	499	0	177	0
N.S.	1	1.00	0.82	0.31	0.51	1.74	0.00	0.62	0.00
time (sec)	N/A	0.095	1.049	1.527	0.303	0.285	0.000	0.310	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	147	790	70	88	90	0	87	0
N.S.	1	1.00	5.37	0.48	0.60	0.61	0.00	0.59	0.00
time (sec)	N/A	0.053	1.099	0.104	0.216	0.259	0.000	0.317	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	316	260	115	148	201	0	201	0
N.S.	1	1.00	0.82	0.36	0.47	0.64	0.00	0.64	0.00
time (sec)	N/A	0.104	1.060	2.417	0.440	0.284	0.000	0.299	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	316	266	116	150	242	0	184	0
N.S.	1	1.00	0.84	0.37	0.47	0.77	0.00	0.58	0.00
time (sec)	N/A	0.104	1.067	2.796	0.297	0.271	0.000	0.296	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	188	188	901	92	117	119	0	120	0
N.S.	1	1.00	4.79	0.49	0.62	0.63	0.00	0.64	0.00
time (sec)	N/A	0.065	1.333	0.116	0.223	0.253	0.000	0.299	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	359	359	218	105	195	723	0	205	0
N.S.	1	1.00	0.61	0.29	0.54	2.01	0.00	0.57	0.00
time (sec)	N/A	0.121	1.094	5.373	0.288	0.300	0.000	0.316	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	78	232	31	43	58	0	32	42
N.S.	1	1.00	2.97	0.40	0.55	0.74	0.00	0.41	0.54
time (sec)	N/A	0.033	0.500	0.071	0.197	0.249	0.000	0.312	8.387

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	368	368	229	109	195	734	0	207	0
N.S.	1	1.00	0.62	0.30	0.53	1.99	0.00	0.56	0.00
time (sec)	N/A	0.131	1.106	3.703	0.289	0.276	0.000	0.325	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	360	360	221	107	193	723	0	199	0
N.S.	1	1.00	0.61	0.30	0.54	2.01	0.00	0.55	0.00
time (sec)	N/A	0.125	1.094	2.812	0.297	0.286	0.000	0.294	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	38	38	267	23	16	48	0	24	34
N.S.	1	1.00	7.03	0.61	0.42	1.26	0.00	0.63	0.89
time (sec)	N/A	0.020	0.589	0.059	0.203	0.256	0.000	0.329	8.368

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	359	359	219	112	191	734	0	198	0
N.S.	1	1.00	0.61	0.31	0.53	2.04	0.00	0.55	0.00
time (sec)	N/A	0.128	1.087	1.869	0.306	0.299	0.000	0.349	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	364	364	211	110	189	719	0	199	0
N.S.	1	1.00	0.58	0.30	0.52	1.98	0.00	0.55	0.00
time (sec)	N/A	0.135	1.087	1.534	0.282	0.279	0.000	0.327	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	223	96	92	132	178	0	109	0
N.S.	1	1.00	0.43	0.41	0.59	0.80	0.00	0.49	0.00
time (sec)	N/A	0.082	1.032	0.117	0.213	0.271	0.000	0.321	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	398	242	137	192	311	0	223	0
N.S.	1	1.00	0.61	0.34	0.48	0.78	0.00	0.56	0.00
time (sec)	N/A	0.146	1.095	2.342	0.288	0.276	0.000	0.322	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	398	234	138	194	352	0	215	0
N.S.	1	1.00	0.59	0.35	0.49	0.88	0.00	0.54	0.00
time (sec)	N/A	0.148	1.113	2.863	0.296	0.277	0.000	0.321	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	269	119	114	163	207	0	143	0
N.S.	1	1.00	0.44	0.42	0.61	0.77	0.00	0.53	0.00
time (sec)	N/A	0.095	1.036	0.135	0.204	0.256	0.000	0.314	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	243	369	0	900	0
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	0.00
time (sec)	N/A	0.096	0.080	0.044	0.205	0.272	0.000	0.360	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	119	159	0	384	0
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	0.00
time (sec)	N/A	0.057	0.061	0.031	0.228	0.265	0.000	0.311	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	35	35	0	83	0
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	0.00
time (sec)	N/A	0.024	0.022	0.025	0.211	0.273	0.000	0.290	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	110	150	115	163	0	375	207
N.S.	1	1.00	0.64	0.87	0.67	0.95	0.00	2.18	1.20
time (sec)	N/A	0.077	0.085	0.273	0.216	0.269	0.000	0.311	8.388

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	79	108	0	235	137
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.054	0.059	0.160	0.203	0.247	0.000	0.286	8.363

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	51	58	54	70	0	132	85
N.S.	1	1.00	0.61	0.69	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.039	0.047	0.114	0.197	0.272	0.000	0.321	8.311

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	31	30	37	0	58	46
N.S.	1	1.00	0.78	0.76	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.019	0.030	0.067	0.203	0.255	0.000	0.316	8.362

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	53	55	211	0	0	0	0	0	0
N.S.	1	1.04	3.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	1758
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	21.70
time (sec)	N/A	0.056	0.041	0.099	0.000	0.285	1.718	0.399	8.891

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	1199
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	19.03
time (sec)	N/A	0.039	0.016	0.072	0.000	0.268	0.841	0.427	8.642

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	174
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58
time (sec)	N/A	0.025	0.009	0.050	0.000	0.284	0.378	0.359	8.245

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	65	0	223	253	66	1362
N.S.	1	1.00	0.96	0.94	0.00	3.23	3.67	0.96	19.74
time (sec)	N/A	0.046	0.018	0.071	0.000	0.288	16.912	0.359	8.766

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	85	0	293	0	93	4281
N.S.	1	1.00	1.03	0.96	0.00	3.29	0.00	1.04	48.10
time (sec)	N/A	0.087	0.022	0.087	0.000	0.307	0.000	0.365	8.782

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	636	636	70	61	0	3402	279	0	4069
N.S.	1	1.00	0.11	0.10	0.00	5.35	0.44	0.00	6.40
time (sec)	N/A	0.751	0.023	0.501	0.000	0.492	147.579	0.000	14.726

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	2882	196	0	2280
N.S.	1	1.00	0.11	0.09	0.00	4.57	0.31	0.00	3.61
time (sec)	N/A	0.645	0.023	0.042	0.000	0.415	53.135	0.000	1.908

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	44	43	0	2314	175	0	2695
N.S.	1	1.00	0.08	0.08	0.00	4.15	0.31	0.00	4.83
time (sec)	N/A	0.325	0.015	0.045	0.000	0.309	1.451	0.000	12.275

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	42	43	0	1542	122	0	2129
N.S.	1	1.00	0.08	0.08	0.00	2.76	0.22	0.00	3.82
time (sec)	N/A	0.335	0.014	0.042	0.000	0.287	0.935	0.000	12.036

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	43	41	0	1798	158	0	1543
N.S.	1	1.00	0.08	0.07	0.00	3.22	0.28	0.00	2.77
time (sec)	N/A	0.293	0.014	0.042	0.000	0.290	0.748	0.000	10.791

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	45	40	0	2206	155	0	2597
N.S.	1	1.00	0.08	0.07	0.00	3.95	0.28	0.00	4.65
time (sec)	N/A	0.318	0.020	0.039	0.000	0.328	3.641	0.000	12.305

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	71	61	0	3225	252	0	2978
N.S.	1	1.00	0.12	0.10	0.00	5.29	0.41	0.00	4.88
time (sec)	N/A	0.500	0.025	0.069	0.000	0.393	2.216	0.000	11.555

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	612	75	62	0	3225	0	0	4063
N.S.	1	1.00	0.12	0.10	0.00	5.27	0.00	0.00	6.64
time (sec)	N/A	0.463	0.023	0.070	0.000	0.422	0.000	0.000	13.755

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	29	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.83	0.77
time (sec)	N/A	0.017	0.005	0.074	0.192	0.247	0.057	0.321	0.029

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	24	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79
time (sec)	N/A	0.012	0.005	0.048	0.200	0.248	0.053	0.293	0.027

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.81
time (sec)	N/A	0.010	0.005	0.046	0.216	0.237	0.046	0.296	0.030

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	21	21	18	17	17	15	19	16
N.S.	1	2.10	2.10	1.80	1.70	1.70	1.50	1.90	1.60
time (sec)	N/A	0.009	0.004	0.042	0.200	0.252	0.047	0.300	0.213

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	23	21	20	24	21
N.S.	1	1.00	1.00	0.81	0.85	0.78	0.74	0.89	0.78
time (sec)	N/A	0.013	0.007	0.056	0.208	0.246	0.064	0.335	8.393

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	28	35	29	36	26
N.S.	1	1.00	1.00	0.79	0.82	1.03	0.85	1.06	0.76
time (sec)	N/A	0.023	0.005	0.067	0.209	0.252	0.077	0.293	8.292

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	35	40	34	41	32
N.S.	1	1.00	1.00	0.80	0.85	0.98	0.83	1.00	0.78
time (sec)	N/A	0.025	0.005	0.070	0.202	0.242	0.090	0.288	0.024

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	118	68	94	102	144	96	124
N.S.	1	1.00	0.95	0.55	0.76	0.82	1.16	0.77	1.00
time (sec)	N/A	0.079	0.047	0.074	0.285	0.265	0.334	0.326	0.141

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	62	92	90	129	94	119
N.S.	1	1.00	0.93	0.51	0.75	0.74	1.06	0.77	0.98
time (sec)	N/A	0.072	0.019	0.061	0.282	0.253	0.328	0.310	0.136

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	63	89	99	134	91	118
N.S.	1	1.00	0.93	0.53	0.75	0.83	1.13	0.76	0.99
time (sec)	N/A	0.056	0.027	0.059	0.297	0.269	0.321	0.304	0.104

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	111	53	85	88	126	87	104
N.S.	1	1.00	0.98	0.47	0.75	0.78	1.12	0.77	0.92
time (sec)	N/A	0.054	0.016	0.056	0.284	0.265	0.315	0.315	0.091

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	107	60	84	106	134	86	114
N.S.	1	1.00	0.96	0.54	0.75	0.95	1.20	0.77	1.02
time (sec)	N/A	0.047	0.016	0.056	0.301	0.273	0.300	0.311	8.350

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	106	54	84	102	110	86	113
N.S.	1	1.00	0.95	0.48	0.75	0.91	0.98	0.77	1.01
time (sec)	N/A	0.047	0.015	0.065	0.291	0.249	0.298	0.303	8.366

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	108	56	84	84	119	86	113
N.S.	1	1.00	0.96	0.50	0.75	0.75	1.06	0.77	1.01
time (sec)	N/A	0.044	0.017	0.051	0.297	0.252	1.207	0.304	8.479

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	107	58	84	124	124	86	110
N.S.	1	1.00	0.96	0.52	0.75	1.11	1.11	0.77	0.98
time (sec)	N/A	0.046	0.015	0.055	0.307	0.253	1.151	0.376	0.127

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	61	89	117	139	91	119
N.S.	1	1.00	0.99	0.51	0.75	0.98	1.17	0.76	1.00
time (sec)	N/A	0.056	0.027	0.070	0.330	0.268	1.167	0.302	8.386

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	59	89	126	128	91	118
N.S.	1	1.00	0.95	0.50	0.75	1.06	1.08	0.76	0.99
time (sec)	N/A	0.053	0.036	0.066	0.323	0.262	1.041	0.313	8.460

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	67	96	112	141	98	124
N.S.	1	1.00	0.94	0.53	0.76	0.89	1.12	0.78	0.98
time (sec)	N/A	0.068	0.029	0.072	0.315	0.252	1.197	0.317	0.111

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	65	96	153	136	98	121
N.S.	1	1.00	0.94	0.52	0.76	1.21	1.08	0.78	0.96
time (sec)	N/A	0.070	0.041	0.079	0.302	0.254	1.035	0.304	8.495

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	59	44	0	300	26	641	320
N.S.	1	1.00	0.14	0.11	0.00	0.73	0.06	1.56	0.78
time (sec)	N/A	0.277	0.010	0.043	0.000	0.273	0.091	0.332	8.731

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.024	0.009	0.044	0.284	0.249	0.054	0.298	0.030

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	41	40	0	295	26	827	304
N.S.	1	1.00	0.10	0.10	0.00	0.72	0.06	2.01	0.74
time (sec)	N/A	0.179	0.014	0.035	0.000	0.276	0.079	0.322	8.612

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	39	40	0	261	24	640	327
N.S.	1	1.00	0.09	0.10	0.00	0.64	0.06	1.56	0.80
time (sec)	N/A	0.179	0.007	0.037	0.000	0.269	0.075	0.328	8.678

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.015	0.007	0.036	0.277	0.252	0.050	0.318	0.023

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	40	38	0	295	26	815	304
N.S.	1	1.00	0.11	0.10	0.00	0.79	0.07	2.17	0.81
time (sec)	N/A	0.156	0.008	0.035	0.000	0.282	0.077	0.308	8.524

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	C	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	375	42	37	0	285	20	632	327
N.S.	1	2.02	0.23	0.20	0.00	1.53	0.11	3.40	1.76
time (sec)	N/A	0.156	0.016	0.030	0.000	0.267	0.083	0.298	8.506

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	33	38	34	41	35	36
N.S.	1	1.00	1.34	0.80	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.026	0.012	0.052	0.272	0.250	0.065	0.302	0.032

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	61	35	0	255	24	829	286
N.S.	1	1.00	0.15	0.08	0.00	0.61	0.06	1.99	0.69
time (sec)	N/A	0.183	0.011	0.058	0.000	0.267	0.086	0.307	8.545

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	65	38	0	337	31	645	324
N.S.	1	1.00	0.16	0.09	0.00	0.81	0.07	1.54	0.78
time (sec)	N/A	0.215	0.010	0.053	0.000	0.247	0.089	0.312	8.811

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	38	43	51	48	45	41
N.S.	1	1.00	1.06	0.79	0.90	1.06	1.00	0.94	0.85
time (sec)	N/A	0.038	0.011	0.055	0.284	0.252	0.075	0.297	0.039

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	54	46	0	338	39	839	318
N.S.	1	1.00	0.13	0.11	0.00	0.80	0.09	1.98	0.75
time (sec)	N/A	0.237	0.013	0.059	0.000	0.270	0.096	0.293	8.722

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	38	33	0	299	24	0	513
N.S.	1	1.00	0.10	0.09	0.00	0.78	0.06	0.00	1.35
time (sec)	N/A	0.258	0.007	0.041	0.000	0.279	0.065	0.000	9.360

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.018	0.010	0.036	0.281	0.252	0.047	0.385	0.030

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	37	36	0	261	24	0	351
N.S.	1	1.00	0.09	0.09	0.00	0.65	0.06	0.00	0.88
time (sec)	N/A	0.192	0.007	0.038	0.000	0.257	0.062	0.000	9.294

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	206	0	0	451	0	0	543
N.S.	1	1.00	0.89	0.00	0.00	1.95	0.00	0.00	2.35
time (sec)	N/A	0.200	0.436	0.000	0.000	0.280	0.000	0.000	9.329

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	166	0	0	367	0	0	315
N.S.	1	1.00	0.97	0.00	0.00	2.15	0.00	0.00	1.84
time (sec)	N/A	0.103	0.306	0.000	0.000	0.288	0.000	0.000	8.694

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	132	0	0	303	0	0	193
N.S.	1	1.00	0.86	0.00	0.00	1.98	0.00	0.00	1.26
time (sec)	N/A	0.090	0.242	0.000	0.000	0.266	0.000	0.000	8.356

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	101	0	0	237	0	96	87
N.S.	1	1.00	0.94	0.00	0.00	2.19	0.00	0.89	0.81
time (sec)	N/A	0.055	0.219	0.000	0.000	0.279	0.000	0.308	8.389

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	87	0	0	197	0	74	72
N.S.	1	1.00	1.05	0.00	0.00	2.37	0.00	0.89	0.87
time (sec)	N/A	0.038	0.294	0.000	0.000	0.264	0.000	0.327	8.489

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	105	0	0	566	0	0	88
N.S.	1	1.00	0.96	0.00	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.071	0.174	0.000	0.000	0.288	0.000	0.000	8.366

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	107	0	0	601	0	0	91
N.S.	1	1.00	0.96	0.00	0.00	5.37	0.00	0.00	0.81
time (sec)	N/A	0.069	0.168	0.000	0.000	0.279	0.000	0.000	8.567

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	91	0	0	215	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.049	0.233	0.000	0.000	0.270	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	259	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	0.064	0.400	0.000	0.000	0.298	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	141	0	0	325	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.101	0.510	0.000	0.000	0.315	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	176	0	0	389	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.152	0.676	0.000	0.000	0.337	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	8.914	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	10.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0	0
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	10.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	10.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	10.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	289	0	0	641	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.248	0.932	0.000	0.000	0.302	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	220	0	0	535	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.143	0.664	0.000	0.000	0.279	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	194	0	0	451	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.122	0.515	0.000	0.000	0.277	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	140	0	0	361	0	170	223
N.S.	1	1.00	0.93	0.00	0.00	2.41	0.00	1.13	1.49
time (sec)	N/A	0.081	0.372	0.000	0.000	0.279	0.000	0.349	8.571

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	114	0	0	297	0	133	115
N.S.	1	1.00	0.92	0.00	0.00	2.40	0.00	1.07	0.93
time (sec)	N/A	0.057	0.641	0.000	0.000	0.256	0.000	0.354	8.525

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	143	0	0	727	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.117	0.513	0.000	0.000	0.347	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	131	0	0	713	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.114	0.459	0.000	0.000	0.350	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	131	0	0	713	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.110	0.509	0.000	0.000	0.322	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	148	0	0	771	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.120	0.670	0.000	0.000	0.328	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	118	0	0	319	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.081	0.638	0.000	0.000	0.304	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	160	0	0	383	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.094	0.974	0.000	0.000	0.354	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	201	0	0	473	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.140	1.329	0.000	0.000	0.445	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	244	0	0	557	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.202	1.533	0.000	0.000	0.488	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0	0
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	10.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0	0
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	10.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	0
N.S.	1	1.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	10.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	10.354	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	10.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	138	0	0	303	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.146	0.275	0.000	0.000	0.275	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	101	0	0	241	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	1.99	0.00	0.00	0.00
time (sec)	N/A	0.072	0.228	0.000	0.000	0.268	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	203	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.058	0.185	0.000	0.000	0.275	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	161	0	59	55
N.S.	1	1.00	1.00	0.00	0.00	2.37	0.00	0.87	0.81
time (sec)	N/A	0.033	0.131	0.000	0.000	0.267	0.000	0.310	8.530

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	41	0	0	118	0	74	34
N.S.	1	1.00	0.95	0.00	0.00	2.74	0.00	1.72	0.79
time (sec)	N/A	0.021	0.079	0.000	0.000	0.262	0.000	0.316	8.622

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	124	0	0	36
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82
time (sec)	N/A	0.025	0.078	0.000	0.000	0.255	0.000	0.000	8.483

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	179	0	0	56
N.S.	1	1.00	1.00	0.00	0.00	2.49	0.00	0.00	0.78
time (sec)	N/A	0.039	0.149	0.000	0.000	0.260	0.000	0.000	8.507

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	91	0	0	221	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.069	0.235	0.000	0.000	0.274	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	110	0	0	263	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.110	0.375	0.000	0.000	0.282	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	192	141	0	0	327	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.160	0.497	0.000	0.000	0.313	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	10.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	10.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0	0
N.S.	1	1.00	2.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	10.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	10.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	170	0	0	591	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.157	0.651	0.000	0.000	0.309	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	131	0	0	459	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.075	0.427	0.000	0.000	0.305	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	95	0	0	387	0	0	84
N.S.	1	1.00	0.79	0.00	0.00	3.22	0.00	0.00	0.70
time (sec)	N/A	0.059	0.388	0.000	0.000	0.283	0.000	0.000	8.611

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	0	68	0	45	38
N.S.	1	1.00	1.00	0.97	0.00	1.74	0.00	1.15	0.97
time (sec)	N/A	0.020	0.239	4.661	0.000	0.265	0.000	0.489	8.649

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	67	0	45	37
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97
time (sec)	N/A	0.018	0.257	2.474	0.000	0.266	0.000	0.401	8.681

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	389	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.052	0.423	0.000	0.000	0.297	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	125	0	0	485	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.086	0.505	0.000	0.000	0.306	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	166	0	0	615	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	3.11	0.00	0.00	0.00
time (sec)	N/A	0.143	0.663	0.000	0.000	0.353	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	256	210	0	0	705	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.189	0.920	0.000	0.000	0.411	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	10.307	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	10.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0	0
N.S.	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	10.525	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	10.437	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1459	449	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.45	4.45	2.57
time (sec)	N/A	0.041	0.617	0.194	0.195	0.263	0.823	0.325	8.493

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.71
time (sec)	N/A	0.014	0.049	0.059	0.195	0.244	0.373	0.336	8.346

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.245	0.000	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	0.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	1.854	0.000	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	1.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	1.737	0.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	6.927	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	162	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	162	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.403	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.415	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.424	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	162	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	164	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.311	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.006	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	24	25	24	31	22	24	25
N.S.	1	1.00	0.80	0.83	0.80	1.03	0.73	0.80	0.83
time (sec)	N/A	0.008	0.018	0.070	0.274	0.250	0.052	0.289	0.026

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	18	23	15	18	18
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.007	0.006	0.036	0.184	0.249	0.036	0.295	8.299

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	24	15	19	21
N.S.	1	1.00	1.00	0.87	0.83	1.04	0.65	0.83	0.91
time (sec)	N/A	0.006	0.008	0.062	0.269	0.242	0.044	0.314	8.309

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.002	0.004	0.030	0.178	0.242	0.037	0.289	0.011

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	23	15	19	20
N.S.	1	1.00	0.87	0.87	0.83	1.00	0.65	0.83	0.87
time (sec)	N/A	0.005	0.006	0.052	0.266	0.244	0.044	0.292	0.015

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.008	0.011	0.053	0.180	0.265	0.051	0.303	8.317

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	25	31	26	25	25
N.S.	1	1.00	1.00	0.83	0.83	1.03	0.87	0.83	0.83
time (sec)	N/A	0.008	0.014	0.081	0.264	0.246	0.066	0.291	0.022

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	33	44	31	33	31
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94
time (sec)	N/A	0.011	0.011	0.060	0.179	0.237	0.065	0.323	0.027

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	30	36	29	31	30
N.S.	1	1.00	0.89	0.76	0.81	0.97	0.78	0.84	0.81
time (sec)	N/A	0.011	0.012	0.103	0.267	0.244	0.081	0.304	0.027

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	34	83	106	90	83	45
N.S.	1	1.00	0.90	0.33	0.80	1.02	0.87	0.80	0.43
time (sec)	N/A	0.038	0.052	0.055	0.273	0.253	0.070	0.308	8.288

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	35	84	103	90	84	47
N.S.	1	1.00	0.94	0.35	0.85	1.04	0.91	0.85	0.47
time (sec)	N/A	0.035	0.048	0.056	0.282	0.263	0.069	0.316	8.271

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	33	82	97	82	82	45
N.S.	1	1.00	0.93	0.34	0.85	1.00	0.85	0.85	0.46
time (sec)	N/A	0.034	0.041	0.054	0.260	0.271	0.067	0.296	0.045

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	35	84	99	83	84	46
N.S.	1	1.00	0.93	0.35	0.85	1.00	0.84	0.85	0.46
time (sec)	N/A	0.036	0.034	0.049	0.265	0.260	0.070	0.296	0.024

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	33	82	101	88	82	44
N.S.	1	1.00	0.94	0.34	0.85	1.04	0.91	0.85	0.45
time (sec)	N/A	0.034	0.033	0.048	0.278	0.249	0.071	0.307	8.150

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	98	41	88	112	97	88	49
N.S.	1	1.00	0.92	0.39	0.83	1.06	0.92	0.83	0.46
time (sec)	N/A	0.037	0.053	0.078	0.269	0.251	0.086	0.312	8.118

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	39	90	122	99	87	51
N.S.	1	1.00	0.91	0.37	0.85	1.15	0.93	0.82	0.48
time (sec)	N/A	0.035	0.051	0.082	0.262	0.253	0.087	0.285	8.261

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	44	95	127	102	96	55
N.S.	1	1.00	0.91	0.39	0.84	1.12	0.90	0.85	0.49
time (sec)	N/A	0.039	0.052	0.084	0.275	0.278	0.095	0.285	0.053

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	42	95	127	102	94	55
N.S.	1	1.00	0.89	0.37	0.84	1.12	0.90	0.83	0.49
time (sec)	N/A	0.036	0.052	0.090	0.266	0.266	0.096	0.327	0.058

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.004	0.132	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	39	35	34	46	34	35	26
N.S.	1	1.00	1.22	1.09	1.06	1.44	1.06	1.09	0.81
time (sec)	N/A	0.009	0.026	0.050	0.180	0.241	0.053	0.286	0.026

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	18	23	15	19	20
N.S.	1	1.00	0.85	0.73	0.69	0.88	0.58	0.73	0.77
time (sec)	N/A	0.009	0.007	0.045	0.186	0.233	0.037	0.325	8.248

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	30	29	40	26	30	21
N.S.	1	1.00	1.32	1.20	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.007	0.010	0.050	0.181	0.248	0.051	0.300	8.439

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.002	0.003	0.036	0.183	0.232	0.031	0.325	0.013

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	30	29	40	26	30	21
N.S.	1	1.00	1.32	1.20	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.006	0.007	0.048	0.182	0.286	0.044	0.303	0.019

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	24	32	19	30	22
N.S.	1	1.00	0.93	0.75	0.86	1.14	0.68	1.07	0.79
time (sec)	N/A	0.011	0.008	0.060	0.180	0.247	0.054	0.308	0.036

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	36	37	54	36	38	26
N.S.	1	1.00	1.28	1.12	1.16	1.69	1.12	1.19	0.81
time (sec)	N/A	0.009	0.015	0.063	0.185	0.234	0.071	0.319	0.027

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	35	50	29	36	32
N.S.	1	1.00	0.95	0.86	0.95	1.35	0.78	0.97	0.86
time (sec)	N/A	0.013	0.010	0.067	0.185	0.231	0.064	0.337	0.030

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	41	42	59	41	42	32
N.S.	1	1.00	1.26	1.05	1.08	1.51	1.05	1.08	0.82
time (sec)	N/A	0.012	0.011	0.069	0.188	0.234	0.086	0.302	0.029

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	29	28	49	32	30	26
N.S.	1	1.00	1.12	0.85	0.82	1.44	0.94	0.88	0.76
time (sec)	N/A	0.008	0.014	0.097	0.262	0.253	0.063	0.310	8.372

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	35	30	29	46	32	31	23
N.S.	1	1.00	1.21	1.03	1.00	1.59	1.10	1.07	0.79
time (sec)	N/A	0.005	0.011	0.090	0.265	0.260	0.064	0.310	0.018

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	28	27	43	26	29	21
N.S.	1	1.00	1.15	1.04	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.006	0.010	0.090	0.275	0.243	0.059	0.284	0.018

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	33	30	29	45	27	31	23
N.S.	1	1.00	1.14	1.03	1.00	1.55	0.93	1.07	0.79
time (sec)	N/A	0.005	0.009	0.088	0.262	0.244	0.066	0.289	0.017

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	28	27	44	31	29	21
N.S.	1	1.00	1.22	1.04	1.00	1.63	1.15	1.07	0.78
time (sec)	N/A	0.004	0.008	0.094	0.266	0.247	0.066	0.293	0.017

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	36	35	55	37	37	26
N.S.	1	1.00	1.11	1.00	0.97	1.53	1.03	1.03	0.72
time (sec)	N/A	0.007	0.016	0.109	0.272	0.240	0.090	0.323	0.025

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	36	37	63	39	34	28
N.S.	1	1.00	1.06	1.00	1.03	1.75	1.08	0.94	0.78
time (sec)	N/A	0.007	0.015	0.123	0.322	0.246	0.085	0.331	8.219

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	41	42	68	44	43	34
N.S.	1	1.00	1.19	0.95	0.98	1.58	1.02	1.00	0.79
time (sec)	N/A	0.008	0.016	0.126	0.319	0.246	0.091	0.299	0.026

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	41	42	68	44	41	34
N.S.	1	1.00	1.00	0.95	0.98	1.58	1.02	0.95	0.79
time (sec)	N/A	0.008	0.014	0.134	0.270	0.261	0.095	0.288	0.026

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.259	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	3916
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	48.35
time (sec)	N/A	0.058	0.037	0.085	0.000	0.279	2.700	1.562	8.922

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	210	139	0	1071	134	2043	5659
N.S.	1	1.00	1.09	0.72	0.00	5.58	0.70	10.64	29.47
time (sec)	N/A	0.225	0.086	0.097	0.000	0.265	2.808	1.923	9.231

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	2654
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	42.13
time (sec)	N/A	0.043	0.018	0.067	0.000	0.265	1.503	1.523	9.035

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	171	105	0	567	76	1034	1220
N.S.	1	1.00	1.08	0.66	0.00	3.57	0.48	6.50	7.67
time (sec)	N/A	0.089	0.058	0.077	0.000	0.261	1.433	2.043	9.181

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	260
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84
time (sec)	N/A	0.025	0.008	0.047	0.000	0.260	0.442	1.662	8.206

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	133	99	0	619	88	1028	1105
N.S.	1	1.00	0.86	0.64	0.00	4.02	0.57	6.68	7.18
time (sec)	N/A	0.074	0.053	0.071	0.000	0.276	2.166	1.893	8.841

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	223	253	68	1690
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.99	24.49
time (sec)	N/A	0.047	0.019	0.080	0.000	0.283	99.709	1.541	8.977

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	75	159	0	1134	153	2055	5451
N.S.	1	1.00	0.41	0.86	0.00	6.16	0.83	11.17	29.62
time (sec)	N/A	0.179	0.024	0.110	0.000	0.275	149.527	1.874	8.971

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	84	0	293	0	94	8817
N.S.	1	1.00	1.03	0.94	0.00	3.29	0.00	1.06	99.07
time (sec)	N/A	0.088	0.023	0.100	0.000	0.388	0.000	1.503	9.218

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	70	63	0	7003	0	0	12709
N.S.	1	1.00	0.18	0.17	0.00	18.38	0.00	0.00	33.36
time (sec)	N/A	0.358	0.028	0.104	0.000	1.020	0.000	0.000	9.507

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	4001	0	0	10382
N.S.	1	1.00	0.19	0.16	0.00	10.64	0.00	0.00	27.61
time (sec)	N/A	0.305	0.027	0.048	0.000	0.414	0.000	0.000	9.700

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	44	43	0	4433	0	0	8033
N.S.	1	1.00	0.14	0.13	0.00	13.64	0.00	0.00	24.72
time (sec)	N/A	0.166	0.018	0.044	0.000	0.326	0.000	0.000	9.344

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	42	43	0	2141	126	0	8169
N.S.	1	1.00	0.13	0.13	0.00	6.59	0.39	0.00	25.14
time (sec)	N/A	0.152	0.017	0.050	0.000	0.297	3.227	0.000	9.486

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	43	43	0	3193	172	0	6067
N.S.	1	1.00	0.14	0.14	0.00	10.14	0.55	0.00	19.26
time (sec)	N/A	0.150	0.017	0.050	0.000	0.289	3.034	0.000	8.960

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	45	40	0	3125	0	0	10337
N.S.	1	1.00	0.14	0.13	0.00	9.92	0.00	0.00	32.82
time (sec)	N/A	0.158	0.023	0.042	0.000	0.329	0.000	0.000	9.418

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	71	63	0	5758	0	0	10509
N.S.	1	1.00	0.20	0.17	0.00	15.86	0.00	0.00	28.95
time (sec)	N/A	0.253	0.025	0.094	0.000	0.527	0.000	0.000	9.010

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	75	62	0	5030	0	0	16497
N.S.	1	1.00	0.21	0.17	0.00	13.78	0.00	0.00	45.20
time (sec)	N/A	0.268	0.029	0.083	0.000	0.619	0.000	0.000	10.981

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	488	0	0	0	0	0	0
N.S.	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	1.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	35	42	35	37
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.95	0.80	0.84
time (sec)	N/A	0.025	0.012	0.096	0.336	0.246	0.079	0.354	0.028

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	98	43	42	40	51	42	43
N.S.	1	1.00	1.81	0.80	0.78	0.74	0.94	0.78	0.80
time (sec)	N/A	0.040	0.140	0.097	0.335	0.228	0.064	0.299	0.029

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.022	0.009	0.078	0.328	0.244	0.061	0.300	0.024

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	94	62	61	61	76	61	51
N.S.	1	1.00	1.25	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.052	0.089	0.088	0.334	0.250	0.097	0.316	0.058

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.016	0.009	0.071	0.317	0.235	0.051	0.302	0.038

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	62	61	61	76	61	51
N.S.	1	1.00	1.05	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.043	0.038	0.076	0.333	0.243	0.094	0.292	8.156

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	133	31	36	32	41	36	34
N.S.	1	1.00	3.41	0.79	0.92	0.82	1.05	0.92	0.87
time (sec)	N/A	0.024	0.049	0.099	0.304	0.241	0.078	0.307	8.198

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	100	44	42	45	53	42	43
N.S.	1	1.00	1.85	0.81	0.78	0.83	0.98	0.78	0.80
time (sec)	N/A	0.036	0.030	0.117	0.283	0.238	0.075	0.332	0.029

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	136	38	41	49	48	46	41
N.S.	1	1.00	2.83	0.79	0.85	1.02	1.00	0.96	0.85
time (sec)	N/A	0.034	0.043	0.125	0.276	0.235	0.088	0.295	0.040

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	107	69	73	84	88	73	62
N.S.	1	1.00	1.20	0.78	0.82	0.94	0.99	0.82	0.70
time (sec)	N/A	0.068	0.059	0.133	0.274	0.251	0.118	0.319	0.023

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	139	92	0	210	192	109	100
N.S.	1	1.00	0.99	0.65	0.00	1.49	1.36	0.77	0.71
time (sec)	N/A	0.070	0.197	0.111	0.000	0.248	0.384	0.304	8.363

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	38
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.43
time (sec)	N/A	0.042	0.018	0.104	0.000	0.261	0.077	0.306	8.423

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	135	89	0	197	197	108	99
N.S.	1	1.00	0.96	0.64	0.00	1.41	1.41	0.77	0.71
time (sec)	N/A	0.072	0.118	0.106	0.000	0.261	0.380	0.297	0.039

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	135	89	0	209	214	108	97
N.S.	1	1.00	0.96	0.64	0.00	1.49	1.53	0.77	0.69
time (sec)	N/A	0.061	0.108	0.094	0.000	0.256	0.389	0.306	8.309

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	40
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.45
time (sec)	N/A	0.036	0.013	0.089	0.000	0.251	0.091	0.339	0.023

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	140	96	0	209	218	113	102
N.S.	1	1.00	0.97	0.66	0.00	1.44	1.50	0.78	0.70
time (sec)	N/A	0.077	0.176	0.127	0.000	0.272	0.388	0.341	0.030

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	148	92	0	237	197	113	104
N.S.	1	1.00	1.01	0.63	0.00	1.61	1.34	0.77	0.71
time (sec)	N/A	0.067	0.223	0.123	0.000	0.263	0.390	0.334	0.017

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	75	0	90	94	100	52
N.S.	1	1.00	0.97	0.77	0.00	0.92	0.96	1.02	0.53
time (sec)	N/A	0.060	0.031	0.138	0.000	0.270	0.112	0.337	0.021

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	171	96	0	232	209	120	110
N.S.	1	1.00	1.11	0.62	0.00	1.51	1.36	0.78	0.71
time (sec)	N/A	0.101	0.248	0.143	0.000	0.267	0.387	0.327	0.019

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	79	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.027	0.009	0.066	0.267	0.239	0.066	0.341	0.030

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	47	48	99	29
N.S.	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51
time (sec)	N/A	0.030	0.012	0.067	0.000	0.236	0.058	0.341	8.263

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.023	0.008	0.054	0.265	0.238	0.063	0.297	8.227

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	98	32	0	153	70	76	53
N.S.	1	1.00	1.20	0.39	0.00	1.87	0.85	0.93	0.65
time (sec)	N/A	0.049	0.098	0.063	0.000	0.277	0.108	0.300	0.031

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.016	0.006	0.053	0.269	0.232	0.057	0.303	8.213

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	32	0	165	70	64	53
N.S.	1	1.00	1.01	0.39	0.00	2.01	0.85	0.78	0.65
time (sec)	N/A	0.039	0.045	0.068	0.000	0.245	0.102	0.303	0.029

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	33	38	34	41	38	36
N.S.	1	1.00	1.34	0.80	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.024	0.014	0.073	0.270	0.234	0.084	0.306	8.175

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	50	49	99	29
N.S.	1	1.00	0.96	0.77	0.00	0.88	0.86	1.74	0.51
time (sec)	N/A	0.027	0.017	0.081	0.000	0.237	0.069	0.335	8.152

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	38	43	51	48	48	41
N.S.	1	1.00	1.06	0.79	0.90	1.06	1.00	1.00	0.85
time (sec)	N/A	0.034	0.015	0.086	0.264	0.248	0.085	0.315	0.038

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	56	46	0	174	83	56	63
N.S.	1	1.00	0.58	0.48	0.00	1.81	0.86	0.58	0.66
time (sec)	N/A	0.062	0.016	0.090	0.000	0.246	0.125	0.324	0.032

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	59	44	0	418	26	254	209
N.S.	1	1.00	0.17	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.228	0.023	0.063	0.000	0.264	1.487	0.329	0.104

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	41	32	0	101	165	205	53
N.S.	1	1.00	0.15	0.12	0.00	0.37	0.60	0.75	0.19
time (sec)	N/A	0.168	0.012	0.061	0.000	0.240	0.098	0.320	0.057

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	39	40	0	393	24	253	474
N.S.	1	1.00	0.11	0.12	0.00	1.13	0.07	0.73	1.37
time (sec)	N/A	0.137	0.013	0.062	0.000	0.260	1.478	0.371	8.246

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	40	40	0	545	26	253	286
N.S.	1	1.00	0.11	0.11	0.00	1.54	0.07	0.71	0.81
time (sec)	N/A	0.140	0.011	0.062	0.000	0.271	1.442	0.362	0.048

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	101	165	205	53
N.S.	1	1.00	0.15	0.11	0.00	0.37	0.60	0.75	0.19
time (sec)	N/A	0.145	0.012	0.054	0.000	0.248	0.099	0.317	0.024

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	61	40	0	555	29	258	253
N.S.	1	1.00	0.17	0.11	0.00	1.54	0.08	0.72	0.70
time (sec)	N/A	0.155	0.016	0.096	0.000	0.257	1.467	0.310	8.313

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	65	38	0	443	31	258	213
N.S.	1	1.00	0.18	0.10	0.00	1.20	0.08	0.70	0.58
time (sec)	N/A	0.160	0.014	0.089	0.000	0.264	1.491	0.314	8.346

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	54	43	0	124	182	217	63
N.S.	1	1.00	0.19	0.15	0.00	0.43	0.63	0.76	0.22
time (sec)	N/A	0.177	0.015	0.086	0.000	0.249	0.131	0.353	8.342

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	54	44	0	428	37	265	486
N.S.	1	1.00	0.14	0.12	0.00	1.14	0.10	0.70	1.29
time (sec)	N/A	0.192	0.016	0.104	0.000	0.255	1.462	0.319	0.038

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	50	62	60	50	64
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03
time (sec)	N/A	0.036	0.029	0.074	0.263	0.233	0.067	0.376	0.080

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	97	42	0	152	54	66	130
N.S.	1	1.00	1.08	0.47	0.00	1.69	0.60	0.73	1.44
time (sec)	N/A	0.104	0.113	0.103	0.000	0.275	0.111	0.347	8.387

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	56	53	45	59
N.S.	1	1.00	0.96	0.60	0.82	1.02	0.96	0.82	1.07
time (sec)	N/A	0.022	0.018	0.071	0.276	0.247	0.060	0.334	8.420

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	34	0	143	49	47	117
N.S.	1	1.00	0.93	0.42	0.00	1.77	0.60	0.58	1.44
time (sec)	N/A	0.059	0.034	0.083	0.000	0.264	0.094	0.329	0.079

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	31	43	42	31	30
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30
time (sec)	N/A	0.019	0.009	0.056	0.270	0.253	0.049	0.333	0.060

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	34	0	159	49	41	125
N.S.	1	1.00	0.99	0.45	0.00	2.12	0.65	0.55	1.67
time (sec)	N/A	0.044	0.027	0.081	0.000	0.250	0.095	0.341	0.032

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	35	51	58	58	51	42
N.S.	1	1.00	0.96	0.61	0.89	1.02	1.02	0.89	0.74
time (sec)	N/A	0.024	0.024	0.074	0.270	0.235	0.074	0.331	8.366

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	65	42	0	171	56	68	130
N.S.	1	1.00	0.73	0.47	0.00	1.92	0.63	0.76	1.46
time (sec)	N/A	0.054	0.017	0.092	0.000	0.250	0.112	0.337	0.034

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	42	56	76	65	63	49
N.S.	1	1.00	0.91	0.64	0.85	1.15	0.98	0.95	0.74
time (sec)	N/A	0.044	0.024	0.088	0.267	0.244	0.089	0.328	8.265

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	48	0	190	65	77	136
N.S.	1	1.00	0.75	0.49	0.00	1.96	0.67	0.79	1.40
time (sec)	N/A	0.092	0.015	0.106	0.000	0.247	0.132	0.375	0.080

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	440	58	46	0	422	29	240	216
N.S.	1	0.96	0.13	0.10	0.00	0.92	0.06	0.52	0.47
time (sec)	N/A	0.280	0.017	0.060	0.000	0.267	0.926	0.370	8.307

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	41	40	0	361	26	239	149
N.S.	1	1.00	0.10	0.09	0.00	0.84	0.06	0.55	0.35
time (sec)	N/A	0.218	0.012	0.063	0.000	0.271	0.932	0.366	8.334

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	39	40	0	373	24	239	454
N.S.	1	1.00	0.09	0.09	0.00	0.83	0.05	0.53	1.01
time (sec)	N/A	0.206	0.013	0.061	0.000	0.257	0.864	0.376	0.125

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	431	40	40	0	525	26	239	275
N.S.	1	1.01	0.09	0.09	0.00	1.23	0.06	0.56	0.64
time (sec)	N/A	0.204	0.010	0.064	0.000	0.250	0.885	0.431	8.286

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	42	37	0	309	26	239	403
N.S.	1	1.00	0.10	0.09	0.00	0.75	0.06	0.58	0.97
time (sec)	N/A	0.188	0.011	0.059	0.000	0.260	0.915	0.327	0.045

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	61	42	0	559	32	244	292
N.S.	1	1.00	0.15	0.10	0.00	1.34	0.08	0.59	0.70
time (sec)	N/A	0.224	0.017	0.092	0.000	0.258	0.940	0.378	8.254

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	65	40	0	451	34	244	492
N.S.	1	1.00	0.14	0.09	0.00	0.97	0.07	0.52	1.06
time (sec)	N/A	0.264	0.016	0.098	0.000	0.267	0.961	0.373	0.118

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	0
N.S.	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.595	0.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	50	62	58	53	64
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03
time (sec)	N/A	0.031	0.215	0.050	0.260	0.235	0.067	0.332	8.450

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	103	67	92	114	170	97	90
N.S.	1	1.00	1.14	0.74	1.02	1.27	1.89	1.08	1.00
time (sec)	N/A	0.057	0.148	0.069	0.268	0.254	0.202	0.341	0.059

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	57	53	48	59
N.S.	1	1.00	0.96	0.60	0.82	1.04	0.96	0.87	1.07
time (sec)	N/A	0.021	0.050	0.045	0.271	0.238	0.058	0.319	0.059

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	91	62	87	109	165	92	77
N.S.	1	1.00	1.12	0.77	1.07	1.35	2.04	1.14	0.95
time (sec)	N/A	0.044	0.068	0.067	0.266	0.256	0.195	0.324	8.372

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	0	43	42	33	30
N.S.	1	1.00	1.65	0.83	0.00	1.87	1.83	1.43	1.30
time (sec)	N/A	0.018	0.024	0.043	0.000	0.248	0.057	0.336	8.636

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	91	62	87	107	165	92	83
N.S.	1	1.00	1.21	0.83	1.16	1.43	2.20	1.23	1.11
time (sec)	N/A	0.031	0.050	0.043	0.269	0.246	0.190	0.321	8.561

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	64	51	59	58	54	42
N.S.	1	1.00	0.96	1.12	0.89	1.04	1.02	0.95	0.74
time (sec)	N/A	0.020	0.050	0.061	0.272	0.241	0.079	0.299	0.242

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	67	92	125	172	97	88
N.S.	1	1.00	1.16	0.75	1.03	1.40	1.93	1.09	0.99
time (sec)	N/A	0.041	0.081	0.076	0.268	0.242	0.221	0.321	0.039

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	61	71	56	76	66	66	49
N.S.	1	1.00	0.92	1.08	0.85	1.15	1.00	1.00	0.74
time (sec)	N/A	0.041	0.063	0.069	0.277	0.242	0.090	0.338	8.589

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	111	72	99	130	199	104	95
N.S.	1	1.00	1.14	0.74	1.02	1.34	2.05	1.07	0.98
time (sec)	N/A	0.062	0.103	0.080	0.271	0.248	0.242	0.333	8.585

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	160	69	0	318	58	148	246
N.S.	1	1.00	0.94	0.41	0.00	1.87	0.34	0.87	1.45
time (sec)	N/A	0.087	0.344	0.121	0.000	0.253	0.735	0.370	8.567

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	160	68	0	261	53	147	147
N.S.	1	1.00	0.96	0.41	0.00	1.56	0.32	0.88	0.88
time (sec)	N/A	0.054	0.184	0.085	0.000	0.260	0.732	0.369	0.124

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	132	60	0	269	49	147	269
N.S.	1	1.00	0.76	0.35	0.00	1.55	0.28	0.85	1.55
time (sec)	N/A	0.052	0.173	0.097	0.000	0.277	0.711	0.368	0.114

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	166	131	64	0	285	53	147	269
N.S.	1	1.14	0.90	0.44	0.00	1.97	0.37	1.01	1.86
time (sec)	N/A	0.040	0.047	0.081	0.000	0.281	0.729	0.345	0.045

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	160	68	0	261	53	147	245
N.S.	1	1.00	0.95	0.40	0.00	1.54	0.31	0.87	1.45
time (sec)	N/A	0.036	0.131	0.077	0.000	0.274	0.734	0.332	8.377

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	174	73	0	327	63	152	250
N.S.	1	1.00	1.01	0.42	0.00	1.90	0.37	0.88	1.45
time (sec)	N/A	0.065	0.241	0.100	0.000	0.280	0.751	0.356	8.311

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	166	73	0	347	63	152	268
N.S.	1	1.00	0.91	0.40	0.00	1.91	0.35	0.84	1.47
time (sec)	N/A	0.086	0.211	0.104	0.000	0.280	0.753	0.358	8.533

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	189	79	0	324	73	159	257
N.S.	1	1.00	1.09	0.46	0.00	1.87	0.42	0.92	1.49
time (sec)	N/A	0.095	0.226	0.108	0.000	0.264	0.775	0.393	8.328

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	189	79	0	352	70	159	291
N.S.	1	1.00	1.00	0.42	0.00	1.86	0.37	0.84	1.54
time (sec)	N/A	0.115	0.242	0.117	0.000	0.282	0.803	0.348	8.385

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	16
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.76
time (sec)	N/A	0.008	0.009	0.048	0.180	0.245	0.047	0.296	0.041

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	19	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85
time (sec)	N/A	0.013	0.010	0.042	0.179	0.248	0.061	0.306	8.318

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.025	0.023	0.055	0.263	0.260	0.067	1.101	8.396

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.015	0.012	0.036	0.271	0.271	0.056	1.101	8.422

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	197	31	36	32	41	33	34
N.S.	1	1.00	5.05	0.79	0.92	0.82	1.05	0.85	0.87
time (sec)	N/A	0.024	0.048	0.098	0.262	0.264	0.078	0.300	0.037

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	208	38	41	49	48	45	41
N.S.	1	1.00	4.33	0.79	0.85	1.02	1.00	0.94	0.85
time (sec)	N/A	0.033	0.047	0.089	0.267	0.287	0.091	0.288	8.305

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	197	31	0	32	41	33	34
N.S.	1	1.00	5.05	0.79	0.00	0.82	1.05	0.85	0.87
time (sec)	N/A	0.023	0.021	0.063	0.000	0.278	0.080	0.285	0.019

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	164	0	466	605	145	183
N.S.	1	1.00	0.95	1.12	0.00	3.17	4.12	0.99	1.24
time (sec)	N/A	0.090	0.122	0.155	0.000	0.290	0.735	0.277	0.095

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	112	128	0	383	498	113	151
N.S.	1	1.00	0.95	1.08	0.00	3.25	4.22	0.96	1.28
time (sec)	N/A	0.071	0.066	0.123	0.000	0.312	0.602	0.283	8.289

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.058	0.073	0.077	0.000	0.285	0.467	0.307	8.318

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.034	0.046	0.044	0.000	0.294	0.336	0.294	0.082

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.023	0.023	0.037	0.000	0.292	0.165	0.309	8.390

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	120	124	34	46
N.S.	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28
time (sec)	N/A	0.022	0.007	0.037	0.000	0.297	0.107	0.286	0.026

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.035	0.052	0.069	0.000	0.291	4.462	0.324	0.273

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	81	0	269	0	79	339
N.S.	1	1.00	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.066	0.066	0.056	0.000	0.295	0.000	0.302	8.619

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	128	0	358	0	105	447
N.S.	1	1.00	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.101	0.096	0.121	0.000	0.315	0.000	0.310	8.586

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	157	0	445	0	136	524
N.S.	1	1.00	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.127	0.076	0.119	0.000	0.328	0.000	0.298	8.599

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	163	238	0	1029	1012	188	382
N.S.	1	1.00	0.83	1.21	0.00	5.25	5.16	0.96	1.95
time (sec)	N/A	0.141	0.183	0.174	0.000	0.292	1.347	0.330	8.686

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	132	198	0	837	842	161	261
N.S.	1	1.00	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.098	0.128	0.159	0.000	0.298	1.072	0.280	8.471

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	169	0	635	729	125	279
N.S.	1	1.00	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.066	0.100	0.106	0.000	0.316	0.758	0.297	8.523

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	387	280	88	135
N.S.	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90
time (sec)	N/A	0.026	0.069	0.075	0.000	0.277	0.317	0.288	8.310

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.021	0.052	0.068	0.000	0.296	0.294	0.293	0.058

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.021	0.050	0.066	0.000	0.275	0.309	0.303	0.048

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	177	0	781	0	126	620
N.S.	1	1.00	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.096	0.122	0.132	0.000	0.343	0.000	0.297	8.703

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	205	0	975	0	171	775
N.S.	1	1.00	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.124	0.177	0.089	0.000	0.401	0.000	0.308	8.847

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	175	255	0	1226	0	229	914
N.S.	1	1.00	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.169	0.259	0.184	0.000	0.458	0.000	0.314	9.119

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	260	401	0	1926	1714	282	705
N.S.	1	1.00	1.09	1.68	0.00	8.09	7.20	1.18	2.96
time (sec)	N/A	0.212	0.238	0.200	0.000	0.294	3.038	0.330	8.593

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	221	357	0	1603	1510	245	620
N.S.	1	1.00	1.16	1.88	0.00	8.44	7.95	1.29	3.26
time (sec)	N/A	0.187	0.197	0.154	0.000	0.287	1.868	0.295	8.820

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	174	260	0	953	547	202	343
N.S.	1	1.00	1.57	2.34	0.00	8.59	4.93	1.82	3.09
time (sec)	N/A	0.047	0.118	0.099	0.000	0.292	0.822	0.308	0.114

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	126	223	0	872	513	163	271
N.S.	1	1.00	1.18	2.08	0.00	8.15	4.79	1.52	2.53
time (sec)	N/A	0.039	0.138	0.096	0.000	0.286	0.663	0.286	8.205

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	131	210	0	887	570	154	313
N.S.	1	1.00	1.14	1.83	0.00	7.71	4.96	1.34	2.72
time (sec)	N/A	0.046	0.092	0.106	0.000	0.280	0.755	0.287	8.410

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	118	0	788	481	135	253
N.S.	1	1.00	0.99	1.15	0.00	7.65	4.67	1.31	2.46
time (sec)	N/A	0.028	0.064	0.092	0.000	0.286	0.604	0.289	8.392

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	101	97	116	0	785	474	136	285
N.S.	1	0.98	0.94	1.13	0.00	7.62	4.60	1.32	2.77
time (sec)	N/A	0.028	0.069	0.093	0.000	0.263	0.697	0.297	8.492

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	178	352	0	1985	0	239	1089
N.S.	1	1.00	0.96	1.90	0.00	10.73	0.00	1.29	5.89
time (sec)	N/A	0.148	0.233	0.185	0.000	0.524	0.000	0.293	8.990

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	221	404	0	2280	0	309	1255
N.S.	1	1.00	0.92	1.69	0.00	9.54	0.00	1.29	5.25
time (sec)	N/A	0.192	0.259	0.133	0.000	0.674	0.000	0.298	9.079

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	27	30	30	34	32	26
N.S.	1	1.00	1.00	0.68	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.015	0.006	0.034	0.183	0.249	0.060	0.295	0.025

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	25	25	27	27	21
N.S.	1	1.00	1.00	0.67	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.013	0.004	0.033	0.184	0.231	0.059	0.307	8.287

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	20	20	20	22	16
N.S.	1	1.00	1.00	0.65	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.009	0.004	0.031	0.182	0.245	0.060	0.279	0.042

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	17	17	17	19	13
N.S.	1	1.00	1.00	0.67	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.008	0.004	0.027	0.186	0.248	0.066	0.282	0.036

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	14	17	17	15	19	8
N.S.	1	1.00	0.91	0.61	0.74	0.74	0.65	0.83	0.35
time (sec)	N/A	0.009	0.003	0.031	0.185	0.248	0.046	0.307	8.486

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	21	21	24	24	17
N.S.	1	1.00	1.00	0.67	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.011	0.014	0.042	0.226	0.298	0.066	0.289	0.056

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.021	0.004	0.048	0.196	0.254	0.072	0.275	0.025

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	31	31	39	36	34	26
N.S.	1	1.00	1.00	0.76	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.023	0.005	0.047	0.189	0.261	0.075	0.323	8.552

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	36	36	44	41	39	32
N.S.	1	1.00	1.00	0.75	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.030	0.006	0.050	0.181	0.275	0.087	0.310	0.027

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	208	249	0	959	0	0	0
N.S.	1	1.00	1.02	1.22	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.154	1.375	0.147	0.000	0.426	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	158	179	0	709	0	0	0
N.S.	1	1.00	1.09	1.23	0.00	4.89	0.00	0.00	0.00
time (sec)	N/A	0.091	0.584	0.067	0.000	0.323	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	131	121	0	590	0	0	100
N.S.	1	1.00	1.25	1.15	0.00	5.62	0.00	0.00	0.95
time (sec)	N/A	0.055	0.194	0.038	0.000	0.320	0.000	0.000	8.525

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	89	88	0	171	0	0	53
N.S.	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.79
time (sec)	N/A	0.027	0.153	0.048	0.000	0.275	0.000	0.000	8.578

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	140	197	0	465	0	231	0
N.S.	1	1.00	1.05	1.48	0.00	3.50	0.00	1.74	0.00
time (sec)	N/A	0.068	0.461	0.083	0.000	0.310	0.000	0.410	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	246	376	0	1081	0	499	0
N.S.	1	1.00	1.12	1.71	0.00	4.91	0.00	2.27	0.00
time (sec)	N/A	0.134	1.199	0.151	0.000	0.407	0.000	0.374	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	32	40	8	8	0	29	134
N.S.	1	1.00	0.44	0.55	0.11	0.11	0.00	0.40	1.84
time (sec)	N/A	0.025	1.016	0.069	0.192	0.251	0.000	0.318	8.388

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	57	0	1059	129	2109	3026
N.S.	1	1.00	1.13	0.32	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.175	0.110	0.048	0.000	0.277	1.233	0.786	8.733

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	2882	196	0	2280
N.S.	1	1.00	0.11	0.09	0.00	4.57	0.31	0.00	3.61
time (sec)	N/A	0.749	0.041	0.059	0.000	0.388	54.067	0.000	10.265

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	4001	0	0	10382
N.S.	1	1.00	0.19	0.16	0.00	10.64	0.00	0.00	27.61
time (sec)	N/A	0.373	0.040	0.047	0.000	0.456	0.000	0.000	9.601

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	105	84	0	0	0	0	0
N.S.	1	1.00	0.99	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.204	0.032	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	55	50	54	53	51	49	44
N.S.	1	1.00	1.38	1.25	1.35	1.32	1.28	1.22	1.10
time (sec)	N/A	0.014	0.030	0.079	0.188	0.285	0.162	0.282	0.027

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	41	23	0	124	45	0
N.S.	1	1.00	0.67	0.55	0.31	0.00	1.65	0.60	0.00
time (sec)	N/A	0.027	0.028	0.091	0.198	0.000	0.397	0.289	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	98	114	84	233	140	0
N.S.	1	1.00	0.85	0.72	0.83	0.61	1.70	1.02	0.00
time (sec)	N/A	0.054	0.057	0.075	0.190	0.262	5.616	0.309	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	93	76	114	61	204	102	0
N.S.	1	1.00	0.68	0.55	0.83	0.45	1.49	0.74	0.00
time (sec)	N/A	0.047	0.022	0.020	0.185	0.284	1.431	0.295	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	67	54	114	32	175	64	0
N.S.	1	1.00	0.49	0.39	0.83	0.23	1.28	0.47	0.00
time (sec)	N/A	0.040	0.018	0.023	0.199	0.258	0.616	0.292	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	88	43	42	114	10	146	26	71
N.S.	1	1.00	0.49	0.48	1.30	0.11	1.66	0.30	0.81
time (sec)	N/A	0.028	0.008	0.086	0.184	0.245	0.478	0.277	8.736

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	65	52	36	33	170	61	0
N.S.	1	1.00	0.44	0.35	0.24	0.22	1.16	0.41	0.00
time (sec)	N/A	0.048	0.023	0.041	0.183	0.262	0.432	0.319	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	72	81	55	113	0	64	0
N.S.	1	1.00	0.55	0.62	0.42	0.87	0.00	0.49	0.00
time (sec)	N/A	0.052	0.029	0.115	0.190	0.270	0.000	0.292	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	56	43	53	136	0	43	53
N.S.	1	1.00	0.41	0.32	0.39	1.01	0.00	0.32	0.39
time (sec)	N/A	0.050	0.055	0.263	0.188	0.278	0.000	0.314	9.067

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	56	43	53	209	0	43	53
N.S.	1	1.00	0.41	0.31	0.39	1.53	0.00	0.31	0.39
time (sec)	N/A	0.053	0.052	0.493	0.186	0.338	0.000	0.291	9.485

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	56	43	53	275	0	43	53
N.S.	1	1.00	0.41	0.31	0.39	2.01	0.00	0.31	0.39
time (sec)	N/A	0.053	0.055	0.779	0.185	0.410	0.000	0.289	9.598

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	56	43	53	343	0	43	53
N.S.	1	1.00	0.41	0.31	0.39	2.50	0.00	0.31	0.39
time (sec)	N/A	0.053	0.054	1.184	0.191	0.577	0.000	0.290	10.060

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	468	468	207	0	362	579	0	1564	777
N.S.	1	1.00	0.44	0.00	0.77	1.24	0.00	3.34	1.66
time (sec)	N/A	0.153	0.308	0.000	0.201	0.425	0.000	0.323	9.607

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	315	315	143	0	198	297	0	745	390
N.S.	1	1.00	0.45	0.00	0.63	0.94	0.00	2.37	1.24
time (sec)	N/A	0.094	0.240	0.000	0.199	0.330	0.000	0.317	8.760

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	83	0	77	110	0	229	138
N.S.	1	1.00	0.58	0.00	0.54	0.77	0.00	1.61	0.97
time (sec)	N/A	0.048	0.137	0.000	0.197	0.302	0.000	0.341	8.439

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	61	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	82	0	0	69
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47
time (sec)	N/A	0.067	0.333	0.000	0.000	0.338	0.000	0.000	8.602

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	93	103	114	147	0	0	0
N.S.	1	1.00	0.53	0.59	0.65	0.84	0.00	0.00	0.00
time (sec)	N/A	0.066	0.093	0.256	0.198	1.994	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	121	163	119	0	0	105	0
N.S.	1	1.00	0.45	0.61	0.44	0.00	0.00	0.39	0.00
time (sec)	N/A	0.097	0.129	5.449	0.199	0.000	0.000	0.304	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	66	65	0	0	0	80	0
N.S.	1	1.00	0.37	0.36	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.062	0.046	0.170	0.000	0.000	0.000	0.443	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	125	113	79	0	0	173	0
N.S.	1	1.00	0.32	0.29	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.126	0.070	0.126	0.209	0.000	0.000	0.325	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	99	91	57	0	0	128	0
N.S.	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.093	0.060	0.036	0.193	0.000	0.000	0.331	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	75	69	30	0	0	79	0
N.S.	1	1.00	0.40	0.37	0.16	0.00	0.00	0.42	0.00
time (sec)	N/A	0.067	0.044	0.030	0.196	0.000	0.000	0.312	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	49	47	10	0	0	34	39
N.S.	1	1.00	0.56	0.53	0.11	0.00	0.00	0.39	0.44
time (sec)	N/A	0.039	0.024	0.023	0.196	0.000	0.000	0.306	8.479

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	86	78	44	0	0	77	0
N.S.	1	1.00	0.45	0.41	0.23	0.00	0.00	0.41	0.00
time (sec)	N/A	0.082	0.040	0.026	0.240	0.000	0.000	0.337	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	126	141	97	0	0	127	0
N.S.	1	1.00	0.42	0.47	0.32	0.00	0.00	0.42	0.00
time (sec)	N/A	0.130	0.097	0.038	0.227	0.000	0.000	0.312	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	152	199	139	0	0	147	0
N.S.	1	1.00	0.37	0.49	0.34	0.00	0.00	0.36	0.00
time (sec)	N/A	0.176	0.096	0.037	0.188	0.000	0.000	0.310	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	98	91	57	0	0	126	0
N.S.	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.096	0.057	0.153	0.181	0.000	0.000	0.310	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	101	91	52	0	0	125	0
N.S.	1	1.00	0.35	0.31	0.18	0.00	0.00	0.43	0.00
time (sec)	N/A	0.092	0.062	0.136	0.188	0.000	0.000	0.314	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	98	141	99	302	0	84	0
N.S.	1	1.00	0.44	0.64	0.45	1.36	0.00	0.38	0.00
time (sec)	N/A	0.088	0.088	1.248	0.187	0.390	0.000	0.314	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	124	117	79	0	0	172	0
N.S.	1	1.00	0.32	0.30	0.20	0.00	0.00	0.44	0.00
time (sec)	N/A	0.121	0.067	0.058	0.179	0.000	0.000	0.427	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	47	45	38	63	0	0
N.S.	1	1.00	0.83	1.02	0.98	0.83	1.37	0.00	0.00
time (sec)	N/A	0.026	0.051	0.663	0.178	0.271	22.442	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	31	32	24	48	0	0
N.S.	1	1.00	0.96	1.11	1.14	0.86	1.71	0.00	0.00
time (sec)	N/A	0.017	0.043	0.650	0.173	0.270	8.957	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	19	15	46	0	0
N.S.	1	1.00	1.00	1.20	1.27	1.00	3.07	0.00	0.00
time (sec)	N/A	0.009	0.024	0.627	0.174	0.258	1.745	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	26	27	22	54	25	20
N.S.	1	1.00	1.09	1.13	1.17	0.96	2.35	1.09	0.87
time (sec)	N/A	0.013	0.037	0.642	0.182	0.272	1.744	0.276	8.710

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	58	58	59	100	0	0
N.S.	1	1.00	0.84	1.02	1.02	1.04	1.75	0.00	0.00
time (sec)	N/A	0.029	0.078	0.625	0.180	0.287	29.315	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	75	71	72	0	0	0
N.S.	1	1.00	0.83	0.99	0.93	0.95	0.00	0.00	0.00
time (sec)	N/A	0.033	0.091	0.615	0.174	0.276	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	76	90	84	85	0	0	0
N.S.	1	1.00	0.82	0.97	0.90	0.91	0.00	0.00	0.00
time (sec)	N/A	0.036	0.103	0.628	0.179	0.264	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	34	54	0	270	0	203	0
N.S.	1	1.00	0.14	0.23	0.00	1.14	0.00	0.86	0.00
time (sec)	N/A	0.144	0.040	0.691	0.000	0.266	0.000	0.321	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	212	0	136	0
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	0.00
time (sec)	N/A	0.090	0.039	0.672	0.000	0.277	0.000	0.287	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	151	0	38	0
N.S.	1	1.00	0.64	1.58	0.00	3.02	0.00	0.76	0.00
time (sec)	N/A	0.025	0.035	0.664	0.000	0.266	0.000	0.297	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	97	0	161	0	0	0
N.S.	1	1.00	0.50	1.43	0.00	2.37	0.00	0.00	0.00
time (sec)	N/A	0.029	0.041	0.670	0.000	0.265	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	34	73	0	171	0	0	0
N.S.	1	1.00	0.19	0.41	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.097	0.039	0.671	0.000	0.266	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	34	73	0	249	0	0	0
N.S.	1	1.00	0.13	0.29	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.152	0.038	0.688	0.000	0.276	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	43	59	0	0	0
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	0.00
time (sec)	N/A	0.026	0.039	0.000	0.199	0.259	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	59	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.027	0.084	0.000	0.000	0.263	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	96	208	74	74	0	0	0
N.S.	1	1.00	0.86	1.86	0.66	0.66	0.00	0.00	0.00
time (sec)	N/A	0.028	0.042	0.113	0.186	0.261	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	70	135	48	48	0	0	0
N.S.	1	1.00	0.62	1.21	0.43	0.43	0.00	0.00	0.00
time (sec)	N/A	0.028	0.024	0.047	0.190	0.259	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	44	64	22	22	0	0	0
N.S.	1	1.00	0.44	0.65	0.22	0.22	0.00	0.00	0.00
time (sec)	N/A	0.021	0.013	0.041	0.185	0.267	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	47	71	32	24	0	0	0
N.S.	1	1.00	0.52	0.79	0.36	0.27	0.00	0.00	0.00
time (sec)	N/A	0.030	0.025	0.060	0.179	0.264	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	37	41	41	0	0	0
N.S.	1	1.00	0.83	0.77	0.85	0.85	0.00	0.00	0.00
time (sec)	N/A	0.018	0.019	0.039	0.182	0.257	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	69	69	0	0	0
N.S.	1	1.00	0.45	0.42	0.78	0.78	0.00	0.00	0.00
time (sec)	N/A	0.035	0.055	0.047	0.190	0.266	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	97	97	0	0	0
N.S.	1	1.00	0.45	0.42	1.10	1.10	0.00	0.00	0.00
time (sec)	N/A	0.034	0.059	0.050	0.180	0.254	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	108	55	99	47	57	0	173	0
N.S.	1	1.00	0.51	0.92	0.44	0.53	0.00	1.60	0.00
time (sec)	N/A	0.029	0.032	0.070	0.183	0.300	0.000	0.283	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	0.00
time (sec)	N/A	0.020	0.021	0.034	0.179	0.266	0.000	0.290	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	0.00
time (sec)	N/A	0.018	0.023	0.029	0.177	0.288	0.000	0.270	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	39	56	19	20	0	25	0
N.S.	1	1.00	0.44	0.64	0.22	0.23	0.00	0.28	0.00
time (sec)	N/A	0.014	0.010	0.028	0.177	0.261	0.000	0.279	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	38	54	13	15	0	0	0
N.S.	1	1.00	0.45	0.64	0.15	0.18	0.00	0.00	0.00
time (sec)	N/A	0.017	0.013	0.035	0.180	0.267	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	42	61	22	23	0	0	0
N.S.	1	1.00	0.45	0.65	0.23	0.24	0.00	0.00	0.00
time (sec)	N/A	0.020	0.025	0.033	0.182	0.284	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	47	61	22	23	0	0	0
N.S.	1	1.00	0.49	0.64	0.23	0.24	0.00	0.00	0.00
time (sec)	N/A	0.021	0.023	0.034	0.176	0.268	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	238	238	90	499	276	390	0	2719	0
N.S.	1	1.00	0.38	2.10	1.16	1.64	0.00	11.42	0.00
time (sec)	N/A	0.068	0.086	0.086	0.188	0.275	0.000	0.413	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	123	146	108	144	0	292	0
N.S.	1	1.00	0.58	0.69	0.51	0.68	0.00	1.38	0.00
time (sec)	N/A	0.045	0.063	0.030	0.178	0.289	0.000	0.310	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	124	145	109	145	0	292	0
N.S.	1	1.00	0.59	0.69	0.52	0.69	0.00	1.38	0.00
time (sec)	N/A	0.042	0.059	0.031	0.179	0.278	0.000	0.311	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	122	138	101	130	0	263	0
N.S.	1	1.00	0.59	0.67	0.49	0.63	0.00	1.28	0.00
time (sec)	N/A	0.036	0.058	0.029	0.176	0.275	0.000	0.297	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	67	127	43	44	0	0	0
N.S.	1	1.00	0.34	0.65	0.22	0.22	0.00	0.00	0.00
time (sec)	N/A	0.035	0.024	0.035	0.270	0.275	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	124	147	101	131	0	0	0
N.S.	1	1.00	0.58	0.69	0.48	0.62	0.00	0.00	0.00
time (sec)	N/A	0.052	0.073	0.043	0.217	0.272	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	124	145	101	134	0	0	0
N.S.	1	1.00	0.57	0.67	0.46	0.61	0.00	0.00	0.00
time (sec)	N/A	0.049	0.070	0.045	0.213	0.289	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	62	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	44	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	45	66	27	22	0	0	0
N.S.	1	1.00	0.53	0.78	0.32	0.26	0.00	0.00	0.00
time (sec)	N/A	0.024	0.021	0.036	0.185	0.263	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	53	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	61	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.014	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	79	104	70	106	0	0	0
N.S.	1	1.00	0.50	0.65	0.44	0.67	0.00	0.00	0.00
time (sec)	N/A	0.055	0.036	0.038	0.195	0.300	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	55	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	79	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.013	0.058	0.000	0.000	0.271	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	45	160	0	0
N.S.	1	1.00	0.74	1.19	0.00	1.05	3.72	0.00	0.00
time (sec)	N/A	0.011	0.111	0.960	0.000	0.275	4.979	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	80	0	0	103	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.036	0.144	0.000	0.000	0.280	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	59	0	0	82	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.030	0.112	0.000	0.000	0.263	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	124	75	0	0	165	0	0	0
N.S.	1	1.06	0.64	0.00	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.048	0.096	0.000	0.000	0.266	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	54	113	59	78	0	0	0
N.S.	1	1.00	0.52	1.10	0.57	0.76	0.00	0.00	0.00
time (sec)	N/A	0.043	0.069	14.742	0.194	0.252	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	97	973	0	353	0	0	0
N.S.	1	1.00	0.87	8.77	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.080	0.245	0.351	0.000	0.269	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	664	0	285	0	0	0
N.S.	1	1.00	0.94	7.63	0.00	3.28	0.00	0.00	0.00
time (sec)	N/A	0.050	0.169	0.287	0.000	0.273	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	402	0	231	0	0	0
N.S.	1	1.00	0.97	5.91	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.034	0.111	0.243	0.000	0.258	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	43	113	0	159	0	39	39
N.S.	1	1.00	1.10	2.90	0.00	4.08	0.00	1.00	1.00
time (sec)	N/A	0.025	0.067	0.198	0.000	0.266	0.000	0.275	8.383

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	90	658	0	333	0	0	0
N.S.	1	1.00	0.92	6.71	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.085	0.216	0.253	0.000	0.283	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	115	958	0	429	0	0	0
N.S.	1	1.00	0.91	7.60	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.125	0.357	0.292	0.000	0.292	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	147	1300	0	522	0	0	0
N.S.	1	1.00	0.90	7.93	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.165	0.317	0.373	0.000	0.279	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	340	280	0	3481	0	0	0
N.S.	1	1.00	0.96	0.79	0.00	9.86	0.00	0.00	0.00
time (sec)	N/A	0.345	0.703	1.036	0.000	0.403	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	526	260	0	2461	0	0	0
N.S.	1	1.00	0.86	0.43	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.736	0.639	0.700	0.000	0.347	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	114	0	801	0	1037	0
N.S.	1	1.00	0.86	0.67	0.00	4.74	0.00	6.14	0.00
time (sec)	N/A	0.137	0.294	0.369	0.000	0.289	0.000	0.677	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	127	268	0	1229	0	0	0
N.S.	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.224	0.530	0.000	0.298	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	699	699	127	534	0	3155	0	0	0
N.S.	1	1.00	0.18	0.76	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	1.000	0.176	1.044	0.000	0.369	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	127	630	0	4375	0	0	0
N.S.	1	1.00	0.31	1.52	0.00	10.57	0.00	0.00	0.00
time (sec)	N/A	0.488	0.170	1.543	0.000	0.436	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	265	0	0	0	0	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	263	0	0	0	0	0	0
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	261	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	0.456	0.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	397	0	259	0	0	224
N.S.	1	1.00	0.97	5.36	0.00	3.50	0.00	0.00	3.03
time (sec)	N/A	0.044	0.158	0.192	0.000	0.285	0.000	0.000	8.493

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	240	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	258	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.387	0.000	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.609	0.000	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.566	0.000	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.509	0.000	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.512	0.000	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	125	0	658	0	0	0
N.S.	1	1.00	0.95	1.05	0.00	5.53	0.00	0.00	0.00
time (sec)	N/A	0.065	0.280	0.115	0.000	0.289	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0	0
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.482	0.000	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.492	0.000	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	469	0	0	0	0	0	0
N.S.	1	1.00	3.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	1.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	475	0	0	0	0	0	0
N.S.	1	1.00	3.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	1.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	471	0	0	0	0	0	0
N.S.	1	1.00	3.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	1.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	466	0	0	0	0	0	0
N.S.	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	1.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	159	209	0	827	0	0	0
N.S.	1	1.00	0.92	1.21	0.00	4.78	0.00	0.00	0.00
time (sec)	N/A	0.107	0.627	0.088	0.000	0.434	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	477	0	0	0	0	0	0
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.975	0.000	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	471	0	0	0	0	0	0
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.971	0.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.205	0.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	148	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.023	0.136	0.000	0.000	0.252	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.678	0.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	0
N.S.	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.703	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	449	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.053	0.655	0.000	0.000	0.338	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0	0
N.S.	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.609	0.000	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3765	273	2303	68491	25656	1734
N.S.	1	1.00	0.75	20.69	1.50	12.65	376.32	140.97	9.53
time (sec)	N/A	0.111	0.560	2.555	0.263	0.330	65.900	0.488	9.427

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1032	152	706	12323	5454	543
N.S.	1	1.00	0.74	8.82	1.30	6.03	105.32	46.62	4.64
time (sec)	N/A	0.050	0.162	0.671	0.232	0.308	16.765	0.330	9.138

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	41	172	65	142	1096	557	83
N.S.	1	1.00	0.71	2.97	1.12	2.45	18.90	9.60	1.43
time (sec)	N/A	0.016	0.097	0.160	0.216	0.283	3.362	0.281	8.933

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.819	0.000	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	1890	0	0	0	0	0	0
N.S.	1	1.00	5.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.609	4.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	615	615	12289	0	0	0	0	0	0
N.S.	1	1.00	19.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.729	7.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	545	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	2.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.708	0.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0	0
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	1.452	0.000	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	150	162	142	142	178	160	141
N.S.	1	1.00	3.26	3.52	3.09	3.09	3.87	3.48	3.07
time (sec)	N/A	0.038	0.047	0.601	0.202	0.268	0.035	0.275	0.054

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	401	494	403	403	559	475	383
N.S.	1	1.00	4.51	5.55	4.53	4.53	6.28	5.34	4.30
time (sec)	N/A	0.126	0.082	0.627	0.206	0.254	0.072	0.296	8.885

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	797	1130	872	872	1314	1079	777
N.S.	1	1.00	5.78	8.19	6.32	6.32	9.52	7.82	5.63
time (sec)	N/A	0.247	0.203	0.641	0.217	0.260	0.130	0.307	9.022

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	154	179	166	166	240	204	164
N.S.	1	1.00	2.80	3.25	3.02	3.02	4.36	3.71	2.98
time (sec)	N/A	0.034	0.019	0.629	0.207	0.265	0.046	0.301	0.046

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	405	566	439	439	722	597	419
N.S.	1	1.00	3.89	5.44	4.22	4.22	6.94	5.74	4.03
time (sec)	N/A	0.107	0.059	0.597	0.217	0.256	0.071	0.299	0.123

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	801	1318	920	920	1654	1330	825
N.S.	1	1.00	5.04	8.29	5.79	5.79	10.40	8.36	5.19
time (sec)	N/A	0.197	0.025	0.638	0.220	0.261	0.148	0.306	8.905

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	219	158	0	1231	178	1315	3988
N.S.	1	1.00	1.13	0.82	0.00	6.38	0.92	6.81	20.66
time (sec)	N/A	0.278	0.106	0.595	0.000	0.285	1.735	0.299	9.178

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	151	0	434	280	126	278
N.S.	1	1.00	0.95	1.86	0.00	5.36	3.46	1.56	3.43
time (sec)	N/A	0.095	0.037	0.635	0.000	0.293	0.921	0.299	8.885

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	175	140	0	703	104	1403	590
N.S.	1	1.00	1.07	0.85	0.00	4.29	0.63	8.55	3.60
time (sec)	N/A	0.112	0.063	0.592	0.000	0.255	0.726	0.294	0.213

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	272	168	52	61
N.S.	1	1.00	1.07	3.00	0.00	6.33	3.91	1.21	1.42
time (sec)	N/A	0.044	0.013	0.083	0.000	0.284	0.566	0.290	8.683

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	128	155	0	468	320	280	2173
N.S.	1	1.00	1.36	1.65	0.00	4.98	3.40	2.98	23.12
time (sec)	N/A	0.089	0.062	0.633	0.000	0.277	13.829	0.357	9.564

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	206	168	0	1339	211	932	3844
N.S.	1	1.00	1.06	0.86	0.00	6.87	1.08	4.78	19.71
time (sec)	N/A	0.216	0.232	0.635	0.000	0.279	2.956	0.307	9.223

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	154	213	0	810	0	101	4950
N.S.	1	1.00	1.27	1.76	0.00	6.69	0.00	0.83	40.91
time (sec)	N/A	0.125	0.096	0.664	0.000	0.295	0.000	0.300	12.240

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	235	188	0	2044	347	1347	5214
N.S.	1	1.00	1.05	0.84	0.00	9.12	1.55	6.01	23.28
time (sec)	N/A	0.308	0.133	0.702	0.000	0.331	104.662	0.304	9.481

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	263	323	0	2454	573	1408	7327
N.S.	1	1.00	0.97	1.20	0.00	9.09	2.12	5.21	27.14
time (sec)	N/A	0.378	0.304	0.630	0.000	0.321	11.780	0.366	10.483

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	100	276	0	1021	495	162	427
N.S.	1	1.00	1.03	2.85	0.00	10.53	5.10	1.67	4.40
time (sec)	N/A	0.086	0.084	0.655	0.000	0.310	2.651	0.304	8.604

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	247	319	0	2474	0	1416	7200
N.S.	1	1.00	0.97	1.26	0.00	9.74	0.00	5.57	28.35
time (sec)	N/A	0.248	0.615	0.615	0.000	0.345	0.000	0.322	9.932

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	96	98	270	0	1042	495	163	417
N.S.	1	0.98	1.00	2.76	0.00	10.63	5.05	1.66	4.26
time (sec)	N/A	0.078	0.089	0.169	0.000	0.300	2.521	0.295	8.619

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	271	364	0	3228	0	1460	9056
N.S.	1	1.00	0.91	1.22	0.00	10.80	0.00	4.88	30.29
time (sec)	N/A	0.466	0.546	0.134	0.000	0.350	0.000	0.297	10.568

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	235	399	0	2476	0	467	11072
N.S.	1	1.00	1.45	2.46	0.00	15.28	0.00	2.88	68.35
time (sec)	N/A	0.200	0.261	0.737	0.000	0.445	0.000	0.401	14.695

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	339	441	0	4330	0	879	10556
N.S.	1	1.00	0.97	1.27	0.00	12.44	0.00	2.53	30.33
time (sec)	N/A	1.058	0.961	0.727	0.000	0.447	0.000	0.353	11.530

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	284	462	0	4562	0	219	12436
N.S.	1	1.00	1.33	2.17	0.00	21.42	0.00	1.03	58.38
time (sec)	N/A	0.247	0.328	0.750	0.000	0.607	0.000	0.292	15.322

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	384	489	0	5734	0	2122	12239
N.S.	1	1.00	0.94	1.20	0.00	14.05	0.00	5.20	30.00
time (sec)	N/A	2.189	1.781	0.781	0.000	0.639	0.000	0.359	13.051

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	328	704	0	6633	0	1802	12677
N.S.	1	1.00	0.96	2.06	0.00	19.45	0.00	5.28	37.18
time (sec)	N/A	0.609	2.600	0.693	0.000	0.452	0.000	0.367	12.046

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	146	544	0	3739	1671	349	1182
N.S.	1	1.00	0.97	3.63	0.00	24.93	11.14	2.33	7.88
time (sec)	N/A	0.135	0.127	0.727	0.000	0.460	7.147	0.345	10.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	382	885	0	7701	0	2447	14584
N.S.	1	1.00	1.05	2.44	0.00	21.21	0.00	6.74	40.18
time (sec)	N/A	0.657	2.857	0.689	0.000	0.624	0.000	0.336	12.329

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	150	147	541	0	3708	1646	349	1157
N.S.	1	0.99	0.97	3.56	0.00	24.39	10.83	2.30	7.61
time (sec)	N/A	0.127	0.119	0.352	0.000	0.426	6.932	0.312	9.812

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	424	1010	0	8554	0	2641	16086
N.S.	1	1.00	0.97	2.31	0.00	19.57	0.00	6.04	36.81
time (sec)	N/A	3.624	4.490	0.328	0.000	0.768	0.000	0.307	12.255

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	391	966	0	9908	0	1045	19440
N.S.	1	1.00	1.53	3.79	0.00	38.85	0.00	4.10	76.24
time (sec)	N/A	0.311	2.658	1.426	0.000	0.951	0.000	0.451	18.815

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	560	1197	0	10260	0	1458	18112
N.S.	1	1.00	1.16	2.47	0.00	21.20	0.00	3.01	37.42
time (sec)	N/A	0.919	6.181	0.902	0.000	1.107	0.000	0.398	16.577

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	491	1141	0	15165	0	379	21465
N.S.	1	1.00	1.51	3.51	0.00	46.66	0.00	1.17	66.05
time (sec)	N/A	0.393	6.158	0.943	0.000	1.608	0.000	0.318	22.778

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	222	162	0	1346	219	1366	4605
N.S.	1	1.00	1.10	0.80	0.00	6.66	1.08	6.76	22.80
time (sec)	N/A	0.221	0.097	0.593	0.000	0.290	1.930	0.303	0.803

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	80	154	0	446	332	158	287
N.S.	1	1.00	0.92	1.77	0.00	5.13	3.82	1.82	3.30
time (sec)	N/A	0.083	0.028	0.616	0.000	0.273	1.010	0.292	0.258

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	178	143	0	799	124	1443	683
N.S.	1	1.00	1.05	0.84	0.00	4.70	0.73	8.49	4.02
time (sec)	N/A	0.113	0.073	0.593	0.000	0.285	0.778	0.328	8.550

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	274	189	61	477
N.S.	1	1.00	1.07	2.95	0.00	6.23	4.30	1.39	10.84
time (sec)	N/A	0.045	0.011	0.076	0.000	0.262	0.594	0.386	0.119

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	131	173	0	474	348	291	2520
N.S.	1	1.00	1.27	1.68	0.00	4.60	3.38	2.83	24.47
time (sec)	N/A	0.090	0.055	0.662	0.000	0.312	17.677	0.400	9.526

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	209	172	0	1477	258	0	4339
N.S.	1	1.00	1.02	0.84	0.00	7.24	1.26	0.00	21.27
time (sec)	N/A	0.215	0.236	0.665	0.000	0.306	3.196	0.000	9.834

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	157	217	0	828	0	356	5947
N.S.	1	1.00	1.18	1.63	0.00	6.23	0.00	2.68	44.71
time (sec)	N/A	0.137	0.091	0.668	0.000	0.378	0.000	0.406	12.005

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	238	192	0	2212	411	1353	5771
N.S.	1	1.00	1.01	0.81	0.00	9.37	1.74	5.73	24.45
time (sec)	N/A	0.296	0.133	0.696	0.000	0.302	105.919	0.294	9.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	266	327	0	2578	641	1474	8025
N.S.	1	1.00	0.95	1.17	0.00	9.24	2.30	5.28	28.76
time (sec)	N/A	0.341	0.283	0.625	0.000	0.319	11.975	0.302	10.242

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	103	280	0	1077	556	202	460
N.S.	1	1.00	1.00	2.72	0.00	10.46	5.40	1.96	4.47
time (sec)	N/A	0.090	0.083	0.685	0.000	0.322	2.768	0.294	8.600

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	250	323	0	2600	0	1482	7835
N.S.	1	1.00	0.95	1.23	0.00	9.89	0.00	5.63	29.79
time (sec)	N/A	0.233	0.599	0.630	0.000	0.304	0.000	0.299	9.982

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	272	0	1066	525	202	442
N.S.	1	1.00	1.01	2.78	0.00	10.88	5.36	2.06	4.51
time (sec)	N/A	0.087	0.079	0.181	0.000	0.298	2.607	0.299	8.549

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	238	403	0	2486	0	489	13434
N.S.	1	1.00	1.37	2.32	0.00	14.29	0.00	2.81	77.21
time (sec)	N/A	0.195	0.277	0.740	0.000	0.543	0.000	0.398	14.694

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	342	445	0	4520	0	1031	12008
N.S.	1	1.00	0.95	1.24	0.00	12.56	0.00	2.86	33.36
time (sec)	N/A	1.033	0.973	0.753	0.000	0.389	0.000	0.342	11.869

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	287	466	0	4604	0	705	14830
N.S.	1	1.00	1.26	2.04	0.00	20.19	0.00	3.09	65.04
time (sec)	N/A	0.239	0.306	0.810	0.000	1.194	0.000	0.414	15.937

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	387	493	0	5954	0	2137	13781
N.S.	1	1.00	0.91	1.17	0.00	14.08	0.00	5.05	32.58
time (sec)	N/A	2.156	1.727	0.801	0.000	0.488	0.000	0.323	13.999

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	331	708	0	6770	0	1958	13840
N.S.	1	1.00	0.94	2.01	0.00	19.18	0.00	5.55	39.21
time (sec)	N/A	0.573	2.605	0.691	0.000	0.464	0.000	0.330	12.207

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	149	548	0	3843	1794	431	1267
N.S.	1	1.00	0.94	3.45	0.00	24.17	11.28	2.71	7.97
time (sec)	N/A	0.129	0.147	0.737	0.000	0.388	7.323	0.330	9.802

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	385	889	0	7838	0	2679	16025
N.S.	1	1.00	1.03	2.37	0.00	20.90	0.00	7.14	42.73
time (sec)	N/A	0.637	2.782	0.698	0.000	0.534	0.000	0.334	12.303

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	148	543	0	3748	1707	429	1199
N.S.	1	1.00	0.97	3.55	0.00	24.50	11.16	2.80	7.84
time (sec)	N/A	0.128	0.115	0.379	0.000	0.556	7.207	0.327	9.757

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	394	970	0	9926	0	1077	22621
N.S.	1	1.00	1.46	3.59	0.00	36.76	0.00	3.99	83.78
time (sec)	N/A	0.304	2.373	0.904	0.000	1.582	0.000	0.432	19.421

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	575	1201	0	10518	0	1704	20580
N.S.	1	1.00	1.15	2.41	0.00	21.08	0.00	3.41	41.24
time (sec)	N/A	0.833	6.144	0.925	0.000	1.136	0.000	0.396	17.022

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	461	1145	0	15231	0	1791	25334
N.S.	1	1.00	1.34	3.34	0.00	44.41	0.00	5.22	73.86
time (sec)	N/A	0.381	6.058	0.999	0.000	4.422	0.000	0.432	24.183

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	398	398	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	104	104	104	107	28	29
N.S.	1	1.00	1.00	3.06	3.06	3.06	3.15	0.82	0.85
time (sec)	N/A	0.024	0.016	0.561	0.210	0.267	0.042	0.353	8.503

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	188	174	174	174	187	46	46
N.S.	1	1.00	3.36	3.11	3.11	3.11	3.34	0.82	0.82
time (sec)	N/A	0.065	0.011	0.587	0.232	0.236	0.068	0.325	0.084

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [411] had the largest ratio of [.800000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.00	15	0.200
5	A	8	8	1.00	11	0.727
6	A	3	2	1.00	26	0.077
7	A	3	2	1.00	26	0.077
8	A	3	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	3	2	1.00	24	0.083
11	A	2	1	1.00	22	0.045
12	A	3	2	1.00	26	0.077
13	A	3	2	1.00	26	0.077
14	A	3	2	1.00	26	0.077
15	A	3	2	1.00	26	0.077
16	A	3	2	1.00	26	0.077
17	A	3	2	1.00	26	0.077
18	A	3	2	1.00	26	0.077
19	A	3	2	1.00	26	0.077
20	A	3	2	1.00	26	0.077
21	A	3	2	1.00	26	0.077
22	A	3	2	1.00	26	0.077
23	A	3	2	1.00	26	0.077
24	A	4	3	1.40	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	26	0.077
26	A	3	2	1.00	26	0.077
27	A	4	3	1.00	26	0.115
28	A	3	2	1.00	26	0.077
29	A	3	2	1.00	26	0.077
30	A	2	2	1.00	26	0.077
31	A	3	2	1.00	24	0.083
32	A	3	2	1.00	22	0.091
33	A	4	3	1.00	26	0.115
34	A	3	2	1.00	26	0.077
35	A	3	2	1.00	26	0.077
36	A	4	3	1.00	26	0.115
37	A	3	2	1.00	26	0.077
38	A	3	2	1.00	26	0.077
39	A	4	3	1.00	26	0.115
40	A	3	2	1.00	26	0.077
41	A	3	2	1.00	26	0.077
42	A	4	3	1.00	26	0.115
43	A	3	2	1.00	26	0.077
44	A	3	2	1.00	26	0.077
45	A	2	2	1.00	26	0.077
46	A	3	2	1.00	26	0.077
47	A	3	2	1.00	26	0.077
48	A	4	4	1.00	26	0.154
49	A	3	2	1.00	26	0.077
50	A	3	2	1.00	26	0.077
51	A	3	2	1.00	26	0.077
52	A	4	3	1.00	26	0.115
53	A	3	2	1.00	26	0.077
54	A	3	2	1.00	26	0.077
55	A	4	3	1.00	26	0.115
56	A	3	2	1.00	26	0.077
57	A	3	2	1.00	26	0.077
58	A	4	3	1.00	26	0.115
59	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
60	A	3	2	1.00	26	0.077
61	A	2	2	1.00	26	0.077
62	A	3	2	1.00	24	0.083
63	A	3	2	1.00	22	0.091
64	A	4	3	1.00	26	0.115
65	A	3	2	1.00	26	0.077
66	A	3	2	1.00	26	0.077
67	A	4	3	1.00	26	0.115
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	26	0.077
70	A	4	3	1.00	26	0.115
71	A	3	2	1.00	26	0.077
72	A	3	2	1.00	26	0.077
73	A	4	3	1.00	26	0.115
74	A	3	2	1.00	26	0.077
75	A	3	2	1.00	26	0.077
76	A	4	3	1.00	26	0.115
77	A	3	2	1.00	26	0.077
78	A	3	2	1.00	26	0.077
79	A	4	3	1.00	26	0.115
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	26	0.077
82	A	2	2	1.00	26	0.077
83	A	3	2	1.00	26	0.077
84	A	3	2	1.00	26	0.077
85	A	4	4	1.00	26	0.154
86	A	3	2	1.00	26	0.077
87	A	3	2	1.00	26	0.077
88	A	5	4	1.00	26	0.154
89	A	8	8	1.00	26	0.308
90	A	8	8	1.00	26	0.308
91	A	3	3	1.00	26	0.115
92	A	7	7	1.00	24	0.292
93	A	7	7	1.00	22	0.318
94	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	8	1.00	26	0.308
96	A	8	8	0.99	26	0.308
97	A	4	3	0.98	26	0.115
98	A	9	9	1.00	26	0.346
99	A	9	9	1.00	26	0.346
100	A	2	2	1.00	26	0.077
101	A	9	8	1.00	24	0.333
102	A	9	8	1.00	22	0.364
103	A	4	3	1.00	26	0.115
104	A	10	9	1.00	26	0.346
105	A	10	9	1.00	26	0.346
106	A	4	3	1.00	26	0.115
107	A	11	9	1.00	26	0.346
108	A	4	3	1.00	26	0.115
109	A	11	9	1.00	26	0.346
110	A	11	9	1.00	26	0.346
111	A	2	2	1.00	26	0.077
112	A	11	8	1.00	24	0.333
113	A	11	8	1.00	22	0.364
114	A	4	3	1.00	26	0.115
115	A	12	9	1.00	26	0.346
116	A	12	9	1.00	26	0.346
117	A	4	3	1.00	26	0.115
118	A	3	2	1.00	28	0.071
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	28	0.071
121	A	2	2	1.00	28	0.071
122	A	2	2	1.00	28	0.071
123	A	2	2	1.00	28	0.071
124	A	2	2	1.00	26	0.077
125	A	4	3	1.00	24	0.125
126	A	4	3	1.00	24	0.125
127	A	4	3	1.00	24	0.125
128	A	2	2	1.00	24	0.083
129	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	24	0.083
131	A	2	2	1.03	22	0.091
132	A	3	3	1.04	20	0.150
133	A	3	3	1.00	24	0.125
134	A	2	2	1.00	24	0.083
135	A	2	2	1.00	24	0.083
136	A	3	3	1.00	24	0.125
137	A	2	2	1.00	24	0.083
138	A	6	6	1.00	18	0.333
139	A	5	5	1.00	18	0.278
140	A	3	3	1.00	18	0.167
141	A	7	7	1.00	18	0.389
142	A	8	7	1.00	18	0.389
143	A	14	8	1.00	18	0.444
144	A	14	8	1.00	18	0.444
145	A	13	7	1.00	18	0.389
146	A	13	7	1.00	18	0.389
147	A	13	7	1.00	16	0.438
148	A	13	7	1.00	14	0.500
149	A	14	8	1.00	18	0.444
150	A	14	8	1.00	18	0.444
151	A	6	4	1.00	16	0.250
152	A	5	4	1.00	16	0.250
153	A	4	3	1.00	16	0.188
154	B	4	3	2.10	16	0.188
155	A	6	5	1.00	16	0.312
156	A	4	3	1.00	16	0.188
157	A	4	3	1.00	16	0.188
158	A	15	10	1.00	16	0.625
159	A	15	10	1.00	16	0.625
160	A	14	9	1.00	16	0.562
161	A	14	9	1.00	16	0.562
162	A	13	8	1.00	16	0.500
163	A	13	8	1.00	16	0.500
164	A	13	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	13	8	1.00	12	0.667
166	A	14	9	1.00	16	0.562
167	A	14	9	1.00	16	0.562
168	A	15	10	1.00	16	0.625
169	A	15	10	1.00	16	0.625
170	A	14	8	1.00	16	0.500
171	A	5	5	1.00	16	0.312
172	A	13	7	1.00	16	0.438
173	A	13	7	1.00	16	0.438
174	A	3	3	1.00	16	0.188
175	A	13	7	1.00	14	0.500
176	C	13	7	2.02	12	0.583
177	A	7	7	1.00	16	0.438
178	A	14	8	1.00	16	0.500
179	A	14	8	1.00	16	0.500
180	A	8	7	1.00	16	0.438
181	A	16	10	1.00	16	0.625
182	A	13	7	1.00	10	0.700
183	A	3	3	1.00	14	0.214
184	A	13	7	1.00	14	0.500
185	A	7	7	1.00	20	0.350
186	A	6	6	1.00	20	0.300
187	A	6	6	1.00	20	0.300
188	A	5	5	1.00	20	0.250
189	A	4	4	1.00	20	0.200
190	A	7	6	1.00	20	0.300
191	A	7	6	1.00	20	0.300
192	A	4	4	1.00	20	0.200
193	A	5	5	1.00	20	0.250
194	A	6	6	1.00	20	0.300
195	A	7	7	1.00	20	0.350
196	A	2	2	1.00	20	0.100
197	A	2	2	1.00	18	0.111
198	A	2	2	1.00	16	0.125
199	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	2	2	1.00	20	0.100
201	A	8	7	1.00	20	0.350
202	A	7	6	1.00	20	0.300
203	A	7	6	1.00	20	0.300
204	A	6	5	1.00	20	0.250
205	A	5	4	1.00	20	0.200
206	A	8	7	1.00	20	0.350
207	A	8	7	1.00	20	0.350
208	A	8	7	1.00	20	0.350
209	A	8	7	1.00	20	0.350
210	A	5	4	1.00	20	0.200
211	A	6	5	1.00	20	0.250
212	A	7	6	1.00	20	0.300
213	A	8	7	1.00	20	0.350
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111
216	A	2	2	1.00	16	0.125
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	6	6	1.00	20	0.300
220	A	5	5	1.00	20	0.250
221	A	5	5	1.00	20	0.250
222	A	4	4	1.00	20	0.200
223	A	3	3	1.00	20	0.150
224	A	3	3	1.00	20	0.150
225	A	4	4	1.00	20	0.200
226	A	5	5	1.00	20	0.250
227	A	6	6	1.00	20	0.300
228	A	7	6	1.00	20	0.300
229	A	2	2	1.00	20	0.100
230	A	2	2	1.00	18	0.111
231	A	2	2	1.00	16	0.125
232	A	2	2	1.00	20	0.100
233	A	2	2	1.00	20	0.100
234	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	5	1.00	20	0.250
236	A	5	5	1.00	20	0.250
237	A	2	2	1.00	20	0.100
238	A	2	2	1.00	20	0.100
239	A	5	5	1.00	20	0.250
240	A	5	5	1.00	20	0.250
241	A	6	6	1.00	20	0.300
242	A	7	6	1.00	20	0.300
243	A	2	2	1.00	20	0.100
244	A	2	2	1.00	18	0.111
245	A	2	2	1.00	16	0.125
246	A	2	2	1.00	20	0.100
247	A	2	2	1.00	20	0.100
248	A	2	1	1.00	20	0.050
249	A	2	1	1.00	18	0.056
250	A	3	2	1.00	20	0.100
251	A	4	3	1.00	20	0.150
252	A	2	2	1.00	22	0.091
253	A	2	2	1.00	22	0.091
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	22	0.091
256	A	2	2	1.00	20	0.100
257	A	4	4	1.00	18	0.222
258	A	3	3	1.00	18	0.167
259	A	2	2	1.00	18	0.111
260	A	2	2	1.00	18	0.111
261	A	2	2	1.00	18	0.111
262	A	2	2	1.00	16	0.125
263	A	2	2	1.00	14	0.143
264	A	3	3	1.00	18	0.167
265	A	2	2	1.00	18	0.111
266	A	2	2	1.00	18	0.111
267	A	3	3	1.00	18	0.167
268	A	2	2	1.00	18	0.111
269	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	3	3	1.00	18	0.167
271	A	2	2	1.00	16	0.125
272	A	5	5	1.00	16	0.312
273	A	4	3	1.00	16	0.188
274	A	4	4	1.00	16	0.250
275	A	2	2	1.00	16	0.125
276	A	4	4	1.00	14	0.286
277	A	4	3	1.00	16	0.188
278	A	5	5	1.00	16	0.312
279	A	4	3	1.00	16	0.188
280	A	6	5	1.00	16	0.312
281	A	12	9	1.00	16	0.562
282	A	11	8	1.00	16	0.500
283	A	11	8	1.00	16	0.500
284	A	11	8	1.00	16	0.500
285	A	11	8	1.00	12	0.667
286	A	12	9	1.00	16	0.562
287	A	12	9	1.00	16	0.562
288	A	13	9	1.00	16	0.562
289	A	13	9	1.00	16	0.562
290	A	2	2	1.00	16	0.125
291	A	5	5	1.00	16	0.312
292	A	4	3	1.00	16	0.188
293	A	4	4	1.00	16	0.250
294	A	2	2	1.00	16	0.125
295	A	4	4	1.00	14	0.286
296	A	4	3	1.00	16	0.188
297	A	5	5	1.00	16	0.312
298	A	4	3	1.00	16	0.188
299	A	6	5	1.00	16	0.312
300	A	6	6	1.00	16	0.375
301	A	5	5	1.00	16	0.312
302	A	5	5	1.00	16	0.312
303	A	5	5	1.00	16	0.312
304	A	5	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	6	1.00	16	0.375
306	A	6	6	1.00	16	0.375
307	A	7	6	1.00	16	0.375
308	A	7	6	1.00	16	0.375
309	A	3	2	1.00	18	0.111
310	A	6	6	1.00	18	0.333
311	A	5	4	1.00	18	0.222
312	A	5	5	1.00	18	0.278
313	A	4	3	1.00	18	0.167
314	A	3	3	1.00	18	0.167
315	A	4	3	1.00	16	0.188
316	A	7	7	1.00	18	0.389
317	A	5	4	1.00	18	0.222
318	A	8	7	1.00	18	0.389
319	A	8	5	1.00	18	0.278
320	A	8	5	1.00	18	0.278
321	A	7	4	1.00	18	0.222
322	A	7	4	1.00	18	0.222
323	A	7	4	1.00	18	0.222
324	A	7	4	1.00	14	0.286
325	A	8	5	1.00	18	0.278
326	A	8	5	1.00	18	0.278
327	A	3	2	1.00	14	0.143
328	A	6	6	1.00	14	0.429
329	A	7	5	1.00	14	0.357
330	A	5	5	1.00	14	0.357
331	A	10	7	1.00	14	0.500
332	A	3	3	1.00	14	0.214
333	A	10	6	1.00	12	0.500
334	A	7	7	1.00	14	0.500
335	A	7	5	1.00	14	0.357
336	A	8	7	1.00	14	0.500
337	A	13	10	1.00	14	0.714
338	A	20	7	1.00	14	0.500
339	A	9	6	1.00	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	19	7	1.00	14	0.500
341	A	19	6	1.00	14	0.429
342	A	9	6	1.00	10	0.600
343	A	20	8	1.00	14	0.571
344	A	20	7	1.00	14	0.500
345	A	12	9	1.00	14	0.643
346	A	22	10	1.00	14	0.714
347	A	3	2	1.00	16	0.125
348	A	6	6	1.00	16	0.375
349	A	5	4	1.00	16	0.250
350	A	5	5	1.00	16	0.312
351	A	10	7	1.00	16	0.438
352	A	3	3	1.00	16	0.188
353	A	10	6	1.00	14	0.429
354	A	7	7	1.00	16	0.438
355	A	5	4	1.00	16	0.250
356	A	8	7	1.00	16	0.438
357	A	13	10	1.00	16	0.625
358	A	20	7	1.00	16	0.438
359	A	19	6	1.00	16	0.375
360	A	19	7	1.00	16	0.438
361	A	19	6	1.00	16	0.375
362	A	19	6	1.00	12	0.500
363	A	22	8	1.00	16	0.500
364	A	20	7	1.00	16	0.438
365	A	22	9	1.00	16	0.562
366	A	22	10	1.00	16	0.625
367	A	3	2	1.00	16	0.125
368	A	5	4	1.00	16	0.250
369	A	5	4	1.00	16	0.250
370	A	4	3	1.00	16	0.188
371	A	4	3	1.00	16	0.188
372	A	3	3	1.00	16	0.188
373	A	4	3	1.00	14	0.214
374	A	6	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	4	1.00	16	0.250
376	A	7	5	1.00	16	0.312
377	A	6	5	1.00	16	0.312
378	A	20	8	0.96	16	0.500
379	A	19	7	1.00	16	0.438
380	A	19	7	1.00	16	0.438
381	A	19	7	1.01	16	0.438
382	A	19	7	1.00	12	0.583
383	A	20	8	1.00	16	0.500
384	A	20	8	1.00	16	0.500
385	A	3	2	1.00	16	0.125
386	A	5	4	1.00	16	0.250
387	A	5	4	1.00	16	0.250
388	A	4	3	1.00	16	0.188
389	A	4	3	1.00	16	0.188
390	A	3	3	1.00	16	0.188
391	A	4	3	1.00	14	0.214
392	A	6	5	1.00	16	0.312
393	A	5	4	1.00	16	0.250
394	A	7	5	1.00	16	0.312
395	A	6	5	1.00	16	0.312
396	A	8	5	1.00	16	0.312
397	A	7	4	1.00	16	0.250
398	A	7	4	1.00	16	0.250
399	A	7	4	1.14	16	0.250
400	A	7	4	1.00	12	0.333
401	A	8	5	1.00	16	0.312
402	A	8	5	1.00	16	0.312
403	A	9	6	1.00	16	0.375
404	A	9	6	1.00	16	0.375
405	A	4	3	1.00	16	0.188
406	A	5	4	1.00	16	0.250
407	A	5	5	1.00	14	0.357
408	A	3	3	1.00	14	0.214
409	A	7	7	1.00	14	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	8	7	1.00	14	0.500
411	A	8	8	1.00	10	0.800
412	A	7	6	1.00	18	0.333
413	A	7	6	1.00	18	0.333
414	A	7	6	1.00	16	0.375
415	A	6	6	1.00	14	0.429
416	A	5	5	1.00	18	0.278
417	A	3	3	1.00	18	0.167
418	A	7	7	1.00	18	0.389
419	A	8	7	1.00	18	0.389
420	A	8	7	1.00	18	0.389
421	A	8	7	1.00	18	0.389
422	A	8	7	1.00	16	0.438
423	A	8	7	1.00	14	0.500
424	A	7	7	1.00	18	0.389
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	18	0.222
427	A	4	4	1.00	18	0.222
428	A	8	7	1.00	18	0.389
429	A	8	7	1.00	18	0.389
430	A	8	7	1.00	18	0.389
431	A	9	8	1.00	14	0.571
432	A	8	8	1.00	18	0.444
433	A	5	4	1.00	18	0.222
434	A	5	5	1.00	18	0.278
435	A	5	5	1.00	18	0.278
436	A	5	5	1.00	18	0.278
437	A	5	4	0.98	18	0.222
438	A	9	8	1.00	18	0.444
439	A	9	8	1.00	18	0.444
440	A	6	4	1.00	18	0.222
441	A	6	4	1.00	16	0.250
442	A	5	4	1.00	14	0.286
443	A	4	3	1.00	18	0.167
444	A	4	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	6	5	1.00	18	0.278
446	A	4	3	1.00	18	0.167
447	A	4	3	1.00	18	0.167
448	A	4	3	1.00	18	0.167
449	A	9	7	1.00	16	0.438
450	A	8	7	1.00	16	0.438
451	A	7	6	1.00	16	0.375
452	A	4	4	1.00	16	0.250
453	A	5	5	1.00	16	0.312
454	A	6	6	1.00	16	0.375
455	A	4	3	1.00	22	0.136
456	A	5	4	1.00	14	0.286
457	A	15	9	1.00	14	0.643
458	A	9	6	1.00	14	0.429
459	A	7	6	1.00	20	0.300
460	A	4	3	1.00	23	0.130
461	A	4	4	1.00	22	0.182
462	A	4	3	1.00	26	0.115
463	A	4	3	1.00	26	0.115
464	A	3	2	1.00	26	0.077
465	A	4	3	1.00	26	0.115
466	A	4	3	1.00	26	0.115
467	A	4	3	1.00	26	0.115
468	A	4	3	1.00	26	0.115
469	A	4	3	1.00	26	0.115
470	A	4	3	1.00	26	0.115
471	A	4	3	1.00	26	0.115
472	A	4	4	1.00	30	0.133
473	A	4	3	1.00	28	0.107
474	A	4	3	1.00	26	0.115
475	A	4	3	1.00	24	0.125
476	A	3	3	1.00	28	0.107
477	A	3	3	1.00	28	0.107
478	C	7	3	1.11	77	0.039
479	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	4	3	1.00	26	0.115
481	A	5	4	1.00	24	0.167
482	A	5	4	1.00	26	0.154
483	A	5	4	1.00	26	0.154
484	A	5	4	1.00	26	0.154
485	A	4	3	1.00	26	0.115
486	A	5	4	1.00	26	0.154
487	A	5	4	1.00	26	0.154
488	A	5	4	1.00	26	0.154
489	A	5	4	1.00	26	0.154
490	A	5	4	1.00	26	0.154
491	A	4	3	1.00	26	0.115
492	A	5	4	1.00	26	0.154
493	A	4	3	1.00	23	0.130
494	A	4	3	1.00	23	0.130
495	A	2	2	1.00	23	0.087
496	A	5	5	1.00	21	0.238
497	A	4	3	1.00	23	0.130
498	A	4	3	1.00	23	0.130
499	A	4	3	1.00	23	0.130
500	A	12	9	1.00	25	0.360
501	A	9	9	1.00	25	0.360
502	A	5	5	1.00	25	0.200
503	A	6	6	1.00	25	0.240
504	A	11	11	1.00	25	0.440
505	A	14	11	1.00	25	0.440
506	A	1	1	1.00	26	0.038
507	A	1	1	1.00	28	0.036
508	A	4	3	1.00	32	0.094
509	A	4	3	1.00	32	0.094
510	A	3	2	1.00	32	0.062
511	A	4	3	1.00	32	0.094
512	A	2	2	1.00	32	0.062
513	A	4	3	1.00	32	0.094
514	A	4	3	1.00	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	5	4	1.00	30	0.133
516	A	3	2	1.00	28	0.071
517	A	3	2	1.00	26	0.077
518	A	2	1	1.00	24	0.042
519	A	3	2	1.00	28	0.071
520	A	3	2	1.00	28	0.071
521	A	3	2	1.00	28	0.071
522	A	9	4	1.00	30	0.133
523	A	3	2	1.00	28	0.071
524	A	3	2	1.00	26	0.077
525	A	3	2	1.00	24	0.083
526	A	4	3	1.00	28	0.107
527	A	3	2	1.00	28	0.071
528	A	3	2	1.00	28	0.071
529	A	2	2	1.00	30	0.067
530	A	2	2	1.00	28	0.071
531	A	2	2	1.00	26	0.077
532	A	2	2	1.00	24	0.083
533	A	5	5	1.00	28	0.179
534	A	2	2	1.00	28	0.071
535	A	2	2	1.00	28	0.071
536	A	2	2	1.00	30	0.067
537	A	2	2	1.00	28	0.071
538	A	2	2	1.00	26	0.077
539	A	2	2	1.00	24	0.083
540	A	4	3	1.00	28	0.107
541	A	2	2	1.00	28	0.071
542	A	2	2	1.00	28	0.071
543	A	2	2	1.00	36	0.056
544	A	2	2	1.00	33	0.061
545	A	3	3	1.00	34	0.088
546	A	3	3	1.00	33	0.091
547	A	3	3	1.06	35	0.086
548	A	4	3	1.00	30	0.100
549	A	7	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	6	6	1.00	24	0.250
551	A	5	5	1.00	24	0.208
552	A	3	3	1.00	22	0.136
553	A	8	7	1.00	24	0.292
554	A	8	7	1.00	24	0.292
555	A	8	7	1.00	24	0.292
556	A	8	5	1.00	26	0.192
557	A	14	8	1.00	26	0.308
558	A	4	3	1.00	26	0.115
559	A	6	5	1.00	26	0.192
560	A	16	10	1.00	26	0.385
561	A	10	7	1.00	26	0.269
562	A	3	2	1.00	20	0.100
563	A	3	2	1.00	18	0.111
564	A	3	2	1.00	16	0.125
565	A	7	7	1.00	20	0.350
566	A	3	2	1.00	20	0.100
567	A	3	2	1.00	20	0.100
568	A	2	2	1.00	22	0.091
569	A	2	2	1.00	22	0.091
570	A	2	2	1.00	20	0.100
571	A	2	2	1.00	18	0.111
572	A	7	6	1.00	22	0.273
573	A	2	2	1.00	22	0.091
574	A	2	2	1.00	22	0.091
575	A	2	2	1.00	22	0.091
576	A	2	2	1.00	22	0.091
577	A	2	2	1.00	20	0.100
578	A	2	2	1.00	18	0.111
579	A	8	7	1.00	22	0.318
580	A	2	2	1.00	22	0.091
581	A	2	2	1.00	22	0.091
582	A	2	2	1.00	22	0.091
583	A	2	2	1.00	22	0.091
584	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	2	2	1.00	18	0.111
586	A	3	3	1.00	22	0.136
587	A	2	2	1.00	22	0.091
588	A	2	2	1.00	22	0.091
589	A	2	2	1.00	22	0.091
590	A	2	2	1.00	22	0.091
591	A	2	2	1.00	20	0.100
592	A	2	2	1.00	18	0.111
593	A	5	5	1.00	22	0.227
594	A	2	2	1.00	22	0.091
595	A	2	2	1.00	22	0.091
596	A	14	3	1.00	22	0.136
597	A	10	3	1.00	22	0.136
598	A	6	3	1.00	20	0.150
599	A	3	2	1.00	22	0.091
600	A	5	3	1.00	22	0.136
601	A	6	4	1.00	22	0.182
602	A	2	2	1.00	24	0.083
603	A	2	2	1.00	24	0.083
604	A	2	2	1.00	24	0.083
605	A	2	2	1.00	24	0.083
606	A	2	2	1.00	22	0.091
607	A	3	2	1.00	28	0.071
608	A	4	3	1.00	30	0.100
609	A	4	3	1.00	30	0.100
610	A	3	2	1.00	31	0.065
611	A	4	3	1.00	33	0.091
612	A	4	3	1.00	33	0.091
613	A	5	4	1.00	30	0.133
614	A	6	6	1.00	30	0.200
615	A	4	3	1.00	30	0.100
616	A	4	4	1.00	28	0.143
617	A	8	8	1.00	30	0.267
618	A	5	4	1.00	30	0.133
619	A	9	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	6	5	1.00	30	0.167
621	A	5	4	1.00	30	0.133
622	A	5	5	1.00	30	0.167
623	A	5	4	1.00	30	0.133
624	A	5	5	0.98	28	0.179
625	A	5	4	1.00	22	0.182
626	A	9	8	1.00	30	0.267
627	A	6	5	1.00	30	0.167
628	A	9	8	1.00	30	0.267
629	A	7	5	1.00	30	0.167
630	A	6	5	1.00	30	0.167
631	A	6	6	1.00	30	0.200
632	A	6	5	1.00	30	0.167
633	A	6	5	0.99	28	0.179
634	A	6	5	1.00	22	0.227
635	A	10	9	1.00	30	0.300
636	A	7	6	1.00	30	0.200
637	A	10	9	1.00	30	0.300
638	A	5	4	1.00	33	0.121
639	A	6	6	1.00	33	0.182
640	A	4	3	1.00	33	0.091
641	A	4	4	1.00	31	0.129
642	A	8	8	1.00	33	0.242
643	A	5	4	1.00	33	0.121
644	A	9	8	1.00	33	0.242
645	A	6	5	1.00	33	0.152
646	A	5	4	1.00	33	0.121
647	A	5	5	1.00	33	0.152
648	A	5	4	1.00	33	0.121
649	A	5	5	1.00	31	0.161
650	A	9	8	1.00	33	0.242
651	A	6	5	1.00	33	0.152
652	A	9	8	1.00	33	0.242
653	A	7	5	1.00	33	0.152
654	A	6	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	6	6	1.00	33	0.182
656	A	6	5	1.00	33	0.152
657	A	6	5	1.00	31	0.161
658	A	10	9	1.00	33	0.273
659	A	7	6	1.00	33	0.182
660	A	10	9	1.00	33	0.273
661	A	7	6	1.00	26	0.231
662	A	10	9	1.00	28	0.321
663	A	3	2	1.00	24	0.083
664	A	4	3	1.00	26	0.115

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \left(ax^3+bx^6\right)^{5/3} dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \left(ax^3+bx^6\right)^{5/3} dx = -\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}+\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}$$

```
[Out] -3/88*a*(b*x^6+a*x^3)^(8/3)/b^2/x^8+1/11*(b*x^6+a*x^3)^(8/3)/b/x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 2039}

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

```
[In] Int[(a*x^3 + b*x^6)^(5/3),x]
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ax^3+bx^6)^{8/3}}{11bx^5} - \frac{(3a)\int \frac{(ax^3+bx^6)^{5/3}}{x^3}\,dx}{11b}\\ &= -\frac{3a(ax^3+bx^6)^{8/3}}{88b^2x^8} + \frac{(ax^3+bx^6)^{8/3}}{11bx^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx = \frac{(x^3(a+bx^3))^{8/3}\,(-3a+8bx^3)}{88b^2x^8}$$

```
[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]
[Out] ((x^3*(a + b*x^3))^(8/3)*(-3*a + 8*b*x^3))/(88*b^2*x^8)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{(b\,x^3+a)\,(-8b\,x^3+3a)\,(b\,x^6+a\,x^3)^{\frac{5}{3}}}{88b^2x^5}$	39
trager	$-\frac{(-8b^3x^9-13b^2x^6a-2a^2b\,x^3+3a^3)\,(b\,x^6+a\,x^3)^{\frac{2}{3}}}{88b^2x^2}$	54
risch	$-\frac{(x^3\,(b\,x^3+a))^{\frac{2}{3}}\,(-8b^3x^9-13b^2x^6a-2a^2b\,x^3+3a^3)}{88x^2b^2}$	54

```
[In] int((b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^6+ax^3)^{\frac{2}{3}}}{88\,b^2x^2}$$

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)
/(b^2*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \int \left(ax^3+bx^6\right)^{\frac{5}{3}} dx$$

```
[In] integrate((b*x**6+a*x**3)**(5/3),x)
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^3+a)^{\frac{2}{3}}}{88\,b^2}$$

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2
```

Giac [F]

$$\int \left(ax^3+bx^6\right)^{5/3} dx = \int \left(bx^6+ax^3\right)^{\frac{5}{3}} dx$$

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^6 + a*x^3)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=-\frac{\left(b\,x^3+a\right)^2\,\left(b\,x^6+a\,x^3\right)^{2/3}\,\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

```
[In] int((a*x^3 + b*x^6)^(5/3),x)
[Out] -((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

3.2 $\int (ax^3+bx^6)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int (ax^3+bx^6)^{2/3}\,dx=\frac{(ax^3+bx^6)^{5/3}}{5bx^5}$$

```
[Out] 1/5*(b*x^6+a*x^3)^(5/3)/b/x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2025}

$$\int (ax^3+bx^6)^{2/3}\,dx=\frac{(ax^3+bx^6)^{5/3}}{5bx^5}$$

```
[In] Int[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{(ax^3+bx^6)^{5/3}}{5bx^5}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax^3+bx^6\right)^{2/3} \, dx = \frac{\left(x^3(a+bx^3)\right)^{5/3}}{5bx^5}$$

```
[In] Integrate[(a*x^3 + b*x^6)^(2/3),x]
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{5 b\,x^{2}}$	29
trager	$\frac{\left(b\,x^{3}+a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{5 b\,x^{2}}$	29
risch	$\frac{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)}{5 x^{2} b}$	29

```
[In] int((b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(ax^3+bx^6\right)^{2/3} \, dx = \frac{\left(bx^6+ax^3\right)^{\frac{2}{3}}(bx^3+a)}{5\,bx^2}$$

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*x**6+a*x**3)**(2/3),x)
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b}$$

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

Giac [F]

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^6 + a*x^3)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\frac{\left(\frac{a}{5\,b}+\frac{x^3}{5}\right)\,{\left(b\,x^6+a\,x^3\right)}^{2/3}}{x^2}$$

```
[In] int((a*x^3 + b*x^6)^(2/3),x)
[Out] ((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

3.3 $\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[Out] -(b*x^6+a*x^3)^(1/3)/a/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2025}

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[In] Int[(a*x^3 + b*x^6)^(-2/3),x]
```

```
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\sqrt[3]{x^3\,(a+bx^3)}}{ax^2}$$

```
[In] Integrate[(a*x^3 + b*x^6)^(-2/3),x]
[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}{a\,x^{2}}$	22
pseudoelliptic	$-\frac{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}}{a\,x^{2}}$	22
gosper	$-\frac{x\left(b\,x^{3}+a\right)}{a\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}$	27
risch	$-\frac{x\left(b\,x^{3}+a\right)}{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}\,a}$	27

```
[In] int(1/(b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] -(b*x^6+a*x^3)^(1/3)/a/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^6+ax^3)^{\frac{1}{3}}}{ax^2}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^3+a)^{\frac{1}{3}}}{ax}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
[Out] -(b + a/x^3)^(1/3)/a
```

Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

```
[In] int(1/(a*x^3 + b*x^6)^(2/3),x)
[Out] -(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

3.4 $\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx=\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

```
[Out] 1/2/a/x^2/(b*x^6+a*x^3)^(2/3)-3/4*(b*x^6+a*x^3)^(1/3)/a^2/x^5+9/4*b*(b*x^6+
a*x^3)^(1/3)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 2041, 2025}

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx=\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

```
[In] Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2026

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1
)/(n - j)], 0] && LtQ[p, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}+\frac{3\int \frac{1}{x^3\left(ax^3+bx^6\right)^{2/3}}\,dx}{a}\\
&= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}-\frac{(9b)\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx}{4a^2}\\
&= \frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx=\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

```
[In] Integrate[(a*x^3 + b*x^6)^(-5/3),x]
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$-\frac{-9 b^{2} x^{6}-6 a b \,x^{3}+a^{2}}{4 x^{2} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}} a^{3}}$	41
gosper	$-\frac{x \left(b \,x^{3}+a \right) \left(-9 b^{2} x^{6}-6 a b \,x^{3}+a^{2}\right)}{4 a^{3} \left(b \,x^{6}+a \,x^{3}\right)^{\frac{5}{3}}}$	46
trager	$-\frac{\left(-9 b^{2} x^{6}-6 a b \,x^{3}+a^{2}\right) \left(b \,x^{6}+a \,x^{3}\right)^{\frac{1}{3}}}{4 \left(b \,x^{3}+a \right) x^{5} a^{3}}$	50
risch	$-\frac{\left(b \,x^{3}+a \right) \left(-7 b \,x^{3}+a \right)}{4 a^{3} x^{2} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}}}+\frac{b^{2} x^{4}}{2 a^{3} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}}}$	62

```
[In] int(1/(b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
[Out] -1/4/x^2*(-9*b^2*x^6-6*a*b*x^3+a^2)/(x^3*(b*x^3+a))^(2/3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx=\frac{(9\,b^2x^6+6\,abx^3-a^2)(bx^6+ax^3)^{\frac{1}{3}}}{4\,(a^3bx^8+a^4x^5)}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x^
5)
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx=\int \frac{1}{(ax^3+bx^6)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(1/(b*x**6+a*x**3)**(5/3),x)
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{9\,b^2x^6+6\,abx^3-a^2}{4\,(bx^3+a)^{\frac{2}{3}}a^3x^4}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{b^2}{2\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}} - \frac{a^9\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}} - 8\,a^9\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{4\,a^{12}}$$

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] 1/2*b^2/(a^3*(b + a/x^3)^(2/3)) - 1/4*(a^9*(b + a/x^3)^(4/3) - 8*a^9*(b + a
/x^3)^(1/3)*b)/a^12
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{\left(b\,x^6+a\,x^3\right)^{1/3}\,\left(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

```
[In] int(1/(a*x^3 + b*x^6)^(5/3),x)
[Out] ((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^3
))
```

3.5 $\int \frac{1}{-x^3+x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{-x^3+x^6} dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] 1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1607, 331, 206, 31, 648, 632, 210, 642}

$$\int \frac{1}{-x^3+x^6} dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x)$$

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3(-1+x^3)}\,dx$$

$$= \frac{1}{2x^2} + \int \frac{1}{-1+x^3}\,dx$$

$$= \frac{1}{2x^2} + \frac{1}{3}\int \frac{1}{-1+x}\,dx + \frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx$$

$$= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx$$

$$= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$

$$= \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{1}{2x^2}+\frac{\ln(x-1)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	36
default	$\frac{1}{2x^2}-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}$	38
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3(-1)^{\frac{1}{3}}}{2x^2}+\frac{x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	78

```
[In] int(1/(x^6-x^3),x,method=_RETURNVERBOSE)
```

[Out] 1/2/x^2+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log\left(x-1\right)-3}{6x^2}$$

[In] integrate(1/(x^6-x^3),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) - 2*x^2*log(x - 1) - 3)/x^2

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

[In] integrate(1/(x**6-x**3),x)

[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + 1/(2*x**2)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

[In] integrate(1/(x^6-x^3),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1) + 1/3*log(x - 1)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{2\,x^2}-\frac{1}{6}\,\log\left(x^2+x+1\right)+\frac{1}{3}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\ln(x-1)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2\,x^2}$$

```
[In] int(-1/(x^3 - x^6),x)
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

3.6 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

```
[Out] 1/6*a*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b*x^9*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}$$

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx^5+b^2x^8)\,dx}{ab+b^2x^3}\\ &= \frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(3ax^6+2bx^9)}{18(a+bx^3)}$$

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(b\,x^3+a)^2\,(-2b\,x^3+a)}{18b^2}$	31
gosper	$\frac{x^6\,(2b\,x^3+3a)\,\sqrt{(b\,x^3+a)^2}}{18b\,x^3+18a}$	36
default	$\frac{x^6\,(2b\,x^3+3a)\,\sqrt{(b\,x^3+a)^2}}{18b\,x^3+18a}$	36
risch	$\frac{a\,x^6\,\sqrt{(b\,x^3+a)^2}}{6b\,x^3+6a}+\frac{b\,x^9\,\sqrt{(b\,x^3+a)^2}}{9b\,x^3+9a}$	54

```
[In] int(x^5*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*csgn(b*x^3+a)*(b*x^3+a)^2*(-2*b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/9*b*x^9 + 1/6*a*x^6
```

Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}ax^3}{6\,b}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a^2}{6\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{9\,b^2}$$

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.29

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{18}\left(2\,bx^9+3\,ax^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,\left(8\,b^2\,\left(a^2+b^2\,x^6\right)-12\,a^2\,b^2+4\,a\,b^3\,x^3\right)}{72\,b^4}$$

```
[In] int(x^5*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4*
a*b^3*x^3))/(72*b^4)
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[Out] 1/5*a*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b*x^8*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx^4+b^2x^7)\,dx}{ab+b^2x^3}\\ &= \frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(8ax^5+5bx^8)}{40(a+bx^3)}$$

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^5(5bx^3+8a)\sqrt{(bx^3+a)^2}}{40bx^3+40a}$	36
default	$\frac{x^5(5bx^3+8a)\sqrt{(bx^3+a)^2}}{40bx^3+40a}$	36
risch	$\frac{ax^5\sqrt{(bx^3+a)^2}}{5bx^3+5a}+\frac{bx^8\sqrt{(bx^3+a)^2}}{8bx^3+8a}$	54

```
[In] int(x^4*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8\text{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ax^5\text{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^4\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int(x^4*((a + b*x^3)^2)^(1/2),x)
[Out] int(x^4*((a + b*x^3)^2)^(1/2), x)
```

3.8 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}$$

```
[Out] 1/4*a*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b*x^7*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx^3+b^2x^6)\,dx}{ab+b^2x^3}\\ &= \frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{\sqrt{(a+bx^3)^2}(7ax^4+4bx^7)}{28(a+bx^3)}$$

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^4(4bx^3+7a)\sqrt{(bx^3+a)^2}}{28bx^3+28a}$	36
default	$\frac{x^4(4bx^3+7a)\sqrt{(bx^3+a)^2}}{28bx^3+28a}$	36
risch	$\frac{ax^4\sqrt{(bx^3+a)^2}}{4bx^3+4a}+\frac{bx^7\sqrt{(bx^3+a)^2}}{7bx^3+7a}$	54

```
[In] int(x^3*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ax^4\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int(x^3*((a + b*x^3)^2)^(1/2),x)
[Out] int(x^3*((a + b*x^3)^2)^(1/2), x)
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[Out] 1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 623}

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \sqrt{a^2+2abx+b^2x^2}\,dx, x, x^3\right)$$
$$= \frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x^3(2a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{-6a^2-6abx^3+6\sqrt{a^2}\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (x^3*(2*a + b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))
/(-6*a^2 - 6*a*b*x^3 + 6*Sqrt[a^2]*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{(b\,x^3+a)^2\,\mathrm{csgn}(b\,x^3+a)}{6b}$	23
default	$\frac{(b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6b}$	24
risch	$\frac{(b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6b}$	24
gosper	$\frac{x^3\,(b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{6b\,x^3+6a}$	35

```
[In] int(x^2*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(b*x^3+a)^2*csgn(b*x^3+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \int x^2\sqrt{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**2*((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x**2*sqrt((a + b*x**3)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(23) = 46.

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{6}\,\sqrt{b^2x^6+2\,abx^3+a^2}x^3+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a}{6\,b}$$

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^
2)*a/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{6}\,\left(bx^6+2\,ax^3\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\left(\frac{a}{6\,b}+\frac{x^3}{6}\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int(x^2*((a + b*x^3)^2)^(1/2),x)
[Out] (a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] 1/2*a*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b*x^5*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1369, 14}

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx+b^2x^4)\,dx}{ab+b^2x^3}\\ &= \frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{\sqrt{(a+bx^3)^2}(5ax^2+2bx^5)}{10(a+bx^3)}$$

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^2(2bx^3+5a)\sqrt{(bx^3+a)^2}}{10bx^3+10a}$	36
default	$\frac{x^2(2bx^3+5a)\sqrt{(bx^3+a)^2}}{10bx^3+10a}$	36
risch	$\frac{ax^2\sqrt{(bx^3+a)^2}}{2bx^3+2a}+\frac{bx^5\sqrt{(bx^3+a)^2}}{5bx^3+5a}$	54

```
[In] int(x*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Sympy [F]

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x\sqrt{(a+bx^3)^2}\,dx$$

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x*sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ax^2\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x\,\sqrt{\left(b\,x^3+a\right)^2}\,dx$$

```
[In] int(x*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x^3)^2)^(1/2), x)
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] a*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1357}

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(2ab+2b^2x^3\right)\,dx}{2ab+2b^2x^3}\\&=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(4ax+bx^4)}{4(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
default	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
risch	$\frac{ax\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{b\,x^4\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	51

```
[In] int(((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{4}\,bx^4+ax$$

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*b*x^4 + a*x
```

Sympy [F]

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \int \sqrt{(a + bx^3)^2}\,dx$$

```
[In] integrate(((b*x**3+a)**2)**(1/2),x)
[Out] Integral(sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \frac{1}{4}\,bx^4 + ax$$

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*b*x^4 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \frac{1}{4}\,(bx^4 + 4\,ax)\mathrm{sgn}\left(bx^3 + a\right)$$

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\,dx = \int \sqrt{(b\,x^3 + a)^2}\,dx$$

```
[In] int(((a + b*x^3)^2)^(1/2),x)
[Out] int(((a + b*x^3)^2)^(1/2), x)
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx = \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] 1/3*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx = \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{ab+b^2x^3}{x}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{ab}{x}+b^2x^2\right)\,dx}{ab+b^2x^3}\\
&= \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 0.69 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int\frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$$
$$=\frac{-2a\sqrt{a^2}bx^3-2\sqrt{a^2}b^2x^6+2abx^3\sqrt{\left(a+bx^3\right)^2}-2a\left(a^2+abx^3-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)}}\right.}{}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (-2*a*Sqrt[a^2]*b*x^3 - 2*Sqrt[a^2]*b^2*x^6 + 2*a*b*x^3*Sqrt[(a + b*x^3)^2]
 - 2*a*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])*ArcTanh[(b*x^3)/(Sqr
t[a^2] - Sqrt[(a + b*x^3)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^2*S
qrt[(a + b*x^3)^2])*Log[x^3] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a
+ b*x^3)^2]] + a*Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2
]] - a^2*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] +
 (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + a*Sqrt[a^2]*b*x
^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - a^2*Sqrt[(a + b*x^3)^2]*L
og[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(6*(a^2 + a*b*x^3 - Sqrt[a^2]*
Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a+a\ln\left(b\,x^{3}\right)\right)}{3}$	26
default	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+3a\ln\left(x\right)\right)}{3b\,x^{3}+3a}$	34
risch	$\frac{b\,x^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3b\,x^{3}+3a}+\frac{a\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	52

```
[In] int(((b*x^3+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/3*csgn(b*x^3+a)*(b*x^3+a+a*ln(b*x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx=\frac{1}{3}\,bx^3+a\log\left(x\right)$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx=\int \frac{\sqrt{\left(a+bx^3\right)^2}}{x}\,dx$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
[Out] Integral(sqrt((a + b*x**3)**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{1}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3+a\right)+a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3} - \frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\,\sqrt{a^2}}{3} + \frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a +
b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}$$

```
[Out] -a*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx = \frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^2}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^2}+b^2x\right)\,dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx = \frac{(-2a+bx^3)\sqrt{(a+bx^3)^2}}{2x\,(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{(-b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{2x(b\,x^3+a)}$	36
default	$-\frac{(-b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{2x(b\,x^3+a)}$	36
risch	$-\frac{a\sqrt{(b\,x^3+a)^2}}{x(b\,x^3+a)}+\frac{b\,x^2\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}$	54

```
[In] int(((b*x^3+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \frac{bx^3 - 2a}{2x}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x^3 - 2*a)/x
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \int \frac{\sqrt{(a + bx^3)^2}}{x^2} \, dx$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
[Out] Integral(sqrt((a + b*x**3)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \frac{bx^3 - 2a}{2x}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] 1/2*(b*x^3 - 2*a)/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \frac{1}{2} bx^2 \mathrm{sgn}\left(bx^3 + a\right) - \frac{a \mathrm{sgn}\left(bx^3 + a\right)}{x}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2}\,dx = \int \frac{\sqrt{{\left(b\,x^3 + a\right)}^2}}{x^2}\,dx$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^2,x)
[Out] int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/2*a*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx = \frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (b*x*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^3}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2+\frac{ab}{x^3}\right)\,dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx=-\frac{\left(a-2bx^3\right)\sqrt{\left(a+bx^3\right)^2}}{2x^2\left(a+bx^3\right)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
[Out] -1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{\left(-2b\,x^{3}+a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x^{2}\left(b\,x^{3}+a\right)}$	34
default	$-\frac{\left(-2b\,x^{3}+a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x^{2}\left(b\,x^{3}+a\right)}$	34
risch	$-\frac{a\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x^{2}\left(b\,x^{3}+a\right)}+\frac{bx\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	51

```
[In] int(((b*x^3+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = \frac{2\,bx^3 - a}{2\,x^2}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = \frac{2\,bx^3 - a}{2\,x^2}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = bx\mathrm{sgn}\left(bx^3 + a\right) - \frac{a\mathrm{sgn}(bx^3 + a)}{2\,x^2}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\,dx = \int \frac{\sqrt{(b\,x^3 + a)^2}}{x^3}\,dx$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^3,x)
[Out] int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/3*a*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx=\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^4}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^4}+\frac{b^2}{x}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{a\sqrt{a^2}-a\sqrt{(a+bx^3)^2}-2abx^3\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)-2\sqrt{a^2}bx^3\log(x^3)+\sqrt{a^2}bx^3\log\left(a\left(\sqrt{a^2}-bx^3-\right.\right.}{6ax^3}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] (a*Sqrt[a^2] - a*Sqrt[(a + b*x^3)^2] - 2*a*b*x^3*ArcTanh[(b*x^3)/(Sqrt[a^2]
 - Sqrt[(a + b*x^3)^2])] - 2*Sqrt[a^2]*b*x^3*Log[x^3] + Sqrt[a^2]*b*x^3*Log
[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])] + Sqrt[a^2]*b*x^3*Log[a*(Sqrt
[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*x^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(-\ln(b\,x^3)b\,x^3+a)}{3x^3}$	28
default	$\frac{\sqrt{(b\,x^3+a)^2}\,(3b\ln(x)x^3-a)}{3x^3(b\,x^3+a)}$	38
risch	$-\frac{a\sqrt{(b\,x^3+a)^2}}{3x^3(b\,x^3+a)}+\frac{b\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	52

```
[In] int(((b*x^3+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*csgn(b*x^3+a)*(-ln(b*x^3)*b*x^3+a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{3\,bx^3\log\left(x\right)-a}{3\,x^3}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{1}{3}\left(-1\right)^{2\,b^2x^3+2\,ab}b\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\left(-1\right)^{2\,abx^3+2\,a^2}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,x^3}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = b\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{bx^3\operatorname{sgn}(bx^3+a)+a\operatorname{sgn}(bx^3+a)}{3\,x^3}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a)
)/x^3
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)\,\sqrt{b^2}}{3} - \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3} - \frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

```
[Out] (log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)^(
1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

```
[Out] -1/4*a*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx=-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -1/4*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^5}\,dx}{ab+b^2x^3} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^5}+\frac{b^2}{x^2}\right)\,dx}{ab+b^2x^3} \\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{\left(4b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{4\left(b\,x^3+a\right)x^4}$	34
default	$-\frac{\left(4b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{4\left(b\,x^3+a\right)x^4}$	34
risch	$\frac{\left(-b\,x^3-\frac{a}{4}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^4\left(b\,x^3+a\right)}$	35

```
[In] int(((b*x^3+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5} \, dx = -\frac{4 \, bx^3 + a}{4 \, x^4}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5} \, dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5} \, dx = -\frac{4 \, bx^3 + a}{4 \, x^4}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5} \, dx = -\frac{4 \, bx^3 \mathrm{sgn}\left(bx^3 + a\right) + a\mathrm{sgn}\left(bx^3 + a\right)}{4 \, x^4}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx=-\frac{(4\,b\,x^3+a)\,\sqrt{(b\,x^3+a)^2}}{4\,x^4\,(b\,x^3+a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^5,x)
[Out] -((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6} \, dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[Out] -1/5*a*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b*((b*x^3+a)^2)^(1/2)/x^2/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6} \, dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -1/5*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^6}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^3}\right)\,dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
[Out] -1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{2}-\frac{a}{5}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^5\left(b\,x^3+a\right)}$	35
gosper	$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$	36
default	$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$	36

```
[In] int(((b*x^3+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
[Out] 1/x^5*(-1/2*b*x^3-1/5*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3 + 2\,a}{10\,x^5}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3 + 2\,a}{10\,x^5}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3\text{sgn}(bx^3 + a) + 2\,a\text{sgn}(bx^3 + a)}{10\,x^5}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{(5\,b\,x^3+2\,a)\,\sqrt{(b\,x^3+a)^2}}{10\,x^5\,(b\,x^3+a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^6,x)
[Out] -((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/6*a*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-1/3*b*((b*x^3+a)^2)^(1/2)/x^3/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -1/6*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{ab+b^2x^3}{x^7}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{ab}{x^7}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int\frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{\sqrt{(a+bx^3)^2}(a+2bx^3)}{6x^6(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(2b\,x^3+a)}{6x^6}$	22
gosper	$-\frac{(2b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6x^6(b\,x^3+a)}$	34
default	$-\frac{(2b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{6x^6(b\,x^3+a)}$	34
risch	$\frac{\left(-\frac{b\,x^3}{3}-\frac{a}{6}\right)\sqrt{(b\,x^3+a)^2}}{x^6(b\,x^3+a)}$	35

```
[In] int(((b*x^3+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*csgn(b*x^3+a)*(2*b*x^3+a)/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = -\frac{2\,bx^3 + a}{6\,x^6}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/6*(2*b*x^3 + a)/x^6
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} \, dx = \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2}{6\,a^2} + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b}{6\,ax^3} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
+ a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx = -\frac{2\,bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
[Out] -1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx = -\frac{(2\,b\,x^3+a)\,\sqrt{(b\,x^3+a)^2}}{6\,x^6\,(b\,x^3+a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^7,x)
[Out] -((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}$$

```
[Out] -1/7*a*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b*((b*x^3+a)^2)^(1/2)/x^4/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -1/7*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^8}\,dx}{ab+b^2x^3} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^5}\right)\,dx}{ab+b^2x^3} \\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{\sqrt{(a+bx^3)^2}(4a+7bx^3)}{28x^7\,(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{4}-\frac{a}{7}\right)\sqrt{(b\,x^3+a)^2}}{x^7(b\,x^3+a)}$	35
gosper	$-\frac{(7b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{28x^7(b\,x^3+a)}$	36
default	$-\frac{(7b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{28x^7(b\,x^3+a)}$	36

```
[In] int(((b*x^3+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
[Out] 1/x^7*(-1/4*b*x^3-1/7*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = -\frac{7\,bx^3 + 4\,a}{28\,x^7}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = -\frac{7\,bx^3 + 4\,a}{28\,x^7}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} \, dx = -\frac{7\,bx^3\text{sgn}(bx^3 + a) + 4\,a\text{sgn}(bx^3 + a)}{28\,x^7}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{\left(7\,b\,x^3+4\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{28\,x^7\,\left(b\,x^3+a\right)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^8,x)
[Out] -((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -1/8*a*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b*((b*x^3+a)^2)^(1/2)/x^5/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -1/8*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^9}\,dx}{ab+b^2x^3} \\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^9}+\frac{b^2}{x^6}\right)\,dx}{ab+b^2x^3} \\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{\sqrt{(a+bx^3)^2}(5a+8bx^3)}{40x^8\,(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))
```

Maple [A] (verified)

Time = 8.96 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{5}-\frac{a}{8}\right)\sqrt{(b\,x^3+a)^2}}{x^8(b\,x^3+a)}$	35
gosper	$-\frac{(8b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{40x^8(b\,x^3+a)}$	36
default	$-\frac{(8b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{40x^8(b\,x^3+a)}$	36

```
[In] int(((b*x^3+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
[Out] 1/x^8*(-1/5*b*x^3-1/8*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3+5\,a}{40\,x^8}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3+5\,a}{40\,x^8}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{8\,bx^3\mathrm{sgn}\left(bx^3+a\right)+5\,a\mathrm{sgn}\left(bx^3+a\right)}{40\,x^8}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = -\frac{(8\,b\,x^3+5\,a)\,\sqrt{(b\,x^3+a)^2}}{40\,x^8\,(b\,x^3+a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^9,x)
[Out] -((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[Out] -1/9*a*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/6*b*((b*x^3+a)^2)^(1/2)/x^6/(b*x
^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -1/9*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{10}}\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^7}\right)dx}{ab+b^2x^3}\\ &= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(3b\,x^{3}+2a\right)}{18x^{9}}$	24
risch	$\frac{\left(-\frac{b\,x^{3}}{6}-\frac{a}{9}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{9}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(3b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18x^{9}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(3b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18x^{9}\left(b\,x^{3}+a\right)}$	36

```
[In] int(((b*x^3+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*csgn(b*x^3+a)*(3*b*x^3+2*a)/x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{3\,bx^3 + 2\,a}{18\,x^9}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^3}{6\,a^3} - \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2}{6\,a^2x^3} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b}{6\,a^3x^6} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}{9\,a^2x^9}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6) -
 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = -\frac{3\,bx^3\mathrm{sgn}(bx^3+a)+2\,a\mathrm{sgn}(bx^3+a)}{18\,x^9}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = -\frac{(3\,b\,x^3+2\,a)\,\sqrt{(b\,x^3+a)^2}}{18\,x^9\,(b\,x^3+a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^10,x)
[Out] -((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}} dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}} dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[Out] -1/10*a*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b*((b*x^3+a)^2)^(1/2)/x^7/(b
*x^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}} dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -1/10*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (b*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{11}}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{11}}+\frac{b^2}{x^8}\right)\,dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(7a+10bx^3)}{70x^{10}\,(a+bx^3)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))
```

Maple [A] (verified)

Time = 11.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{7}-\frac{a}{10}\right)\sqrt{(b\,x^3+a)^2}}{x^{10}(b\,x^3+a)}$	35
gosper	$-\frac{(10b\,x^3+7a)\sqrt{(b\,x^3+a)^2}}{70x^{10}(b\,x^3+a)}$	36
default	$-\frac{(10b\,x^3+7a)\sqrt{(b\,x^3+a)^2}}{70x^{10}(b\,x^3+a)}$	36

```
[In] int(((b*x^3+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
[Out] 1/x^10*(-1/7*b*x^3-1/10*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}}\,dx = -\frac{10\,bx^3 + 7\,a}{70\,x^{10}}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}}\,dx = -\frac{10\,bx^3 + 7\,a}{70\,x^{10}}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}}\,dx = -\frac{10\,bx^3\mathrm{sgn}(bx^3 + a) + 7\,a\mathrm{sgn}(bx^3 + a)}{70\,x^{10}}$$

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}}\,dx = -\frac{(10\,b\,x^3 + 7\,a)\,\sqrt{(b\,x^3 + a)^2}}{70\,x^{10}\,(b\,x^3 + a)}$$

```
[In] int(((a + b*x^3)^2)^(1/2)/x^11,x)
[Out] -((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))
```

3.23 $\int x^9(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

```
[Out] 1/10*a^3*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/13*a^2*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+3/16*a*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/19*b^3*x^19*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9(ab+b^2x^3)^3\,dx}{b^2(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^3b^3x^9+3a^2b^4x^{12}+3ab^5x^{15}+b^6x^{18})\,dx}{b^2(ab+b^2x^3)}\\ &= \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}\\ &\quad+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16(a+bx^3)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^{10}\sqrt{(a+bx^3)^2}(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9)}{19760(a+bx^3)}$$

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 104
0*b^3*x^9))/(19760*(a + b*x^3))
```

Maple [A] (verified)

Time = 8.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{10}\left(1040 b^{3} x^{9}+3705 b^{2} x^{6} a+4560 a^{2} b \,x^{3}+1976 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{19760 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{10}\left(1040 b^{3} x^{9}+3705 b^{2} x^{6} a+4560 a^{2} b \,x^{3}+1976 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{19760 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{10} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}+\frac{3 a^{2} b \,x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{16} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{16 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{19} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{19 b \,x^{3}+19 a}$	116

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+
a)^2)^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Sympy [F]

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^{9}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{19}\, b^3 x^{19} + \frac{3}{16}\, a b^2 x^{16} + \frac{3}{13}\, a^2 b x^{13} + \frac{1}{10}\, a^3 x^{10}$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{19}\, b^3 x^{19} \mathrm{sgn}\left(bx^3+a\right) + \frac{3}{16}\, a b^2 x^{16} \mathrm{sgn}\left(bx^3+a\right) + \frac{3}{13}\, a^2 b x^{13} \mathrm{sgn}\left(bx^3+a\right) + \frac{1}{10}\, a^3 x^{10} \mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b*
x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^9\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2} \,d x$$

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.24 $\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^3}-\frac{2a(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^3}+\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}$$

```
[Out] 1/12*a^2*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^3-2/15*a*(b*x^3+a)^4*((b*x^3+a)^
2)^(1/2)/b^3+1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18(a+bx^3)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}$$

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(18*(a + b*x^3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^3\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(a^3b^3x^2+3a^2b^4x^3+3ab^5x^4+b^6x^5\right)\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}\\ &\quad+\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^9\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{\left(a+bx^3\right)^2}\right)\right)}{180\left(-a^2-abx^3+\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)}$$

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9)*(Sqrt[a^2]*b*x^3 +
 a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(180*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqr
t[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}\left(10 b^{2} x^{6}-4 a b\,x^{3}+a^{2}\right)}{180 b^{3}}$	42
gosper	$\frac{x^{9}\left(10 b^{3} x^{9}+36 b^{2} x^{6} a+45 a^{2} b\,x^{3}+20 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{9}\left(10 b^{3} x^{9}+36 b^{2} x^{6} a+45 a^{2} b\,x^{3}+20 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3} x^{9}}{9 b\,x^{3}+9 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2} b\,x^{12}}{4 b\,x^{3}+4 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2} a\,x^{15}}{5 b\,x^{3}+5 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3} x^{18}}{18 b\,x^{3}+18 a}$	116

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*csgn(b*x^3+a)*(b*x^3+a)^4*(10*b^2*x^6-4*a*b*x^3+a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^8 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{18}\, b^3x^{18} + \frac{1}{5}\, ab^2x^{15} + \frac{1}{4}\, a^2bx^{12} + \frac{1}{9}\, a^3x^9$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [F]

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x^8 \left((a + bx^3)^2\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}} a^2 x^3}{12\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} x^3}{18\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}} a^3}{12\,b^3} - \frac{7\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} a}{90\,b^3}$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3 -
 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{18} b^3 x^{18} \operatorname{sgn}(bx^3 + a) + \frac{1}{5} a b^2 x^{15} \operatorname{sgn}(bx^3 + a) + \frac{1}{4} a^2 b x^{12} \operatorname{sgn}(bx^3 + a) + \frac{1}{9} a^3 x^9 \operatorname{sgn}(bx^3 + a)$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^
12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \int x^8 \, \left(a^2 + 2\, a\, b\, x^3 + b^2\, x^6\right)^{3/2} \, dx$$

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.25 $\int x^7(a^2 + 2abx^3 + b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

```
[Out] 1/8*a^3*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/11*a^2*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/14*a*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/17*b^3*x^17*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3x^7+3a^2b^4x^{10}+3ab^5x^{13}+b^6x^{16}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}\\ &\quad+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9\right)}{10472\left(a+bx^3\right)}$$

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*
b^3*x^9))/(10472*(a + b*x^3))
```

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{8}\left(616 b^{3} x^{9}+2244 b^{2} x^{6} a+2856 a^{2} b\,x^{3}+1309 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{8}\left(616 b^{3} x^{9}+2244 b^{2} x^{6} a+2856 a^{2} b\,x^{3}+1309 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3} x^{8} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 b\,x^{3}+8 a}+\frac{3 a^{2} b\,x^{11} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{11 \left(b\,x^{3}+a\right)}+\frac{3 a\,b^{2} x^{14} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{14 \left(b\,x^{3}+a\right)}+\frac{b^{3} x^{17} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{17 b\,x^{3}+17 a}$	116

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Sympy [F]

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \int x^{7}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{1}{17}\,b^3x^{17} + \frac{3}{14}\,ab^2x^{14} + \frac{3}{11}\,a^2bx^{11} + \frac{1}{8}\,a^3x^8$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{1}{17}\,b^3x^{17}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{14}\,ab^2x^{14}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b*
x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \int x^7\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2} \,d x$$

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.26 $\int x^6(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}$$

```
[Out] 1/7*a^3*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/10*a^2*b*x^10*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/13*a*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/16*b^3*x^16*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6(ab+b^2x^3)^3\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^3b^3x^6+3a^2b^4x^9+3ab^5x^{12}+b^6x^{15})\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)} \\
&\quad+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^7\sqrt{(a+bx^3)^2}(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9)}{7280(a+bx^3)}$$

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*
b^3*x^9))/(7280*(a + b*x^3))
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{7}\left(455 b^{3} x^{9}+1680 b^{2} x^{6} a+2184 a^{2} b \,x^{3}+1040 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{7280 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{7}\left(455 b^{3} x^{9}+1680 b^{2} x^{6} a+2184 a^{2} b \,x^{3}+1040 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{7280 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{7} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{7 b \,x^{3}+7 a}+\frac{3 a^{2} b \,x^{10} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{16} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{16 b \,x^{3}+16 a}$	116

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{16} \, b^3x^{16} + \frac{3}{13} \, ab^2x^{13} + \frac{3}{10} \, a^2bx^{10} + \frac{1}{7} \, a^3x^7$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Sympy [F]

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^{6} \left(\left(a + bx^{3}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{16} b^3x^{16} + \frac{3}{13} ab^2x^{13} + \frac{3}{10} a^2bx^{10} + \frac{1}{7} a^3x^7$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{16} b^3x^{16}\operatorname{sgn}\left(bx^3+a\right) + \frac{3}{13} ab^2x^{13}\operatorname{sgn}\left(bx^3+a\right) + \frac{3}{10} a^2bx^{10}\operatorname{sgn}\left(bx^3+a\right) + \frac{1}{7} a^3x^7\operatorname{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b*
x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^6 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2} dx$$

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.27 $\int x^5(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=-\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{\left(a+bx^3\right)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

```
[Out] -1/12*a*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^2+1/15*(b*x^3+a)^4*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\left(a+bx^3\right)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/12*(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^3\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right)dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{\left(a+bx^3\right)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx=\frac{x^6\left(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9\right)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{\left(a+bx^3\right)^2}\right)\right)}{60\left(-a^2-abx^3+\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)}$$

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^6*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)*(Sqrt[a^2]*b*x^3 +
a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(60*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[
(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}\left(-4b\,x^{3}+a\right)}{60b^{2}}$	31
gosper	$\frac{x^{6}\left(4b^{3}x^{9}+15b^{2}x^{6}a+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{6}\left(4b^{3}x^{9}+15b^{2}x^{6}a+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}x^{6}}{6b\,x^{3}+6a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b\,x^{9}}{3b\,x^{3}+3a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}a\,x^{12}}{4b\,x^{3}+4a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}x^{15}}{15b\,x^{3}+15a}$	116

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/60*csgn(b*x^3+a)*(b*x^3+a)^4*(-4*b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [F]

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = -\frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}ax^3}{12\,b} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}a^2}{12\,b^2} + \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}{15\,b^2}$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{60} \left(4\,b^3x^{15} + 15\,ab^2x^{12} + 20\,a^2bx^9 + 10\,a^3x^6\right)\mathrm{sgn}\left(bx^3 + a\right)$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a
)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}\left(-a^2 + 3\,a\,b\,x^3 + 4\,b^2\,x^6\right)}{60\,b^2}$$

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

3.28 $\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

```
[Out] 1/5*a^3*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/8*a^2*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/11*a*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^3*x^14*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4(ab+b^2x^3)^3\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^3b^3x^4+3a^2b^4x^7+3ab^5x^{10}+b^6x^{13})\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} \\
&\quad +\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx = \frac{x^5\sqrt{(a+bx^3)^2}(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9)}{3080(a+bx^3)}$$

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^
3*x^9))/(3080*(a + b*x^3))
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{5}\left(220 b^{3} x^{9}+840 b^{2} x^{6} a+1155 a^{2} b \,x^{3}+616 a^{3}\right)\left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{3080 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{5}\left(220 b^{3} x^{9}+840 b^{2} x^{6} a+1155 a^{2} b \,x^{3}+616 a^{3}\right)\left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{3080 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 b \,x^{3}+5 a}+\frac{3 a^{2} b \,x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 b \,x^{3}+14 a}$	116

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [F]

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}\,ab^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^3x^5\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*x
^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\int x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.29 $\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

```
[Out] 1/4*a^3*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/7*a^2*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/10*a*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13*b^3*x^13*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3(ab+b^2x^3)^3\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^3b^3x^3+3a^2b^4x^6+3ab^5x^9+b^6x^{12})\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} \\
&\quad +\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{x^4\sqrt{(a+bx^3)^2}(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9)}{1820(a+bx^3)}$$

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3
*x^9))/(1820*(a + b*x^3))
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{4}\left(140 b^{3} x^{9}+546 b^{2} x^{6} a+780 a^{2} b\,x^{3}+455 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{4}\left(140 b^{3} x^{9}+546 b^{2} x^{6} a+780 a^{2} b\,x^{3}+455 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3} x^{4} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{4 b\,x^{3}+4 a}+\frac{3 a^{2} b\,x^{7} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{7 \left(b\,x^{3}+a\right)}+\frac{3 a\,b^{2} x^{10} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{10 \left(b\,x^{3}+a\right)}+\frac{b^{3} x^{13} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{13 b\,x^{3}+13 a}$	116

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{13} \, b^3 x^{13} + \frac{3}{10} \, a b^2 x^{10} + \frac{3}{7} \, a^2 b x^7 + \frac{1}{4} \, a^3 x^4$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^{3} \left(\left(a + b x^{3}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{13} b^3x^{13} + \frac{3}{10} ab^2x^{10} + \frac{3}{7} a^2bx^7 + \frac{1}{4} a^3x^4$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{13} b^3x^{13} \text{sgn}(bx^3 + a) + \frac{3}{10} ab^2x^{10} \text{sgn}(bx^3 + a) + \frac{3}{7} a^2bx^7 \text{sgn}(bx^3 + a) + \frac{1}{4} a^3x^4 \text{sgn}(bx^3 + a)$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*x
^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} \,dx$$

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.30 $\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[Out] 1/12*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 623}

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx, x, x^3\right)$$
$$= \frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{x^3(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{12\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^3*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9)*(Sqrt[a^2]*b*x^3 + a*(Sq
rt[a^2] - Sqrt[(a + b*x^3)^2])))/(12*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a +
b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{(b\,x^3+a)^4\,\text{csgn}(b\,x^3+a)}{12b}$	23
default	$\frac{(b\,x^3+a)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12b}$	24
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(b\,x^3+a)^3}{12b}$	26
gosper	$\frac{x^3\left(b^3x^9+4b^2x^6a+6a^2b\,x^3+4a^3\right)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12(b\,x^3+a)^3}$	57

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(b*x^3+a)^4*csgn(b*x^3+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{12}\,b^3x^{12} + \frac{1}{3}\,ab^2x^9 + \frac{1}{2}\,a^2bx^6 + \frac{1}{3}\,a^3x^3$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [F]

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^2 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{12} \left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}x^3 + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a}{12\,b}$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(3/2)*a/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{12}\left(2\left(bx^6+2\,ax^3\right)a^2+\left(bx^6+2\,ax^3\right)^2b\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\left(b^2\,x^3+a\,b\right)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{12\,b^2}$$

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

3.31 $\int x(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

```
[Out] 1/2*a^3*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/5*a^2*b*x^5*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/8*a*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^3*x^11*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1369, 276}

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x+3a^2b^4x^4+3ab^5x^7+b^6x^{10}\right)\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}\\ &\quad+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*
x^9))/(440*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{2}\left(40 b^{3} x^{9}+165 b^{2} x^{6} a+264 a^{2} b \,x^{3}+220 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{2}\left(40 b^{3} x^{9}+165 b^{2} x^{6} a+264 a^{2} b \,x^{3}+220 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{3 a^{2} b \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 b \,x^{3}+11 a}$	116

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x(a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{11} b^3x^{11} + \frac{3}{8} ab^2x^8 + \frac{3}{5} a^2bx^5 + \frac{1}{2} a^3x^2$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x(a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \frac{1}{11} b^3x^{11}\mathrm{sgn}(bx^3 + a) + \frac{3}{8} ab^2x^8\mathrm{sgn}(bx^3 + a) + \frac{3}{5} a^2bx^5\mathrm{sgn}(bx^3 + a) + \frac{1}{2} a^3x^2\mathrm{sgn}(bx^3 + a)$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5
*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x(a^2 + 2abx^3 + b^2x^6)^{3/2} \, dx = \int x\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} \,dx$$

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.32 $\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3} + \frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3} + \frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3} + \frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3}$$

```
[Out] a^3*x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/4*a^2*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/7*a*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b
*x^3+a)^3+1/10*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1357, 200}

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{3ab^2x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{7\left(a+bx^3\right)^3} + \frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3} + \frac{b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{10\left(a+bx^3\right)^3} + \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a^2+2abx^3+b^2x^6)^{3/2}\int (2ab+2b^2x^3)^3\,dx}{(2ab+2b^2x^3)^3}\\
&= \frac{(a^2+2abx^3+b^2x^6)^{3/2}\int (8a^3b^3+24a^2b^4x^3+24ab^5x^6+8b^6x^9)\,dx}{(2ab+2b^2x^3)^3}\\
&= \frac{a^3x(a^2+2abx^3+b^2x^6)^{3/2}}{(a+bx^3)^3}+\frac{3a^2bx^4(a^2+2abx^3+b^2x^6)^{3/2}}{4\,(a+bx^3)^3}\\
&\quad+\frac{3ab^2x^7(a^2+2abx^3+b^2x^6)^{3/2}}{7\,(a+bx^3)^3}+\frac{b^3x^{10}(a^2+2abx^3+b^2x^6)^{3/2}}{10\,(a+bx^3)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x\sqrt{(a+bx^3)^2}(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9)}{140\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9
))/(140*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x\left(14 b^{3} x^{9}+60 b^{2} x^{6} a+105 a^{2} b \,x^{3}+140 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{140 \left(b \,x^{3}+a \right)^{3}}$	56
default	$\frac{x\left(14 b^{3} x^{9}+60 b^{2} x^{6} a+105 a^{2} b \,x^{3}+140 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{140 \left(b \,x^{3}+a \right)^{3}}$	56
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{3} x^{10}}{10 b \,x^{3}+10 a}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a \,b^{2} x^{7}}{7 \left(b \,x^{3}+a \right)}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{2} b \,x^{4}}{4 \left(b \,x^{3}+a \right)}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{3} x}{b \,x^{3}+a}$	113

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int \left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{10}\,b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}\,ab^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}\,a^2bx^4\mathrm{sgn}\left(bx^3+a\right)+a^3x\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4
*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx=\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] a^2*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^2*x^6*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+1/9*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^3*ln(x)*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx=\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x}\,dx}{b^2\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x}\,dx, x, x^3\right)}{3b^2\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)\,dx, x, x^3\right)}{3b^2\,(ab+b^2x^3)}$$

$$= \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)} + \frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x} \, dx = \frac{\sqrt{(a + bx^3)^2}(bx^3(18a^2 + 9abx^3 + 2b^2x^6) + 18a^3 \log(x))}{18\,(a + bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x
]))/(18*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^3+a)\,(2b^3x^9+9b^2x^6a+18a^2b\,x^3+6a^3\ln(b\,x^3)+11a^3)}{18}$	54
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(2b^3x^9+9b^2x^6a+18a^2b\,x^3+18a^3\ln(x))}{18(b\,x^3+a)^3}$	57
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b(\frac{1}{9}b^2x^9+\frac{1}{2}ab\,x^6+a^2x^3)}{b\,x^3+a}+\frac{a^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	73

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*csgn(b*x^3+a)*(2*b^3*x^9+9*b^2*x^6*a+18*a^2*b*x^3+6*a^3*ln(b*x^3)+11*a
^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x} \, dx = \frac{1}{9}\,b^3x^9 + \frac{1}{2}\,ab^2x^6 + a^2bx^3 + a^3\log\left(x\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}abx^3 + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}a^2 + \frac{1}{9}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*
a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 + 1/9
*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{2}\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + a^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b
*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

3.34 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

```
[Out] -a^3*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+3/2*a^2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+3/5*a*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^3*x^8*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^2}\,dx}{b^2\,(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x+3ab^5x^4+b^6x^7\right)dx}{b^2\,(ab+b^2x^3)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}\\ &\quad+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{\sqrt{(a+bx^3)^2}(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9)}{40x\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-5 b^{3} x^{9}-24 b^{2} x^{6} a-60 a^{2} b\,x^{3}+40 a^{3}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b\,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(-5 b^{3} x^{9}-24 b^{2} x^{6} a-60 a^{2} b\,x^{3}+40 a^{3}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b\,x^{3}+a \right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b \left(\frac{1}{8} b^{2} x^{8}+\frac{3}{5} a b\,x^{5}+\frac{3}{2} a^{2} x^{2}\right)}{b\,x^{3}+a}-\frac{a^{3} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{x \left(b\,x^{3}+a \right)}$	76

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(
b*x^3+a)^3

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx=\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)

[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3+a\right) - \frac{a^3\mathrm{sgn}(bx^3+a)}{x}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*s
gn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^2}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

3.35 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

```
[Out] -1/2*a^3*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+3*a^2*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+3/4*a*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^3*x^7*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx = \frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (3*a^2*b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^3}\,dx}{b^2(ab+b^2x^3)} \\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3a^2b^4+\frac{a^3b^3}{x^3}+3ab^5x^3+b^6x^6\right)dx}{b^2(ab+b^2x^3)} \\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} \\ &\quad +\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^3)^2}(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9)}{28x^2(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-4 b^{3} x^{9}-21 b^{2} x^{6} a -84 a^{2} b \,x^{3}+14 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 x^{2} \left(b \,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(-4 b^{3} x^{9}-21 b^{2} x^{6} a -84 a^{2} b \,x^{3}+14 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 x^{2} \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \left(\frac{1}{7} b^{2} x^{7}+\frac{3}{4} a b \,x^{4}+3 a^{2} x \right)}{b \,x^{3}+a}-\frac{a^{3} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	74

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2
/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx=\frac{4\,b^{3}x^{9}+21\,ab^{2}x^{6}+84\,a^{2}bx^{3}-14\,a^{3}}{28\,x^{2}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx=\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \frac{4\,b^3x^9 + 21\,ab^2x^6 + 84\,a^2bx^3 - 14\,a^3}{28\,x^2}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3 + a\right) + 3\,a^2bx\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^3\mathrm{sgn}\left(bx^3 + a\right)}{2\,x^2}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b
*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^3} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

3.36 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/3*a^3*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+a*b^2*x^3*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+1/6*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a^2*b*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4} dx = \frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (a*b^2*x^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^4}\,dx}{b^2(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\frac{(ab+b^2x)^3}{x^2}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}\\
&\quad+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx=\frac{\sqrt{(a+bx^3)^2}(-2a^3+6ab^2x^6+b^3x^9+18a^2bx^3\log(x))}{6x^3\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x])
)/(6*x^3*(a + b*x^3))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}x^{9}+6b^{2}x^{6}a+18a^{2}b\ln\left(x\right)x^{3}-2a^{3}\right)}{6x^{3}\left(b\,x^{3}+a\right)^{3}}$	59
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\frac{b^{3}x^{9}}{2}-3b^{2}x^{6}a-3\ln\left(b\,x^{3}\right)a^{2}b\,x^{3}-\frac{5a^{2}b\,x^{3}}{2}+a^{3}\right)}{3x^{3}}$	59
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(b\,x^{3}+3a\right)^{2}}{6b\,x^{3}+6a}-\frac{a^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3x^{3}\left(b\,x^{3}+a\right)}+\frac{3a^{2}b\ln\left(x\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	92

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*b^2*x^6*a+18*a^2*b*ln(x)*x^3-2*a^3)/x^3/
(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx=\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log\left(x\right)-2\,a^3}{6\,x^3}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^2x^3 + (-1)^{2\,b^2x^3+2\,ab}a^2b\log\left(2\,b^2x^3+2\,ab\right) - (-1)^{2\,abx^3+2\,a^2}a^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}ab - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{3\,x^3}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2*
b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs(x
) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^3+a\right) + ab^2x^3\mathrm{sgn}\left(bx^3+a\right) + 3\,a^2b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{3\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{3\,x^3}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^4}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] -1/4*a^3*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-3*a^2*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+3/2*a*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b^3*x^5*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] -1/4*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^5}\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3b^3}{x^5}+\frac{3a^2b^4}{x^2}+3ab^5x+b^6x^4\right)dx}{b^2(ab+b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} \\
&\quad+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$
\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5}\,dx=\frac{\sqrt{(a+bx^3)^2}(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9)}{20x^4(a+bx^3)}
$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-4 b^{3} x^{9}-30 b^{2} x^{6} a+60 a^{2} b \,x^{3}+5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b \,x^{3}+a \right)^{3} x^{4}}$	58
default	$-\frac{\left(-4 b^{3} x^{9}-30 b^{2} x^{6} a+60 a^{2} b \,x^{3}+5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 \left(b \,x^{3}+a \right)^{3} x^{4}}$	58
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{5} b \,x^{5}+\frac{3}{2} a \,x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-3 a^{2} b \,x^{3}-\frac{1}{4} a^{3}\right)}{\left(b \,x^{3}+a \right) x^{4}}$	78

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/(b*x
^3+a)^3/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx=\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} \, dx = \frac{4\,b^3x^9 + 30\,ab^2x^6 - 60\,a^2bx^3 - 5\,a^3}{20\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} \, dx = \frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{12\,a^2bx^3\mathrm{sgn}\left(bx^3 + a\right) + a^3\mathrm{sgn}\left(bx^3 + a\right)}{4\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x
^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^5} \,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

3.38 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx=-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

```
[Out] -1/5*a^3*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-3/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+3*a*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^3*x^4*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx=\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^6}\,dx}{b^2(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(3ab^5+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^3}+b^6x^3\right)dx}{b^2(ab+b^2x^3)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}\\ &\quad+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9)}{20x^5(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-5 b^{3} x^{9}-60 b^{2} x^{6} a+30 a^{2} b\, x^{3}+4 a^{3}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20 \left(b\, x^{3}+a\right)^{3} x^{5}}$	58
default	$-\frac{\left(-5 b^{3} x^{9}-60 b^{2} x^{6} a+30 a^{2} b\, x^{3}+4 a^{3}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20 \left(b\, x^{3}+a\right)^{3} x^{5}}$	58
risch	$\frac{\sqrt{\left(b\, x^{3}+a\right)^{2}}\, b^{2} \left(\frac{1}{4} b\, x^{4}+3 a x\right)}{b\, x^{3}+a}+\frac{\sqrt{\left(b\, x^{3}+a\right)^{2}}\, \left(-\frac{3}{2} a^{2} b\, x^{3}-\frac{1}{5} a^{3}\right)}{\left(b\, x^{3}+a\right) x^{5}}$	76

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x
^3+a)^3/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx=\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} \, dx = \frac{5 \, b^3 x^9 + 60 \, a b^2 x^6 - 30 \, a^2 b x^3 - 4 \, a^3}{20 \, x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} \, dx = \frac{1}{4} \, b^3 x^4 \mathrm{sgn}\left(b x^3 + a\right) + 3 \, a b^2 x \mathrm{sgn}\left(b x^3 + a\right) - \frac{15 \, a^2 b x^3 \mathrm{sgn}\left(b x^3 + a\right) + 2 \, a^3 \mathrm{sgn}\left(b x^3 + a\right)}{10 \, x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*
sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} \, dx = \int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^6} \,d x$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7} \, dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/6*a^3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-a^2*b*((b*x^3+a)^2)^(1/2)/x^3/(b
*x^3+a)+1/3*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a*b^2*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7} \, dx = -\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/6*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*L
og[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^7}\,dx}{b^2(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\frac{(ab+b^2x)^3}{x^3}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)}\\
&\quad+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 612 vs. 2(162) = 324.

Time = 0.82 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.78

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^7} dx = \frac{4a^4\sqrt{a^2} + 28a^3\sqrt{a^2}bx^3 + 35(a^2)^{3/2}b^2x^6 + 3a\sqrt{a^2}b^3x^9 - 8\sqrt{a^2}b^4x^{12} - 4a^4\sqrt{(a}}{}$$

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]

[Out] (4*a^4*Sqrt[a^2] + 28*a^3*Sqrt[a^2]*b*x^3 + 35*(a^2)^(3/2)*b^2*x^6 + 3*a*Sqrt[a^2]*b^3*x^9 - 8*Sqrt[a^2]*b^4*x^12 - 4*a^4*Sqrt[(a + b*x^3)^2] - 24*a^3*b*x^3*Sqrt[(a + b*x^3)^2] - 11*a^2*b^2*x^6*Sqrt[(a + b*x^3)^2] + 8*a*b^3*x^9*Sqrt[(a + b*x^3)^2] - 24*a*b^2*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 24*b^2*x^6*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^2*Sqrt[(a + b*x^3)^2])*Log[x^3] + 12*(a^2)^(3/2)*b^2*x^6*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 12*a*Sqrt[a^2]*b^3*x^9*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - 12*a^2*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 12*(a^2)^(3/2)*b^2*x^6*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + 12*a*Sqrt[a^2]*b^3*x^9*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - 12*a^2*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(24*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(-2b^3x^9-6\ln(b\,x^3)\,a\,b^2x^6-2b^2x^6a+6a^2b\,x^3+a^3)}{6x^6}$	59
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(2b^3x^9+18b^2a\ln(x)x^6-6a^2b\,x^3-a^3)}{6(b\,x^3+a)^3x^6}$	60
risch	$\frac{b^3x^3\sqrt{(b\,x^3+a)^2}}{3b\,x^3+3a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-a^2b\,x^3-\frac{1}{6}a^3)}{(b\,x^3+a)x^6}+\frac{3a\,b^2\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	97

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/6*csgn(b*x^3+a)*(-2*b^3*x^9-6*ln(b*x^3)*a*b^2*x^6-2*b^2*x^6*a+6*a^2*b*x^3+a^3)/x^6

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{2\,b^3x^9+18\,ab^2x^6\log\left(x\right)-6\,a^2bx^3-a^3}{6\,x^6}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.36

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3x^3}{2\,a} + (-1)^{2\,b^2x^3+2\,ab}ab^2\log\left(2\,b^2x^3+2\,ab\right) - (-1)^{2\,abx^3+2\,a^2}ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^2 + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b}{6\,ax^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{6\,a^2x^6}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a*
b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/6*(
b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{9\,ab^2x^6\mathrm{sgn}(bx^3+a)+6\,a^2bx^3\mathrm{sgn}(bx^3+a)+a^3\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*
b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^7}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

3.40 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[Out] -1/7*a^3*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-3/4*a^2*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-3*a*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b^3*x^2*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx = -\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} + \frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -1/7*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^8}\,dx}{b^2\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^5}+\frac{3ab^5}{x^2}+b^6x\right)dx}{b^2\,(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}\\
&\quad-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx^3)^2}\,(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9)}{28x^7\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^
9))/(x^7*(a + b*x^3))
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-14 b^{3} x^{9}+84 b^{2} x^{6} a+21 a^{2} b \,x^{3}+4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 \left(b \,x^{3}+a \right)^{3} x^{7}}$	58
default	$-\frac{\left(-14 b^{3} x^{9}+84 b^{2} x^{6} a+21 a^{2} b \,x^{3}+4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{28 \left(b \,x^{3}+a \right)^{3} x^{7}}$	58
risch	$\frac{b^{3} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-3 b^{2} x^{6} a -\frac{3}{4} a^{2} b \,x^{3}-\frac{1}{7} a^{3}\right)}{\left(b \,x^{3}+a \right) x^{7}}$	78

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*
x^3+a)^3/x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{84\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 21\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 4\,a^3\mathrm{sgn}(bx^3 + a)}{28\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^8}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

3.41 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/8*a^3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-3/5*a^2*b*((b*x^3+a)^2)^(1/2)/x^
5/(b*x^3+a)-3/2*a*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b^3*x*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9} dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -1/8*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^9}\,dx}{b^2(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^6+\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^3}\right)dx}{b^2(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}\\
&\quad-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.38

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9}\,dx=-\frac{\sqrt{(a+bx^3)^2}(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9)}{40x^8(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^
9))/(x^8*(a + b*x^3))
```

Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-40 b^{3} x^{9}+60 b^{2} x^{6} a+24 a^{2} b\,x^{3}+5 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 \left(b\,x^{3}+a\right)^{3} x^{8}}$	58
default	$-\frac{\left(-40 b^{3} x^{9}+60 b^{2} x^{6} a+24 a^{2} b\,x^{3}+5 a^{3}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{40 \left(b\,x^{3}+a\right)^{3} x^{8}}$	58
risch	$\frac{b^{3} x \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-\frac{3}{2} b^{2} x^{6} a-\frac{3}{5} a^{2} b\,x^{3}-\frac{1}{8} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{8}}$	75

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/(b*
x^3+a)^3/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx=\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^9} \, dx = \frac{40 \, b^3 x^9 - 60 \, ab^2 x^6 - 24 \, a^2 b x^3 - 5 \, a^3}{40 \, x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^9} \, dx = b^3 x \operatorname{sgn}(bx^3 + a) - \frac{60 \, ab^2 x^6 \operatorname{sgn}(bx^3 + a) + 24 \, a^2 b x^3 \operatorname{sgn}(bx^3 + a) + 5 \, a^3 \operatorname{sgn}(bx^3 + a)}{40 \, x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn
(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^9} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^9} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)
```

3.42 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}} \, dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)} - \frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)} + \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/9*a^3*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-a*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^3*ln(x)*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}} \, dx = -\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)} - \frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)} + \frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/9*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^4}\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6\left(a+bx^3\right)}\\
&\quad -\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.65

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{2a^3\sqrt{a^2}+9(a^2)^{3/2}\,bx^3+18a\sqrt{a^2}b^2x^6-2a^3\sqrt{(a+bx^3)^2}-7a^2bx^3\sqrt{(a+bx^3}}{}$$

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]

[Out] (2*a^3*Sqrt[a^2] + 9*(a^2)^(3/2)*b*x^3 + 18*a*Sqrt[a^2]*b^2*x^6 - 2*a^3*Sqrt[(a + b*x^3)^2] - 7*a^2*b*x^3*Sqrt[(a + b*x^3)^2] - 11*a*b^2*x^6*Sqrt[(a + b*x^3)^2] - 12*a*b^3*x^9*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 12*Sqrt[a^2]*b^3*x^9*Log[x^3] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(36*a*x^9)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^3+a)\,(6\ln(b\,x^3)b^3x^9-18b^2x^6a-9a^2b\,x^3-2a^3)}{18x^9}$	52
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(18b^3\ln(x)x^9-18b^2x^6a-9a^2b\,x^3-2a^3)}{18(b\,x^3+a)^3x^9}$	60
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-b^2x^6a-\frac{1}{2}a^2b\,x^3-\frac{1}{9}a^3)}{(b\,x^3+a)x^9}+\frac{b^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	76

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)

[Out] 1/18*csgn(b*x^3+a)*(6*ln(b*x^3)*b^3*x^9-18*b^2*x^6*a-9*a^2*b*x^3-2*a^3)/x^9

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{18\,b^3x^9\log(x)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")

[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(113) = 226.

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.57

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3}{2\,a} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2x^3} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{18\,a^3x^6} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{9\,a^2x^9}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a
 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^
3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{11\,b^3x^9\mathrm{sgn}(bx^3+a)+18\,ab^2x^6\mathrm{sgn}(bx^3+a)+9\,a^2bx^3\mathrm{sgn}(bx^3+a)+2\,a^3\mathrm{sgn}(bx^3+a)}{18\,x^9}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2
*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^
9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{10}} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^{10}} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

3.43 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

```
[Out] -1/10*a^3*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-3/7*a^2*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-3/4*a*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b^3*((b*x^3+a)^2)
^(1/2)/x/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/10*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(x*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^{11}}\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3b^3}{x^{11}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^5}+\frac{b^6}{x^2}\right)dx}{b^2(ab+b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} \\
&\quad -\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx=-\frac{\sqrt{(a+bx^3)^2}(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9)}{140x^{10}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^
3*x^9))/(x^10*(a + b*x^3))
```

Maple [A] (verified)

Time = 12.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-b^3x^9-\frac{3}{4}b^2x^6a-\frac{3}{7}a^2b\,x^3-\frac{1}{10}a^3\right)}{(b\,x^3+a)x^{10}}$	57
gosper	$-\frac{(140b^3x^9+105b^2x^6a+60a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{140x^{10}(b\,x^3+a)^3}$	58
default	$-\frac{(140b^3x^9+105b^2x^6a+60a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{140x^{10}(b\,x^3+a)^3}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-b^3*x^9-3/4*b^2*x^6*a-3/7*a^2*b*x^3-1/10*a^
3)/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx = -\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}} \, dx = -\frac{140\,b^3x^9 + 105\,ab^2x^6 + 60\,a^2bx^3 + 14\,a^3}{140\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}} \, dx = -\frac{140\,b^3x^9\mathrm{sgn}(bx^3 + a) + 105\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 60\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 14\,a^3\mathrm{sgn}(bx^3 + a)}{140\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*
b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}} \, dx = -\frac{a^3\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3 + a\right)} - \frac{b^3\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{x\,\left(b\,x^3 + a\right)} - \frac{3\,a\,b^2\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{4\,x^4\,\left(b\,x^3 + a\right)} - \frac{3\,a^2\,b\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{7\,x^7\,\left(b\,x^3 + a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^7*(a + b*x^3))
```

3.44 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx=-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -1/11*a^3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-3/8*a^2*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-3/5*a*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b^3*((b*x^3+a
)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx=-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -1/11*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^11*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(2*x^2*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^{12}}\,dx}{b^2\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^9}+\frac{3ab^5}{x^6}+\frac{b^6}{x^3}\right)dx}{b^2\,(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}\\
&\quad-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx=-\frac{\sqrt{(a+bx^3)^2}(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9)}{440x^{11}\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b
^3*x^9))/(x^11*(a + b*x^3))
```

Maple [A] (verified)

Time = 13.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{2}b^{3}x^{9}-\frac{3}{5}b^{2}x^{6}a-\frac{3}{8}a^{2}b\,x^{3}-\frac{1}{11}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{11}}$	57
gosper	$-\frac{\left(220b^{3}x^{9}+264b^{2}x^{6}a+165a^{2}b\,x^{3}+40a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{440x^{11}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(220b^{3}x^{9}+264b^{2}x^{6}a+165a^{2}b\,x^{3}+40a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{440x^{11}\left(b\,x^{3}+a\right)^{3}}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/2*b^3*x^9-3/5*b^2*x^6*a-3/8*a^2*b*x^3-1/1
1*a^3)/x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{12}}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9 + 264\,ab^2x^6 + 165\,a^2bx^3 + 40\,a^3}{440\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{220\,b^3x^9\mathrm{sgn}(bx^3 + a) + 264\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 165\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 40\,a^3\mathrm{sgn}(bx^3 + a)}{440\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2
*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{12}}\,dx = -\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)} - \frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)} - \frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)} - \frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

3.45 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}} dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}} dx = -\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12ax^{12}}$$

```
[Out] -1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^12
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 270}

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}} dx = -\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12ax^{12}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/12*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^12)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{13}}\,dx}{b^2(ab+b^2x^3)} \\ &= -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9)}{12x^{12}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
[Out] -1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(
x^12*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)(2b\,x^3+a)(2b^2x^6+2ab\,x^3+a^2)}{12x^{12}}$	41
gosper	$-\frac{(4b^3x^9+6b^2x^6a+4a^2b\,x^3+a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12x^{12}(b\,x^3+a)^3}$	56
default	$-\frac{(4b^3x^9+6b^2x^6a+4a^2b\,x^3+a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12x^{12}(b\,x^3+a)^3}$	56
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{3}b^3x^9-\frac{1}{2}b^2x^6a-\frac{1}{3}a^2b\,x^3-\frac{1}{12}a^3\right)}{(b\,x^3+a)x^{12}}$	57

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
[Out] -1/12*csgn(b*x^3+a)*(2*b*x^3+a)*(2*b^2*x^6+2*a*b*x^3+a^2)/x^12
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(28) = 56.

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.61

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx &= \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{12\,a^4} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{12\,a^3x^3} \\ &- \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{12\,a^4x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{12\,a^3x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{12\,a^2x^{12}}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^
4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{4\,b^3x^9\mathrm{sgn}(bx^3 + a) + 6\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 4\,a^2bx^3\mathrm{sgn}(bx^3 + a) + a^3\mathrm{sgn}(bx^3 + a)}{12\,x^{12}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3*
sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.68

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{a^3\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3 + a\right)} - \frac{b^3\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{3\,x^3\,\left(b\,x^3 + a\right)} - \frac{a\,b^2\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{2\,x^6\,\left(b\,x^3 + a\right)} - \frac{a^2\,b\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{3\,x^9\,\left(b\,x^3 + a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(3*x^9*(a + b*x^3))
```

3.46 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}} dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

```
[Out] -1/13*a^3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-3/10*a^2*b*((b*x^3+a)^2)^(1/2)
/x^10/(b*x^3+a)-3/7*a*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b^3*((b*x^3
+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}} dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/13*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^13*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{14}}\,dx}{b^2(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{11}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^5}\right)dx}{b^2(ab+b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} \\
&\quad -\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9)}{1820x^{13}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455
*b^3*x^9))/(x^13*(a + b*x^3))
```

Maple [A] (verified)

Time = 16.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{4}b^{3}x^{9}-\frac{3}{7}b^{2}x^{6}a-\frac{3}{10}a^{2}b\,x^{3}-\frac{1}{13}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{13}}$	57
gosper	$-\frac{\left(455b^{3}x^{9}+780b^{2}x^{6}a+546a^{2}b\,x^{3}+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820x^{13}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(455b^{3}x^{9}+780b^{2}x^{6}a+546a^{2}b\,x^{3}+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{1820x^{13}\left(b\,x^{3}+a\right)^{3}}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/4*b^3*x^9-3/7*b^2*x^6*a-3/10*a^2*b*x^3-1/
13*a^3)/x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx=-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{14}}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{455\,b^3x^9\mathrm{sgn}(bx^3+a)+780\,ab^2x^6\mathrm{sgn}(bx^3+a)+546\,a^2bx^3\mathrm{sgn}(bx^3+a)+140\,a^3\mathrm{sgn}(bx^3+a)}{1820\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^
2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

3.47 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}} dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

```
[Out] -1/14*a^3*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-3/11*a^2*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-3/8*a*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b^3*((b*x^3
+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}} dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -1/14*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^14*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{15}}\,dx}{b^2\left(ab+b^2x^3\right)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{15}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^9}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^3\right)} \\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)} \\
&\quad -\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{15}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9\right)}{3080x^{14}\left(a+bx^3\right)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 61
6*b^3*x^9))/(x^14*(a + b*x^3))
```

Maple [A] (verified)

Time = 19.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{14}a^{3}-\frac{3}{11}a^{2}b\,x^{3}-\frac{3}{8}b^{2}x^{6}a-\frac{1}{5}b^{3}x^{9}\right)}{\left(b\,x^{3}+a\right)x^{14}}$	57
gosper	$-\frac{\left(616b^{3}x^{9}+1155b^{2}x^{6}a+840a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080x^{14}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(616b^{3}x^{9}+1155b^{2}x^{6}a+840a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080x^{14}\left(b\,x^{3}+a\right)^{3}}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/14*a^3-3/11*a^2*b*x^3-3/8*b^2*x^6*a-1/5*b
^3*x^9)/x^14
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}}\,dx = -\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\, dx = -\frac{616\, b^3x^9 + 1155\, ab^2x^6 + 840\, a^2bx^3 + 220\, a^3}{3080\, x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\, dx = -\frac{616\, b^3x^9\mathrm{sgn}(bx^3 + a) + 1155\, ab^2x^6\mathrm{sgn}(bx^3 + a) + 840\, a^2bx^3\mathrm{sgn}(bx^3 + a) + 220\, a^3\mathrm{sgn}(bx^3 + a)}{3080\, x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a
^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}}\, dx = -\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)} - \frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)} - \frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)} - \frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx = -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

```
[Out] -1/15*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^15+1/60*b*(b*x^3+a)^3*((b*x^3+a)^
2)^(1/2)/a^2/x^12
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1369, 272, 47, 37}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx = \frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}} - \frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/15*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^15) + (b*(a + b*
x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^6}\,dx, x, x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx, x, x^3\right)}{15ab\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9)}{60x^{15}\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
[Out] -1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^
9))/(x^15*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\left(\frac{5}{2}b^3x^9+5b^2x^6a+\frac{15}{4}a^2b\,x^3+a^3\right)\operatorname{csgn}\left(b\,x^3+a\right)}{15x^{15}}$	44
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-\frac{1}{15}a^3-\frac{1}{4}a^2b\,x^3-\frac{1}{3}b^2x^6a-\frac{1}{6}b^3x^9\right)}{\left(b\,x^3+a\right)x^{15}}$	57
gosper	$-\frac{\left(10b^3x^9+20b^2x^6a+15a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{60x^{15}\left(b\,x^3+a\right)^3}$	58
default	$-\frac{\left(10b^3x^9+20b^2x^6a+15a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{60x^{15}\left(b\,x^3+a\right)^3}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
[Out] -1/15*(5/2*b^3*x^9+5*b^2*x^6*a+15/4*a^2*b*x^3+a^3)*csgn(b*x^3+a)/x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(58) = 116$.

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.13

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5}{12\,a^5} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{12\,a^4x^3} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{12\,a^5x^6} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{12\,a^4x^9} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{12\,a^3x^{12}} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{15\,a^2x^{15}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a
^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(5/2)/(a^2*x^15)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{10\,b^3x^9\mathrm{sgn}(bx^3+a)+20\,ab^2x^6\mathrm{sgn}(bx^3+a)+15\,a^2bx^3\mathrm{sgn}(bx^3+a)+4\,a^3\mathrm{sgn}(bx^3+a)}{60\,x^{15}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{15\,x^{15}\,\left(b\,x^3+a\right)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{12}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(4*x^12*(a + b*x^3))
```

3.49 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}} dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[Out] -1/16*a^3*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-3/13*a^2*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-3/10*a*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b^3*((b*x
^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}} dx = -\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -1/16*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^16*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^3}{x^{17}}\,dx}{b^2\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^3b^3}{x^{17}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{11}}+\frac{b^6}{x^8}\right)dx}{b^2\,(ab+b^2x^3)}\\
&= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\,(a+bx^3)}\\
&\quad-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\,(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx=-\frac{\sqrt{(a+bx^3)^2}\,(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9)}{7280x^{16}\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
[Out] -1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1
040*b^3*x^9))/(x^16*(a + b*x^3))
```

Maple [A] (verified)

Time = 24.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{16}a^{3}-\frac{3}{13}a^{2}b\,x^{3}-\frac{3}{10}b^{2}x^{6}a-\frac{1}{7}b^{3}x^{9}\right)}{\left(b\,x^{3}+a\right)x^{16}}$	57
gosper	$-\frac{\left(1040b^{3}x^{9}+2184b^{2}x^{6}a+1680a^{2}b\,x^{3}+455a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(1040b^{3}x^{9}+2184b^{2}x^{6}a+1680a^{2}b\,x^{3}+455a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^{3}+a\right)^{3}}$	58

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/16*a^3-3/13*a^2*b*x^3-3/10*b^2*x^6*a-1/7*
b^3*x^9)/x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx=-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{1040\,b^3x^9 + 2184\,ab^2x^6 + 1680\,a^2bx^3 + 455\,a^3}{7280\,x^{16}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{1040\,b^3x^9\mathrm{sgn}(bx^3 + a) + 2184\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 1680\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 455\,a^3\mathrm{sgn}(bx^3 + a)}{7280\,x^{16}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680
*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{17}}\,dx = -\frac{a^3\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3 + a\right)} - \frac{b^3\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{7\,x^7\,\left(b\,x^3 + a\right)} - \frac{3\,a\,b^2\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3 + a\right)} - \frac{3\,a^2\,b\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3 + a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

3.50 $\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}+\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\,(a+bx^3)}$$

```
[Out] 1/14*a^5*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a^4*b*x^17*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+1/2*a^3*b^2*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/23*a^2*b^3
*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/26*a*b^4*x^26*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/29*b^5*x^29*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\,(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}$$

```
[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^5b^5x^{13}+5a^4b^6x^{16}+10a^3b^7x^{19}+10a^2b^8x^{22}+5ab^9x^{25}+b^{10}x^{28})\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26(a+bx^3)}+\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{x^{14}\sqrt{(a+bx^3)^2}(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}}{2063698(a+bx^3)}$$

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*
x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a
 + b*x^3))

Maple [A] (verified)

Time = 14.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{14}\left(71162b^{5}x^{15}+396865a\,b^{4}x^{12}+897260a^{2}b^{3}x^{9}+1031849a^{3}b^{2}x^{6}+606970a^{4}b\,x^{3}+147407a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{2063698\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{14}\left(71162b^{5}x^{15}+396865a\,b^{4}x^{12}+897260a^{2}b^{3}x^{9}+1031849a^{3}b^{2}x^{6}+606970a^{4}b\,x^{3}+147407a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{2063698\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5}x^{14}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{14b\,x^{3}+14a}+\frac{5a^{4}b\,x^{17}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{17\left(b\,x^{3}+a\right)}+\frac{a^{3}b^{2}x^{20}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2b\,x^{3}+2a}+\frac{10a^{2}b^{3}x^{23}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{23\left(b\,x^{3}+a\right)}+\frac{5a\,b^{4}x^{26}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{26\left(b\,x^{3}+a\right)}+\frac{b^{5}x^{29}}{29}$

[In] int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+1031849
*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14

Sympy [F]

$$\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x^{13}\left((a+bx^3)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{29}\,b^5x^{29}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{26}\,ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{23}\,a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,a^4bx^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*b
^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^17*
sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{13}\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.51 $\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\,(a+bx^3)}$$

```
[Out] 1/13*a^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a^4*b*x^16*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/19*a^3*b^2*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^2*b^
3*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*a*b^4*x^25*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/28*b^5*x^28*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\,(a+bx^3)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}$$

```
[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^5b^5x^{12}+5a^4b^6x^{15}+10a^3b^7x^{18}+10a^2b^8x^{21}+5ab^9x^{24}+b^{10}x^{27})\,dx}{b^4(ab+b^2x^3)}\\
&= \frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19(a+bx^3)}\\
&\quad+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{x^{13}\sqrt{(a+bx^3)^2}(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}+}{1521520(a+bx^3)}$$

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x
^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a
+ b*x^3))
```

Maple [A] (verified)

Time = 13.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{13}\left(54340b^{5}x^{15}+304304a\,b^{4}x^{12}+691600a^{2}b^{3}x^{9}+800800a^{3}b^{2}x^{6}+475475a^{4}b\,x^{3}+117040a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1521520\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{13}\left(54340b^{5}x^{15}+304304a\,b^{4}x^{12}+691600a^{2}b^{3}x^{9}+800800a^{3}b^{2}x^{6}+475475a^{4}b\,x^{3}+117040a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1521520\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5}x^{13}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{13b\,x^{3}+13a}+\frac{5a^{4}b\,x^{16}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{16\left(b\,x^{3}+a\right)}+\frac{10a^{3}b^{2}x^{19}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{19\left(b\,x^{3}+a\right)}+\frac{5a^{2}b^{3}x^{22}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{11\left(b\,x^{3}+a\right)}+\frac{a\,b^{4}x^{25}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{5b\,x^{3}+5a}+\frac{b^{5}x^{28}\sqrt{}}{28b\,x}$

```
[In] int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800*
a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Sympy [F]

$$\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \int x^{12}\left((a+bx^3)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{1}{28}\,b^5x^{28} + \frac{1}{5}\,ab^4x^{25} + \frac{5}{11}\,a^2b^3x^{22} + \frac{10}{19}\,a^3b^2x^{19} + \frac{5}{16}\,a^4bx^{16} + \frac{1}{13}\,a^5x^{13}$$

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{1}{28}\,b^5x^{28}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{5}\,ab^4x^{25}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{11}\,a^2b^3x^{22}\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{19}\,a^3b^2x^{19}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{16}\,a^4bx^{16}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{13}\,a^5x^{13}\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^3
*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^16*
sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{12}\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.52 $\int x^{11}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a^3(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}-\frac{a(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{8b^4}+\frac{(a+bx^3)^8\sqrt{a^2+2abx^3+b^2x^6}}{27b^4}$$

```
[Out] -1/18*a^3*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^4+1/7*a^2*(b*x^3+a)^6*((b*x^3+a
)^2)^(1/2)/b^4-1/8*a*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^4+1/27*(b*x^3+a)^8*(
(b*x^3+a)^2)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^5}{18b^4}$$

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/18*(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^4 + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{11}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int x^3\left(ab+b^2x\right)^5\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int\left(-\frac{a^3\left(ab+b^2x\right)^5}{b^3}+\frac{3a^2\left(ab+b^2x\right)^6}{b^4}-\frac{3a\left(ab+b^2x\right)^7}{b^5}+\frac{\left(ab+b^2x\right)^8}{b^6}\right)dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^3\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}\\
&\quad-\frac{a\left(a+bx^3\right)^7\sqrt{a^2+2abx^3+b^2x^6}}{8b^4}+\frac{\left(a+bx^3\right)^8\sqrt{a^2+2abx^3+b^2x^6}}{27b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{12}(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}\right.\right.}{1512\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (x^12*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(1512*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(b\,x^3+a)^6\,(-56b^3x^9+21b^2x^6a-6a^2b\,x^3+a^3)}{1512b^4}$
gosper	$\frac{x^{12}\,(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1512(b\,x^3+a)^5}$
default	$\frac{x^{12}\,(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1512(b\,x^3+a)^5}$
risch	$\frac{5\sqrt{(b\,x^3+a)^2}\,b^4a\,x^{24}}{24(b\,x^3+a)}+\frac{\sqrt{(b\,x^3+a)^2}\,b^5x^{27}}{27b\,x^3+27a}+\frac{5\sqrt{(b\,x^3+a)^2}\,a^3b^2x^{18}}{9(b\,x^3+a)}+\frac{10\sqrt{(b\,x^3+a)^2}\,a^2b^3x^{21}}{21(b\,x^3+a)}+\frac{\sqrt{(b\,x^3+a)^2}\,a^5x^{12}}{12b\,x^3+12a}$

[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/1512*csgn(b*x^3+a)*(b*x^3+a)^6*(-56*b^3*x^9+21*a*b^2*x^6-6*a^2*b*x^3+a^3)/b^4

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^4}{18\,b^4}+\frac{83\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a^2}{1512\,b^4}$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3/b
^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{27}\,b^5x^{27}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*b
^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15*s
gn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.53 $\int x^{10}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{10}(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}$$

```
[Out] 1/11*a^5*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a^4*b*x^14*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/17*a^3*b^2*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^2*b^3
*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/23*a*b^4*x^23*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/26*b^5*x^26*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^{10}(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}$$

```
[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^{10}+5a^4b^6x^{13}+10a^3b^7x^{16}+10a^2b^8x^{19}+5ab^9x^{22}+b^{10}x^{25}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}\\
&\quad+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{x^{11}\sqrt{\left(a+bx^3\right)^2}\left(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+\right.}{782782\left(a+bx^3\right)}$$

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a + b*x^3))

Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{11}\left(30107 b^{5} x^{15}+170170 a \,b^{4} x^{12}+391391 a^{2} b^{3} x^{9}+460460 a^{3} b^{2} x^{6}+279565 a^{4} b \,x^{3}+71162 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{782782 \left(b \,x^{3}+a \right)^{5}}$
default	$\frac{x^{11}\left(30107 b^{5} x^{15}+170170 a \,b^{4} x^{12}+391391 a^{2} b^{3} x^{9}+460460 a^{3} b^{2} x^{6}+279565 a^{4} b \,x^{3}+71162 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{782782 \left(b \,x^{3}+a \right)^{5}}$
risch	$\frac{a^{5} x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 b \,x^{3}+11 a}+\frac{5 a^{4} b \,x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 \left(b \,x^{3}+a \right)}+\frac{10 a^{3} b^{2} x^{17} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{17 \left(b \,x^{3}+a \right)}+\frac{a^{2} b^{3} x^{20} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{5 a \,b^{4} x^{23} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{23 \left(b \,x^{3}+a \right)}+\frac{b^{5} x^{26} \sqrt{}}{26 b \,x}$

[In] int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*a^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5/14*a^4*b*x^14 + 1/11*a^5*x^11

Sympy [F]

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{10}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{26}\,b^5x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{23}\,ab^4x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^14*
sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{10}\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.54 $\int x^9(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}$$

```
[Out] 1/10*a^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/13*a^4*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/8*a^3*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/19*a^2*b^3
*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/22*a*b^4*x^22*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/25*b^5*x^25*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{1369, 276\}$

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^9+5a^4b^6x^{12}+10a^3b^7x^{15}+10a^2b^8x^{18}+5ab^9x^{21}+b^{10}x^{24}\right)dx}{b^4\left(ab+b^2x^3\right)}\\&=\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}\\&\quad+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{10}\sqrt{(a+bx^3)^2}(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+}{543400\,(a+bx^3)}$$

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^
6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a +
b*x^3))
```

Maple [A] (verified)

Time = 8.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{10}\left(21736b^5x^{15}+123500a\,b^4x^{12}+286000a^2b^3x^9+339625a^3b^2x^6+209000a^4b\,x^3+54340a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{543400(b\,x^3+a)^5}$
default	$\frac{x^{10}\left(21736b^5x^{15}+123500a\,b^4x^{12}+286000a^2b^3x^9+339625a^3b^2x^6+209000a^4b\,x^3+54340a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{543400(b\,x^3+a)^5}$
risch	$\frac{a^5x^{10}\sqrt{(b\,x^3+a)^2}}{10b\,x^3+10a}+\frac{5a^4b\,x^{13}\sqrt{(b\,x^3+a)^2}}{13(b\,x^3+a)}+\frac{5a^3b^2x^{16}\sqrt{(b\,x^3+a)^2}}{8(b\,x^3+a)}+\frac{10a^2b^3x^{19}\sqrt{(b\,x^3+a)^2}}{19(b\,x^3+a)}+\frac{5a\,b^4x^{22}\sqrt{(b\,x^3+a)^2}}{22(b\,x^3+a)}+\frac{b^5x^2}{2}$

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a
^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Sympy [F]

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{25}\,b^5x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*
sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^9\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.55 $\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{24b^3}$$

```
[Out] 1/18*a^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3-2/21*a*(b*x^3+a)^6*((b*x^3+a)^
2)^(1/2)/b^3+1/24*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^5}{18b^3}$$

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2(ab+b^2x)^5\,dx,x,x^3\right)}{3b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^5}{b^2}-\frac{2a(ab+b^2x)^6}{b^3}+\frac{(ab+b^2x)^7}{b^4}\right)dx,x,x^3\right)}{3b^4(ab+b^2x^3)}\\ &= \frac{a^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}\\ &\quad+\frac{(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{24b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{x^9(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\right.\right.}{504\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 120*a*b^
4*x^12 + 21*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]
)))/(504*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(21 b^{2} x^{6}-6 a b\,x^{3}+a^{2}\right)}{504 b^{3}}$
gosper	$\frac{x^{9}\left(21 b^{5} x^{15}+120 a\,b^{4} x^{12}+280 a^{2} b^{3} x^{9}+336 a^{3} b^{2} x^{6}+210 a^{4} b\,x^{3}+56 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{9}\left(21 b^{5} x^{15}+120 a\,b^{4} x^{12}+280 a^{2} b^{3} x^{9}+336 a^{3} b^{2} x^{6}+210 a^{4} b\,x^{3}+56 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{5} x^{24}}{24 b\,x^{3}+24 a}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{4} a\,x^{21}}{21\left(b\,x^{3}+a\right)}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2} b^{3} x^{18}}{9\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5} x^{9}}{9 b\,x^{3}+9 a}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,a^{4} x^{12}}{12\left(b\,x^{3}+a\right)}+$ 2[illegible]

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/504*csgn(b*x^3+a)*(b*x^3+a)^6*(21*b^2*x^6-6*a*b*x^3+a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \,dx=\int x^8 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} \,dx$$

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \,dx=\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^3}{24\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3}{18\,b^3}-\frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3 -
 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \,dx=\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sg
n(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^8\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.56 $\int x^7(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=&\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}\\&+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}\\&+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}\\&+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}\end{aligned}$$

```
[Out] 1/8*a^5*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^4*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+5/7*a^3*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/17*a^2*b^3*x
^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*a*b^4*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/23*b^5*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{1369, 276\}$

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)} + \frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)} + \frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)} + \frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)} + \frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^7\left(ab+b^2x^3\right)^5 \, dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^7+5a^4b^6x^{10}+10a^3b^7x^{13}+10a^2b^8x^{16}+5ab^9x^{19}+b^{10}x^{22}\right) dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}\\
&\quad+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^8\sqrt{(a+bx^3)^2}\left(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+1\right.}{240856\,(a+bx^3)}$$

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6
 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*
x^3))
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{8}\left(10472 b^{5} x^{15}+60214 a\,b^{4} x^{12}+141680 a^{2} b^{3} x^{9}+172040 a^{3} b^{2} x^{6}+109480 a^{4} b\,x^{3}+30107 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856 \left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{8}\left(10472 b^{5} x^{15}+60214 a\,b^{4} x^{12}+141680 a^{2} b^{3} x^{9}+172040 a^{3} b^{2} x^{6}+109480 a^{4} b\,x^{3}+30107 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856 \left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{8} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 b\,x^{3}+8 a}+\frac{5 a^{4} b\,x^{11} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{11 \left(b\,x^{3}+a\right)}+\frac{5 a^{3} b^{2} x^{14} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{7 \left(b\,x^{3}+a\right)}+\frac{10 a^{2} b^{3} x^{17} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{17 \left(b\,x^{3}+a\right)}+\frac{a\,b^{4} x^{20} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{4 b\,x^{3}+4 a}+\frac{b^{5} x^{23}}{23 b}$

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3
*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Sympy [F]

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \int x^7 \left((a + bx^3)^2 \right)^{\frac{5}{2}} \, dx$$

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{23} b^5 x^{23} + \frac{1}{4} ab^4 x^{20} + \frac{10}{17} a^2 b^3 x^{17} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{11} a^4 b x^{11} + \frac{1}{8} a^5 x^8$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{23} b^5 x^{23} \operatorname{sgn}(bx^3 + a) + \frac{1}{4} ab^4 x^{20} \operatorname{sgn}(bx^3 + a) + \frac{10}{17} a^2 b^3 x^{17} \operatorname{sgn}(bx^3 + a) + \frac{5}{7} a^3 b^2 x^{14} \operatorname{sgn}(bx^3 + a) + \frac{5}{11} a^4 b x^{11} \operatorname{sgn}(bx^3 + a) + \frac{1}{8} a^5 x^8 \operatorname{sgn}(bx^3 + a)$$

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^
3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*s
gn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \int x^7 \, \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} \, dx$$

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.57 $\int x^6(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}$$

```
[Out] 1/7*a^5*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^4*b*x^10*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+10/13*a^3*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^2*b^3*x^
16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/19*a*b^4*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/22*b^5*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^6\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^6+5a^4b^6x^9+10a^3b^7x^{12}+10a^2b^8x^{15}+5ab^9x^{18}+b^{10}x^{21}\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}\\
&\quad+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^6\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx=\frac{x^7\sqrt{\left(a+bx^3\right)^2}\left(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+6916\ldots\right.}{152152\left(a+bx^3\right)}$$

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
+ 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3
))
```

Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{7} \left(6916 b^{5} x^{15}+40040 a \,b^{4} x^{12}+95095 a^{2} b^{3} x^{9}+117040 a^{3} b^{2} x^{6}+76076 a^{4} b \,x^{3}+21736 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{152152 \left(b \,x^{3}+a \right)^{5}}$
default	$\frac{x^{7} \left(6916 b^{5} x^{15}+40040 a \,b^{4} x^{12}+95095 a^{2} b^{3} x^{9}+117040 a^{3} b^{2} x^{6}+76076 a^{4} b \,x^{3}+21736 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{152152 \left(b \,x^{3}+a \right)^{5}}$
risch	$\frac{a^{5} x^{7} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{7 b \,x^{3}+7 a}+\frac{a^{4} b \,x^{10} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{10 a^{3} b^{2} x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 \left(b \,x^{3}+a \right)}+\frac{5 a^{2} b^{3} x^{16} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 \left(b \,x^{3}+a \right)}+\frac{5 a \,b^{4} x^{19} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{19 \left(b \,x^{3}+a \right)}+\frac{b^{5} x^{22} \sqrt{(}}{22 b \,x^{3}}$

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b
^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{22} \, b^5 x^{22} + \frac{5}{19} \, a b^4 x^{19} + \frac{5}{8} \, a^2 b^3 x^{16} + \frac{10}{13} \, a^3 b^2 x^{13} + \frac{1}{2} \, a^4 b x^{10} + \frac{1}{7} \, a^5 x^7$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Sympy [F]

$$\int x^6(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^6(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^6(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^5x^7\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*s
gn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^6\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.58 $\int x^5(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

```
[Out] -1/18*a*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^2+1/21*(b*x^3+a)^6*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/18*(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x(ab+b^2x)^5\,dx,x,x^3\right)}{3b^4(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^5}{b}+\frac{(ab+b^2x)^6}{b^2}\right)dx,x,x^3\right)}{3b^4(ab+b^2x^3)} \\
&= -\frac{a(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

$$\int x^5(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{x^6(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15})\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+\cdots}\right.\right.}{126\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^6*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*b^3*x^9 + 35*a*b^4*x
^12 + 6*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/
(126*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(-6b\,x^{3}+a\right)}{126b^{2}}$
gosper	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x^{6}}{6b\,x^{3}+6a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,a^{4}x^{9}}{9\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{12}}{6\left(b\,x^{3}+a\right)}+\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{15}}{3\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{4}a\,x^{18}}{18\left(b\,x^{3}+a\right)}$ -

[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/126*csgn(b*x^3+a)*(b*x^3+a)^6*(-6*b*x^3+a)/b^2

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9*a^4*b*x^9 + 1/6*a^5*x^6

Sympy [F]

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = -\frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} ax^3}{18\,b} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} a^2}{18\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{21\,b^2}$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{126} \left(6\,b^5x^{21} + 35\,ab^4x^{18} + 84\,a^2b^3x^{15} + 105\,a^3b^2x^{12} + 70\,a^4bx^9 + 21\,a^5x^6\right) \mathrm{sgn}\left(bx^3 + a\right)$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 70
*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \int x^5 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} \,dx$$

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.59 $\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\,(a+bx^3)}$$

```
[Out] 1/5*a^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^4*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+10/11*a^3*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^2*b^3*x^1
4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a*b^4*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)+1/20*b^5*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{1369, 276\}$

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)} + \frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)} + \frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)} + \frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)} + \frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(ab+b^2x^3\right)^5 \, dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x^4+5a^4b^6x^7+10a^3b^7x^{10}+10a^2b^8x^{13}+5ab^9x^{16}+b^{10}x^{19}\right) dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)} + \frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)} + \frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)} + \frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)} + \frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^5\sqrt{(a+bx^3)^2}(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618}{52360\,(a+bx^3)}$$

[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 + 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^5\left(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5}$
default	$\frac{x^5\left(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^5\sqrt{\left(b\,x^3+a\right)^2}}{5b\,x^3+5a}+\frac{5a^4b\,x^8\sqrt{\left(b\,x^3+a\right)^2}}{8\left(b\,x^3+a\right)}+\frac{10a^3b^2x^{11}\sqrt{\left(b\,x^3+a\right)^2}}{11\left(b\,x^3+a\right)}+\frac{5a^2b^3x^{14}\sqrt{\left(b\,x^3+a\right)^2}}{7\left(b\,x^3+a\right)}+\frac{5a\,b^4x^{17}\sqrt{\left(b\,x^3+a\right)^2}}{17\left(b\,x^3+a\right)}+\frac{b^5x^{20}}{20b}$

[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5/8*a^4*b*x^8 + 1/5*a^5*x^5

Sympy [F]

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^4 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^5x^5\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sg
n(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int x^4 \, \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} \, dx$$

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.60 $\int x^3(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

```
[Out] 1/4*a^5*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^4*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+a^3*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/13*a^2*b^3*x^13*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a*b^4*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1
/19*b^5*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^3+5a^4b^6x^6+10a^3b^7x^9+10a^2b^8x^{12}+5ab^9x^{15}+b^{10}x^{18}\right)dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx = \frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x\right.}{27664\left(a+bx^3\right)}$$

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664(b\,x^3+a)^5}$
default	$\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664(b\,x^3+a)^5}$
risch	$\frac{a^5x^4\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}+\frac{5a^4b\,x^7\sqrt{(b\,x^3+a)^2}}{7(b\,x^3+a)}+\frac{a^3b^2x^{10}\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{10a^2b^3x^{13}\sqrt{(b\,x^3+a)^2}}{13(b\,x^3+a)}+\frac{5a\,b^4x^{16}\sqrt{(b\,x^3+a)^2}}{16(b\,x^3+a)}+\frac{b^5x^{19}\sqrt{(b}}{19b\,x^3}$

[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2*x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a^4*b*x^7 + 1/4*a^5*x^4

Sympy [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{19} b^5 x^{19} + \frac{5}{16} ab^4 x^{16} + \frac{10}{13} a^2 b^3 x^{13} + a^3 b^2 x^{10} + \frac{5}{7} a^4 b x^7 + \frac{1}{4} a^5 x^4$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \frac{1}{19} b^5 x^{19} \text{sgn}(bx^3 + a) + \frac{5}{16} ab^4 x^{16} \text{sgn}(bx^3 + a) + \frac{10}{13} a^2 b^3 x^{13} \text{sgn}(bx^3 + a) + a^3 b^2 x^{10} \text{sgn}(bx^3 + a) + \frac{5}{7} a^4 b x^7 \text{sgn}(bx^3 + a) + \frac{1}{4} a^5 x^4 \text{sgn}(bx^3 + a)$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(b*
x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx = \int x^3 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2} \,dx$$

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.61 $\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

```
[Out] 1/18*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 623}

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2}\,dx, x, x^3\right)$$
$$= \frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{18b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{x^3\sqrt{(a+bx^3)^2}\left(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15}\right)}{18\,(a+bx^3)}$$

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{(b\,x^3+a)^6 \operatorname{csgn}(b\,x^3+a)}{18b}$	23
default	$\frac{(b\,x^3+a)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18b}$	24
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(b\,x^3+a)^5}{18b}$	26
gosper	$\frac{x^3\left(b^5x^{15}+6a\,b^4x^{12}+15a^2b^3x^9+20a^3b^2x^6+15a^4b\,x^3+6a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18(b\,x^3+a)^5}$	79

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/18*(b*x^3+a)^6*csgn(b*x^3+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{18} \, b^5 x^{18} + \frac{1}{3} \, a b^4 x^{15} + \frac{5}{6} \, a^2 b^3 x^{12} + \frac{10}{9} \, a^3 b^2 x^9 + \frac{5}{6} \, a^4 b x^6 + \frac{1}{3} \, a^5 x^3$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*
a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [F]

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int x^{2} \left(\left(a + b x^{3}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{18} \, {\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}} x^{3} + \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}} a}{18 \, b}$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(5/2)*a/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{1}{18}\left(3\left(bx^6+2\,ax^3\right)a^4+3\left(bx^6+2\,ax^3\right)^2a^2b+\left(bx^6+2\,ax^3\right)^3b^2\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a*
x^3)^3*b^2)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{(b^2\,x^3+a\,b)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{18\,b^2}$$

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

3.62 $\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 252

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}$$

```
[Out] 1/2*a^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^4*b*x^5*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+5/4*a^3*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/11*a^2*b^3*x^11*((b
*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a*b^4*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/
17*b^5*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1369, 276}

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5x+5a^4b^6x^4+10a^3b^7x^7+10a^2b^8x^{10}+5ab^9x^{13}+b^{10}x^{16}\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15}\right)}{5236\left(a+bx^3\right)}$$

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 47
60*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{2}\left(308 b^{5} x^{15}+1870 a\, b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b\, x^{3}+2618 a^{5}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b\, x^{3}+a\right)^{5}}$
default	$\frac{x^{2}\left(308 b^{5} x^{15}+1870 a\, b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b\, x^{3}+2618 a^{5}\right) \left(\left(b\, x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b\, x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{2} \sqrt{\left(b\, x^{3}+a\right)^{2}}}{2 b\, x^{3}+2 a}+\frac{a^{4} b\, x^{5} \sqrt{\left(b\, x^{3}+a\right)^{2}}}{b\, x^{3}+a}+\frac{5 a^{3} b^{2} x^{8} \sqrt{\left(b\, x^{3}+a\right)^{2}}}{4 \left(b\, x^{3}+a\right)}+\frac{10 a^{2} b^{3} x^{11} \sqrt{\left(b\, x^{3}+a\right)^{2}}}{11 \left(b\, x^{3}+a\right)}+\frac{5 a\, b^{4} x^{14} \sqrt{\left(b\, x^{3}+a\right)^{2}}}{14 \left(b\, x^{3}+a\right)}+\frac{b^{5} x^{17} \sqrt{(b}}{17 b\, x^{3}+}$

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6+
5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}\mathrm{sgn}(bx^3+a)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}(bx^3+a)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}(bx^3+a)+\frac{5}{4}\,a^3b^2x^8\mathrm{sgn}(bx^3+a)+a^4bx^5\mathrm{sgn}(bx^3+a)+\frac{1}{2}\,a^5x^2\mathrm{sgn}(bx^3+a)$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b*x
^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.63 $\int (a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\begin{aligned}\int (a^2+2abx^3+b^2x^6)^{5/2}\,dx &= \frac{a^5x(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5} \\ &+ \frac{5a^4bx^4(a^2+2abx^3+b^2x^6)^{5/2}}{4\,(a+bx^3)^5}+\frac{10a^3b^2x^7(a^2+2abx^3+b^2x^6)^{5/2}}{7\,(a+bx^3)^5} \\ &+ \frac{a^2b^3x^{10}(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5} \\ &+ \frac{5ab^4x^{13}(a^2+2abx^3+b^2x^6)^{5/2}}{13\,(a+bx^3)^5}+\frac{b^5x^{16}(a^2+2abx^3+b^2x^6)^{5/2}}{16\,(a+bx^3)^5}\end{aligned}$$

```
[Out] a^5*x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/4*a^4*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(5/2)/(b*x^3+a)^5+10/7*a^3*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)
/(b*x^3+a)^5+a^2*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/13*a*
b^4*x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+1/16*b^5*x^16*(b^2*x^6+2
*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{1357, 200\}$

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5} + \frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5} + \frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5} + \frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5} + \frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5} + \frac{10a^3b^2x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{7\left(a+bx^3\right)^5}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(2ab+2b^2x^3\right)^5\,dx}{\left(2ab+2b^2x^3\right)^5}$$

$$= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(32a^5b^5+160a^4b^6x^3+320a^3b^7x^6+320a^2b^8x^9+160ab^9x^{12}+32b^{10}x^{15}\right)\,d}{\left(2ab+2b^2x^3\right)^5}$$

$$= \frac{a^5x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5} + \frac{5a^4bx^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{4\left(a+bx^3\right)^5} + \frac{10a^3b^2x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{7\left(a+bx^3\right)^5} + \frac{a^2b^3x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{\left(a+bx^3\right)^5} + \frac{5ab^4x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{13\left(a+bx^3\right)^5} + \frac{b^5x^{16}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{16\left(a+bx^3\right)^5}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.33

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x\sqrt{(a+bx^3)^2}(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15})}{1456\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456
*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5}$
default	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^5x}{b\,x^3+a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b\,a^4x^4}{4\left(b\,x^3+a\right)}+\frac{10\sqrt{\left(b\,x^3+a\right)^2}\,a^3b^2x^7}{7\left(b\,x^3+a\right)}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^2b^3x^{10}}{b\,x^3+a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b^4a\,x^{13}}{13\left(b\,x^3+a\right)}+\frac{\sqrt{\left(b\,x^3+a\right.}}{16b\,x^3}$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820
*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int \left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}\left(bx^3+a\right)+a^5x\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^1
0*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^
3 + a) + a^5*x*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.64 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15(a+bx^3)} + \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] 5/3*a^4*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/3*a^3*b^2*x^6*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/9*a^2*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/12*a*b^4*x^
12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/15*b^5*x^15*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+a^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used

$= \{1369, 272, 45\}$

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\,(a+bx^3)} + \frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\,(a+bx^3)} + \frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)} + \frac{a^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)} + \frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x}\,dx}{b^4\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x}\,dx,x,x^3\right)}{3b^4\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^{10}x^4\right)\,dx,x,x^3\right)}{3b^4\,(ab+b^2x^3)}$$

$$= \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\,(a+bx^3)}+\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15\,(a+bx^3)}+\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx^3)^2}(bx^3(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12})+18}{180\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75
*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^3+a\right)\left(\frac{b^5x^{15}}{5}+\frac{5a\,b^4x^{12}}{4}+\frac{10a^2b^3x^9}{3}+5a^3b^2x^6+5a^4b\,x^3+a^5\ln\left(b\,x^3\right)+\frac{137a^5}{60}\right)}{3}$	75
default	$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{15}+75a\,b^4x^{12}+200a^2b^3x^9+300a^3b^2x^6+300a^4b\,x^3+180a^5\ln\left(x\right)\right)}{180\left(b\,x^3+a\right)^5}$	79
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,b\left(\frac{1}{15}b^4x^{15}+\frac{5}{12}a\,b^3x^{12}+\frac{10}{9}a^2b^2x^9+\frac{5}{3}a^3b\,x^6+\frac{5}{3}a^4x^3\right)}{b\,x^3+a}+\frac{a^5\ln\left(x\right)\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}$	96

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/3*csgn(b*x^3+a)*(1/5*b^5*x^15+5/4*a*b^4*x^12+10/3*a^2*b^3*x^9+5*a^3*b^2*x
^6+5*a^4*b*x^3+a^5*ln(b*x^3)+137/60*a^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \frac{1}{15}\,b^5x^{15} + \frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{3}\,a^3b^2x^6 + \frac{5}{3}\,a^4bx^3 + a^5\log\left(x\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \frac{1}{6}\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3bx^3 + \frac{1}{3}\,(-1)^{2\,b^2x^3 + 2\,ab}\,a^5\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3 + 2\,a^2}\,a^5\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{1}{12}\,\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}abx^3 + \frac{1}{2}\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^4 + \frac{7}{36}\,\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}a^2 + \frac{1}{15}\,\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b*x
/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b*
x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{1}{15}\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{12}\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right) + a^5\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b
*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

3.65 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}$$

```
[Out] -a^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a^4*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+2*a^3*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/4*a^2*b^3*x^8*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/11*a*b^4*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^5
*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^3\right)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^2}+5a^4b^6x+10a^3b^7x^4+10a^2b^8x^7+5ab^9x^{10}+b^{10}x^{13}\right)dx}{b^4\left(ab+b^2x^3\right)} \\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} \\
&\quad +\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$
\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+2\right.}{308x\left(a+bx^3\right)}
$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-22 b^{5} x^{15}-140 a \,b^{4} x^{12}-385 a^{2} b^{3} x^{9}-616 a^{3} b^{2} x^{6}-770 a^{4} b \,x^{3}+308 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-22 b^{5} x^{15}-140 a \,b^{4} x^{12}-385 a^{2} b^{3} x^{9}-616 a^{3} b^{2} x^{6}-770 a^{4} b \,x^{3}+308 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \left(\frac{1}{14} b^{4} x^{14}+\frac{5}{11} a \,b^{3} x^{11}+\frac{5}{4} a^{2} b^{2} x^{8}+2 a^{3} b \,x^{5}+\frac{5}{2} a^{4} x^{2}\right)}{b \,x^{3}+a}-\frac{a^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{x \left(b \,x^{3}+a \right)}$	98

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")

[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 770*a^4*b*x^3 - 308*a^5)/x

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2} dx = \frac{22 b^5 x^{15} + 140 ab^4 x^{12} + 385 a^2 b^3 x^9 + 616 a^3 b^2 x^6 + 770 a^4 b x^3 - 308 a^5}{308 x}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2} dx = \frac{1}{14} b^5 x^{14} \mathrm{sgn}(bx^3 + a) + \frac{5}{11} ab^4 x^{11} \mathrm{sgn}(bx^3 + a) + \frac{5}{4} a^2 b^3 x^8 \mathrm{sgn}(bx^3 + a) + 2 a^3 b^2 x^5 \mathrm{sgn}(bx^3 + a) + \frac{5}{2} a^4 b x^2 \mathrm{sgn}(bx^3 + a) - \frac{a^5 \mathrm{sgn}(bx^3 + a)}{x}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3
*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^
3 + a) - a^5*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2} dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^2} \,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

3.66 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

```
[Out] -1/2*a^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a^4*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+5/2*a^3*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/7*a^2*b^3*x^7*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^4*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13
*b^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] -1/2*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (5*a^4*b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
2*(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^3}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5a^4b^6+\frac{a^5b^5}{x^3}+10a^3b^7x^3+10a^2b^8x^6+5ab^9x^9+b^{10}x^{12}\right)dx}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^3)^2}(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b}{182x^2(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-14 b^{5} x^{15}-91 a \,b^{4} x^{12}-260 a^{2} b^{3} x^{9}-455 a^{3} b^{2} x^{6}-910 a^{4} b \,x^{3}+91 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 x^{2} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-14 b^{5} x^{15}-91 a \,b^{4} x^{12}-260 a^{2} b^{3} x^{9}-455 a^{3} b^{2} x^{6}-910 a^{4} b \,x^{3}+91 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 x^{2} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \left(\frac{1}{13} b^{4} x^{13}+\frac{1}{2} a \,b^{3} x^{10}+\frac{10}{7} a^{2} b^{2} x^{7}+\frac{5}{2} a^{3} b \,x^{4}+5 a^{4} x \right)}{b \,x^{3}+a}-\frac{a^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	96

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*
b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} \, dx = \frac{14 \, b^5 x^{15} + 91 \, ab^4 x^{12} + 260 \, a^2 b^3 x^9 + 455 \, a^3 b^2 x^6 + 910 \, a^4 b x^3 - 91 \, a^5}{182 \, x^2}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} \, dx = \frac{1}{13} \, b^5 x^{13} \mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{2} \, ab^4 x^{10} \mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{7} \, a^2 b^3 x^7 \mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{2} \, a^3 b^2 x^4 \mathrm{sgn}\left(bx^3 + a\right) + 5 \, a^4 bx \mathrm{sgn}\left(bx^3 + a\right) - \frac{a^5 \mathrm{sgn}\left(bx^3 + a\right)}{2 \, x^2}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3
+ a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} \, dx = \int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{5/2}}{x^3} \,d x$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

3.67 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/3*a^5*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+10/3*a^3*b^2*x^3*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/3*a^2*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/9*a*b^4*x^9*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/12*b^5*x^12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a^4*b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx = \frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{5a^4b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -1/3*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (10*a^3*b^2*
x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right)\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}\\
&\quad+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{\sqrt{(a+bx^3)^2}(-12a^5+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}+180a^4bx^3}{36x^3(a+bx^3)}$$

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\left(-\frac{b^5x^{15}}{4}-\frac{5a\,b^4x^{12}}{3}-5a^2b^3x^9-10a^3b^2x^6-5\ln(b\,x^3)a^4b\,x^3-\frac{77a^4b\,x^3}{12}+a^5\right)}{3x^3}$	81
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(3b^5x^{15}+20a\,b^4x^{12}+60a^2b^3x^9+120a^3b^2x^6+180b\,a^4\ln(x)x^3-12a^5)}{36x^3(b\,x^3+a)^5}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2\left(\frac{1}{12}b^3x^{12}+\frac{5}{9}a\,b^2x^9+\frac{5}{3}a^2b\,x^6+\frac{10}{3}a^3x^3\right)}{b\,x^3+a}-\frac{a^5\sqrt{(b\,x^3+a)^2}}{3x^3(b\,x^3+a)}+\frac{5a^4b\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	117

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*csgn(b*x^3+a)*(-1/4*b^5*x^15-5/3*a*b^4*x^12-5*a^2*b^3*x^9-10*a^3*b^2*x^6-5*ln(b*x^3)*a^4*b*x^3-77/12*a^4*b*x^3+a^5)/x^3

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log(x)-12\,a^5}{36\,x^3}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")

[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a^4*b*x^3*log(x) - 12*a^5)/x^3

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx &= \frac{5}{6}\sqrt{b^2x^6+2\,abx^3+a^2}a^2b^2x^3 \\ &+ \frac{5}{3}(-1)^{2\,b^2x^3+2\,ab}a^4b\log\left(2\,b^2x^3+2\,ab\right) - \frac{5}{3}(-1)^{2\,abx^3+2\,a^2}a^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{5}{12}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2x^3 + \frac{5}{2}\sqrt{b^2x^6+2\,abx^3+a^2}a^3b \\ &+ \frac{35}{36}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ab - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{3\,x^3}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2
)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx &= \frac{1}{12}b^5x^{12}\mathrm{sgn}\left(bx^3+a\right) \\ &+ \frac{5}{9}ab^4x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}a^2b^3x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{3}a^3b^2x^3\mathrm{sgn}\left(bx^3+a\right) \\ &+ 5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{5\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{3\,x^3}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*s
gn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^4} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^4} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}$$

```
[Out] -1/4*a^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a^4*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+5*a^3*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+2*a^2*b^3*x^5*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/8*a*b^4*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^5*x
^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = \frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -1/4*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a +
 b*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^3\right)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^2}+10a^3b^7x+10a^2b^8x^4+5ab^9x^7+b^{10}x^{10}\right)dx}{b^4\left(ab+b^2x^3\right)} \\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} \\
&\quad +\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^1\right.}{88x^4\left(a+bx^3\right)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-8 b^{5} x^{15}-55 a \,b^{4} x^{12}-176 a^{2} b^{3} x^{9}-440 a^{3} b^{2} x^{6}+440 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{88 x^{4} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-8 b^{5} x^{15}-55 a \,b^{4} x^{12}-176 a^{2} b^{3} x^{9}-440 a^{3} b^{2} x^{6}+440 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{88 x^{4} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{11} b^{3} x^{11}+\frac{5}{8} a \,b^{2} x^{8}+2 a^{2} b \,x^{5}+5 a^{3} x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-5 a^{4} b \,x^{3}-\frac{1}{4} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{4}}$	100

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*
x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right) + 2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{4\,x^4}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5
*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^5}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

3.69 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}$$

```
[Out] -1/5*a^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a^4*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+10*a^3*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/2*a^2*b^3*x^4*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a*b^4*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/10*
b^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -1/5*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3
))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^6}\,dx}{b^4\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^3b^7+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^3}+10a^2b^8x^3+5ab^9x^6+b^{10}x^9\right)dx}{b^4\,(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{\sqrt{(a+bx^3)^2}(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^1}{70x^5\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7 b^{5} x^{15}-50 a \,b^{4} x^{12}-175 a^{2} b^{3} x^{9}-700 a^{3} b^{2} x^{6}+175 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 x^{5} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-7 b^{5} x^{15}-50 a \,b^{4} x^{12}-175 a^{2} b^{3} x^{9}-700 a^{3} b^{2} x^{6}+175 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{70 x^{5} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{10} b^{3} x^{10}+\frac{5}{7} a \,b^{2} x^{7}+\frac{5}{2} a^{2} b \,x^{4}+10 a^{3} x \right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{5}{2} a^{4} b \,x^{3}-\frac{1}{5} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{5}}$	98

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*
x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx=\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\, dx = \frac{7\,b^5x^{15} + 50\,ab^4x^{12} + 175\,a^2b^3x^9 + 700\,a^3b^2x^6 - 175\,a^4bx^3 - 14\,a^5}{70\,x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\, dx = \frac{1}{10}\, b^5x^{10}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{7}\, ab^4x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{2}\, a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right) + 10\, a^3b^2x\mathrm{sgn}\left(bx^3+a\right) - \frac{25\, a^4bx^3\mathrm{sgn}(bx^3 + a) + 2\, a^5\mathrm{sgn}(bx^3 + a)}{10\, x^5}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x
^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*
x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\, dx = \int \frac{(a^2 + 2\, a\, b\, x^3 + b^2\, x^6)^{5/2}}{x^6}\, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

3.70 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/6*a^5*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a^4*b*((b*x^3+a)^2)^(1/2)/x^
3/(b*x^3+a)+10/3*a^2*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/6*a*b^4*x^6*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b^5*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
3*b^2*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] -1/6*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(9*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^7}\,dx}{b^4\,(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^3}\,dx,x,x^3\right)}{3b^4\,(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right)\,dx,x,x^3\right)}{3b^4\,(ab+b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\,(a+bx^3)} \\
&\quad+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\,(a+bx^3)} \\
&\quad+\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\,\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{1}{3}\left(\frac{(12a^5+120a^4bx^3+57a^3b^2x^6-240a^2b^3x^9-60ab^4x^{12}-8b^5x^{15})\left(\sqrt{a^2}bx\right.}{24x^6\left(a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}\right.$$

$$-10a^3b^2\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)$$

$$-10\left(a^2\right)^{3/2}b^2\log\left(x^3\right)+5\left(a^2\right)^{3/2}b^2\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+5\left(a^2\right)^{3/2}b^2\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3\right.\right.$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] (((12*a^5 + 120*a^4*b*x^3 + 57*a^3*b^2*x^6 - 240*a^2*b^3*x^9 - 60*a*b^4*x^1
2 - 8*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(2
4*x^6*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])) - 10*a^3*b^2*ArcTanh
[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 10*(a^2)^(3/2)*b^2*Log[x^3] +
 5*(a^2)^(3/2)*b^2*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 5*(a^2)^(
3/2)*b^2*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/3
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left((b x^3+a)^2\right)^{\frac{5}{2}} (2b^5x^{15}+15a b^4x^{12}+60a^2b^3x^9+180a^3b^2\ln(x)x^6-30a^4b x^3-3a^5)}{18(b x^3+a)^5 x^6}$	82
pseudoelliptic	$-\frac{\mathrm{csgn}(b x^3+a)\left(-\frac{2b^5x^{15}}{3}-5a b^4x^{12}-20a^2b^3x^9-20\ln(b x^3)a^3b^2x^6-\frac{47a^3b^2x^6}{3}+10a^4b x^3+a^5\right)}{6x^6}$	83
risch	$\frac{\sqrt{(b x^3+a)^2}\, b^3 \left(\frac{1}{9}b^2x^9+\frac{5}{6}ab x^6+\frac{10}{3}a^2x^3\right)}{b x^3+a}+\frac{\sqrt{(b x^3+a)^2}\,\left(-\frac{5}{3}a^4 b x^3-\frac{1}{6}a^5\right)}{(b x^3+a)x^6}+\frac{10a^3b^2\ln(x)\sqrt{(b x^3+a)^2}}{b x^3+a}$	119

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*b
^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log(x)-30\,a^4bx^3-3\,a^5}{18\,x^6}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
- 30*a^4*b*x^3 - 3*a^5)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx &= \frac{5}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}ab^3x^3 \\ &+ \frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3b^2\log\left(2\,b^2x^3+2\,ab\right) \\ &- \frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3x^3}{6\,a} \\ &+ 5\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2b^2+\frac{35}{18}\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2 \\ &+ \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{6\,a^2}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{2\,ax^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{6\,a^2x^6}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2*l
```

```
og(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(
3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/a
^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(7/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{30\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+10\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^
3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*
x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^7} \,d x$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

3.71 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

```
[Out] -1/7*a^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a^4*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-10*a^3*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^2*b^3*x^2*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)+a*b^4*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^5*x^8
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8} dx = \frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a +
 b*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^8}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^2}+10a^2b^8x+5ab^9x^4+b^{10}x^7\right)dx}{b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}\\
&\quad+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{\sqrt{(a+bx^3)^2}(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15}}{56x^7\left(a+bx^3\right)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7 b^{5} x^{15}-56 a \,b^{4} x^{12}-280 a^{2} b^{3} x^{9}+560 a^{3} b^{2} x^{6}+70 a^{4} b \,x^{3}+8 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{56 \left(b \,x^{3}+a \right)^{5} x^{7}}$	80
default	$-\frac{\left(-7 b^{5} x^{15}-56 a \,b^{4} x^{12}-280 a^{2} b^{3} x^{9}+560 a^{3} b^{2} x^{6}+70 a^{4} b \,x^{3}+8 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{56 \left(b \,x^{3}+a \right)^{5} x^{7}}$	80
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{3} \left(\frac{1}{8} b^{2} x^{8}+a b \,x^{5}+5 a^{2} x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-10 a^{3} b^{2} x^{6}-\frac{5}{4} a^{4} b \,x^{3}-\frac{1}{7} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{7}}$	99

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x
^3+8*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^3+a\right)+ab^4x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+35\,a^4bx^3\mathrm{sgn}(bx^3+a)+4\,a^5\mathrm{sgn}(bx^3+a)}{28\,x^7}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(b
*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 +
 a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^8}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

3.72 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9} dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} + \frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

```
[Out] -1/8*a^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a^4*b*((b*x^3+a)^2)^(1/2)/x^5/(b
*x^3+a)-5*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+10*a^2*b^3*x*((b*x^3+a)
^2)^(1/2)/(b*x^3+a)+5/4*a*b^4*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^5*x^7
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9} dx = \frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] -1/8*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a
+ b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^9}\,dx}{b^4\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^2b^8+\frac{a^5b^5}{x^9}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^3}+5ab^9x^3+b^{10}x^6\right)dx}{b^4\,(ab+b^2x^3)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5\,(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2\,(a+bx^3)}\\
&\quad+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}+\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{\sqrt{(a+bx^3)^2}(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15}}{56x^8\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-8b^{5}x^{15}-70a\,b^{4}x^{12}-560a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+56a^{4}b\,x^{3}+7a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56\left(b\,x^{3}+a\right)^{5}x^{8}}$	80
default	$-\frac{\left(-8b^{5}x^{15}-70a\,b^{4}x^{12}-560a^{2}b^{3}x^{9}+280a^{3}b^{2}x^{6}+56a^{4}b\,x^{3}+7a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56\left(b\,x^{3}+a\right)^{5}x^{8}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}\left(\frac{1}{7}b^{2}x^{7}+\frac{5}{4}ab\,x^{4}+10a^{2}x\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-5a^{3}b^{2}x^{6}-a^{4}b\,x^{3}-\frac{1}{8}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{8}}$	98

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x
^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx=\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^9}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^3+a\right) + 10\,a^2b^3x\mathrm{sgn}\left(bx^3+a\right)-\frac{40\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+8\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{8\,x^8}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sg
n(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 +
 a) + a^5*sgn(b*x^3 + a))/x^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^9}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)
```

3.73 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}} \, dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/9*a^5*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/6*a^4*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-10/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+5/3*a*b^4*x^3*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/6*b^5*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
2*b^3*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}} \, dx = \frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -1/9*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(6*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}\\
&\quad -\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}\\
&\quad +\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.11

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{1}{3}\left(\frac{(4a^5+30a^4bx^3+120a^3b^2x^6+53a^2b^3x^9-60ab^4x^{12}-6b^5x^{15})\left(\sqrt{a^2}bx^3+\right.}{12x^9\left(a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}\right.$$

$$-10a^2b^3\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)-10a\sqrt{a^2}b^3\log\left(x^3\right)$$

$$\left.+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+5a\sqrt{a^2}b^3\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)\right)$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] (((4*a^5 + 30*a^4*b*x^3 + 120*a^3*b^2*x^6 + 53*a^2*b^3*x^9 - 60*a*b^4*x^12
- 6*b^5*x^15)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(12*
x^9*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])) - 10*a^2*b^3*ArcTanh[(
b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 10*a*Sqrt[a^2]*b^3*Log[x^3] + 5
*a*Sqrt[a^2]*b^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 5*a*Sqrt[a^
2]*b^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/3
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(3b^5x^{15}+30a\,b^4x^{12}+180a^2b^3\ln(x)x^9-60a^3b^2x^6-15a^4b\,x^3-2a^5)}{18(b\,x^3+a)^5x^9}$	82
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\left(-\frac{3b^5x^{15}}{2}-15a\,b^4x^{12}-30\ln(b\,x^3)a^2b^3x^9-\frac{27a^2b^3x^9}{2}+30a^3b^2x^6+\frac{15a^4b\,x^3}{2}+a^5\right)}{9x^9}$	83
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^3(b\,x^3+5a)^2}{6b\,x^3+6a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{10}{3}a^3b^2x^6-\frac{5}{6}a^4b\,x^3-\frac{1}{9}a^5)}{(b\,x^3+a)x^9}+\frac{10a^2b^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	118

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-60
*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log(x)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
- 15*a^4*b*x^3 - 2*a^5)/x^9
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx &= \frac{5}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3 \\ &+ \frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^2b^3\log\left(2\,b^2x^3+2\,ab\right) \\ &- \frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4x^3}{6\,a^2} \\ &+ 5\,\sqrt{b^2x^6+2\,abx^3+a^2}ab^3+\frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{18\,a^3} \\ &- \frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{18\,a^2x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{9\,a^2x^9}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)
*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*log
```

```
(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/
2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/
a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/
2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^3+a\right) + 10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{110\,a^2b^3x^9\mathrm{sgn}(bx^3+a) + 60\,a^3b^2x^6\mathrm{sgn}(bx^3+a) + 15\,a^4bx^3\mathrm{sgn}(bx^3+a) + 2\,a^5\mathrm{sgn}(bx^3+a)}{18\,x^9}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(
abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^
9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{10}} \,d x$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

3.74 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

```
[Out] -1/10*a^5*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a^4*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-5/2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-10*a^2*b^3*((b*
x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a*b^4*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5
*b^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -1/10*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x
^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{11}}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^5}+\frac{10a^2b^8}{x^2}+5ab^9x+b^{10}x^4\right)dx}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15})}{70x^{10}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^
2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))
```

Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-14 b^{5} x^{15}-175 a\,b^{4} x^{12}+700 a^{2} b^{3} x^{9}+175 a^{3} b^{2} x^{6}+50 a^{4} b\,x^{3}+7 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{70 \left(b\,x^{3}+a\right)^{5} x^{10}}$	80
default	$-\frac{\left(-14 b^{5} x^{15}-175 a\,b^{4} x^{12}+700 a^{2} b^{3} x^{9}+175 a^{3} b^{2} x^{6}+50 a^{4} b\,x^{3}+7 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{70 \left(b\,x^{3}+a\right)^{5} x^{10}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{4} \left(\frac{1}{5} b\,x^{5}+\frac{5}{2} a\,x^{2}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-10 a^{2} b^{3} x^{9}-\frac{5}{2} a^{3} b^{2} x^{6}-\frac{5}{7} a^{4} b\,x^{3}-\frac{1}{10} a^{5}\right)}{\left(b\,x^{3}+a\right) x^{10}}$	100

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b
*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx=\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{1}{5}\,b^5x^5\mathrm{sgn}(bx^3+a)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}(bx^3+a) - \frac{700\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+175\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+50\,a^4bx^3\mathrm{sgn}(bx^3+a)+7\,a^5\mathrm{sgn}(bx^3+a)}{70\,x^{10}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
[Out] 1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*b
^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn(b
*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{11}}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)
```

3.75 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}} dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} + \frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

```
[Out] -1/11*a^5*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a^4*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5*a^2*b^3*((b*x^3
+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a*b^4*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^5*
x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}} dx = \frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/11*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^11*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5ab^9+\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^3}+b^{10}x^3\right)dx}{b^4\left(ab+b^2x^3\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)} -\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2\left(a+bx^3\right)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15}\right)}{88x^{11}\left(a+bx^3\right)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^
2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))
```

Maple [A] (verified)

Time = 12.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-22b^{5}x^{15}-440a\,b^{4}x^{12}+440a^{2}b^{3}x^{9}+176a^{3}b^{2}x^{6}+55a^{4}b\,x^{3}+8a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88\left(b\,x^{3}+a\right)^{5}x^{11}}$	80
default	$-\frac{\left(-22b^{5}x^{15}-440a\,b^{4}x^{12}+440a^{2}b^{3}x^{9}+176a^{3}b^{2}x^{6}+55a^{4}b\,x^{3}+8a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{88\left(b\,x^{3}+a\right)^{5}x^{11}}$	80
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{4}\left(\frac{1}{4}b\,x^{4}+5ax\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-5a^{2}b^{3}x^{9}-2a^{3}b^{2}x^{6}-\frac{5}{8}a^{4}b\,x^{3}-\frac{1}{11}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{11}}$	98

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b
*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx=\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{12}}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^3+a\right) - \frac{440\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+176\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+55\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+8\,a^5\mathrm{sgn}\left(bx^3+a\right)}{88\,x^{11}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
[Out] 1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*x
^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x^3
 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{12}} \,d x$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}} dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} + \frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/12*a^5*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-5/9*a^4*b*((b*x^3+a)^2)^(1/2)/
x^9/(b*x^3+a)-5/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-10/3*a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+1/3*b^5*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a*b^4*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}} dx = \frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -1/12*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^12*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{13}}\,dx}{b^4(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^5}\,dx,x,x^3\right)}{3b^4(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,x,x^3\right)}{3b^4(ab+b^2x^3)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)}\\
&\quad -\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(252) = 504$.

Time = 0.94 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.76

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{48a^6\sqrt{a^2}+368a^5\sqrt{a^2}bx^3+1280a^4\sqrt{a^2}b^2x^6+2880a^3\sqrt{a^2}b^3x^9+2677(a^2)^{3/2}}{}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] (48*a^6*Sqrt[a^2] + 368*a^5*Sqrt[a^2]*b*x^3 + 1280*a^4*Sqrt[a^2]*b^2*x^6 +
2880*a^3*Sqrt[a^2]*b^3*x^9 + 2677*(a^2)^(3/2)*b^4*x^12 + 565*a*Sqrt[a^2]*b^
5*x^15 - 192*Sqrt[a^2]*b^6*x^18 - 48*a^6*Sqrt[(a + b*x^3)^2] - 320*a^5*b*x^
3*Sqrt[(a + b*x^3)^2] - 960*a^4*b^2*x^6*Sqrt[(a + b*x^3)^2] - 1920*a^3*b^3*
x^9*Sqrt[(a + b*x^3)^2] - 757*a^2*b^4*x^12*Sqrt[(a + b*x^3)^2] + 192*a*b^5*
x^15*Sqrt[(a + b*x^3)^2] - 960*a*b^4*x^12*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(
a + b*x^3)^2])*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - 960*b^4
*x^12*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^2*Sqrt[(a + b*x^3)^2])*Log[x^3]
+ 480*(a^2)^(3/2)*b^4*x^12*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 4
80*a*Sqrt[a^2]*b^5*x^15*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - 480*
a^2*b^4*x^12*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2
]] + 480*(a^2)^(3/2)*b^4*x^12*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]]
+ 480*a*Sqrt[a^2]*b^5*x^15*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - 4
80*a^2*b^4*x^12*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3
)^2]])/(576*x^12*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\left(-4b^5x^{15}-20\ln\left(b\,x^3\right)a\,b^4x^{12}-4a\,b^4x^{12}+40a^2b^3x^9+20a^3b^2x^6+\frac{20a^4b\,x^3}{3}+a^5\right)\operatorname{csgn}\left(b\,x^3+a\right)}{12x^{12}}$	81
default	$\frac{\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}\left(12b^5x^{15}+180b^4a\ln(x)x^{12}-120a^2b^3x^9-60a^3b^2x^6-20a^4b\,x^3-3a^5\right)}{36\left(b\,x^3+a\right)^5x^{12}}$	82
risch	$\frac{b^5x^3\sqrt{\left(b\,x^3+a\right)^2}}{3b\,x^3+3a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-\frac{10}{3}a^2b^3x^9-\frac{5}{3}a^3b^2x^6-\frac{5}{9}a^4b\,x^3-\frac{1}{12}a^5\right)}{\left(b\,x^3+a\right)x^{12}}+\frac{5a\,b^4\ln(x)\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}$	119

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-4*b^5*x^15-20*ln(b*x^3)*a*b^4*x^12-4*a*b^4*x^12+40*a^2*b^3*x^9+20*a
^3*b^2*x^6+20/3*a^4*b*x^3+a^5)*csgn(b*x^3+a)/x^12
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{12\,b^5x^{15}+180\,ab^4x^{12}\log(x)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^
6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx &= \frac{5\sqrt{b^2x^6+2\,abx^3+a^2}b^5x^3}{6\,a} \\ &+ \frac{5}{3}(-1)^{2\,b^2x^3+2\,ab}\,ab^4\log\left(2\,b^2x^3+2\,ab\right) - \frac{5}{3}(-1)^{2\,abx^3+2\,a^2}\,ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5x^3}{12\,a^3} + \frac{5}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^4 \\ &+ \frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{36\,a^2} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^4}{9\,a^4} - \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{9\,a^3x^3} \\ &- \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{9\,a^4x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{36\,a^3x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{12\,a^2x^{12}}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*
b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/a^
4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)
*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{125\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+120\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+60\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+20\,a^4bx^3\mathrm{sgn}(bx^3+a)+3\,a^5\mathrm{s}}{36\,x^{12}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125
*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^
6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{13}}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

3.77 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

```
[Out] -1/13*a^5*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a^4*b*((b*x^3+a)^2)^(1/2)/
x^10/(b*x^3+a)-10/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/2*a^2*b^3*(
(b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a*b^4*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/
2*b^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -1/13*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^13*(a + b*x^3)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^5}{x^{14}}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^5}+\frac{5ab^9}{x^2}+b^{10}x\right)dx}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int\frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15})}{182x^{13}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
[Out] -1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*
a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))
```

Maple [A] (verified)

Time = 17.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-91 b^{5} x^{15}+910 a \,b^{4} x^{12}+455 a^{2} b^{3} x^{9}+260 a^{3} b^{2} x^{6}+91 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 \left(b \,x^{3}+a \right)^{5} x^{13}}$	80
default	$-\frac{\left(-91 b^{5} x^{15}+910 a \,b^{4} x^{12}+455 a^{2} b^{3} x^{9}+260 a^{3} b^{2} x^{6}+91 a^{4} b \,x^{3}+14 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{182 \left(b \,x^{3}+a \right)^{5} x^{13}}$	80
risch	$\frac{b^{5} x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-5 a \,b^{4} x^{12}-\frac{5}{2} a^{2} b^{3} x^{9}-\frac{10}{7} a^{3} b^{2} x^{6}-\frac{1}{2} a^{4} b \,x^{3}-\frac{1}{13} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{13}}$	100

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*
b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx=\frac{91\,b^{5}x^{15}-910\,ab^{4}x^{12}-455\,a^{2}b^{3}x^{9}-260\,a^{3}b^{2}x^{6}-91\,a^{4}bx^{3}-14\,a^{5}}{182\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx=\int \frac{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^3+a\right)$$
$$-\frac{910\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+455\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+260\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+91\,a^4bx^3\mathrm{sgn}(bx^3+a)+14\,a}{182\,x^{13}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2
*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn
(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{14}}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

3.78 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}} dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)} - \frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/14*a^5*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a^4*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-5/4*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-2*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+
b^5*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}} dx = \frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} - \frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -1/14*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^14*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{15}}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}+\frac{a^5b^5}{x^{15}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^3}\right)dx}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15})}{308x^{14}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
[Out] -1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616
*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))
```

Maple [A] (verified)

Time = 20.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-308 b^{5} x^{15}+770 a \,b^{4} x^{12}+616 a^{2} b^{3} x^{9}+385 a^{3} b^{2} x^{6}+140 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x^{14} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-308 b^{5} x^{15}+770 a \,b^{4} x^{12}+616 a^{2} b^{3} x^{9}+385 a^{3} b^{2} x^{6}+140 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{308 x^{14} \left(b \,x^{3}+a \right)^{5}}$	80
risch	$\frac{b^{5} x \sqrt{\left(b \,x^{3}+a \right)^{2}}}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{5}{2} a \,b^{4} x^{12}-2 a^{2} b^{3} x^{9}-\frac{5}{4} a^{3} b^{2} x^{6}-\frac{5}{11} a^{4} b \,x^{3}-\frac{1}{14} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{14}}$	97

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x,method=_RETURNVERBOSE)
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^
4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx=\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}}\,dx=\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = \frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = b^5x\mathrm{sgn}(bx^3+a) - \frac{770\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+616\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+385\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+140\,a^4bx^3\mathrm{sgn}(bx^3+a)+22}{308\,x^{14}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x
^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^
3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{15}}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)
```

3.79 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

```
[Out] -1/15*a^5*((b*x^3+a)^2)^(1/2)/x^15/(b*x^3+a)-5/12*a^4*b*((b*x^3+a)^2)^(1/2)
/x^12/(b*x^3+a)-10/9*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/3*a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a*b^4*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+
a)+b^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -1/15*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^15*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a
 + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}\\
&\quad-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{1}{360}\left(-\frac{\sqrt{(a+bx^3)^2}(12a^4+63a^3bx^3+137a^2b^2x^6+163ab^3x^9+137b^4x^{12})}{x^{15}} + \frac{\sqrt{a^2}(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12})}{x^{15}} - 120b^5\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right) - \frac{120\sqrt{a^2}b^5\log(x^3)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a}\right)$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
[Out] (-((Sqrt[(a + b*x^3)^2]*(12*a^4 + 63*a^3*b*x^3 + 137*a^2*b^2*x^6 + 163*a*b^
3*x^9 + 137*b^4*x^12))/x^15) + (Sqrt[a^2]*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*
b^2*x^6 + 300*a*b^3*x^9 + 300*b^4*x^12))/x^15 - 120*b^5*ArcTanh[(b*x^3)/(Sq
rt[a^2] - Sqrt[(a + b*x^3)^2])] - (120*Sqrt[a^2]*b^5*Log[x^3])/a + (60*Sqrt
[a^2]*b^5*Log[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])])/a + (60*Sqrt[a^
2]*b^5*Log[a*(Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/a)/360
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\left(-5\ln(b\,x^3)b^5x^{15}+a\left(25b^4x^{12}+25a\,b^3x^9+\frac{50}{3}a^2b^2x^6+\frac{25}{4}a^3b\,x^3+a^4\right)\right)}{15x^{15}}$	72
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}\left(180b^5\ln(x)x^{15}-300a\,b^4x^{12}-300a^2b^3x^9-200a^3b^2x^6-75a^4b\,x^3-12a^5\right)}{180(b\,x^3+a)^5x^{15}}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{1}{15}a^5-\frac{5}{12}a^4b\,x^3-\frac{10}{9}a^3b^2x^6-\frac{5}{3}a^2b^3x^9-\frac{5}{3}a\,b^4x^{12}\right)}{(b\,x^3+a)x^{15}}+\frac{b^5\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	98

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*csgn(b*x^3+a)*(-5*ln(b*x^3)*b^5*x^15+a*(25*b^4*x^12+25*a*b^3*x^9+50/3
*a^2*b^2*x^6+25/4*a^3*b*x^3+a^4))/x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{180\,b^5x^{15}\log\left(x\right)-300\,ab^4x^{12}-300\,a^2b^3x^9-200\,a^3b^2x^6-75\,a^4bx^3-12\,a^5}{180\,x^{15}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(175) = 350.

Time = 0.23 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx &= \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^6x^3}{6\,a^2} \\ &+ \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^6x^3}{12\,a^4}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^5}{2\,a}+\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^5}{36\,a^3} \\ &- \frac{2\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^5}{45\,a^5}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^4}{9\,a^4x^3}+\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{45\,a^5x^6} \\ &- \frac{11\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^2}{180\,a^4x^9}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b}{20\,a^3x^{12}}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{15\,a^2x^{15}}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^
6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 + 2*a
*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/a^
5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7
/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^12) - 1
/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.49

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = b^5\log(|x|)\,\mathrm{sgn}(bx^3+a) - \frac{137\,b^5x^{15}\mathrm{sgn}(bx^3+a)+300\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+300\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+200\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+75\,a}{180\,x^{15}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a
*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6
*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^{16}}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

3.80 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

```
[Out] -1/16*a^5*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-5/13*a^4*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-a^3*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-10/7*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a*b^4*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-
b^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/16*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^16*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{17}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{17}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^5}+\frac{b^{10}}{x^2}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15})}{1456x^{16}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2
080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))
```

Maple [A] (verified)

Time = 25.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{16}a^5-\frac{5}{13}a^4b\,x^3-a^3b^2x^6-\frac{10}{7}a^2b^3x^9-\frac{5}{4}a\,b^4x^{12}-b^5x^{15}\right)}{(b\,x^3+a)x^{16}}$	79
gosper	$-\frac{(1456b^5x^{15}+1820a\,b^4x^{12}+2080a^2b^3x^9+1456a^3b^2x^6+560a^4b\,x^3+91a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456x^{16}(b\,x^3+a)^5}$	80
default	$-\frac{(1456b^5x^{15}+1820a\,b^4x^{12}+2080a^2b^3x^9+1456a^3b^2x^6+560a^4b\,x^3+91a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456x^{16}(b\,x^3+a)^5}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/16*a^5-5/13*a^4*b*x^3-a^3*b^2*x^6-10/7*a^
2*b^3*x^9-5/4*a*b^4*x^12-b^5*x^15)/x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*
x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")

[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{1456\,b^5x^{15}\mathrm{sgn}(bx^3+a)+1820\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+2080\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+1456\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+}{1456\,x^{16}}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")

[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 2080*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(13*x^13*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1
/2))/(7*x^7*(a + b*x^3)) - (a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x^1
0*(a + b*x^3))
```

3.81 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[Out] -1/17*a^5*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/14*a^4*b*((b*x^3+a)^2)^(1/2)
/x^14/(b*x^3+a)-10/11*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/4*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a*b^4*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
-1/2*b^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -1/17*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^17*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a +
 b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^5}{x^{18}}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{15}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^3}\right)dx}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int\frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15})}{5236x^{17}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
[Out] -1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 +
 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))
```

Maple [A] (verified)

Time = 27.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{17}a^{5}-\frac{5}{14}a^{4}b\,x^{3}-\frac{10}{11}a^{3}b^{2}x^{6}-\frac{5}{4}a^{2}b^{3}x^{9}-a\,b^{4}x^{12}-\frac{1}{2}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{17}}$	79
gosper	$-\frac{\left(2618b^{5}x^{15}+5236a\,b^{4}x^{12}+6545a^{2}b^{3}x^{9}+4760a^{3}b^{2}x^{6}+1870a^{4}b\,x^{3}+308a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236x^{17}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(2618b^{5}x^{15}+5236a\,b^{4}x^{12}+6545a^{2}b^{3}x^{9}+4760a^{3}b^{2}x^{6}+1870a^{4}b\,x^{3}+308a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236x^{17}\left(b\,x^{3}+a\right)^{5}}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x,method=_RETURNVERBOSE)
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/17*a^5-5/14*a^4*b*x^3-10/11*a^3*b^2*x^6-5
/4*a^2*b^3*x^9-a*b^4*x^12-1/2*b^5*x^15)/x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{18}}\,dx = -\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{2618\,b^5x^{15}\mathrm{sgn}(bx^3+a)+5236\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+6545\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+4760\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+}{5236\,x^{17}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 65
45*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*
b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)} -\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^5\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)} -\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^8\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(x^5*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(14*x^14*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(4*x^8*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(1
1*x^11*(a + b*x^3))
```

3.82 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}} dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}} dx = -\frac{(a+bx^3)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[Out] -1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^18
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 270}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}} dx = -\frac{(a+bx^3)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -1/18*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^18)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{19}}\,dx}{b^4\,(ab+b^2x^3)}$$
$$= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15})}{18x^{18}\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3
*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\,(2b\,x^3+a)\,(b^2x^6+ab\,x^3+a^2)\,(3b^2x^6+3ab\,x^3+a^2)}{18x^{18}}$	58
gosper	$-\frac{(6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18x^{18}(b\,x^3+a)^5}$	78
default	$-\frac{(6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18x^{18}(b\,x^3+a)^5}$	78
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{18}a^5-\frac{1}{3}a^4b\,x^3-\frac{5}{6}a^3b^2x^6-\frac{10}{9}a^2b^3x^9-\frac{5}{6}a\,b^4x^{12}-\frac{1}{3}b^5x^{15}\right)}{(b\,x^3+a)x^{18}}$	79

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

[Out] -1/18*csgn(b*x^3+a)*(2*b*x^3+a)*(b^2*x^6+a*b*x^3+a^2)*(3*b^2*x^6+3*a*b*x^3+a^2)/x^18

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{6\,b^5x^{15} + 15\,ab^4x^{12} + 20\,a^2b^3x^9 + 15\,a^3b^2x^6 + 6\,a^4bx^3 + a^5}{18\,x^{18}}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")

[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4*b*x^3 + a^5)/x^18

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 5.12

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^6}{18\,a^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^5}{18\,a^5x^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^4}{18\,a^6x^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^3}{18\,a^5x^9} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{12}} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b}{18\,a^3x^{15}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{18\,a^2x^{18}}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")

[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^18)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(28) = 56.

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.59

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{6\,b^5x^{15}\mathrm{sgn}(bx^3 + a) + 15\,ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 20\,a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 15\,a^3b^2x^6\mathrm{sgn}(bx^3 + a) + 6\,a^4bx^3\mathrm{sgn}}{18\,x^{18}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^
3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.63

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{a^5\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3 + a\right)} - \frac{b^5\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{3\,x^3\,\left(b\,x^3 + a\right)} - \frac{5\,a\,b^4\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{6\,x^6\,\left(b\,x^3 + a\right)} - \frac{a^4\,b\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3 + a\right)} - \frac{10\,a^2\,b^3\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{9\,x^9\,\left(b\,x^3 + a\right)} - \frac{5\,a^3\,b^2\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3 + a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b*x
^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
6*x^12*(a + b*x^3))
```

3.83 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

```
[Out] -1/19*a^5*((b*x^3+a)^2)^(1/2)/x^19/(b*x^3+a)-5/16*a^4*b*((b*x^3+a)^2)^(1/2)
/x^16/(b*x^3+a)-10/13*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a*b^4*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a
)-1/4*b^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -1/19*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^19*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^5}{x^{20}}\,dx}{b^4\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{17}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{11}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^5}\right)dx}{b^4\,(ab+b^2x^3)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\,(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\,(a+bx^3)}\\
&\quad-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\,(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int\frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx=$$
$$-\frac{\sqrt{(a+bx^3)^2}(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15})}{27664x^{19}\,(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^
6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x^3
))
```

Maple [A] (verified)

Time = 33.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{19}a^5-\frac{5}{16}a^4b\,x^3-\frac{10}{13}a^3b^2x^6-a^2b^3x^9-\frac{5}{7}a\,b^4x^{12}-\frac{1}{4}b^5x^{15}\right)}{(b\,x^3+a)x^{19}}$	79
gosper	$-\frac{\left(6916b^5x^{15}+19760a\,b^4x^{12}+27664a^2b^3x^9+21280a^3b^2x^6+8645a^4b\,x^3+1456a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664x^{19}(b\,x^3+a)^5}$	80
default	$-\frac{\left(6916b^5x^{15}+19760a\,b^4x^{12}+27664a^2b^3x^9+21280a^3b^2x^6+8645a^4b\,x^3+1456a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664x^{19}(b\,x^3+a)^5}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/19*a^5-5/16*a^4*b*x^3-10/13*a^3*b^2*x^6-a
^2*b^3*x^9-5/7*a*b^4*x^12-1/4*b^5*x^15)/x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx = -\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}}\, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{20}}\, dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}}\, dx = -\frac{6916\, b^5x^{15} + 19760\, ab^4x^{12} + 27664\, a^2b^3x^9 + 21280\, a^3b^2x^6 + 8645\, a^4bx^3 + 1456\, a^5}{27664\, x^{19}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}}\, dx = -\frac{6916\, b^5x^{15}\mathrm{sgn}(bx^3 + a) + 19760\, ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 27664\, a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 21280\, a^3b^2x^6\mathrm{sgn}(bx^3 + a}{27664\, x^{19}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*
a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(x^10*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
13*x^13*(a + b*x^3))
```

3.84 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

```
[Out] -1/20*a^5*((b*x^3+a)^2)^(1/2)/x^20/(b*x^3+a)-5/17*a^4*b*((b*x^3+a)^2)^(1/2)
/x^17/(b*x^3+a)-5/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-10/11*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a*b^4*((b*x^3+a)^2)^(1/2)/x^8/(b*x
^3+a)-1/5*b^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/20*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^20*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5
*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{21}}\,dx}{b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{21}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{15}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^6}\right)dx}{b^4(ab+b^2x^3)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)}\\ &\quad-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15})}{52360x^{20}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x
^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b*x
^3))
```

Maple [A] (verified)

Time = 38.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^{3}+a)^{2}}\,\left(-\frac{1}{20}a^{5}-\frac{5}{17}a^{4}b\,x^{3}-\frac{5}{7}a^{3}b^{2}x^{6}-\frac{10}{11}a^{2}b^{3}x^{9}-\frac{5}{8}a\,b^{4}x^{12}-\frac{1}{5}b^{5}x^{15}\right)}{(b\,x^{3}+a)\,x^{20}}$	79
gosper	$-\frac{\left(10472b^{5}x^{15}+32725a\,b^{4}x^{12}+47600a^{2}b^{3}x^{9}+37400a^{3}b^{2}x^{6}+15400a^{4}b\,x^{3}+2618a^{5}\right)\left((b\,x^{3}+a)^{2}\right)^{\frac{5}{2}}}{52360x^{20}(b\,x^{3}+a)^{5}}$	80
default	$-\frac{\left(10472b^{5}x^{15}+32725a\,b^{4}x^{12}+47600a^{2}b^{3}x^{9}+37400a^{3}b^{2}x^{6}+15400a^{4}b\,x^{3}+2618a^{5}\right)\left((b\,x^{3}+a)^{2}\right)^{\frac{5}{2}}}{52360x^{20}(b\,x^{3}+a)^{5}}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/20*a^5-5/17*a^4*b*x^3-5/7*a^3*b^2*x^6-10/
11*a^2*b^3*x^9-5/8*a*b^4*x^12-1/5*b^5*x^15)/x^20
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{21}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**21, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15} + 32725\,ab^4x^{12} + 47600\,a^2b^3x^9 + 37400\,a^3b^2x^6 + 15400\,a^4bx^3 + 2618\,a^5}{52360\,x^{20}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{10472\,b^5x^{15}\mathrm{sgn}(bx^3 + a) + 32725\,ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 47600\,a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 37400\,a^3b^2x^6\mathrm{sgn}(bx^3}{52360\,x^{20}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) +
 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 1540
0*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{20\,x^{20}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(20*x^20*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(7*x^14*(a + b*x^3))
```

3.85 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

```
[Out] -1/21*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^21+1/126*b*(b*x^3+a)^5*((b*x^3+a)
^2)^(1/2)/a^2/x^18
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1369, 272, 47, 37}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}} - \frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/21*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^21) + (b*(a + b*
x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^7}\,dx,x,x^3\right)}{21ab^3\left(ab+b^2x^3\right)}$$

$$= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15})}{126x^{21}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^
2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)(21b^5x^{15}+70a\,b^4x^{12}+105a^2b^3x^9+84a^3b^2x^6+35a^4b\,x^3+6a^5)}{126x^{21}}$	68
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{1}{21}a^5-\frac{5}{18}a^4b\,x^3-\frac{2}{3}a^3b^2x^6-\frac{5}{6}a^2b^3x^9-\frac{5}{9}a\,b^4x^{12}-\frac{1}{6}b^5x^{15})}{(b\,x^3+a)x^{21}}$	79
gosper	$-\frac{(21b^5x^{15}+70a\,b^4x^{12}+105a^2b^3x^9+84a^3b^2x^6+35a^4b\,x^3+6a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{126x^{21}(b\,x^3+a)^5}$	80
default	$-\frac{(21b^5x^{15}+70a\,b^4x^{12}+105a^2b^3x^9+84a^3b^2x^6+35a^4b\,x^3+6a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{126x^{21}(b\,x^3+a)^5}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/126*csgn(b*x^3+a)*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*
x^6+35*a^4*b*x^3+6*a^5)/x^21
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}} dx = -\frac{21 b^5x^{15} + 70 ab^4x^{12} + 105 a^2b^3x^9 + 84 a^3b^2x^6 + 35 a^4bx^3 + 6 a^5}{126 x^{21}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}} dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{22}} dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(58) = 116.

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.87

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}} dx = & -\frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} b^7}{18\,a^7} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} b^6}{18\,a^6x^3} \\ & + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}} b^5}{18\,a^7x^6} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}} b^4}{18\,a^6x^9} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}} b^3}{18\,a^5x^{12}} \\ & - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}} b^2}{18\,a^4x^{15}} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}} b}{18\,a^3x^{18}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{21\,a^2x^{21}}\end{aligned}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a
^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^
18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{21\,b^5x^{15}\mathrm{sgn}(bx^3+a)+70\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+105\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+84\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+35\,a^4b\cdots}{126\,x^{21}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2
*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(
b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.75

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)} - \frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)} - \frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(3*x^15*(a + b*x^3))
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\,(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\,(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\,(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\,(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\,(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}$$

```
[Out] -1/22*a^5*((b*x^3+a)^2)^(1/2)/x^22/(b*x^3+a)-5/19*a^4*b*((b*x^3+a)^2)^(1/2)
/x^19/(b*x^3+a)-5/8*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-10/13*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^10/(b*
x^3+a)-1/7*b^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\,(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\,(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\,(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\,(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\,(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -1/22*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^22*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\frac{(ab+b^2x^3)^5}{x^{23}}\,dx}{b^4(ab+b^2x^3)} \\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(\frac{a^5b^5}{x^{23}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{17}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{11}}+\frac{b^{10}}{x^8}\right)dx}{b^4(ab+b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}(a+bx^3)} \\
&\quad-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int\frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15})}{152152x^{22}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*
x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a + b
*x^3))
```

Maple [A] (verified)

Time = 46.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{22}a^{5}-\frac{5}{19}a^{4}b\,x^{3}-\frac{5}{8}a^{3}b^{2}x^{6}-\frac{10}{13}a^{2}b^{3}x^{9}-\frac{1}{2}a\,b^{4}x^{12}-\frac{1}{7}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{22}}$	79
gosper	$-\frac{\left(21736b^{5}x^{15}+76076a\,b^{4}x^{12}+117040a^{2}b^{3}x^{9}+95095a^{3}b^{2}x^{6}+40040a^{4}b\,x^{3}+6916a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152x^{22}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(21736b^{5}x^{15}+76076a\,b^{4}x^{12}+117040a^{2}b^{3}x^{9}+95095a^{3}b^{2}x^{6}+40040a^{4}b\,x^{3}+6916a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{152152x^{22}\left(b\,x^{3}+a\right)^{5}}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/22*a^5-5/19*a^4*b*x^3-5/8*a^3*b^2*x^6-10/
13*a^2*b^3*x^9-1/2*a*b^4*x^12-1/7*b^5*x^15)/x^22
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{21736\,b^5x^{15}\mathrm{sgn}(bx^3+a)+76076\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+117040\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+95095\,a^3b^2x^6\mathrm{sgn}(bx^3}{152152\,x^{22}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
+ 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40
040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{22\,x^{22}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)} -\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^{10}\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)} -\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^{16}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(22*x^22*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^10*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(13*x^13*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2
))/(8*x^16*(a + b*x^3))
```

3.87 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

```
[Out] -1/23*a^5*((b*x^3+a)^2)^(1/2)/x^23/(b*x^3+a)-1/4*a^4*b*((b*x^3+a)^2)^(1/2)/
x^20/(b*x^3+a)-10/17*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/7*a^2*b^3
*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a*b^4*((b*x^3+a)^2)^(1/2)/x^11/(b*
x^3+a)-1/8*b^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -1/23*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^23*(a + b*x^3)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{24}}\,dx}{b^4\,(ab+b^2x^3)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{21}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{15}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^9}\right)dx}{b^4\,(ab+b^2x^3)}\\
&= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\,(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\,(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\,(a+bx^3)}\\
&\quad-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\,(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\,(a+bx^3)}-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx =$$

$$-\frac{\sqrt{(a+bx^3)^2}(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15})}{240856x^{23}\,(a+bx^3)}$$

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]

[Out] -1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(a + b*x^3))

Maple [A] (verified)

Time = 50.94 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{23}a^{5}-\frac{1}{4}a^{4}b\,x^{3}-\frac{10}{17}a^{3}b^{2}x^{6}-\frac{5}{7}a^{2}b^{3}x^{9}-\frac{5}{11}a\,b^{4}x^{12}-\frac{1}{8}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{23}}$	79
gosper	$-\frac{\left(30107b^{5}x^{15}+109480a\,b^{4}x^{12}+172040a^{2}b^{3}x^{9}+141680a^{3}b^{2}x^{6}+60214a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856x^{23}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(30107b^{5}x^{15}+109480a\,b^{4}x^{12}+172040a^{2}b^{3}x^{9}+141680a^{3}b^{2}x^{6}+60214a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856x^{23}\left(b\,x^{3}+a\right)^{5}}$	80

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(-1/23*a^5-1/4*a^4*b*x^3-10/17*a^3*b^2*x^6-5/7*a^2*b^3*x^9-5/11*a*b^4*x^12-1/8*b^5*x^15)/x^23

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")

[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{30107\,b^5x^{15}\mathrm{sgn}(bx^3+a)+109480\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+172040\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+141680\,a^3b^2x^6\mathrm{sgn}(bx}{240856\,x^{23}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a)
 + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) +
60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{23\,x^{23}\,\left(b\,x^3+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)} - \frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{20}\,\left(b\,x^3+a\right)} - \frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)} - \frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(23*x^23*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(4*x^20*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(7*x^14*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(17*x^17*(a + b*x^3))
```

3.88 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}} \, dx = -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{504a^3x^{18}}$$

```
[Out] -1/24*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^24+1/84*b*(b*x^3+a)^5*((b*x^3+a)^
2)^(1/2)/a^2/x^21-1/504*b^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a^3/x^18
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1369, 272, 47, 37}

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}} \, dx = -\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{24ax^{24}} + \frac{b\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{84a^2x^{21}} - \frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^5}{504a^3x^{18}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -1/24*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^24) + (b*(a + b*
x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^5}{x^{25}}\,dx}{b^4\left(ab+b^2x^3\right)}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}\\
&= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}}-\frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx,x,x^3\right)}{12ab^3\left(ab+b^2x^3\right)}\\
&= -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}}+\frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}}\\
&\quad+\frac{\sqrt{a^2+2abx^3+b^2x^6}\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx,x,x^3\right)}{84a^2b^2\left(ab+b^2x^3\right)}
\end{aligned}$$

$$= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{504a^3x^{18}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15})}{504x^{24}(a+bx^3)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
[Out] -1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336
*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 4.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b x^3+a)\left(\frac{8}{3}b^5x^{15}+10a b^4x^{12}+16a^2b^3x^9+\frac{40}{3}a^3b^2x^6+\frac{40}{7}a^4b x^3+a^5\right)}{24x^{24}}$	66
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{1}{24}a^5-\frac{5}{9}a^3b^2x^6-\frac{2}{3}a^2b^3x^9-\frac{5}{12}a b^4x^{12}-\frac{1}{9}b^5x^{15}-\frac{5}{21}a^4b x^3\right)}{(b x^3+a)x^{24}}$	79
gosper	$-\frac{(56b^5x^{15}+210a b^4x^{12}+336a^2b^3x^9+280a^3b^2x^6+120a^4b x^3+21a^5)\left((b x^3+a)^2\right)^{\frac{5}{2}}}{504x^{24}(b x^3+a)^5}$	80
default	$-\frac{(56b^5x^{15}+210a b^4x^{12}+336a^2b^3x^9+280a^3b^2x^6+120a^4b x^3+21a^5)\left((b x^3+a)^2\right)^{\frac{5}{2}}}{504x^{24}(b x^3+a)^5}$	80

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
[Out] -1/24*csgn(b*x^3+a)*(8/3*b^5*x^15+10*a*b^4*x^12+16*a^2*b^3*x^9+40/3*a^3*b^2
*x^6+40/7*a^4*b*x^3+a^5)/x^24
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(89) = 178.

Time = 0.21 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.12

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^8}{18\,a^8} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}b^7}{18\,a^7x^3} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^6}{18\,a^8x^6} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^5}{18\,a^7x^9} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^4}{18\,a^6x^{12}} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^3}{18\,a^5x^{15}} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b^2}{18\,a^4x^{18}} + \frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}b}{56\,a^3x^{21}} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}}{24\,a^2x^{24}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(a^
```

```
8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x
^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}\mathrm{sgn}(bx^3+a)+210\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+336\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+120\,a}{504\,x^{24}}$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^
2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*s
gn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{24\,x^{24}\,\left(b\,x^3+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)} - \frac{2\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^{18}\,\left(b\,x^3+a\right)}$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(9*x^18*(a + b*x^3))
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/2*x^2*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/6*a^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)/((b*x^3+
a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

$= \{1369, 327, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{x^4}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a(ab+b^2x^3)\right)\int \frac{x}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a^{2/3}(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad - \frac{\left(a^{2/3}(ab+b^2x^3)\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}(ab+b^2x^3)\right)\int\frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a(ab+b^2x^3)\right)\int\frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{2b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}(ab+b^2x^3)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int\frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(3b^{2/3}x^2+2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6b^{5/3}\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 \left(b \,x^{3}+a \right) b}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, a \left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{3} b +a \right)}\frac{\ln \left(x -\_R \right)}{\_R}\right)}{3 \left(b \,x^{3}+a \right) b^{2}}$	77
default	$\frac{\left(b \,x^{3}+a \right) \left(3 x^{2} b \left(\frac{a}{b}\right)^{\frac{1}{3}}+2 \arctan \left(\frac{\sqrt{3}\, \left(-2 x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) \sqrt{3}\, a +2 \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) a -\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) a \right)}{6 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$	113

```
[In] int(x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b
^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{3\,x^{2}-2\,\sqrt{3}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(a\right.}{6\,b}$$

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{x^4}{\sqrt{(a + bx^3)^2}}\,dx$$

```
[In] integrate(x**4/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x**4/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \frac{x^2}{2\,b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \frac{x^2\mathrm{sgn}(bx^3 + a)}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}(bx^3 + a)}{3\,b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}(bx^3 + a)}{3\,b^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}(bx^3 + a)}{6\,b^3}$$

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(
b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^4}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(x^4/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^3)^2)^(1/2), x)
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 235

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] x*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)-1/3*a^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*
x)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/6*a^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(1/3)*(b*x^3+a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used

= {1369, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt[3]{a}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(ab+b^2x^3\right)\int \frac{x^3}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{\left(\sqrt[3]{a}\left(ab+b^2x^3\right)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(\sqrt[3]{a}(ab+b^2x^3)\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a^{2/3}(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{2b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(6\sqrt[3]{b}x+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.\right.}{6b^{4/3}\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{x\sqrt{(b\,x^{3}+a)^{2}}}{(b\,x^{3}+a)b}-\frac{\sqrt{(b\,x^{3}+a)^{2}}\,a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3(b\,x^{3}+a)b^{2}}$	74
default	$\frac{(b\,x^{3}+a)\left(6xb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{(b\,x^{3}+a)^{2}}\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	110

```
[In] int(x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b^2*a*s
um(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b}$$

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^3}{\sqrt{(a+bx^3)^2}}\,dx$$

```
[In] integrate(x**3/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x**3/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x}{b} - \frac{\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}(bx^3+a)}{3\,b} + \frac{x\mathrm{sgn}(bx^3+a)}{b} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}(bx^3+a)}{3\,b^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}(bx^3+a)}{6\,b^2}$$

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
+ a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^3}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(x^3/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3/((a + b*x^3)^2)^(1/2), x)
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1366, 622, 31}

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^3\right)\\ &= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{2\text{arctanh}\left(\frac{\frac{\sqrt{a^2}}{b}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b}}{x^3}\right)}{3b}$$

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (-2*ArcTanh[(Sqrt[a^2]/b - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/b)/x^3])/(3*b)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$\frac{\ln(b\,x^3+a)\,\text{csgn}(b\,x^3+a)}{3b}$	22
default	$\frac{(b\,x^3+a)\ln(b\,x^3+a)}{3b\sqrt{(b\,x^3+a)^2}}$	32
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)b}$	34

```
[In] int(x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(b*x^3+a)/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(bx^3+a\right)}{3\,b}$$

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(b*x^3 + a)/b
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx$$

```
[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x**2/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b}$$

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*log(x^3 + a/b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(|bx^3+a|\right)\mathrm{sgn}(bx^3+a)}{3\,b}$$

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\ln\left(b^2\,x^3+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^3+2\,a\,b)}{3\,\sqrt{b^2}}$$

```
[In] int(x^2/((a + b*x^3)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/
6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)/((b*x
^3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(1/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1369, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{x}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{(ab+b^2x^3)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3\sqrt[3]{a}b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3\sqrt[3]{a}b\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6\sqrt[3]{a}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad + \frac{(ab+b^2x^3)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{2b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad + \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3\left(b\,x^{3}+a\right)b}$	47
default	$-\frac{\left(b\,x^{3}+a\right)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	97

```
[In] int(x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \left[\frac{3\,\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\,\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\right.}{6\,ab^2}\right.$$

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
```

```
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{x}{\sqrt{(a + bx^3)^2}}\,dx$$

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(x/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}\left(bx^3+a\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}\left(bx^3+a\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}\left(bx^3+a\right)}{3\,a}$$

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3
 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x
^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*
x^3 + a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(x/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x/((a + b*x^3)^2)^(1/2), x)
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 202

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-1/6
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)/((b*x^
3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(2/3)/b^(1/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1357, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
```

```
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(2ab+2b^2x^3)\int \frac{1}{2ab+2b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(2ab+2b^2x^3)\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}b^{2/3}x}\,dx}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2ab+2b^2x^3)\int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{2}b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{3\,2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(2ab+2b^2x^3)\int \frac{-2^{2/3}\sqrt[3]{a}b+2\,2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{12a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2ab+2b^2x^3)\int \frac{1}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2ab+2b^2x^3)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

[Out] -1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.88 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{3(b\,x^3+a)b}$	47
default	$\frac{(b\,x^3+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{(b\,x^3+a)^2}\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	97

[In] int(1/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2\right.}{6\,a^2b}\right.$$

[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^

```
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{1}{\sqrt{(a + bx^3)^2}}\,dx$$

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{ab}\right.$$

$$+ a)$$

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1
/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(1/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)^2)^(1/2), x)
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (b*x^3+a)*ln(x)/a/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a/((b*x^3+a
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1369, 272, 36, 29, 31}

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log(x)(a+bx^3)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{1}{x(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-2a\log\left(x^3\right)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx^3-\sqrt{\left(a+bx^3\right)^2}\right)+a\log\left(\sqrt{a^2}+bx^3-\sqrt{\left(a+bx^3\right)^2}\right)+\sqrt{a^2}\,\text{lo}}{6a\sqrt{a^2}}$$

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] (-2*a*Log[x^3] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2
]] + a*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + Sqrt[a^2]*Log[a*(Sqrt
[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*Sqrt[a^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$-\frac{\left(\ln \left(b\,x^{3}+a\right)-\ln \left(b\,x^{3}\right)\right) \operatorname{csgn}\left(b\,x^{3}+a\right)}{3 a}$	31
default	$\frac{\left(b\,x^{3}+a\right) \left(3 \ln \left(x\right)-\ln \left(b\,x^{3}+a\right)\right)}{3 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a}$	39
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \ln \left(x\right)}{\left(b\,x^{3}+a\right) a}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \ln \left(b\,x^{3}+a\right)}{3 \left(b\,x^{3}+a\right) a}$	61

```
[In] int(1/x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(ln(b*x^3+a)-ln(b*x^3))*csgn(b*x^3+a)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.22

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{\log \left(bx^3+a\right)-3\,\log \left(x\right)}{3\,a}$$

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x\sqrt{\left(a+bx^3\right)^2}}\,dx$$

```
[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right)}{3\,a}$$

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = -\frac{1}{3}\left(\frac{\log\left(|bx^3 + a|\right)}{a} - \frac{3\,\log\left(|x|\right)}{a}\right)\operatorname{sgn}\left(bx^3 + a\right)$$

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = -\frac{\ln\left(a\,b + \frac{a^2}{x^3} + \frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

```
[In] int(1/(x*((a + b*x^3)^2)^(1/2)),x)
[Out] -log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/(
3*(a^2)^(1/2))
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 238

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (-b*x^3-a)/a/x/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)
*x)/a^(4/3)/((b*x^3+a)^2)^(1/2)-1/6*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)/((b*x^3+a)^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

$= \{1369, 331, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{1}{x^2(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\int \frac{x}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad - \frac{(ab+b^2x^3)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6a^{4/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{2a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(6\sqrt[3]{a}-2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6a^{4/3}x\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)^2]
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.74 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{(b\,x^{3}+a)^{2}}}{(b\,x^{3}+a)ax}+\frac{\sqrt{(b\,x^{3}+a)^{2}}\left(\sum_{\_R=\operatorname{RootOf}\left(a^{4}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}+3b\right)x-a^{3}\_R^{2}\right)\right)}{3b\,x^{3}+3a}$	93
default	$-\frac{(b\,x^{3}+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{(b\,x^{3}+a)^{2}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}ax}$	111

```
[In] int(1/x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R
*ln((-4*_R^3*a^4+3*b)*x-a^3*_R^2),_R=RootOf(_Z^3*a^4-b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{2\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\right.\right.}{6\,ax}$$

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^3)^2}}\,dx$$

```
[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(1/(x**2*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} + \frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b} - \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}} -\right.}{a^2b}\right.$$

$$+ a)$$

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn(b*
x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^2\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 243

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad+\frac{b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad-\frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad+\frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 1/2*(-b*x^3-a)/a/x^2/((b*x^3+a)^2)^(1/2)-1/3*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/6*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(2/3)*(b*x^3+a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)/((b*x^3
+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used

$= \{1369, 331, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/2*(a + b*x^3)/(a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*
x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{1}{x^3(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\int \frac{1}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad - \frac{\left(\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{6a^{5/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(b^{2/3}(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{2a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(3a^{2/3}-2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\right.\right.}{6a^{5/3}x^2\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^
2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a + b
*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2\left(b\,x^{3}+a\right)a\,x^{2}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}-3b^{2}\right)x-a^{2}b\_R\right)\right)}{3b\,x^{3}+3a}$	94
default	$-\frac{\left(b\,x^{3}+a\right)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x^{2}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x^{2}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^{2}+3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,x^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	118

```
[In] int(1/x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x^2+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*
sum(_R*ln((-4*_R^3*a^5-3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^5+b^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x^2\left(-\right.}{6ax^2}$$

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx^3)^2}}\,dx$$

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{1}{2\,ax^2}$$

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{a^2}\right.$$

$$+a)$$

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(
1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 +
a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

```
[In] int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(-b*x^3-a)/a/x^3/((b*x^3+a)^2)^(1/2)-b*(b*x^3+a)*ln(x)/a^2/((b*x^3+a)^2
)^(1/2)+1/3*b*(b*x^3+a)*ln(b*x^3+a)/a^2/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 46}

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/3*(a + b*x^3)/(a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*L
og[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3
])/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^3)\int \frac{1}{x^4(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{a^2-\sqrt{a^2}\sqrt{(a+bx^3)^2}+2abx^3\log\left(x^3\right)+\left(-a+\sqrt{a^2}\right)bx^3\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)-abx^3\log\left(\sqrt{\right.}}{6\left(a^2\right)^{3/2}x^3}$$

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] (a^2 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + 2*a*b*x^3*Log[x^3] + (-a + Sqrt[a^2]
)*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - a*b*x^3*Log[Sqrt[a^2
] + b*x^3 - Sqrt[(a + b*x^3)^2]] - Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] + b*x^3 -
Sqrt[(a + b*x^3)^2]])/(6*(a^2)^(3/2)*x^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-\frac{\left(\ln \left(b\,x^{3}\right) b\,x^{3}-b \ln \left(b\,x^{3}+a \right) x^{3}+a \right) \operatorname{csgn}\left(b\,x^{3}+a \right)}{3 a^{2} x^{3}}$	44
default	$-\frac{\left(b\,x^{3}+a \right) \left(3 b \ln \left(x \right) x^{3}-b \ln \left(b\,x^{3}+a \right) x^{3}+a \right)}{3 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a^{2} x^{3}}$	51
risch	$-\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}}{3 \left(b\,x^{3}+a \right) a\,x^{3}}-\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b \ln \left(x \right)}{\left(b\,x^{3}+a \right) a^{2}}+\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b \ln \left(-b\,x^{3}-a \right)}{3 \left(b\,x^{3}+a \right) a^{2}}$	95

```
[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(ln(b*x^3)*b*x^3-b*ln(b*x^3+a)*x^3+a)*csgn(b*x^3+a)/a^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{bx^{3}\log\left(bx^{3}+a\right)-3\,bx^{3}\log\left(x\right)-a}{3\,a^{2}x^{3}}$$

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^{4}\sqrt{\left(a+bx^{3}\right)^{2}}}\,dx$$

```
[In] integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
[Out] Integral(1/(x**4*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2} - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^2
 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{3}\left(\frac{b\log\left(|bx^3+a|\right)}{a^2} - \frac{3\,b\log\left(|x|\right)}{a^2} + \frac{bx^3-a}{a^2x^3}\right)\operatorname{sgn}\left(bx^3+a\right)$$

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3
))*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,{\left(a^2\right)}^{3/2}} - \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

```
[In] int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)))
)/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

3.98 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 280

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}} \, dx = \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/9*x^2/a/b/((b*x^3+a)^2)^(1/2)-1/6*x^2/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/54*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)/((b*x^3
+a)^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(4/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used

$= \{1369, 294, 296, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{x^4}{(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{x}{(ab+b^2x^3)^2}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{x}{ab+b^2x^3}\,dx}{9ab\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad - \frac{\left(ab+b^2x^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{27a^{4/3}b^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{4/3}b^2\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad + \frac{\left(ab+b^2x^3\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{54a^{4/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{18ab^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{4/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad - \frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad + \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$
\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-3a^{4/3}b^{2/3}x^2+6\sqrt[3]{a}b^{5/3}x^5-2\sqrt{3}\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(a+bx^3\right)^2}{}
$$

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*
x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.86 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{x^5}{9a}-\frac{x^2}{18b}\right)}{(b x^3+a)^3}+\frac{\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b x^3+a)b^2 a}$
default	$-\frac{\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6-6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5+4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-\right.}{3}\right.}{}$

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/9/a*x^5-1/18/b*x^2)+1/27*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)/b^2/a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{6\,ab^3x^5-3\,a^2b^2x^2+3\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab-}{}\right.}{}\right.$$

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
```

```
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{2\,bx^5-ax^2}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log(x
+ (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,ab\mathrm{sgn}\,(bx^3+a)} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\,(bx^3+a)} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\mathrm{sgn}\,(bx^3+a)} + \frac{2\,bx^5-ax^2}{18\,(bx^3+a)^2ab\mathrm{sgn}\,(bx^3+a)}$$

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b*sgn(b*x^
3 + a)) - 1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*sgn(b*x^3 + a
)) - 1/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(a^2*b^3*sgn(b*x^3 + a)) + 1/18*(2*b*x^5 - a*x^2)/((b*x^3 + a)^
2*a*b*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.99 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 276

$$\begin{aligned}\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{x}{6b\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad +\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 1/18*x/a/b/((b*x^3+a)^2)^(1/2)-1/6*x/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/27*(
b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/54*(b*
x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)/((b*x^3+a)
^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used

= {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{1}{ab+b^2x^3}\,dx}{9ab\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{27a^{5/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{5/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{54a^{5/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{18a^{4/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-6a^{5/3}\sqrt[3]{b}x+3a^{2/3}b^{4/3}x^4-2\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(a+bx^3)}{}$$

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^
```

```
(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^
3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{x^{4}}{18 a}-\frac{x}{9 b}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27\left(b\,x^{3}+a\right) b^{2} a}$
default	$\frac{\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2 x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{2} x^{6}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{2} x^{6}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{2} x^{6}+3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2} x^{4}-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x\cdots\right)}{3\left(\frac{a}{b}\cdots\right)}\right)\cdots\right.}{\cdots}$

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/18/a*x^4-1/9/b*x)+1/27*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)/b^2/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\left[\frac{3\,a^{2}b^{2}x^{4}-6\,a^{3}bx+3\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,\left(a^{2}b\cdots\right.}{\cdots}\right.\cdots}{\cdots}\right.$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
```

```
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{bx^4-2\,ax}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x +
(a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\left(bx^3+a\right)} + \frac{bx^4-2\,ax}{18\left(bx^3+a\right)^2ab\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*sgn(b*x^3 + a)) - 1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^2*b*sgn(b*x^3 + a)) + 1/18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b*sgn(
b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.100 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}} dx = -\frac{1}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/6/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 621}

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}} dx = -\frac{1}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*1/(b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^3\right)$$
$$= -\frac{1}{6b\,(a+bx^3)\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(38) = 76$.

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.76

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{x^3\left(2a^4+a^3bx^3-ab^3x^9+a\sqrt{a^2}bx^3\sqrt{(a+bx^3)^2}-\sqrt{a^2}\sqrt{(a+bx^3)^2}(2a^2+b^2x^6)\right)}{6a^4\,(a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}$$

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*(x^3*(2*a^4 + a^3*b*x^3 - a*b^3*x^9 + a*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^
3)^2] - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*(2*a^2 + b^2*x^6)))/(a^4*(a + b*x^3)*
(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)}{6(b\,x^3+a)^2 b}$	23
gosper	$-\frac{b\,x^3+a}{6b\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^3+a}{6b\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{6(b\,x^3+a)^3 b}$	26

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/6/(b*x^3+a)^2/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=-\frac{1}{6\,(b^3x^6+2\,ab^2x^3+a^2b)}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2b^3}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/6/((x^3 + a/b)^2*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\,(bx^3+a)^2b\mathrm{sgn}\,(bx^3+a)}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] -1/6/((b*x^3 + a)^2*b*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b\,(b\,x^3+a)^3}$$

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

3.101 $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 277

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{6a\,(a+bx^3)\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 2/9*x^2/a^2/((b*x^3+a)^2)^(1/2)+1/6*x^2/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/27*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)/((b*x^3
+a)^2)^(1/2)-2/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(7/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1369, 296, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x^2}{6a\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2b(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2(ab+b^2x^3))\int \frac{x}{ab+b^2x^3}\,dx}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(2(ab+b^2x^3))\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{27a^{7/3}b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2(ab+b^2x^3))\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{7/3}b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int\frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{7/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int\frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{9a^2b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2(ab+b^2x^3))\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{7/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int\frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{21a^{4/3}b^{2/3}x^2+12\sqrt[3]{a}b^{5/3}x^5-4\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4(a+bx^3)$$

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(bx^3+a)^2}\left(\frac{2bx^5}{9a^2}+\frac{7x^2}{18a}\right)}{(bx^3+a)^3}+\frac{2\sqrt{(bx^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(bx^3+a)a^2b}$
default	$-\frac{\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6-12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5+8\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(\cdots}{\cdots}\right.\right.}{\cdots}$

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(2/9*b/a^2*x^5+7/18*x^2/a)+2/27*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)/a^2/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.86

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{12\,ab^3x^5+21\,a^2b^2x^2+6\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
```

```
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F]

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{4\,bx^5+7\,ax^2}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) + 1/
27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^2\mathrm{sgn}\,(bx^3+a)} - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\,(bx^3+a)} - \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2\mathrm{sgn}\,(bx^3+a)} + \frac{4\,bx^5+7\,ax^2}{18\,(bx^3+a)^2a^2\mathrm{sgn}\,(bx^3+a)}$$

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/27*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*sgn(b*x^
3 + a)) - 2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a))
 - 2/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^3*b^2*sgn(b*x^3 + a)) + 1/18*(4*b*x^5 + 7*a*x^2)/((b*x^3 + a)^
2*a^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.102 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 286

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}}+\frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}}-\frac{5(a+bx^3)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{3/2}}+\frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{3/2}}-\frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\,(a^2+2abx^3+b^2x^6)^{3/2}}$$

```
[Out] 1/6*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)+5/18*x*(b*x^3+a)^2/a^2/(b^2
*x^6+2*a*b*x^3+a^2)^(3/2)+5/27*(b*x^3+a)^3*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^
(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/54*(b*x^3+a)^3*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/27*(b*x
^3+a)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)/(
b^2*x^6+2*a*b*x^3+a^2)^(3/2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

= {1357, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{5x(a+bx^3)^2}{18a^2(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{x(a+bx^3)}{6a(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2ab+2b^2x^3\right)^3 \int \frac{1}{\left(2ab+2b^2x^3\right)^3}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right) \int \frac{1}{\left(2ab+2b^2x^3\right)^2}\,dx}{12ab\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right) \int \frac{1}{2ab+2b^2x^3}\,dx}{36a^2b^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x\left(a+bx^3\right)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right) \int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}b^{2/3}x}\,dx}{108\ 2^{2/3}a^{8/3}b^{8/3}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{\left(5\left(2ab+2b^2x^3\right)^3\right) \int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{2}b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{108\ 2^{2/3}a^{8/3}b^{8/3}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

$$
\begin{aligned}
&= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&+ \frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&- \frac{\left(5(2ab+2b^2x^3)^3\right)\int\frac{-2^{2/3}\sqrt[3]{a}b+2\ 2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{432a^{8/3}b^{10/3}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&+ \frac{\left(5(2ab+2b^2x^3)^3\right)\int\frac{1}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{72\sqrt[3]{2}a^{7/3}b^{7/3}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&+ \frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} - \frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&+ \frac{\left(5(2ab+2b^2x^3)^3\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{72a^{8/3}b^{10/3}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&= \frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&- \frac{5(a+bx^3)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} + \frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \\
&- \frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.82

$$
\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{24a^{5/3}\sqrt[3]{b}x+15a^{2/3}b^{4/3}x^4-10\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10(a+b
$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) +
```

```
b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b
*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(
a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{5 b\,x^{4}}{18 a^{2}}+\frac{4 x}{9 a}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} b+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27\left(b\,x^{3}+a\right) a^{2} b}$
default	$\frac{\left(-10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{2}x^{6}+10\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{2}x^{6}-5\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{2}x^{6}+15\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}x^{4}-20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{\ldots}\right.\right.}{\ldots}$

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(5/18*b/a^2*x^4+4/9*x/a)+5/27*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)/a^2/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{3/2}}\,dx=\left[\frac{15\,a^{2}b^{2}x^{4}+24\,a^{3}bx+15\,\sqrt{\frac{1}{3}}\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3}-3\ldots}{\ldots}\right.}{\ldots}\right.$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
```

```
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\int \frac{1}{(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{5\,bx^4+8\,ax}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)}+\frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x +
(a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,bx^4+8\,ax}{18\left(bx^3+a\right)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a)) + 5/27*s
qrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(a^3*b*sgn(b*x^3 + a)) + 5/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^3*b*sgn(b*x^3 + a)) + 1/18*(5*b*x^4 + 8*a*x)/((b*x^3 + a)^2*
a^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.103 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}} dx = \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3/a^2/((b*x^3+a)^2)^(1/2)+1/6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^3+a)*l
n(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^3/((b*x^3+a)^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 46}

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}} dx = \frac{1}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\log(x)(a+bx^3)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{1}{x(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^3}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad +\frac{\left(a+bx^3\right)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 790 vs. 2(147) = 294.

Time = 1.10 (sec) , antiderivative size = 790, normalized size of antiderivative = 5.37

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{4a^4bx^3+3a^3b^2x^6-ab^4x^{12}-4\left(a^2\right)^{3/2}bx^3\sqrt{\left(a+bx^3\right)^2}+a\sqrt{a^2}b^2x^6\sqrt{\left(a+b\right.}}{}$$

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (4*a^4*b*x^3 + 3*a^3*b^2*x^6 - a*b^4*x^12 - 4*(a^2)^(3/2)*b*x^3*Sqrt[(a + b
*x^3)^2] + a*Sqrt[a^2]*b^2*x^6*Sqrt[(a + b*x^3)^2] - Sqrt[a^2]*b^3*x^9*Sqrt
[(a + b*x^3)^2] + 2*((a^2)^(3/2)*b^2*x^6 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x^3
)^2]) + a^3*b*x^3*(2*Sqrt[a^2] - Sqrt[(a + b*x^3)^2]))*ArcTanh[(b*x^3)/(Sqr
t[a^2] - Sqrt[(a + b*x^3)^2])] - 2*(a^5 + 2*a^4*b*x^3 - (a^2)^(3/2)*b*x^3*S
qrt[(a + b*x^3)^2] + a^3*(b^2*x^6 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))*Log[x^3
] + a^5*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 2*a^4*b*x^3*Log[Sqrt
[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + a^3*b^2*x^6*Log[Sqrt[a^2] - b*x^3 -
Sqrt[(a + b*x^3)^2]] - a^3*Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*
x^3 - Sqrt[(a + b*x^3)^2]] - (a^2)^(3/2)*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt
[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + a^5*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a
+ b*x^3)^2]] + 2*a^4*b*x^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + a
^3*b^2*x^6*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - a^3*Sqrt[a^2]*Sqr
t[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - (a^2)^(3/2)
*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(3
*a^3*Sqrt[a^2]*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

method	result	si
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}+\ln\left(b\,x^{3}\right)\left(b\,x^{3}+a\right)^{2}+a b\,x^{3}+\frac{3 a^{2}}{2}\right)}{3\left(b\,x^{3}+a\right)^{2} a^{3}}$	7
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{b\,x^{3}}{3 a^{2}}+\frac{1}{2 a}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{3}+a\right) a^{3}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right) a^{3}}$	9
default	$\frac{\left(6\ln\left(x\right) b^{2} x^{6}-2\ln\left(b\,x^{3}+a\right) b^{2} x^{6}+12\ln\left(x\right) a b\,x^{3}-4\ln\left(b\,x^{3}+a\right) a b\,x^{3}+2 a b\,x^{3}+6 a^{2}\ln\left(x\right)-2\ln\left(b\,x^{3}+a\right) a^{2}+3 a^{2}\right)\left(b\,x^{3}+a\right)}{6 a^{3}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	1

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/3*csgn(b*x^3+a)*(-ln(b*x^3+a)*(b*x^3+a)^2+ln(b*x^3)*(b*x^3+a)^2+a*b*x^3+3/2*a^2)/(b*x^3+a)^2/a^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{2\,abx^3+3\,a^2-2\left(b^2x^6+2\,abx^3+a^2\right)\log\left(bx^3+a\right)+6\left(b^2x^6+2\,abx^3+a^2\right)}{6\left(a^3b^2x^6+2\,a^4bx^3+a^5\right)}$$

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3} + \frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2} + \frac{1}{6\left(x^3+\frac{a}{b}\right)^2ab^2}$$

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3 + 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{3\,b^2x^6+8\,abx^3+6\,a^2}{6\,\left(bx^3+a\right)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/(a^3*sgn(b*x^3 + a)) + log(abs(x))/(a^3*sgn(b*x^3
+ a)) + 1/6*(3*b^2*x^6 + 8*a*b*x^3 + 6*a^2)/((b*x^3 + a)^2*a^3*sgn(b*x^3 +
a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{1}{x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.104 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{14\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{7\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 7/18/a^2/x/((b*x^3+a)^2)^(1/2)+1/6/a/x/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-14/9*(
b*x^3+a)/a^3/x/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/
3)*x)/a^(10/3)/((b*x^3+a)^2)^(1/2)-7/27*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+
a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)/((b*x
^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1369, 296, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}} \, dx = \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)} + \frac{14\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^3))\int \frac{1}{x^2(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^2}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(14\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)}\,dx}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(14b\left(ab+b^2x^3\right)\right)\int \frac{x}{ab+b^2x^3}\,dx}{9a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(14\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(14\left(ab+b^2x^3\right)\right)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(7\left(ab+b^2x^3\right)\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{10/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(7\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{9a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14\left(a+bx^3\right)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(14\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{10/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-54a^{7/3}-147a^{4/3}bx^3-84\sqrt[3]{a}b^2x^6+28\sqrt{3}\sqrt[3]{b}x(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)$$

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)*
x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*x*
(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\sqrt{(bx^3+a)^2}\left(-\frac{14b^2x^6}{9a^3}-\frac{49bx^3}{18a^2}-\frac{1}{a}\right)}{(bx^3+a)^3x}+\frac{14\sqrt{(bx^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(a^{10}\_Z^3-b)}\_R\ln\left(\left(-4\_R^3a^{10}+3b\right)x-a^7\_R^2\right)\right)}{27(bx^3+a)}$
default	$\left(28\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^7+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^7-14\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^7-84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^6+56\sqrt{3}\arctan\left(\right.\right.$

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-14/9*b^2/a^3*x^6-49/18*b/a^2*x^3-1/a)/x+1
4/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^10+3*b)*x-a^7*_R^2)
,_R=RootOf(_Z^3*a^10-b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}(b^2x^7+2\,abx^4+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\,(b^2x^7+2\,abx^4+}{54\,(a^3b^2x^7+2\,a^4b}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*
(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7 +
 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)
) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) +
 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^2\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{28\,b^2x^6+49\,abx^3+18\,a^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)} - \frac{14\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x
) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*
(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)
) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)} + \frac{14\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b\mathrm{sgn}\left(bx^3+a\right)} - \frac{7\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b\mathrm{sgn}\left(bx^3+a\right)} - \frac{10\,b^2x^5+13\,abx^2}{18\left(bx^3+a\right)^2a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{1}{a^3x\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 14/27*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) + 14/2
7*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*b*sgn(b*x^3 + a)) - 7/27*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a^4*b*sgn(b*x^3 + a)) - 1/18*(10*b^2*x^5 + 13*a*b*x^2)/((b*
x^3 + a)^2*a^3*sgn(b*x^3 + a)) - 1/(a^3*x*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}} \,dx$$

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.105 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{20b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{10b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 4/9/a^2/x^2/((b*x^3+a)^2)^(1/2)+1/6/a/x^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-10/
9*(b*x^3+a)/a^3/x^2/((b*x^3+a)^2)^(1/2)-20/27*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+
b^(1/3)*x)/a^(11/3)/((b*x^3+a)^2)^(1/2)+10/27*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/((b*x^3+a)^2)^(1/2)+20/27*b^(2/3)*(
b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1369, 296, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}\,(a + bx^3)} + \frac{20b^{2/3}(a + bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{20b^{2/3}(a + bx^3)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{10b^{2/3}(a + bx^3)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(a + bx^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^3)) \int \frac{1}{x^3(ab+b^2x^3)^3}\, dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(4b(ab+b^2x^3))\int \frac{1}{x^3(ab+b^2x^3)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(20(ab+b^2x^3))\int \frac{1}{x^3(ab+b^2x^3)}\,dx}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(20b(ab+b^2x^3))\int \frac{1}{ab+b^2x^3}\,dx}{9a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(20\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(20\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(10(ab+b^2x^3))\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{27a^{11/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(10b^{2/3}(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{9a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(20(ab+b^2x^3))\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{20b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-27a^{8/3}-96a^{5/3}bx^3-60a^{2/3}b^2x^6+40\sqrt{3}b^{2/3}x^2(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}}\right)\cdots}{\cdots}$$

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*x
^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)*x
^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{10b^2x^6}{9a^3}-\frac{16b\,x^3}{9a^2}-\frac{1}{2a}\right)}{(b\,x^3+a)^3x^2}+\frac{20\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3a^{11}-3b^2\right)x-a^4b\_R\right)\right)}{27(b\,x^3+a)}$
default	$-\frac{\left(-40\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^8+40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^8-20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^8+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^6-80\sqrt{3}\,\arctan\left(\right.}{}$

[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-10/9*b^2/a^3*x^6-16/9*b/a^2*x^3-1/2/a)/x^2+20/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^11-3*b^2)*x-a^4*b*_R),_R=RootOf(_Z^3*a^11+b^2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{60\,b^2x^6+96\,abx^3-40\,\sqrt{3}(b^2x^8+2\,abx^5+a^2x^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+20\,(b^2x^8+2\,abx^5+}{54\,(a^3b}$$

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] -1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)

Sympy [F]

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\int \frac{1}{x^3\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=-\frac{20\,b^2x^6+32\,abx^3+9\,a^2}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)}-\frac{20\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^
2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3
*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/
3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{10\,\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{20\,b^2x^6+32\,abx^3+9\,a^2}{18\,\left(bx^4+ax\right)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 20/27*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) - 20/2
7*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*sgn(b*x^3 + a)) - 10/27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^4*sgn(b*x^3 + a)) - 1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)
/((b*x^4 + a*x)^2*a^3*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \int \frac{1}{x^3 (a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}} \, dx$$

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.106 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b(a+bx^3)\log(x)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -2/3*b/a^3/((b*x^3+a)^2)^(1/2)-1/6*b/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/3*
(-b*x^3-a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-3*b*(b*x^3+a)*ln(x)/a^4/((b*x^3+a)^2
)^(1/2)+b*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 46}

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{1}{x^4(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right)dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b(a+bx^3)\log(x)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 901 vs. 2(188) = 376.

Time = 1.33 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.79

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx =$$

$$-\frac{-2a^6 - 5a^5bx^3 + 2a^4b^2x^6 + 4a^3b^3x^9 - ab^5x^{15} + 2a^4\sqrt{a^2}\sqrt{(a+bx^3)^2} + 3a^3\sqrt{a^2}bx^3\sqrt{(a+bx^3)^2} - 5(a^2)^{3/2}}{}$$

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] -1/3*(-2*a^6 - 5*a^5*b*x^3 + 2*a^4*b^2*x^6 + 4*a^3*b^3*x^9 - a*b^5*x^15 + 2
*a^4*Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + 3*a^3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^3)
^2] - 5*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3)^2] + a*Sqrt[a^2]*b^3*x^9*Sqrt[
(a + b*x^3)^2] - Sqrt[a^2]*b^4*x^12*Sqrt[(a + b*x^3)^2] + 6*b*x^3*((a^2)^(3
/2)*b^2*x^6 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]) + a^3*b*x^3*(2*Sqrt[a^2
] - Sqrt[(a + b*x^3)^2]))*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])
] - 6*b*x^3*(a^5 + 2*a^4*b*x^3 - (a^2)^(3/2)*b*x^3*Sqrt[(a + b*x^3)^2] + a^
3*(b^2*x^6 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))*Log[x^3] + 3*a^5*b*x^3*Log[Sqr
t[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 6*a^4*b^2*x^6*Log[Sqrt[a^2] - b*x^3
 - Sqrt[(a + b*x^3)^2]] + 3*a^3*b^3*x^9*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b
*x^3)^2]] - 3*a^3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3
 - Sqrt[(a + b*x^3)^2]] - 3*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqr
t[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 3*a^5*b*x^3*Log[Sqrt[a^2] + b*x^3 -
 Sqrt[(a + b*x^3)^2]] + 6*a^4*b^2*x^6*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x
^3)^2]] + 3*a^3*b^3*x^9*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - 3*a^
3*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x
^3)^2]] - 3*(a^2)^(3/2)*b^2*x^6*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] + b*x^3 -
 Sqrt[(a + b*x^3)^2]])/(a^4*Sqrt[a^2]*x^3*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(
a + b*x^3)^2])^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

method	result
pseudoelliptic	$-\frac{\left(-3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}+a\right)+3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}\right)+a\left(3b^{2}x^{6}+\frac{9}{2}ab\,x^{3}+a^{2}\right)\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)^{2}a^{4}x^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\left(-\frac{b^{2}x^{6}}{a^{3}}-\frac{3b\,x^{3}}{2a^{2}}-\frac{1}{3a}\right)}{\left(b\,x^{3}+a\right)^{3}x^{3}}-\frac{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{4}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(-b\,x^{3}-a\right)}{\left(b\,x^{3}+a\right)a^{4}}$
default	$-\frac{(18b^{3}\ln(x)x^{9}-6\ln\left(b\,x^{3}+a\right)b^{3}x^{9}+36b^{2}a\ln(x)x^{6}-12\ln\left(b\,x^{3}+a\right)a\,b^{2}x^{6}+6b^{2}x^{6}a+18a^{2}b\ln(x)x^{3}-6\ln\left(b\,x^{3}+a\right)a^{2}b\,x^{3}+9a^{2}b}{6x^{3}a^{4}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-3*b*x^3*(b*x^3+a)^2*ln(b*x^3+a)+3*b*x^3*(b*x^3+a)^2*ln(b*x^3)+a*(3*b
^2*x^6+9/2*a*b*x^3+a^2))*csgn(b*x^3+a)/(b*x^3+a)^2/a^4/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(bx^3+a\right)+18\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(x\right)}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{a^4} - \frac{b}{\sqrt{b^2x^6+2\,abx^3+a^2}a^3} - \frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^2b} - \frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}a^2x^3}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] (-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 - b
/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/(s
qrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{b\log\left(|bx^3+a|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)} - \frac{3\,b\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)} - \frac{9\,b^3x^6+22\,ab^2x^3+14\,a^2b}{6\left(bx^3+a\right)^2a^4\mathrm{sgn}\left(bx^3+a\right)} + \frac{3\,bx^3-a}{3\,a^4x^3\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] b*log(abs(b*x^3 + a))/(a^4*sgn(b*x^3 + a)) - 3*b*log(abs(x))/(a^4*sgn(b*x^3
 + a)) - 1/6*(9*b^3*x^6 + 22*a*b^2*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4*sgn(b
*x^3 + a)) + 1/3*(3*b*x^3 - a)/(a^4*x^3*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}} \,d x$$

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.107 $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 359

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 5/486*x/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^4/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)-1/27*x/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/162*x/a/b^2/(b*x^3+a)/((b*
x^3+a)^2)^(1/2)+5/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(7/3)/((b*x
^3+a)^2)^(1/2)-5/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\begin{aligned}\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}\\ &+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &+\frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{x^6}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(b^2(ab+b^2x^3)\right)\int \frac{x^3}{(ab+b^2x^3)^4}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{1}{(ab+b^2x^3)^3}\,dx}{27\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad +\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(5(ab+b^2x^3)\right)\int \frac{1}{(ab+b^2x^3)^2}\,dx}{162ab\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad +\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(5(ab+b^2x^3)\right)\int \frac{1}{ab+b^2x^3}\,dx}{243a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad +\frac{\left(5(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{729a^{8/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(5(ab+b^2x^3)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{8/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad -\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad +\frac{5\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(5(ab+b^2x^3)\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{1458a^{8/3}b^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad +\frac{\left(5(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{486a^{7/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(5(ab+b^2x^3))\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{8/3}b^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a\sqrt[3]{b}x - 351\sqrt[3]{b}x(a+bx^3) + \frac{18\sqrt[3]{b}x(a+bx^3)^2}{a} + \frac{30\sqrt[3]{b}x(a+bx^3)^3}{a^2} + \right.}{291$$

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(
a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a +
b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{5b x^{10}}{486a^2}+\frac{x^7}{27a}-\frac{25x^4}{324b}-\frac{5ax}{243b^2}\right)}{(b x^3+a)^5}+\frac{5\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3 b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729(b x^3+a)a^2b^3}$
default	$-\frac{\left(20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-20\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+10\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-30\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}+80\sqrt{3}\,\mathrm{arct}}{}$

```
[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/486*b/a^2*x^10+1/27/a*x^7-25/324/b*x^4-5
/243*a/b^2*x)+5/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^2/b^3*sum(1/_R^2*ln(x-_
R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{30\,a^2b^4x^{10}+108\,a^3b^3x^7-225\,a^4b^2x^4-60\,a^5bx+30\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a^2b^4x^9}{}\right.$$

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x +
 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 +
 a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)
^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^
```

```
5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7
 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9
+ 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 10*
(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(
2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

Sympy [F]

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^6}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12
+ 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) -
 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) + 5/72
9*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2b\mathrm{sgn}\left(bx^3+a\right)} - \frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^3b^2\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,\left(bx^3+a\right)^4a^2b^2\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -5/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b*sgn(
b*x^3 + a)) - 5/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2*sgn(b*
x^3 + a)) + 5/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^3*b^3*sgn(b*x^3 + a)) + 1/972*(10*b^3*x^10 + 36*a*b^
2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/((b*x^3 + a)^4*a^2*b^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^6}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

```
[In] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.108 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/12*a/b^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-1/9/b^2/(b*x^3+a)^2/((b*x^3+a)^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 45}

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{a}{12b^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^4(ab+b^2x^3))\int \frac{x^5}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(b^4(ab+b^2x^3))\,\text{Subst}\left(\int \frac{x}{(ab+b^2x)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(b^4(ab+b^2x^3))\,\text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{a}{12b^2\,(a+bx^3)^3\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2\,(a+bx^3)^2\,\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(78) = 156.

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx =$$

$$-\frac{x^6\left(3\sqrt{a^2}b^6x^{18}+3a^3b^3x^9\sqrt{(a+bx^3)^2}-3a^2b^4x^{12}\sqrt{(a+bx^3)^2}+3ab^5x^{15}\sqrt{(a+bx^3)^2}+a^4b^2x^6\left(\sqrt{a^2}-3\right.\right.}{36a^7\,(a+bx^3)^3\left(a^2+abx^3-\sqrt{a^2}\sqrt{(\right.}$$

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/36*(x^6*(3*Sqrt[a^2]*b^6*x^18 + 3*a^3*b^3*x^9*Sqrt[(a + b*x^3)^2] - 3*a^
2*b^4*x^12*Sqrt[(a + b*x^3)^2] + 3*a*b^5*x^15*Sqrt[(a + b*x^3)^2] + a^4*b^2
*x^6*(Sqrt[a^2] - 3*Sqrt[(a + b*x^3)^2]) + 6*a^6*(Sqrt[a^2] - Sqrt[(a + b*x
^3)^2]) + 2*a^5*b*x^3*(2*Sqrt[a^2] + Sqrt[(a + b*x^3)^2])))/(a^7*(a + b*x^3
)^3*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

method	result	size
pseudoelliptic	$-\frac{\left(4 b \,x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{36 b^{2} \left(b \,x^{3}+a \right)^{4}}$	31
gosper	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
default	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{x^{3}}{9 b}-\frac{a}{36 b^{2}}\right)}{\left(b \,x^{3}+a \right)^{5}}$	37

```
[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(4*b*x^3+a)*csgn(b*x^3+a)/b^2/(b*x^3+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=-\frac{4\,bx^{3}+a}{36\,\left(b^{6}x^{12}+4\,ab^{5}x^{9}+6\,a^{2}b^{4}x^{6}+4\,a^{3}b^{3}x^{3}+a^{4}b^{2}\right)}$$

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3
 + a^4*b^2)
```

Sympy [F]

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^5}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2} + \frac{a}{12\,\left(x^3+\frac{a}{b}\right)^4 b^6}$$

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{4\,bx^3+a}{36\,(bx^3+a)^4 b^2 \mathrm{sgn}\,(bx^3+a)}$$

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] -1/36*(4*b*x^3 + a)/((b*x^3 + a)^4*b^2*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{(4\,b\,x^3+a)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,(b\,x^3+a)^5}$$

```
[In] int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

3.109 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 368

$$\begin{aligned}\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} dx &= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 7/243*x^2/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x^2/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+1/54*x^2/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+7/324*x^2/a^2/b/(b*x^3+a)/
((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(5/3)/
((b*x^3+a)^2)^(1/2)+7/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1369, 294, 296, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{7x^2}{324a^2b\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{x^4}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2(ab+b^2x^3)\right)\int\frac{x}{(ab+b^2x^3)^4}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b(ab+b^2x^3)\right)\int\frac{x}{(ab+b^2x^3)^3}\,dx}{54a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7(ab+b^2x^3)\right)\int\frac{x}{(ab+b^2x^3)^2}\,dx}{81a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7(ab+b^2x^3)\right)\int\frac{x}{ab+b^2x^3}\,dx}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(7(ab+b^2x^3)\right)\int\frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{729a^{10/3}b^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7(ab+b^2x^3)\right)\int\frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{10/3}b^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7(ab+b^2x^3)\right)\int\frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{1458a^{10/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7(ab+b^2x^3)\right)\int\frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{486a^3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(7(ab+b^2x^3))\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{10/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(a+bx^3\right)\left(-243a^{10/3}b^{2/3}x^2+54a^{7/3}b^{2/3}x^2(a+bx^3)+63a^{4/3}b^{2/3}x^2(a+bx^3)^2\right.}{}$$

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3
) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^
3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b
*x^3)^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.70 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{7b^2x^{11}}{243a^3}+\frac{35b x^8}{324a^2}+\frac{4x^5}{27a}-\frac{7x^2}{486b}\right)}{(b x^3+a)^5}+\frac{7\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{729(b x^3+a)a^3b^2}$
default	$-\frac{\left(28\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-14\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{11}+112\sqrt{3}\,\mathrm{arc}\right.}{}$

[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(7/243*b^2/a^3*x^11+35/324*b/a^2*x^8+4/27/a
*x^5-7/486/b*x^2)+7/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R*ln(x
-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.99

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{84\,ab^5x^{11}+315\,a^2b^4x^8+432\,a^3b^3x^5-42\,a^4b^2x^2+42\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a^2b^4x}{}\right.$$

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2
 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)
^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*

```
a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 315*a^2*b^4
*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*
b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*ar
ctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^
4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^4}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)}+\frac{7\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*x
^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
 + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 7/
729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{7\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{1}{3}}\,a^3b\mathrm{sgn}\,(bx^3+a)} - \frac{7\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\,(bx^3+a)} - \frac{7\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b^3\mathrm{sgn}\,(bx^3+a)} + \frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,(bx^3+a)^4a^3b\mathrm{sgn}\,(bx^3+a)}$$

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -7/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b*sgn(
b*x^3 + a)) - 7/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*sgn(b*x^
3 + a)) - 7/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^4*b^3*sgn(b*x^3 + a)) + 1/972*(28*b^3*x^11 + 105*a*b^2
*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^4}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.110 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 360

$$\begin{aligned}\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad-\frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad+\frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad+\frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 5/243*x/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
1/108*x/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/81*x/a^2/b/(b*x^3+a)/((b*x^3+
a)^2)^(1/2)+10/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(4/3)/((b*x^3
+a)^2)^(1/2)-5/729*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
1/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-10/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\begin{aligned}\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{10(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^4}\,dx}{12\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(2b\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^3}\,dx}{27a\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(5\left(ab+b^2x^3\right)\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{81a^2\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(10\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{\left(10\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{729a^{11/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(10\left(ab+b^2x^3\right)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{11/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{10\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(5\left(ab+b^2x^3\right)\right)\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{11/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\
&\quad+\frac{\left(5\left(ab+b^2x^3\right)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{243a^{10/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(10(ab+b^2x^3))\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{11/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{11/3}\sqrt[3]{b}x+27a^{8/3}\sqrt[3]{b}x(a+bx^3)+36a^{5/3}\sqrt[3]{b}x(a+bx^3)^2+6\right.}{}$$

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 4
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
+ 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{5b^2x^{10}}{243a^3}+\frac{2b x^7}{27a^2}+\frac{31x^4}{324a}-\frac{10x}{243b}\right)}{(b x^3+a)^5}+\frac{10\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729(b x^3+a)a^3b^2}$
default	$\frac{\left(-40\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}-160\sqrt{3}\,\mathrm{arc}\right.}{}$

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/243*b^2/a^3*x^10+2/27*b/a^2*x^7+31/324/a
*x^4-10/243/b*x)+10/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{60\,a^2b^4x^{10}+216\,a^3b^3x^7+279\,a^4b^2x^4-120\,a^5bx+60\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a^2b^4x}{}\right.$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
+ 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
+ a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
+ 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)
^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^
3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b
)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b
```

```
^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^
7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x
^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sq
rt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) -
20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^
3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a
^9*b^2)]
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^3}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.54

$$\begin{aligned}\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)}\\&\quad+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12
+ 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) -
5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10/729
*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\left(-ab^2\right)^{\frac{2}{3}}a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{5\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\left(-ab^2\right)^{\frac{2}{3}}a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\left(bx^3+a\right)} + \frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\left(bx^3+a\right)^4a^3b\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*
b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 5/729*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 10/729*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^4*b*sgn(b*x^3 + a)) + 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 +
 93*a^2*b*x^4 - 40*a^3*x)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.111 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12b(a+bx^3)(a^2+2abx^3+b^2x^6)^{3/2}}$$

```
[Out] -1/12/b/(b*x^3+a)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 621}

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12b(a+bx^3)(a^2+2abx^3+b^2x^6)^{3/2}}$$

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/12*1/(b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, x^3\right)$$
$$= -\frac{1}{12b\,(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{3/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 267 vs. $2(38) = 76$.

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 7.03

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{x^3\left(-\sqrt{a^2}b^7x^{21} - a^3b^4x^{12}\sqrt{(a+bx^3)^2} + a^2b^5x^{15}\sqrt{(a+bx^3)^2} - ab^6x^{18}\sqrt{(a+bx^3)^2} + 4a^7\left(\sqrt{a^2} - \sqrt{(a+\right.\right.}{12a^8\,(a+bx^3)^3\left(a^2+\right.}$$

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/12*(x^3*(-(Sqrt[a^2]*b^7*x^21) - a^3*b^4*x^12*Sqrt[(a + b*x^3)^2] + a^2*
b^5*x^15*Sqrt[(a + b*x^3)^2] - a*b^6*x^18*Sqrt[(a + b*x^3)^2] + 4*a^7*(Sqrt
[a^2] - Sqrt[(a + b*x^3)^2]) + 2*a^5*b^2*x^6*(2*Sqrt[a^2] - Sqrt[(a + b*x^3
)^2]) + 2*a^6*b*x^3*(3*Sqrt[a^2] - Sqrt[(a + b*x^3)^2]) + a^4*b^3*x^9*(Sqrt
[a^2] + Sqrt[(a + b*x^3)^2])))/(a^8*(a + b*x^3)^3*(a^2 + a*b*x^3 - Sqrt[a^2
]*Sqrt[(a + b*x^3)^2]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)}{12(b\,x^3+a)^4 b}$	23
gosper	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{12(b\,x^3+a)^5 b}$	26

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12/(b*x^3+a)^4/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(b^5x^{12}+4\,ab^4x^9+6\,a^2b^3x^6+4\,a^3b^2x^3+a^4b)}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\left(x^3+\frac{a}{b}\right)^4 b^5}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/12/((x^3 + a/b)^4*b^5)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(bx^3+a)^4 b\,\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] -1/12/((b*x^3 + a)^4*b*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,b\,\left(b\,x^3+a\right)^5}$$

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

3.112 $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 359

$$\begin{aligned}\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{35x^2}{324a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&-\frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 35/243*x^2/a^4/((b*x^3+a)^2)^(1/2)+1/12*x^2/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/
2)+5/54*x^2/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+35/324*x^2/a^3/(b*x^3+a)/((
b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)/(
(b*x^3+a)^2)^(1/2)+35/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1369, 296, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(b^4(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(5b^3(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad+\frac{(35b^2(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^3}\,dx}{54a^2\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

$$=\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(35b(ab+b^2x^3))\int\frac{x}{(ab+b^2x^3)^2}\,dx}{81a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(35(ab+b^2x^3))\int\frac{x}{ab+b^2x^3}\,dx}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$
$$-\frac{(35(ab+b^2x^3))\int\frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{729a^{13/3}b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(35(ab+b^2x^3))\int\frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{13/3}b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$
$$-\frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(35(ab+b^2x^3))\int\frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{1458a^{13/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{(35(ab+b^2x^3))\int\frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{486a^4b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$
$$-\frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$
$$+\frac{(35(ab+b^2x^3))\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{13/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35x^2}{324a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.61

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{10/3}x^2+270a^{7/3}x^2(a+bx^3)+315a^{4/3}x^2(a+bx^3)^2+420\sqrt[3]{a}x\right.}{}$$

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*
x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(a +
 b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^
2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.87 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(\frac{35b^3x^{11}}{243a^4}+\frac{175b^2x^8}{324a^3}+\frac{20b\,x^5}{27a^2}+\frac{104x^2}{243a}\right)}{(b\,x^3+a)^5}+\frac{35\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R}\right)}{729(b\,x^3+a)b\,a^4}$
default	$\frac{\left(-140\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-140\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+70\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+420\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{11}-560\sqrt{3}\right.}{}$

[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(35/243/a^4*b^3*x^11+175/324*b^2/a^3*x^8+20/27*b/a^2*x^5+104/243*x^2/a)+35/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b/a^4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.04

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \left[\frac{420\,ab^5x^{11}+1575\,a^2b^4x^8+2160\,a^3b^3x^5+1248\,a^4b^2x^2+210\,\sqrt{\frac{1}{3}}(ab^5x^{12}+}{}\right.$$

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a

```
)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*
b^3*x^3 + a^9*b^2)]
```

Sympy [F]

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \frac{140\,b^3x^{11} + 525\,ab^2x^8 + 720\,a^2bx^5 + 416\,a^3x^2}{972\,(a^4b^4x^{12} + 4\,a^5b^3x^9 + 6\,a^6b^2x^6 + 4\,a^7bx^3 + a^8)} + \frac{35\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{35\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{35\,\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] 1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) +
35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{35\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,\left(-ab^2\right)^{\frac{1}{3}}\,a^4\mathrm{sgn}\left(bx^3+a\right)} - \frac{35\,\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)} - \frac{35\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b^2\mathrm{sgn}\left(bx^3+a\right)} + \frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\,\left(bx^3+a\right)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -35/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^4*sgn(b
*x^3 + a)) - 35/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^3
+ a)) - 35/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^5*b^2*sgn(b*x^3 + a)) + 1/972*(140*b^3*x^11 + 525*a*b^2
*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.113 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 364

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \\ &+ \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \\ &+ \frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}} - \frac{110(a+bx^3)^5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \\ &+ \frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}} - \frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\end{aligned}$$

```
[Out] 1/12*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/108*x*(b*x^3+a)^2/a^2/(
b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/81*x*(b*x^3+a)^3/a^3/(b^2*x^6+2*a*b*x^3+a^2
)^(5/2)+55/243*x*(b*x^3+a)^4/a^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+110/729*(b*x
^3+a)^5*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)-55/729*(b*x^3+a)^5*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(
1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)-110/729*(b*x^3+a)^5*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1357, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{11x(a+bx^3)^2}{108a^2(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{x(a+bx^3)}{12a(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{110(a+bx^3)^5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{55x(a+bx^3)^4}{243a^4(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3(a^2+2abx^3+b^2x^6)^{5/2}}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(2ab+2b^2x^3)^5 \int \frac{1}{(2ab+2b^2x^3)^5}\,dx}{(a^2+2abx^3+b^2x^6)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\,(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{\left(11(2ab+2b^2x^3)^5\right)\int \frac{1}{(2ab+2b^2x^3)^4}\,dx}{24ab\,(a^2+2abx^3+b^2x^6)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11(2ab+2b^2x^3)^5\right)\int \frac{1}{(2ab+2b^2x^3)^3}\,dx}{54a^2b^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(55(2ab+2b^2x^3)^5\right)\int \frac{1}{(2ab+2b^2x^3)^2}\,dx}{648a^3b^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(55(2ab+2b^2x^3)^5\right)\int \frac{1}{2ab+2b^2x^3}\,dx}{1944a^4b^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(55(2ab+2b^2x^3)^5\right)\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}b^{2/3}x}\,dx}{5832\ 2^{2/3}a^{14/3}b^{14/3}\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(55(2ab+2b^2x^3)^5\right)\int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{2}b^{2/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{5832\ 2^{2/3}a^{14/3}b^{14/3}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$=\frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{\left(55(2ab+2b^2x^3)^5\right)\int\frac{-2^{2/3}\sqrt[3]{a}b+2\,2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{23328a^{14/3}b^{16/3}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{\left(55(2ab+2b^2x^3)^5\right)\int\frac{1}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}bx+2^{2/3}b^{4/3}x^2}\,dx}{3888\sqrt[3]{2}a^{13/3}b^{13/3}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$=\frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{\left(55(2ab+2b^2x^3)^5\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3888a^{14/3}b^{16/3}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$=\frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{110(a+bx^3)^5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{11/3}x+297a^{8/3}x(a+bx^3)+396a^{5/3}x(a+bx^3)^2+660a^{2/3}x\right.}{}$$

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]

[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(bx^3+a)^2}\left(\frac{55b^3x^{10}}{243a^4}+\frac{22b^2x^7}{27a^3}+\frac{341bx^4}{324a^2}+\frac{133x}{243a}\right)}{(bx^3+a)^5}+\frac{110\sqrt{(bx^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729(bx^3+a)ba^4}$
default	$\frac{\left(-440\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+440\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-220\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+660\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}-1760\right.}{}$

[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(55/243/a^4*b^3*x^10+22/27*b^2/a^3*x^7+341/324*b/a^2*x^4+133/243*x/a)+110/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b/a^4*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \left[\frac{660\,a^2b^4x^{10}+2376\,a^3b^3x^7+3069\,a^4b^2x^4+1596\,a^5bx+660\,\sqrt{\frac{1}{3}}(ab^5x^{12}+4\,a\right.$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*
b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2
*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 +
6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^
3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +
4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)
/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*
x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a
*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^
9*b^2*x^3 + a^10*b)]
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{(a^2+2abx^3+b^2x^6)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)} + \frac{110\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{55\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{110\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4*
x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3)) -
55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{110\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{110\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{55\,(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\,(bx^3+a)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -110/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^3 + a)) + 110
/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^5*b*sgn(b*x^3 + a)) + 55/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^5*b*sgn(b*x^3 + a)) + 1/972*(220*b^3*x^10 + 792*a*b^
2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{1}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}\, dx$$

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.114 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log(x)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3/a^4/((b*x^3+a)^2)^(1/2)+1/12/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+1/9/a^2/
(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/6/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^
3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^5/((b*x^3+a)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1369, 272, 46}

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{1}{x(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^4(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{\left(b^4(ab+b^2x^3)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx)^2}-\frac{1}{a^5b^4(a+bx)}\right)dx,\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\\
&\quad+\frac{\left(a+bx^3\right)\log(x)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{a(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9)+36(a+bx^3)^4\log(x)-12(a+bx^3)^4\log(a+bx^3)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]

[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*L
og[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*
x^3)^2])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$\frac{\left(-\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}+\ln\left(b\,x^{3}\right)\left(b\,x^{3}+a\right)^{4}+a\,b^{3}x^{9}+\frac{7a^{2}b^{2}x^{6}}{2}+\frac{13a^{3}b\,x^{3}}{3}+\frac{25a^{4}}{12}\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)^{4}a^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\left(\frac{b^{3}x^{9}}{3a^{4}}+\frac{7b^{2}x^{6}}{6a^{3}}+\frac{13b\,x^{3}}{9a^{2}}+\frac{25}{36a}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{5}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)a^{5}}$
default	$\frac{\left(36\ln\left(x\right)b^{4}x^{12}-12\ln\left(b\,x^{3}+a\right)b^{4}x^{12}+144\ln\left(x\right)a\,b^{3}x^{9}-48\ln\left(b\,x^{3}+a\right)a\,b^{3}x^{9}+12a\,b^{3}x^{9}+216\ln\left(x\right)a^{2}b^{2}x^{6}-72\ln\left(b\,x^{3}+a\right)a^{2}b^{2}\right.}{36a^{5}\left(\left(b\,x^{3}+a\right)^{2}\right.}$

[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(-ln(b*x^3+a)*(b*x^3+a)^4+ln(b*x^3)*(b*x^3+a)^4+a*b^3*x^9+7/2*a^2*b^2*x
^6+13/3*a^3*b*x^3+25/12*a^4)*csgn(b*x^3+a)/(b*x^3+a)^4/a^5

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{12\,ab^3x^9+42\,a^2b^2x^6+52\,a^3bx^3+25\,a^4-12\,(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+}{36\,(a^5b^4x^{12}+4\,a^6b^3}$$

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
+ 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4

```
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x
^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=&-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5}\\&+\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}+\frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}a^4}\\&+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^3b^2}+\frac{1}{12\left(x^3+\frac{a}{b}\right)^4ab^4}\end{aligned}$$

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
+ 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b*x
^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=&-\frac{\log\left(|bx^3+a|\right)}{3\,a^5\mathrm{sgn}\left(bx^3+a\right)}+\frac{\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx^3+a\right)}\\&+\frac{25\,b^4x^{12}+112\,ab^3x^9+192\,a^2b^2x^6+152\,a^3bx^3+50\,a^4}{36\left(bx^3+a\right)^4a^5\mathrm{sgn}\left(bx^3+a\right)}\end{aligned}$$

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 + a))/(a^5*sgn(b*x^3 + a)) + log(abs(x))/(a^5*sgn(b*x^3
+ a)) + 1/36*(25*b^4*x^12 + 112*a*b^3*x^9 + 192*a^2*b^2*x^6 + 152*a^3*b*x^3
 + 50*a^4)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.115 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 398

$$\begin{aligned}\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{12ax(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{65}{324a^3x(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{455\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 455/972/a^4/x/((b*x^3+a)^2)^(1/2)+1/12/a/x/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
13/108/a^2/x/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+65/324/a^3/x/(b*x^3+a)/((b*x^3
+a)^2)^(1/2)-455/243*(b*x^3+a)/a^5/x/((b*x^3+a)^2)^(1/2)+455/729*b^(1/3)*(b
*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/((b*x^3+a)^2)^(1/2)-455/1458*b^(1/3)
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/((b*x^3+a)^2)
^(1/2)+455/729*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(16/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2} + \frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3} + \frac{455\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^4(ab+b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^5} \, dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{(13b^3(ab+b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^4} \, dx}{12a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{(65b^2(ab+b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^3} \, dx}{54a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x(a+bx^3) \sqrt{a^2+2abx^3+b^2x^6}} + \frac{(455b(ab+b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^2} \, dx}{324a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x(a+bx^3) \sqrt{a^2+2abx^3+b^2x^6}} + \frac{(455(ab+b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)} \, dx}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x(a+bx^3) \sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(455b(ab+b^2x^3)) \int \frac{x}{ab+b^2x^3} \, dx}{243a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax(a+bx^3)^3 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x(a+bx^3)^2 \sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x(a+bx^3) \sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(455(ab+b^2x^3)) \int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x} \, dx}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(455(ab+b^2x^3)) \int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2} \, dx}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(455(ab+b^2x^3))\int \frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{1458a^{16/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(455\sqrt[3]{b}(ab+b^2x^3)\right)\int \frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{486a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(455(ab+b^2x^3))\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{16/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455(a+bx^3)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{10/3}bx^2 - 594a^{7/3}bx^2(a+bx^3) - 1179a^{4/3}bx^2(a+bx^3)^2\right.}{}$$

[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]

[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{455b^4x^{12}}{243a^5}-\frac{2275b^3x^9}{324a^4}-\frac{260b^2x^6}{27a^3}-\frac{1352b\,x^3}{243a^2}-\frac{1}{a}\right)}{(b\,x^3+a)^5 x}+\frac{455\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3a^{16}+\right.\right.\right.}{729(b\,x^3+a)}$
default	$-\frac{\left(-1820\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{13}-1820\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{13}+910\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{13}+5460\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{12}-\right.}{}$

[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-455/243/a^5*b^4*x^12-2275/324*b^3/a^4*x^9-260/27*b^2/a^3*x^6-1352/243*b/a^2*x^3-1/a)/x+455/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^16+3*b)*x-a^11*_R^2),_R=RootOf(_Z^3*a^16-b))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx =$$
$$-\frac{5460\,b^4x^{12}+20475\,ab^3x^9+28080\,a^2b^2x^6+16224\,a^3bx^3+2916\,a^4+1820\,\sqrt{3}(b^4x^{13}+4\,ab^3x^{10}+6\,a^2b^2x^7+}{}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*
x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*
a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(
b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 +
4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a
*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4
 + a^9*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^2 \left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = -\frac{1820 \, b^4 x^{12} + 6825 \, a b^3 x^9 + 9360 \, a^2 b^2 x^6 + 5408 \, a^3 b x^3 + 972 \, a^4}{972 \, (a^5 b^4 x^{13} + 4 \, a^6 b^3 x^{10} + 6 \, a^7 b^2 x^7 + 4 \, a^8 b x^4 + a^9 x)} - \frac{455 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 \, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{455 \, \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 \, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{455 \, \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 \, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
+ 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 + a
^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(
a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{455 \, b\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 \, a^6 \mathrm{sgn}\left(bx^3 + a\right)} + \frac{455 \sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 \, a^6 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{455 \, (-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 \, a^6 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{1}{a^5 x \mathrm{sgn}\left(bx^3 + a\right)} - \frac{848 \, b^4 x^{11} + 2937 \, a b^3 x^8 + 3528 \, a^2 b^2 x^5 + 1520 \, a^3 b x^2}{972 \, (bx^3 + a)^4 a^5 \mathrm{sgn}\left(bx^3 + a\right)}$$

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 455/729*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) + 45
5/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^6*b*sgn(b*x^3 + a)) - 455/1458*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^6*b*sgn(b*x^3 + a)) - 1/(a^5*x*sgn(b*x^3 + a)) -
1/972*(848*b^4*x^11 + 2937*a*b^3*x^8 + 3528*a^2*b^2*x^5 + 1520*a^3*b*x^2)/(
(b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)

[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)

3.116 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 398

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{770b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}\\
&+\frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] 154/243/a^4/x^2/((b*x^3+a)^2)^(1/2)+1/12/a/x^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+7/54/a^2/x^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+77/324/a^3/x^2/(b*x^3+a)/(
(b*x^3+a)^2)^(1/2)-385/243*(b*x^3+a)/a^5/x^2/((b*x^3+a)^2)^(1/2)-770/729*b^
(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/((b*x^3+a)^2)^(1/2)+385/729*
b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)/((b*x^
3+a)^2)^(1/2)+770/729*b^(2/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(17/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2} + \frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3} + \frac{770b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b^3\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(77b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^3}\,dx}{54a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(154b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)^2}\,dx}{81a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(770\left(ab+b^2x^3\right)\right)\int \frac{1}{x^3\left(ab+b^2x^3\right)}\,dx}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(770b\left(ab+b^2x^3\right)\right)\int \frac{1}{ab+b^2x^3}\,dx}{243a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{385\left(a+bx^3\right)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(770\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(770\sqrt[3]{b}\left(ab+b^2x^3\right)\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{b}-b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(385(ab+b^2x^3))\int\frac{-\sqrt[3]{a}b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{729a^{17/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(385b^{2/3}(ab+b^2x^3)\right)\int\frac{1}{a^{2/3}b^{2/3}-\sqrt[3]{a}bx+b^{4/3}x^2}\,dx}{243a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(770(ab+b^2x^3))\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243a^{17/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$=\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{770b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{11/3}bx-621a^{8/3}bx(a+bx^3)-1314a^{5/3}bx(a+bx^3)^2-3\right.}{}$$

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3
)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b
*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3
)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\sqrt{(bx^3+a)^2}\left(-\frac{385b^4x^{12}}{243a^5}-\frac{154b^3x^9}{27a^4}-\frac{2387b^2x^6}{324a^3}-\frac{931bx^3}{243a^2}-\frac{1}{2a}\right)}{(bx^3+a)^5x^2}+\frac{770\sqrt{(bx^3+a)^2}\left(\sum_{\_R=\text{RootOf}(a^{17}\_Z^3+b^2)}\_R\ln\left(\left(-4\_R^3a^{17}-\right.\right.\right.}{729(bx^3+a)}$
default	$-\frac{\left(-3080\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{14}+3080\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{14}-1540\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{14}+4620\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{12}-1\right.}{}$

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-385/243/a^5*b^4*x^12-154/27*b^3/a^4*x^9-2
387/324*b^2/a^3*x^6-931/243*b/a^2*x^3-1/2/a)/x^2+770/729*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^17-3*b^2)*x-a^6*b*_R),_R=RootOf(_Z^3*a^17+
b^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx =$$

$$-\frac{4620\, b^4x^{12} + 16632\, ab^3x^9 + 21483\, a^2b^2x^6 + 11172\, a^3bx^3 + 1458\, a^4 - 3080\, \sqrt{3}(b^4x^{14} + 4\, ab^3x^{11} + 6\, a^2b^2x^8}{}$$

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*
x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a
^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) +
 a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b
^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^3 \left((a + bx^3)^2\right)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx = -\frac{1540 b^4x^{12} + 5544 ab^3x^9 + 7161 a^2b^2x^6 + 3724 a^3bx^3 + 486 a^4}{972 (a^5b^4x^{14} + 4 a^6b^3x^{11} + 6 a^7b^2x^8 + 4 a^8bx^5 + a^9x^2)} - \frac{770 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{385 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{770 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
+ 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a
^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*
(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx = \frac{770 b\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 a^6 \mathrm{sgn}(bx^3 + a)} - \frac{770 \sqrt{3}(-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^6 \mathrm{sgn}(bx^3 + a)} - \frac{385 (-ab^2)^{\frac{1}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^6 \mathrm{sgn}(bx^3 + a)} - \frac{1}{2 a^5x^2 \mathrm{sgn}(bx^3 + a)} - \frac{1054 b^4x^{10} + 3600 ab^3x^7 + 4245 a^2b^2x^4 + 1780 a^3bx}{972 (bx^3 + a)^4 a^5 \mathrm{sgn}(bx^3 + a)}$$

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 770/729*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) - 77
0/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
```

```
^(1/3))/(a^6*sgn(b*x^3 + a)) - 385/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^6*sgn(b*x^3 + a)) - 1/2/(a^5*x^2*sgn(b*x^3 + a)) - 1
/972*(1054*b^4*x^10 + 3600*a*b^3*x^7 + 4245*a^2*b^2*x^4 + 1780*a^3*b*x)/((b
*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x^3\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.117 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{5b(a+bx^3)\log(x)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b(a+bx^3)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

```
[Out] -4/3*b/a^5/((b*x^3+a)^2)^(1/2)-1/12*b/a^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-2
/9*b/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-1/2*b/a^4/(b*x^3+a)/((b*x^3+a)^2)^
(1/2)+1/3*(-b*x^3-a)/a^5/x^3/((b*x^3+a)^2)^(1/2)-5*b*(b*x^3+a)*ln(x)/a^6/((
b*x^3+a)^2)^(1/2)+5/3*b*(b*x^3+a)*ln(b*x^3+a)/a^6/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{1369, 272, 46\}$

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{b}{12a^2 \left(a+bx^3\right)^3 \sqrt{a^2+2abx^3+b^2x^6}} - \frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5b(a+bx^3)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}} - \frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{1}{x^4(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(b^4(ab+b^2x^3))\,\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(b^4(ab+b^2x^3))\,\text{Subst}\left(\int \left(\frac{1}{a^5b^5x^2} - \frac{5}{a^6b^4x} + \frac{1}{a^2b^3(a+bx)^5} + \frac{2}{a^3b^3(a+bx)^4} + \frac{3}{a^4b^3(a+bx)^3} + \frac{4}{a^5b^3(a+bx)^2} + \frac{5}{a^6b^3(a}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{12a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{9a^3(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{2a^4(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5b(a+bx^3)\log(x)}{a^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5b(a+bx^3)\log(a+bx^3)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{-a(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12}) - 180bx^3(a+bx^3}{36a^6x^3(a+bx^3)^3\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^1
2)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3
])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.42

method	result
pseudoelliptic	$-\frac{\left(-5bx^3(bx^3+a)^4\ln(bx^3+a)+5bx^3(bx^3+a)^4\ln(bx^3)+a\left(5b^4x^{12}+\frac{35}{2}ab^3x^9+\frac{65}{3}a^2b^2x^6+\frac{125}{12}a^3bx^3+a^4\right)\right)\operatorname{csgn}(bx^3+a)}{3(bx^3+a)^4a^6x^3}$
risch	$\frac{\sqrt{(bx^3+a)^2}\left(-\frac{1}{3a}-\frac{125bx^3}{36a^2}-\frac{65b^2x^6}{9a^3}-\frac{35b^3x^9}{6a^4}-\frac{5b^4x^{12}}{3a^5}\right)}{(bx^3+a)^5x^3} - \frac{5\sqrt{(bx^3+a)^2}\,b\ln(x)}{(bx^3+a)a^6} + \frac{5\sqrt{(bx^3+a)^2}\,b\ln(-bx^3-a)}{3(bx^3+a)a^6}$
default	$\frac{(60\ln(bx^3+a)b^5x^{15}-180b^5\ln(x)x^{15}+240\ln(bx^3+a)ab^4x^{12}-720b^4a\ln(x)x^{12}-60ab^4x^{12}+360\ln(bx^3+a)a^2b^3x^9-1080a^2b^3}{36}$

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-5*b*x^3*(b*x^3+a)^4*ln(b*x^3+a)+5*b*x^3*(b*x^3+a)^4*ln(b*x^3)+a*(5*b
^4*x^12+35/2*a*b^3*x^9+65/3*a^2*b^2*x^6+125/12*a^3*b*x^3+a^4))*csgn(b*x^3+a
)/(b*x^3+a)^4/a^6/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\,(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4}{36\,(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*
b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^
3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x
^9 + 4*a^9*b*x^6 + a^10*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^4 \left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

$$\begin{aligned}\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{5\,(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^6} \\ &\quad - \frac{5\,b}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^3} - \frac{5\,b}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^5} \\ &\quad - \frac{1}{3\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2x^3} - \frac{5}{6\,\left(x^3+\frac{a}{b}\right)^2a^4b} - \frac{1}{12\,\left(x^3+\frac{a}{b}\right)^4a^2b^3}\end{aligned}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^6
 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 + 2*
a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) - 5/6
/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{5 \, b \log \left(|bx^3 + a| \right)}{3 \, a^6 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{5 \, b \log \left(|x| \right)}{a^6 \mathrm{sgn}\left(bx^3 + a\right)} + \frac{5 \, bx^3 - a}{3 \, a^6 x^3 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{125 \, b^5 x^{12} + 548 \, ab^4 x^9 + 912 \, a^2 b^3 x^6 + 688 \, a^3 b^2 x^3 + 202 \, a^4 b}{36 \, (bx^3 + a)^4 a^6 \mathrm{sgn}\left(bx^3 + a\right)}$$

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/3*b*log(abs(b*x^3 + a))/(a^6*sgn(b*x^3 + a)) - 5*b*log(abs(x))/(a^6*sgn(b
*x^3 + a)) + 1/3*(5*b*x^3 - a)/(a^6*x^3*sgn(b*x^3 + a)) - 1/36*(125*b^5*x^1
2 + 548*a*b^4*x^9 + 912*a^2*b^3*x^6 + 688*a^3*b^2*x^3 + 202*a^4*b)/((b*x^3
+ a)^4*a^6*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^4 \left(a^2 + 2 \, a \, b \, x^3 + b^2 \, x^6\right)^{5/2}} \, dx$$

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal result

Integrand size = 28, antiderivative size = 313

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)} \\ &+ \frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)} + \frac{10a^3b^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\left(a+bx^3\right)} \\ &+ \frac{10a^2b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\left(a+bx^3\right)} \\ &+ \frac{5ab^4(dx)^{13+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{13}(13+m)\left(a+bx^3\right)} + \frac{b^5(dx)^{16+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{16}(16+m)\left(a+bx^3\right)}\end{aligned}$$

```
[Out] a^5*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+5*a^4*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+10*a^3*b^2*(d*x)^(7+m)*((b*x^3+a)^2)
^(1/2)/d^7/(7+m)/(b*x^3+a)+10*a^2*b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10
/(10+m)/(b*x^3+a)+5*a*b^4*(d*x)^(13+m)*((b*x^3+a)^2)^(1/2)/d^13/(13+m)/(b*x
^3+a)+b^5*(d*x)^(16+m)*((b*x^3+a)^2)^(1/2)/d^16/(16+m)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used

$= \{1369, 276\}$

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+16}}{d^{16}(m+16)\left(a+bx^3\right)} + \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+13}}{d^{13}(m+13)\left(a+bx^3\right)} + \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)} + \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)} + \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)} + \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}$$

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^5 dx}{b^4\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5(dx)^m + \frac{5a^4b^6(dx)^{3+m}}{d^3} + \frac{10a^3b^7(dx)^{6+m}}{d^6} + \frac{10a^2b^8(dx)^{9+m}}{d^9} + \frac{5ab^9(dx)^{12+m}}{d^{12}} + \frac{b^{10}(d}{}\right.}{b^4\left(ab+b^2x^3\right)}$$

$$
\begin{aligned}
&= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)} + \frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)} \\
&\quad + \frac{10a^3b^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\,(a+bx^3)} + \frac{10a^2b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\,(a+bx^3)} \\
&\quad + \frac{5ab^4(dx)^{13+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{13}(13+m)\,(a+bx^3)} + \frac{b^5(dx)^{16+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{16}(16+m)\,(a+bx^3)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.35

$$
\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^{5/2}\left(\frac{a^5}{1+m}+\frac{5a^4bx^3}{4+m}+\frac{10a^3b^2x^6}{7+m}+\frac{10a^2b^3x^9}{10+m}+\frac{5ab^4x^{12}}{13+m}+\frac{b^5x^{15}}{16+m}\right)}{\left(a+bx^3\right)^5}
$$

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10
*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m)
 + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.45

method	result
gosper	$\frac{x\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5x^{15}+10\right.}{}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+364\right.}{}$

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5*
m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*x^
15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b^4*
m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b
^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+582
40*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+1239
20*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m
^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m
```

```
^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(1+m)/(4+m)/(7+m)/(10
+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.18

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \frac{((b^5m^5 + 35\,b^5m^4 + 445\,b^5m^3 + 2485\,b^5m^2 + 5714\,b^5m + 3640\,b^5)x^{16} + 5\,(ab^4m^5 + 38\,ab^4m^4 -}{}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Sympy [F]

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \int (dx)^m \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \frac{((m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640)b^5d^mx^{16} + 5\,(m^5 + 38\,m^4 + 511\,m^3 + 29}{}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
+ 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 4
1*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44
*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m
^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 +
955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m
^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(247) = 494$.

Time = 0.36 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.88

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
 + 45474*m^2 + 95064*m + 58240)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int (d\,x)^m \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$

Optimal result

Integrand size = 28, antiderivative size = 205

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)} + \frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)} + \frac{3ab^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\,(a+bx^3)} + \frac{b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\,(a+bx^3)}$$

```
[Out] a^3*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+3*a^2*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+3*a*b^2*(d*x)^(7+m)*((b*x^3+a)^2)^(1
/2)/d^7/(7+m)/(b*x^3+a)+b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10/(10+m)/(b
*x^3+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 276}

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)\,(a+bx^3)} + \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)\,(a+bx^3)} + \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)\,(a+bx^3)} + \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)\,(a+bx^3)}$$

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{3+m}}{d^3}+\frac{3ab^5(dx)^{6+m}}{d^6}+\frac{b^6(dx)^{9+m}}{d^9}\right)dx}{b^2\left(ab+b^2x^3\right)}\\ &= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}\\ &\quad+\frac{3ab^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\left(a+bx^3\right)}+\frac{b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int (dx)^m\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x(dx)^m\sqrt{\left(a+bx^3\right)^2}\left(a^3\left(280+138m+21m^2+m^3\right)+3a^2b\left(70+87m+18m^2+m^3\right)x^3+3ab\right.}{(1+m)(4+m)(7+m)(10+m)\,(a}$$

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*
(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b
^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a +
b*x^3))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{x\left(b^{3} m^{3} x^{9}+12 b^{3} m^{2} x^{9}+39 m\,x^{9} b^{3}+3 a\,b^{2} m^{3} x^{6}+28 b^{3} x^{9}+45 a\,b^{2} m^{2} x^{6}+162 m\,x^{6} b^{2} a+3 a^{2} b\,m^{3} x^{3}+120 b^{2} x^{6} a+54 a^{2} b\,m^{2} x^{3}+261 m\,x^{3} a^{2}}{(10+m)(7+m)(4+m)(1+m)\left(b\,x^{3}+a\right)^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b^{3} m^{3} x^{9}+12 b^{3} m^{2} x^{9}+39 m\,x^{9} b^{3}+3 a\,b^{2} m^{3} x^{6}+28 b^{3} x^{9}+45 a\,b^{2} m^{2} x^{6}+162 m\,x^{6} b^{2} a+3 a^{2} b\,m^{3} x^{3}+120 b^{2} x^{6} a+54 a^{2} b\,m^{2} x^{3}}{\left(b\,x^{3}+a\right)(10+m)(7+m)(4+m)(1+m)}$

[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(4+m)/(1+m)/(b*x^3+a)^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{\left((b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3)x^{10}+3\,(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2)x^7+3\,(a}{m^4+22\,m^3+159\,m}$$

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \frac{((m^3 + 12\,m^2 + 39\,m + 28)b^3d^mx^{10} + 3\,(m^3 + 15\,m^2 + 54\,m + 40)ab^2d^mx^7 + 3\,(m^3 + 18\,m^2 -}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m +}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b
^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 1
38*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(161) = 322.

Time = 0.31 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.87

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \frac{(dx)^m\, b^3m^3x^{10}\mathrm{sgn}(bx^3 + a) + 12\,(dx)^m\, b^3m^2x^{10}\mathrm{sgn}(bx^3 + a) + 39\,(dx)^m\, b^3mx^{10}\mathrm{sgn}(bx^3 + a) +}{}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x
^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn
(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*
x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*
b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m
*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)
^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x
)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 97

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)(a+bx^3)} + \frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)(a+bx^3)}$$

```
[Out] a*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+b*(d*x)^(4+m)*((b*x^3+a
)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 14}

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{a\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}$$

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(ab+b^2x^3\right)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(ab(dx)^m+\frac{b^2(dx)^{3+m}}{d^3}\right)\,dx}{ab+b^2x^3}\\ &= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x(dx)^m\sqrt{\left(a+bx^3\right)^2}\left(a(4+m)+b(1+m)x^3\right)}{(1+m)(4+m)\left(a+bx^3\right)}$$

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m
)*(a + b*x^3))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)(dx)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{(4+m)(1+m)\left(b\,x^{3}+a\right)}$	56
risch	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)(dx)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{(4+m)(1+m)\left(b\,x^{3}+a\right)}$	56

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{((bm + b)x^4 + (am + 4\,a)x)(dx)^m}{m^2 + 5\,m + 4}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

Sympy [F]

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \int (dx)^m \sqrt{(a + bx^3)^2}\, dx$$

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{(bd^m(m + 1)x^4 + ad^m(m + 4)x)x^m}{m^2 + 5\,m + 4}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx$$
$$=\frac{(dx)^m\,bmx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,bx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,amx\mathrm{sgn}(bx^3+a)+4\,(dx)^m\,ax\mathrm{sgn}(bx^3+a)}{m^2+5\,m+4}$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*
m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(dx)^{1+m}\left(a+bx^3\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/d/(1
+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\left(a+bx^3\right)(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(ab+b^2x^3)\int \frac{(dx)^m}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(dx)^{1+m}(a+bx^3)\ {}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x(dx)^m(a+bx^3)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*
x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{b^2x^6+2ab\,x^3+a^2}}dx$$

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
[Out] integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(a+bx^3)^2}}\,dx$$

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
[Out] Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^6+2\,abx^3+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}} \,dx$$

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.122 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(dx)^{1+m}(a+bx^3)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(a+bx^3)(dx)^{m+1}\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{(dx)^m}{(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(dx)^{1+m}\,(a+bx^3)\;{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x(dx)^m\,(a+bx^3)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3(1+m)\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6+2ab\,x^3+a^2)^{\frac{3}{2}}}dx$$

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}\,dx$$

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(dx)^{1+m}(a+bx^3)\,\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([5, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^5/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)(dx)^{m+1}\,\text{Hypergeometric2F1}\left(5,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{(dx)^m}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(dx)^{1+m}\,(a+bx^3)\;{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{x(dx)^m\,(a+bx^3)\,\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5(1+m)\sqrt{(a+bx^3)^2}}$$

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6+2ab\,x^3+a^2)^{\frac{5}{2}}}dx$$

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15 +
 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{(d\,x)^m}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}} \,dx$$

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,\frac{4+m}{3},-\frac{bx^3}{a}\right)}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2*p, 1/3+1/3*m],[4/3+1/3*
m],-b*x^3/a)/d/(1+m)/((1+b*x^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1370, 371}

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{3},-2p,\frac{m+4}{3},-\frac{bx^3}{a}\right)}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3, -
2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int(dx)^m\left(1+\frac{bx^3}{a}\right)^{2p}dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1+m}{3},-2p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int(dx)^m\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$= \frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{1+m}$$

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] (x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int(dx)^m\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int (dx)^m \left(\left(a+bx^3\right)^2\right)^p \, dx$$

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral((d*x)**m*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int (d\,x)^m \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \,dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.125 $\int x^{11}(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a^3(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a(a+bx^3)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(3+2p)}+\frac{(a+bx^3)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(2+p)}$$

```
[Out] -1/3*a^3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+2*p)+1/2*a^2*(b*x^3+a)^
2*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(p+1)-a*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)
^p/b^4/(3+2*p)+1/6*(b*x^3+a)^4*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(2+p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1370, 272, 45}

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{\left(a+bx^3\right)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(p+2)}-\frac{a\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(2p+3)}+\frac{a^2\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(p+1)}-\frac{a^3\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(2p+1)}$$

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -1/3*(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(1 + 2*p)) + (a^2
*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x
^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2
+ 2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^{11}\left(1+\frac{bx^3}{a}\right)^{2p}\,dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right)^{1+2p}}{b^3}-\frac{3a^3\left(1+\frac{bx}{a}\right)^{2+2p}}{b^3}+\frac{a^3\left(1+\frac{bx}{a}\right)^{3+2p}}{b^3}\right)dx,x,x^3\right)$$
$$=-\frac{a^3(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a(a+bx^3)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(3+2p)}+\frac{(a+bx^3)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(2+p)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(1+2p)x^3-3ab^2(1+3p+2p^2)x^6+b^3(3+11p+12p^2+4p^3)x^9\right)}{6b^4(1+p)(2+p)(1+2p)(3+2p)}$$

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{(b^2x^6+2ab\,x^3+a^2)^p\left(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3b^3x^9+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3b^2x^6a-6a^2bp\,x^3-3a^2b\,x^3+3a^3\right)(b\,x^3}{6b^4(4p^4+20p^3+35p^2+25p+6)}$
risch	$-\frac{\left(-4b^4p^3x^{12}-12b^4p^2x^{12}-11b^4p\,x^{12}-4a\,b^3p^3x^9-3b^4x^{12}-6a\,b^3p^2x^9-2ap\,x^9b^3+6a^2b^2p^2x^6+3a^2p\,x^6b^2-6a^3p\,x^3b+3a^4\right)\left((b\right.}{6(3+2p)(2+p)(1+p)(1+2p)b^4}$
parallelrisch	$\frac{4x^{12}(b^2x^6+2ab\,x^3+a^2)^pb^4p^3+12x^{12}(b^2x^6+2ab\,x^3+a^2)^pb^4p^2+11x^{12}(b^2x^6+2ab\,x^3+a^2)^pb^4p+3x^{12}(b^2x^6+2ab\,x^3+a^2)^pb^4+4}{}$

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9-
3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b*x
^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^{12}+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^9+6\,a^3bpx^3-3\,(2\,a^2b^2p^2+a^2b^2p)x^6}{6\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3*
a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^
6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p
^2 + 25*b^4*p + 6*b^4)
```

Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(x - (-a/b)**(1/3))/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*a**3*l
og(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b*
*5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 +
 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(x
```

```
- (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b*
*7*x**9) + 18*a**2*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))
/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*
*2*b*x**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b*
*7*x**9) + 27*a**2*b*x**3/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**
6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(4*x*
*2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3
 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x**6*log(2)/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6/(18*a**3
*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9*lo
g(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 1
8*b**7*x**9) + 6*b**3*x**9*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3)
)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*b
**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**
7*x**9), Eq(p, -2)), (Integral(x**11/((a + b*x**3)**2)**(3/2), x), Eq(p, -3
/2)), (6*a**3*log(x - (-a/b)**(1/3))/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(
4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**4 + 6*b**5*x**3) - 12
*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3/(6*a*b**4 + 6*b**5*x**3) + 6
*a**2*b*x**3*log(x - (-a/b)**(1/3))/(6*a*b**4 + 6*b**5*x**3) + 6*a**2*b*x**
3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**4 + 6*b**5*x**3
) - 12*a**2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*a*b**2*x**6/(6*a*b**
4 + 6*b**5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3), Eq(p, -1)), (Integra
l(x**11/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**
3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*
p + 36*b**4) + 6*a**3*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*
p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 6*a**2*b**2*
p**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3
+ 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 3*a**2*b**2*p*x**6*(a**2 + 2*a*b*
x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b*
*4*p + 36*b**4) + 4*a*b**3*p**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24
*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 6*a*b*
*3*p**2*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p*
*3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 2*a*b**3*p*x**9*(a**2 + 2*a*b*
x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b*
*4*p + 36*b**4) + 4*b**4*p**3*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*
b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 12*b**4
*p**2*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**
3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 11*b**4*p*x**12*(a**2 + 2*a*b*x
**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**
4*p + 36*b**4) + 3*b**4*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p
**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{((4\,p^3+12\,p^2+11\,p+3)b^4x^{12}+2\,(2\,p^3+3\,p^2+p)ab^3x^9-3\,(2\,p^2+p)a^2b^2x^6+6\,a^3bpx^3-3\,a^4)(bx^3+a)}{6\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^4}$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^9
 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/((4
*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.

Time = 0.31 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.18

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{4\,(b^2x^6+2\,abx^3+a^2)^pb^4p^3x^{12}+12\,(b^2x^6+2\,abx^3+a^2)^pb^4p^2x^{12}+11\,(b^2x^6+2\,abx^3+a^2)^pb^4px^{12}+4\,(b^2}{}$$

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p^2*
x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4*p^4 +
20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^{12}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{6\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{a^4}{2\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a^3\,p\,x^3}{b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a\,p\,x^9\,\left(2\,p^2+3\,p+1\right)}{3\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{a^2\,p\,x^6\,\left(2\,p+1\right)}{2\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
+ 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*p^
4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*x^9
*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p*x^6
*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

3.126 $\int x^8(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int x^8(a^2+2abx^3+b^2x^6)^p\,dx=\frac{a^2(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^3(1+2p)}-\frac{a(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{3b^3(1+p)}+\frac{(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{3b^3(3+2p)}$$

```
[Out] 1/3*a^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+2*p)-1/3*a*(b*x^3+a)^2*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(p+1)+1/3*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)^
p/b^3/(3+2*p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1370, 272, 45}

$$\int x^8(a^2+2abx^3+b^2x^6)^p\,dx=\frac{(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{3b^3(2p+3)}-\frac{a(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{3b^3(p+1)}+\frac{a^2(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^3(2p+1)}$$

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^8\left(1+\frac{bx^3}{a}\right)^{2p}dx$$

$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^3\right)$$

$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{1+2p}}{b^2}+\frac{a^2\left(1+\frac{bx}{a}\right)^{2+2p}}{b^2}\right)dx,x,x^3\right)$$

$$=\frac{a^2(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{(a+bx^3)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(3+2p)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(a+bx^3)\left((a+bx^3)^2\right)^p \left(a^2-ab(1+2p)x^3+b^2\left(1+3p+2p^2\right)x^6\right)}{3b^3(1+p)(1+2p)(3+2p)}$$

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*
p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{\left(b\,x^3+a\right)\left(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-ab\,x^3+a^2\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3b^3\left(4p^3+12p^2+11p+3\right)}$
risch	$\frac{\left(2b^3p^2x^9+3b^3p\,x^9+b^3x^9+2a\,b^2p^2x^6+a\,b^2p\,x^6-2a^2bp\,x^3+a^3\right)\left(\left(b\,x^3+a\right)^2\right)^p}{3(1+p)(3+2p)(1+2p)b^3}$
parallelrisch	$\frac{2x^9\left(b^2x^6+2ab\,x^3+a^2\right)^p a\,b^3p^2+3x^9\left(b^2x^6+2ab\,x^3+a^2\right)^p a\,b^3p+x^9\left(b^2x^6+2ab\,x^3+a^2\right)^p a\,b^3+2x^6\left(b^2x^6+2ab\,x^3+a^2\right)^p a^2b^2p^2+x}{3(3+2p)(1+p)(1+2p)a\,b^3}$

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{\left((2\,b^3p^2+3\,b^3p+b^3)x^9-2\,a^2bpx^3+(2\,ab^2p^2+ab^2p)x^6+a^3\right)(b^2x^6+2\,abx^3+a^2)^p}{3\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2
*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

Sympy [F]

$$\int x^8 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx$$

$$= \begin{cases} \frac{x^9 (a^2)^p}{9} \\ \int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{3}{2}}} \, dx \\ -\frac{2a^2 \log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2 \log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2}{3ab^3+3b^4x^3} + \frac{4a^2\log(2)}{3ab^3+3b^4x^3} - \frac{2abx^3 \log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2abx^3 \log}{} \\ \int \frac{x^8}{\sqrt{(a+bx^3)^2}} \, dx \\ \frac{a^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} - \frac{2a^2bpx^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2ab^2p^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{ab^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2b^3p^2x^9(a}{12b^3p^3+36} \end{cases}$$

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*
x**3) - 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**3
+ 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b**3
+ 3*b**4*x**3) - 2*a*b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*x**3)
 - 2*a*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**3 +
 3*b**4*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**6/(3*a
*b**3 + 3*b**4*x**3), Eq(p, -1)), (Integral(x**8/sqrt((a + b*x**3)**2), x),
 Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*
b**3*p**2 + 33*b**3*p + 9*b**3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2
*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*a*b**2*p**
2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33
*b**3*p + 9*b**3) + a*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b*
*3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*b**3*p**2*x**9*(a**2 + 2*a
*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3)
+ 3*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*
p**2 + 33*b**3*p + 9*b**3) + b**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(
12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{((2\,p^2+3\,p+1)b^3x^9+(2\,p^2+p)ab^2x^6-2\,a^2bpx^3+a^3)(bx^3+a)^{2\,p}}{3\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^
3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{2\,(b^2x^6+2\,abx^3+a^2)^pb^3p^2x^9+3\,(b^2x^6+2\,abx^3+a^2)^pb^3px^9+(b^2x^6+2\,abx^3+a^2)^pb^3x^9+2\,(b^2x^6+2\,ab}{3\,(4\,b^3p^3+12\,b^3p}$$

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x^6
- 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,\left(\frac{x^9\,\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^6\,\left(2\,p+1\right)}{3\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*b^
2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2 +
4*p^3 + 3)))
```

3.127 $\int x^5(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

```
[Out] -1/3*a*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^2/(1+2*p)+1/6*(b*x^3+a)^2*(b^2
*x^6+2*a*b*x^3+a^2)^p/b^2/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1370, 272, 45}

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -1/3*(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^2*(1 + 2*p)) + ((a +
b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^5\left(1+\frac{bx^3}{a}\right)^{2p}dx\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^3\right)\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}\right.\right.\\
&\qquad\left.\left.+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)dx,x,x^3\right)\\
&= -\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-a+b(1+2p)x^3\right)}{6b^2(1+p)(1+2p)}$$

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
2*p))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\left(-2b^{2}p\,x^{6}-b^{2}x^{6}-2abp\,x^{3}+a^{2}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{p}}{6b^{2}(1+p)(1+2p)}$	58
gosper	$-\frac{\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}\left(-2x^{3}pb-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)}{6b^{2}(2p^{2}+3p+1)}$	60
norman	$\frac{x^{6}e^{p\ln\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)}}{6p+6}-\frac{a^{2}e^{p\ln\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)}}{6b^{2}(2p^{2}+3p+1)}+\frac{pa\,x^{3}e^{p\ln\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)}}{3b(2p^{2}+3p+1)}$	120
parallelrisch	$\frac{2x^{6}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}b^{2}p+x^{6}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}b^{2}+2x^{3}\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}abp-\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}a^{2}}{6b^{2}(2p^{2}+3p+1)}$	128

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
[Out] -1/6*(-2*b^2*p*x^6-b^2*x^6-2*a*b*p*x^3+a^2)/b^2/(1+p)/(1+2*p)*((b*x^3+a)^2)
^p
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{((2\,b^2p+b^2)x^6+2\,abpx^3-a^2)(b^2x^6+2\,abx^3+a^2)^p}{6\,(2\,b^2p^2+3\,b^2p+b^2)}$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F]

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$=\begin{cases}\frac{x^6(a^2)^p}{6}\\ \frac{a\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3}+\frac{a\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3}-\frac{2a\log(2)}{3ab^2+3b^3x^3}+\frac{a}{3ab^2+3b^3x^3}+\frac{bx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3}+\frac{bx^3\log\left(4x^2+4x\right.}{3ab^2}\\ \int\frac{x^5}{\sqrt{(a+bx^3)^2}}\,dx\\ -\frac{a^2(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2abpx^3(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2b^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{b^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}\end{cases}$$

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(x - (-a/b)**(1/3))/(3*a*b**2
 + 3*b**3*x**3) + a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*
b**2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3
*b**3*x**3) + b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) + b*x*
*3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**3*x**
3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -1)), (Integral(x**5/s
qrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2*a*b*x**
3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x**6*(a**2
 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + b**2*x*
*6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \,dx = \frac{\left(b^2(2\,p+1)x^6+2\,abpx^3-a^2\right)\left(bx^3+a\right)^{2\,p}}{6\,(2\,p^2+3\,p+1)b^2}$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \,dx = \frac{2\,(b^2x^6+2\,abx^3+a^2)^pb^2px^6+(b^2x^6+2\,abx^3+a^2)^pb^2x^6+2\,(b^2x^6+2\,abx^3+a^2)^pabpx^3-(b^2x^6+2\,abx^3}{6\,(2\,b^2p^2+3\,b^2p+b^2)}$$

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*b*
x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \left(\frac{x^6\,(2\,p+1)}{6\,\left(2\,p^2+3\,p+1\right)} - \frac{a^2}{6\,b^2\,\left(2\,p^2+3\,p+1\right)} + \frac{a\,p\,x^3}{3\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2/
(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

3.128 $\int x^4(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

```
[Out] 1/5*x^5*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([5/3, -2*p],[8/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1370, 371}

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x
^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^4\left(1+\frac{bx^3}{a}\right)^{2p}dx\\ &= \frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] (x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int x^4\left(b^2x^6+2ab\,x^3+a^2\right)^p dx$$

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx=\int (b^2x^6+2\,abx^3+a^2)^p x^4\,dx$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Sympy [F]

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx=\int x^4\left((a+bx^3)^2\right)^p\,dx$$

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral(x**4*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx=\int (b^2x^6+2\,abx^3+a^2)^p x^4\,dx$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Giac [F]

$$\int x^4(a^2+2abx^3+b^2x^6)^p\,dx=\int (b^2x^6+2\,abx^3+a^2)^p x^4\,dx$$

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x^4\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.129 $\int x^3(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

```
[Out] 1/4*x^4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([4/3, -2*p],[7/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1370, 371}

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x
^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^3\left(1+\frac{bx^3}{a}\right)^{2p}dx\\ &= \frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{4}x^4\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] (x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int x^3\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x^3 \, dx$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Sympy [F]

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \int x^3 \left((a + bx^3)^2\right)^p \, dx$$

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral(x**3*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x^3 \, dx$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Giac [F]

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x^3 \, dx$$

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a^2+2abx^3+b^2x^6)^p\,dx=\int x^3\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.130 $\int x^2(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}$$

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1366, 623}

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(2p+1)}$$

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^p \, dx, x, x^3\right)$$
$$= \frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(a+bx^3)\left((a+bx^3)^2\right)^p}{3b(1+2p)}$$

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(b\,x^3+a)\left((b\,x^3+a)^2\right)^p}{3b(1+2p)}$	31
gosper	$\frac{(b\,x^3+a)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3b(1+2p)}$	40
parallelrisch	$\frac{x^3\left(b^2x^6+2ab\,x^3+a^2\right)^p ab+\left(b^2x^6+2ab\,x^3+a^2\right)^p a^2}{3a(1+2p)b}$	67
norman	$\frac{x^3 e^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{6p+3}+\frac{a\,e^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{3b(1+2p)}$	71

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x^3+a)/b/(1+2*p)*((b*x^3+a)^2)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(bx^3+a)(b^2x^6+2\,abx^3+a^2)^p}{3\,(2\,bp+b)}$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

Sympy [F]

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\begin{cases}\frac{x^3}{3\sqrt{a^2}} & \text{for } b=0\wedge p=-\frac{1}{2}\\ \frac{x^3(a^2)^p}{3} & \text{for } b=0\\ \int\frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b}+\frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3,
 Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a**
2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(6*b*p + 3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(bx^3+a)(bx^3+a)^{2\,p}}{3\,b(2\,p+1)}$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(b^2x^6+2\,abx^3+a^2)^pbx^3+(b^2x^6+2\,abx^3+a^2)^pa}{3\,(2\,bp+b)}$$

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)
/(2*b*p + b)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\frac{x^3}{3\,\left(2\,p+1\right)}+\frac{a}{3\,b\,\left(2\,p+1\right)}\right)\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p$$

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] (x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

3.131 $\int x(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int x(a^2+2abx^3+b^2x^6)^p\,dx = \frac{x^2(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(1,\frac{5}{3}+2p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 5/3+2*p],[5/3],-b
*x^3/a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1370, 371}

$$\int x(a^2+2abx^3+b^2x^6)^p\,dx = \frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x
^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x\left(1+\frac{bx^3}{a}\right)^{2p}dx\\ &= \frac{1}{2}x^2\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] (x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int x\left(b^2x^6+2ab\,x^3+a^2\right)^p dx$$

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x \, dx$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Sympy [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int x\left((a + bx^3)^2\right)^p \, dx$$

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral(x*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x \, dx$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Giac [F]

$$\int x(a^2 + 2abx^3 + b^2x^6)^p \, dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x \, dx$$

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.132 $\int (a^2+2abx^3+b^2x^6)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = \frac{x(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(1,\frac{4}{3}+2p,\frac{4}{3},-\frac{bx^3}{a}\right)}{a}$$

```
[Out] x*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 4/3+2*p],[4/3],-b*x^3/a
)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1357, 252, 251}

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{1}{3},-2p,\frac{4}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^3
)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(2ab+2b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\left(2ab+2b^2x^3\right)^{2p}\,dx\\ &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= x\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.98

$$\int\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$=\frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p\text{AppellF1}\left(1+2p,-2p,\right.}{\sqrt[3]{b}(1+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p,
 -2*p, 2*(1 + p), -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (-1
)^(1/3))*a^(1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3
])])/(4^p*b^(1/3)*(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3)))^(2*p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2*p))
```

Maple [F]

$$\int \left(b^2x^6 + 2ab\,x^3 + a^2\right)^p dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Sympy [F]

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

Maxima [F]

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Giac [F]

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p \, dx$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")

[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p \, dx$$

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)

3.133 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x} dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x} dx = -\frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

```
[Out] -1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*x
^3/a)/a/(1+2*p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1370, 272, 67}

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x} dx = -\frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -1/3*((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1 + 2*
p, 2*(1 + p), 1 + (b*x^3)/a])/(a*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x}\,dx\\ &= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx,x,x^3\right)\\ &= -\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a(1+2p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int\frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x}\,dx = -\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\,\text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
[Out] -1/3*((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
1 + (b*x^3)/a])/(a*(1 + 2*p))
```

Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x} dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x}\, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x}\, dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x}\, dx = \int \frac{\left((a + bx^3)^2\right)^p}{x}\, dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
[Out] Integral(((a + b*x**3)**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x}\, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x}\, dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x} \, dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p}{x} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x, x)
```

3.134 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

```
[Out] -(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-1/3, -2*p],[2/3],-b*x^3/a)/x/((1+b*x
^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1370, 371}

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2} dx = -\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^
3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^2}\,dx$$

$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int\frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^2}\,dx$$
$$= -\frac{\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2*p)))
```

Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^2}dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^2}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^2}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
[Out] Integral(((a + b*x**3)**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^2}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^2} \, dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p}{x^2} \, dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2, x)
```

3.135 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p} (a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

```
[Out] -1/2*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2/3, -2*p],[1/3],-b*x^3/a)/x^2/(
(1+b*x^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1370, 371}

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx = -\frac{\left(\frac{bx^3}{a}+1\right)^{-2p} (a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b
*x^3)/a)])/(x^2*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^3}\,dx$$

$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^3}\,dx$$

$$= -\frac{\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
[Out] -1/2*(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/(
x^2*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^3}dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^3}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
[Out] Integral(((a + b*x**3)**2)**p/x**3, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^3}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^3}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \frac{b(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

```
[Out] 1/3*b*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b*
x^3/a)/a^2/(1+2*p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1370, 272, 67}

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \frac{b(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(2,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^4}\,dx\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx,x,x^3\right)\\
&= \frac{b(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int\frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^4}\,dx = \frac{b(a+bx^3)\left((a+bx^3)^2\right)^p\text{Hypergeometric2F1}\left(2,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
[Out] (b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Maple [F]

$$\int \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p}{x^4}dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^4}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
[Out] Integral(((a + b*x**3)**2)**p/x**4, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^4}\,dx$$

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")

[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^4}\,dx$$

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4, x)

3.137 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

```
[Out] -1/4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-4/3, -2*p],[-1/3],-b*x^3/a)/x^4/
((1+b*x^3/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1370, 371}

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5} dx = -\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((
b*x^3)/a)])/(x^4*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^5}\,dx$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int\frac{\left(a^2+2abx^3+b^2x^6\right)^p}{x^5}\,dx$$
$$= -\frac{\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
[Out] -1/4*(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])/
(x^4*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]

$$\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{x^5}dx$$

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^5}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{\left((a+bx^3)^2\right)^p}{x^5}\,dx$$

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
[Out] Integral(((a + b*x**3)**2)**p/x**5, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^5}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^5}\,dx$$

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^5}\,dx$$

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^8}{a+bx^3+cx^6} dx = \frac{x^3}{3c} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^3+cx^6)}{6c^2}$$

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1371, 717, 648, 632, 212, 642}

$$\int \frac{x^8}{a+bx^3+cx^6} dx = -\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^3+cx^6)}{6c^2} + \frac{x^3}{3c}$$

```
[In] Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\
&= \frac{x^3}{3c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}\\
&= \frac{x^3}{3c}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}\\
&= \frac{x^3}{3c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx=\frac{2cx^3+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]

[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^3}{3c}+\frac{-\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$
risch	$\frac{x^3}{3c}-\frac{2\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^3+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{3c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(\right.\right.}{}$

[In] int(x^8/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3/c+1/3/c*(-1/2*b/c*ln(c*x^6+b*x^3+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx$$
$$=\left[\frac{2\left(b^2c-4\,ac^2\right)x^3-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^6\right.}{6\left(b^2c^2-4\,ac^3\right)}\right.$$

[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b
*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1

```
/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(75) = 150$.

Time = 1.72 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \left(-\frac{b}{6c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(}{6c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \left(-\frac{b}{6c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(}{6c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \frac{x^3}{3c}$$

```
[In] integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*
log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b
 - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a
*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx=\frac{x^3}{3\,c}-\frac{b\log\left(cx^6+bx^3+a\right)}{6\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*
c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 1758, normalized size of antiderivative = 21.70

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

```
[In] int(x^8/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b^
2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*a*
b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c
^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^
3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36
*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (2
7*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)
^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)
*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*b
^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2
*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((
45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a
*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/2)) + (((6*a^2*
c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 3
6*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(
```

```
2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2))))/(4*
a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)
 - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((36*a^2*c^5 - 7
2*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*
(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2
*a*c - b^2))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 -
9*b^2*c^2)) - (((15*a*b^3*c^2 - 12*a^2*b*c^3)/c^3 - ((3*b^3 - 12*a*b*c)*((3
6*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 -
 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^
2)^(1/2)) + (a*b*(2*a*c - b^2)^3)/(2*c^3*(4*a*c - b^2)^(3/2))))/(a^2*(b^6 -
 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)) + (b*c^2*(4*a*c - b^2)^(3/2)*((a*
b^4 - a^2*b^2*c)/c^3 + ((3*b^3 - 12*a*b*c)*((15*a*b^3*c^2 - 12*a^2*b*c^3)/c
^3 - ((3*b^3 - 12*a*b*c)*((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*
b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(
36*a*c^3 - 9*b^2*c^2)) + (((((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*
(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/
2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3
*a*b*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(2*c*(4*a*c - b^2)*(36*a*c^3 - 9*b
^2*c^2))))/(a^2*(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)))*(2*a*c - b
^2))/(3*c^2*(4*a*c - b^2)^(1/2))
```

3.139 $\int \frac{x^5}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1371, 648, 632, 212, 642}

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[In] Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^3\right) \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} \\
&= \frac{\log\left(a+bx^3+cx^6\right)}{6c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6),x]
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^3 + c*x^6])/(6*c)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^6+b\,x^3+a\right)}{6c}-\frac{b\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^3+2\sqrt{-b^2(4ac-b^2)}\,a\right)a}{3(4ac-b^2)}-\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^3+2\sqrt{-b^2(4ac-b^2)}\,a\right)b^2}{6c(4ac-b^2)}+$

```
[In] int(x^5/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+(b^2-4\,ac)\log\left(cx^6+bx^3+a\right)}{6\,(b^2c-4\,ac^2)}, \frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{arc}}{}\right.$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)} + \frac{1}{6c}\right)\log\left(x^3 + \frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6c\,(4ac-b^2)} + \frac{1}{6c}\right)\log\left(x^3 + \frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)$$

```
[In] integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c
*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b
*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2
)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
6*log(c*x^6 + b*x^3 + a)/c
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 1199, normalized size of antiderivative = 19.03

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(x^5/(a + b*x^3 + c*x^6),x)
```

```
[Out] (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*ata
n((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2
))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12*a*
c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3*(12
*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^
2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*
a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*c)*(4*a
*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)) + ((12
*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9
*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2))/(2*(36*a
*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b*(12*b^
2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(1
2*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^(1/2))))/(4*a
^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c - b^2)^(3/2)*
(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12*a*c - 3*b^2))/(
2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2)
)/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c -
 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))
)/(a^2*b^2*c) + ((2*a*c - b^2)*(4*a*c - b^2)*((a*b^4)/(2*(4*a*c - b^2)^(3/2
)) + (((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
```

```
)/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9
*b^2*c)*(4*a*c - b^2)^(1/2)))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) +
(b*(((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(12*
a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c))/(6*c*(4*a*c - b^2)^(1/2
))))/(a^2*b^3*c)))/(3*c*(4*a*c - b^2)^(1/2))
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^2}{a+bx^3+cx^6} \, dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[Out] -2/3*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1366, 632, 212}

$$\int \frac{x^2}{a+bx^3+cx^6} \, dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[In] Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \frac{2\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6),x]
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^3-2a\right)}{3\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^3+2a\right)}{3\sqrt{-4ac+b^2}}$	70

```
[In] int(x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,(b^2-4\,ac)}\right]$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(37) = 74.

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

```
[In] integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + bx^3 + cx^6} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{a + bx^3 + cx^6} \, dx = \frac{2 \arctan\left(\frac{2\,cx^3 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \frac{x^2}{a + bx^3 + cx^6} \, dx = -\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,\left(4\,a\,c-b^2\right)^4}{2}+a\,b\,\left(4\,a\,c-b^2\right)^3+a\,b^3\,\left(4\,a\,c-b^2\right)^2+b^2\,x^3\,\left(4\,a\,c-b^2\right)^3+\frac{b^4\,x^3\,\left(4\,a\,c-b^2\right)^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^2/(a + b*x^3 + c*x^6),x)
```

```
[Out] -(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b^
2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^3*
c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*(4*
a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))
```

3.141 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^3+cx^6)}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log(a+bx^3+cx^6)}{6a}$$

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{x(a+bx^3+cx^6)}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log(a+bx^3+cx^6)}{6a}+\frac{\log(x)}{a}$$

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}
\end{aligned}$$

$$= \frac{\log(x)}{a} - \frac{\log(a+bx^3+cx^6)}{6a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a+bx^3+cx^6)}{6a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3+cx^6)}\,dx$$

$$= \frac{\log(x)}{a} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\frac{\ln(c\,x^6+b\,x^3+a)}{2} + \frac{b\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3a}$	65
risch	$\frac{\ln(x)}{a} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)} \_R\ln\left(\left(\left(-14ac+4b^2\right)\_R-7c\right)x^3+ab\_R-3b\right)\right)}{3}$	76

```
[In] int(1/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a-1/3/a*(1/2*ln(c*x^6+b*x^3+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)
/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4\,ac\right)\log}{6\left(ab^2-4\,a^2c\right)}\right.$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 16.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) + \frac{\log\left(x\right)}{a}$$

```
[In] integrate(1/x/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-b*
```

```
sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*s
qrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^6+bx^3+a\right)}{6\,a} + \frac{\log\left(|x|\right)}{a}$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 1362, normalized size of antiderivative = 19.74

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c
)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(27*
```

```
b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(216*a
^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 - 36*
a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(9*a*b
^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c^3 - (
27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*b^2 - 3
6*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*(4*a*c -
 b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)^3*(27*b^
3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*(9*a*
b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(12*a*c - 3*
b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))
))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(12*a*c - 3*b^2
)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(49*a*c - 12*b^
2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(63*b^2*c^4 - ((
108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(
216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3
*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (b^3*(108*b^4*
c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c
 - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c -
3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^2)/(8*a*(9*a*b^2 - 36*a^
2*c)^2*(4*a*c - b^2)^(1/2))))/(16*a^4*c^3*(49*a*c - 12*b^2)) - ((4*b^5 + 29
*a^2*b*c^2 - 23*a*b^3*c)*(((63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12
*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^3)/(8*(9*a*b^2 -
36*a^2*c)^3) - (b^4*(108*b^4*c^3 - 378*a*b^2*c^4))/(1296*a^4*(4*a*c - b^2)^
2) + (b^2*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2)^2)/(48*a^2*(9*a*b^
2 - 36*a^2*c)^2*(4*a*c - b^2)) - (b^2*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b
^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2))/(24*a
^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(4
9*a*c - 12*b^2)))*(4*a*c - b^2)^2)/(b^3*c^3)))/(3*a*(4*a*c - b^2)^(1/2))
```

3.142 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx = -\frac{1}{3ax^3}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} -\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}$$

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-1/3*(-2*a*c+b^2)*arctanh
((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx = -\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} +\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(
3*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, x^3\right)$$
$$= -\frac{1}{3ax^3} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^3\right)}{3a}$$
$$= -\frac{1}{3ax^3} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, x^3\right)}{3a}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^3+cx^6)}{6a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a^2}$$
$$= -\frac{1}{3ax^3} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^3+cx^6)}{6a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a^2}$$

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2
)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{3a\,x^{3}}-\frac{b\ln(x)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}$
risch	$-\frac{1}{3a\,x^{3}}-\frac{b\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-14a^{3}c+4a^{2}b^{2}\right)\_R^{2}+6\_Rabc-3c^{2}\right)x^{3}+\right.\right.}{3}$

[In] int(1/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/3/a/x^3-b*ln(x)/a^2-1/3/a^2*(-1/2*b*ln(c*x^6+b*x^3+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+}{6\,(a^2b^2-4\,a^3c)x^3}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+6\,(b^3-}{6\,(a^2b^2-4\,a^3c)x^3}$$

[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} dx = \frac{b \log (cx^6 + bx^3 + a)}{6 a^2} - \frac{b \log (|x|)}{a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx^3 + b}{\sqrt{-b^2 + 4 ac}}\right)}{3 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^3 - a}{3 a^2 x^3}$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*ar
ctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^
3 - a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 4281, normalized size of antiderivative = 48.10

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (atan((48*a^8*x^3*(((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c)
*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4
 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*
c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c
- b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 33*a^2*
b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 -
((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c -
 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b
^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((252*a^4*b*c^
5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^
2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*
b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*
(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c -
 b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a
^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/
(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((42*a^3*
c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^
3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^
4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (12*b*c^6)/a^3))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^3*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(432*a^10*(4*a*c -
b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(16*a^4*c^3*(a^2*c^2 - 12*b^4 + 48*a*b
^2*c)) + ((4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^4*c)*(((3*b^3 - 12*a
*b*c)*(((((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 1
2*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)
)))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^
4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2
*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2
*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(3
6*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - c^7/a^4 - ((3*b^3 - 12
*a*b*c)*(((3*b^3 - 12*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 -
 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108
*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^
3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (12*b*c^6)/a^3))/(2*(36*a^
3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((2
```

52*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1
/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4
))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*
a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))
)*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ ((2*a*c - b^2)^4*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(1296*a^12*(4*a*c -
b^2)^2)))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 12*b^4 + 48*a*b^2*c))
)*(4*a*c - b^2)^2)/(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5) + (
3*a^4*(4*a*c - b^2)^2*(4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*
c)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^
3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2
*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (
(2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((2
7*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*
(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)
^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*
b^3 - 12*a*b*c)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(
(27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(
2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((((((27*a^3*b^4*c^
3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c -
9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3
- 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)
))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c -
b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + (b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^
2)^3)/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(c^3*(a^2*c^2 -
12*b^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5))
+ (3*a^4*(4*a*c - b^2)^(3/2)*(4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^
4*c)*(((3*b^3 - 12*a*b*c)*((((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*
a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6
a^2(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4
a(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*
c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*
a*c - b^2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (b*c^6)/a
^4 + ((3*b^3 - 12*a*b*c)*((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)
*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3
- 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9
*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(
36*a^3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4
*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c

```
 - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*
b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b
^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b
^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 -
(27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c
 - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
+ (b^3*c^3*(2*a*c - b^2)^4)/(48*a^7*(4*a*c - b^2)^2)))/(c^3*(a^2*c^2 - 12*b
^4 + 48*a*b^2*c)*(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5)))*(2*
a*c - b^2))/(3*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^3 +
 c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)) - 1/(3*a*x^3)
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 636

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$- \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x^2/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-
b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-
4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)
/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/
c^(5/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-
4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3
```

```
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arct
an(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2
*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1381, 1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

$$= \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[In] Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6}\,dx}{2c}\\ &= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}\end{aligned}$$

$$= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\ 2^{2/3} c^{4/3} \sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{3\ 2^{2/3} c^{4/3} \sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\ 2^{2/3} c^{4/3} \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{3\ 2^{2/3} c^{4/3} \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{4c^{4/3}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{4c^{4/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$
\begin{aligned}
&= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

$$= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{3x^2 - 2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

```
[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{x^2}{2c}-\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^6+\_Z^3 b+a\right)}\frac{\left(\_R^4 b+\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	61
risch	$\frac{x^2}{2c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^6+\_Z^3 b+a\right)}\frac{\left(-\_R^4 b-\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	63

```
[In] int(x^7/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6
*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3402 vs. 2(498) = 996.

Time = 0.49 (sec) , antiderivative size = 3402, normalized size of antiderivative = 5.35

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/6*((1/2)^(1/3)*(sqrt(-3)*c - c)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 -
4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2
*c^5 - 4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2
 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + sqrt(-3)*(b^10 - 12*a*b^8*c + 52*a^2*b
^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) - (b^8*c^5 - 13*a*b^6*c^6 + 60*a^
2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 + sqrt(-3)*(b^8*c^5 - 13*a*b^6*c^6
 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9))*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^
5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25
*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^2*c^5 - 4*a*c^6))^(2/3) + 4*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) - (
1/2)^(1/3)*(sqrt(-3)*c + c)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^
6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 -
```

4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*
a^3*b^4*c^3 + 60*a^4*b^2*c^4 - sqrt(-3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2
 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) - (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*
c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 - sqrt(-3)*(b^8*c^5 - 13*a*b^6*c^6 + 60*
a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9))*sqrt((b^10 - 10*a*b^8*c + 35*a
^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 4
8*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*
a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b
^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c
^5 - 4*a*c^6))^(2/3) + 4*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) + 2*(1/2)
^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*lo
g((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4
*b^2*c^4 - (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*
a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^
4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*
a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 2*(a^3*b^5 -
5*a^4*b^3*c + 5*a^5*b*c^2)*x) + (1/2)^(1/3)*(sqrt(-3)*c - c)*((b^4 - 3*a*b^
2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 -
12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + sqrt(-3)*(b
^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) + (b^8
*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 + sqrt(
-3)*(b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9
))*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*
a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 4*(a^3*b^5 - 5*a^4*
b^3*c + 5*a^5*b*c^2)*x) - (1/2)^(1/3)*(sqrt(-3)*c + c)*((b^4 - 3*a*b^2*c +
a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50
*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3)*log(-(1/2)^(2/3)*(b^10 - 12*a*b
^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - sqrt(-3)*(b^10 -
12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4) + (b^8*c^5 -
 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9 - sqrt(-3)*(b
^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9))*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*
c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c

```
^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) + 4*(a^3*b^5 - 5*a^4*b^3*c
+ 5*a^5*b*c^2)*x) + 2*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5
- 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a
^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b
^2*c^5 - 4*a*c^6))^(1/3)*log((1/2)^(2/3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^
2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*
c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^
2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sq
rt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*
c^6))^(2/3) + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x) + 3*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 147.58 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.44

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^3c^8-34992a^2b^2c^7+8748ab^4c^6-729b^6c^5\right)+t^3\cdot\left(432a^4c^4-1512a^3b^2c^3+1107a^2b\right.\right.$$
$$+\frac{x^2}{2c}$$

```
[In] integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**
4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_t**
5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 315
9*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*
a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*
_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5)
))) + x**2/(2*c)
```

Maxima [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \int \frac{x^7}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^7}{a + bx^3 + cx^6} \, dx = \int \frac{x^7}{cx^6 + bx^3 + a} \, dx$$

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 4069, normalized size of antiderivative = 6.40

$$\int \frac{x^7}{a + bx^3 + cx^6} \, dx = \text{Too large to display}$$

```
[In] int(x^7/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1
/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))
^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*
c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*
b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4
 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^
6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)*(-(b^8 +
16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
)^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c
- b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))
/6 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 +
16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)
*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a
```

^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2
*b^2*c^7)))^(1/3) + x^2/(2*c) - log((a^4*x*(a*c - b^2))/c^2 - (2^(1/3)*(3^(
1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*
b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4
*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2
*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8
*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36)*
((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a*c - b^2))/c^2 - (2^
(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(b^4 + 8*a^2*
c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b
^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4
+ b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b
^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2
) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(
2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 -
b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) - log((a^4*x*(a*c - b^2))
/c^2 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^
4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2
- 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 1
6*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3
+ 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c -
b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a

```
*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a
*c - b^2))/c^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a
^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^
2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1
/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*
(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^
3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 1
2*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(
4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^
2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5
*(4*a*c - b^2)^3))^(2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 -
b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
 - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 631

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$
$$=\frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
```

```
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1381, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{x}{c}$$

```
[In] Int[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}\\ &= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}\end{aligned}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$
\begin{aligned}
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + 2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} \\
&+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\left(b - \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b - \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\left(b + \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b + \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}
$$

$$= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

```
[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\left(-\_R^3b-a\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3c}$	59

```
[In] int(x^6/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. $2(495) = 990$.

Time = 0.41 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] -1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^
8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1
/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*
c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2
 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5*c^4 -
 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*
c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*(sqrt(-3
)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a*b^4 - 4*a^2*b
^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3
*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3) - (b^5*c^4 -
```

```
 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^
6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b
^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c +
 (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(
b^2*c^4 - 4*a*c^5))^(1/3)) + (1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c
- (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/
(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2
)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a
*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3) + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^
6 + sqrt(-3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48
*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt(
(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 -
12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))
 - (1/2)^(1/3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 -
 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)
*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c +
 18*a^2*b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 -
8*a^3*c^3) + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*
a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*
b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*
c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 2*(1/2)^(1/3)*c*(-(b^3
- 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16
*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^
3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2
)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4
- 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*
c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 2*(1/2)^(1/3)*c*(-(b
^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64
*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c
^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 1
6*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a
^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 6*x)/c
```

Sympy [A] (verification not implemented)

Time = 53.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$
$$=\mathrm{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5c\right.\right.$$
$$+\frac{x}{c}$$

```
[In] integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4))) +
 x/c
```

Maxima [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\int \frac{x^6}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\int \frac{x^6}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(x^6/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
```

```
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

3.145 $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/2
))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(
1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(
b-(-4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(1
/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*c
+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)-1/6*ln(2^(1/3
)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(2/3)*2^(1
/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^2
)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))^
(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1388, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

```
[In] Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right) \\
&\quad +\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx \\
&= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&\quad -\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&\quad -\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&\quad +\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt{b^2-4ac}}
\end{aligned}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4\sqrt[3]{c}} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4\sqrt[3]{c}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$=\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$
$$-\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$
$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$
$$+\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$
$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$
$$+\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

$$
= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$
\int \frac{x^4}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1^2}{b+2c\#1^3}\&\right]
$$

```
[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}\right)}{3}$	43

```
[In] int(x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. 2(421) = 842.

Time = 0.31 (sec) , antiderivative size = 2314, normalized size of antiderivative = 4.15

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c
 + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/
(b^2*c^2 - 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 +
 sqrt(-3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5 + sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 +
 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)
/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) - 1/6*(1/2)^(1/3)*(sqr
t(-3) + 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c
^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))
^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - sqrt(-3)*(b^5 - 6*a
*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5 - sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(
(b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64
*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*
c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3)
```

```
)^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log((1/2)^(2/3
)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + sqrt(-3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)
 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5 + sqrt(-3)*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^
2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2
*a^2*c)*x) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8
*a^2*b*c^2 - sqrt(-3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) + (b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5 - sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4
 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) + 1/3*(1/2)
^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 -
 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/
3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - (b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^
4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 -
 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x) + 1/
3*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6
*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3
))^(1/3)*log(-(1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 + (b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(
b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(((b^2*c^2 - 4*a*c^
3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(a*b^2 - 2*a^2*c)*x)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.31

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^3c^5-34992a^2b^2c^4+8748ab^4c^3-729b^6c^2\right)+t^3\left(-432a^2bc^2+216ab^3c-27b^5\right)+a\right.$$

```
[In] integrate(x**4/(c*x**6+b*x**3+a),x)
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2,
```

```
Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 +
 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*
_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

Maxima [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \int \frac{x^4}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \int \frac{x^4}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 2695, normalized size of antiderivative = 4.83

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(x^4/(a + b*x^3 + c*x^6),x)
[Out] log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2
)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 +
 a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
 - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3
 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
```

*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^
3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a
^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(
1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - 45*a^2*b^2*c^2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4
*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2
*(4*a*c - b^2)^3))^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4
)))^(1/3) + log((2^(1/3)*(3^(1/2)*1i + 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(
1/2)*1i - 1)*(54*a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i
+ 1)*(4*a*c - b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)
*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 +
12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*
c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^
2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c
^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3)
)^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5 - b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((
2^(1/3)*(3^(1/2)*1i + 1)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3
)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(1/2)*1i - 1)*(54*
a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)

```
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)*((3^(1/2)*1i)/2
- 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*
a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48
*a^2*b^2*c^4)))^(1/3)
```

3.146 $\int \frac{x^3}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/
2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^
(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*
(b-(-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)+1/6*ln(2^(1/
3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(1/3)*2^(
2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)
*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^
2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1
/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))
^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$
$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

```
[In] Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
```

```
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
```

```
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right) + \frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}\sqrt{b^2-4ac}}$$

$$-\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

$$+\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}\sqrt{b^2-4ac}}$$

$$
\begin{aligned}
&= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&+ \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&+ \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&- \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&- \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} \\
&+ \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}
\end{aligned}
$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.08

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1}{b+2c\#1^3}\&\right]$$

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3 b+a\right)}\frac{\_R^3 \ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3 b+a\right)}\frac{\_R^3 \ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}\right)}{3}$	43

```
[In] int(x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. 2(421) = 842.

Time = 0.29 (sec) , antiderivative size = 1542, normalized size of antiderivative = 2.76

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] -1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log
(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2)^(1/3
)*(sqrt(-3) - 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 - sqrt(-3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^
2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1
)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*sqrt(b^2/(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)
*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2
```

```
*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((b^2*c -
4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) -
 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3 - sqrt(-3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a
*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))
^(1/3) + 2*b*x) + 1/3*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*l
og((1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(
1/3) + b*x) + 1/3*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a
*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log(
-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1
/3) + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$
$$= \operatorname{RootSum}\Big(t^6\cdot\left(46656a^3c^4-34992a^2b^2c^3+8748ab^4c^2-729b^6c\right)+t^3\cdot\left(432a^2c^2-216ab^2c+27b^4\right)+a,\Big(t$$

```
[In] integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t
, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**
4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

Maxima [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \int \frac{x^3}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \int \frac{x^3}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 2129, normalized size of antiderivative = 3.82

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(x^3/(a + b*x^3 + c*x^6),x)
[Out] log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/
(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(
4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3)^(1
/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6 + 3*a
*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) +
log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2^(2/3)*b
*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b
^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a
^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c - b^2)^3)^(
1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/6 + 3*a*c^
2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b
^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + lo
g((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(3^(1/2)
*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)
*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16
*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-(b*(-(4*a*c - b^2)^
3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3
*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2)
 + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3)))^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^
2*b*c^3 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
 b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c
```

```
 - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3)
)/36)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a
*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + log((2^(2/3)*(3^(1/2)*1
i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(81*a*c^3*x*(4*a*c - b^2)^2 -
 (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3
)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(3^(
1/2)*1i + 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(
c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*
c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*
((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8
*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3)
- log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (2^(1/3)*(81*
a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c
- b^2)^3))^(1/3))/4)*(3^(1/2)*1i - 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 -
3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/
2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)))^(1/3)
```

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 558

$$\int \frac{x}{a+bx^3+cx^6}\,dx = -\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*2^(1/3)*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-
(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^
(2/3))*2^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(1/3)*
c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(
1/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/3*2^(1/3)*c
```

```
^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c
^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(1/3)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/3*2^(1/3)*c^(1/3)*arctan
(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1389, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
```

```
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

```
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{c\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(\sqrt[3]{2}c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{c^{2/3}\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\sqrt{b^2-4ac}}$$

$$-\frac{c^{2/3}\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\sqrt{b^2-4ac}}$$

$$+\frac{\sqrt[3]{c}\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\sqrt[3]{c}\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$
\begin{aligned}
&= -\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt[3]{c}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&- \frac{\sqrt[3]{c}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&+ \frac{\left(\sqrt[3]{2}\sqrt[3]{c}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&- \frac{\left(\sqrt[3]{2}\sqrt[3]{c}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[3]{c}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[3]{c}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1+2c\#1^4}\&\right]$$

```
[In] Integrate[x/(a + b*x^3 + c*x^6),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	41

```
[In] int(x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(421) = 842$.

Time = 0.29 (sec) , antiderivative size = 1798, normalized size of antiderivative = 3.22

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log
(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b^2*c) - (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*b^6 - 12*a^2*
b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3)) - 1/
6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(4
*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c - sqrt(-3)*(b^4 - 4*a*b^2*c) - (a*b^6
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 - sqrt(-3)*(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3)) + 1/6*
(1/2)^(1/3)*(sqrt(-3) - 1)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b
^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(4*b*
c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b^2*c) + (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*b^6 - 12*a^2*b^4*c
 + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3)) - 1/6*(1/2
)^(1/3)*(sqrt(-3) + 1)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
```

```
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(4*b*c*x
- (1/2)^(2/3)*(b^4 - 4*a*b^2*c - sqrt(-3)*(b^4 - 4*a*b^2*c) + (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 - sqrt(-3)*(a*b^6 - 12*a^2*b^4*c + 4
8*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1
/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4
 - 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^
2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^
2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/
(a*b^2 - 4*a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c
))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c
 + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.28

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^4c^3-34992a^3b^2c^2+8748a^2b^4c-729ab^6\right)+t^3\left(-432a^2c^2+216ab^2c-27b^4\right)+c,\ \left(\right.\right.$$

```
[In] integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_
t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_
t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*
c + 9*_t**2*b**4)/(b*c))))
```

Maxima [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \int \frac{x}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \int \frac{x}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 1543, normalized size of antiderivative = 2.77

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(x/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*
(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c
)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((b*(-(4*
a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3*x*(b^4 + 8
*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^
2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c -
b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)
^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*
a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b*(-(4*a*
c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3
))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(54*a*(4
*a*c - b^2)^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4
+ 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2)))^(1/3) - log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (2
7*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^
(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(
4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^
```

```
3))*((3^(1/2)*1i)/2 + 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2
+ 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/
3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b
*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)
*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/
(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3)
```

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 558

$$\int \frac{1}{a+bx^3+cx^6}\,dx = -\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] 1/3*2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-(
-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*
x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(
```

```
2/3))*2^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c
^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1
/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c^
(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^
(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/3*2^(2/3)*c^(2/3)*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{a+bx^3+cx^6}\,dx = -\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
```

```
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{c^{2/3}\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(2^{2/3}c^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.08

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1^2+2c\#1^5}\&\right]$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-1),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+\_Z^{3}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}\right)}{3}$	40

```
[In] int(1/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(421) = 842$.

Time = 0.33 (sec) , antiderivative size = 2206, normalized size of antiderivative = 3.95

$$\int \frac{1}{a + bx^3 + cx^6}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] -1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c
 + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/
(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4 -
6*a*b^2*c + 8*a^2*c^2 + sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2) - (a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^
2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 -
4*a^3*c))^(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a^3*c)*sqr
t((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 -
64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (
1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 - sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2
*c^2) - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt(-3)*(a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*
b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)
) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-((a^2
*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c
+ 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2
*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + sqrt(-3)*(b^4
- 6*a*b^2*c + 8*a^2*c^2) + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(-3)
*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)
```

```
/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b
^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/6*(1/2)^(1/3)*(s
qrt(-3) - 1)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4
*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c
))^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*
c^2 - sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2) + (a^2*b^5 - 8*a^3*b^3*c + 16*
a^4*b*c^2 - sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2))*sqrt((b^4 - 4*
a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)
))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
+ 1/3*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a
^3*c))^(1/3)*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*
a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^
2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 4
8*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) + 1/3*(1/2)^(
1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 1
2*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)
*log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + (a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^
4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))
```

Sympy [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{1}{a+bx^3+cx^6}\,dx$$
$$=\text{RootSum}\left(t^6\cdot\left(46656a^5c^3-34992a^4b^2c^2+8748a^3b^4c-729a^2b^6\right)+t^3\cdot\left(432a^2bc^2-216ab^3c+27b^5\right)+c^2,\right.$$

```
[In] integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, L
ambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*
_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 -
 b**2*c))))
```

Maxima [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \int \frac{1}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \int \frac{1}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 2597, normalized size of antiderivative = 4.65

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b*x^3 + c*x^6),x)
[Out] log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)
*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1
/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x + (2
^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b
^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a
*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c
 - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c
```

```
^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3
- 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x - (2^(2/3)*(3^(1/2)*
1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c -
2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4 - 3
6*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4
*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2
/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) - log(6*c^5*x - (2^(2/3
)*(3^(1/2)*1i + 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36
*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*c^3*x*(4*a*c - b^2)^2
- (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c -
b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*
x - (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3)
)^(1/3)*(9*b^2*c^4 - 36*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*
c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^
2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) -
 log(6*c^5*x - (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*
c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*
c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^
2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)))^(1/3)
```

3.149 $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 610

$$\int \frac{1}{x^2 (a+bx^3+cx^6)} \, dx$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(1+b/
(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(
2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a
*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1
/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1
/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(
1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
```

```
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/
3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*
a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1382, 1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$$
$$= \frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{ax}+\frac{\int \frac{x(-b-cx^3)}{a+bx^3+cx^6}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}\end{aligned}$$

$$= -\frac{1}{ax} + \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$
\begin{aligned}
= & -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
& + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
& - \frac{\left(c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4a} \\
& - \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4a} \\
& - \frac{\left(\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
& - \frac{\left(\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3 b+a\right)}\frac{\left(\_R^4 c+\_R b\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^3a^7-48b^2c^2a^6+12b^4ca^5-b^6a^4\right)\_Z^6+\left(-32bc^3a^3+32b^3c^2a^2-10b^5ca+b^7\right)\_Z^3+c^4\right)}\_R\ln\left(\left(\left(224c^3a^7-176b^2\right.\right.\right.\right.}{}$

```
[In] int(1/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b
+a))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. 2(471) = 942.

Time = 0.39 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^
4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)
^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 -
 (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^
6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*
b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*
a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^
2 - 4*a^5*c))^(2/3) + 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + 2*(1/2)^(1
/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b
^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log((1/2)^(2/3)*(b^9 - 1
1*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + (a^4*b^8 - 13*
a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*
```

c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b
^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(
2/3) + 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) + (1/2)^(1/3)*(sqrt(-3)*a*x
 - a*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b
^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(-(1/2)^(2/3)*(b^9 -
11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 + sqrt(-3)*(b^9
 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4) - (a^4*b^8
- 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4 + sqrt(-3)*(
a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4))*sq
rt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6
 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b
^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a
^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2
 - 4*a^5*c))^(2/3) + 4*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x) - (1/2)^(1/3)
*(sqrt(-3)*a*x + a*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*log(-(1/2)
^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4 -
 sqrt(-3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^
4) - (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^
4 - sqrt(-3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3 + 6
4*a^8*c^4))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4
*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*
a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 4*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*x)
 + (1/2)^(1/3)*(sqrt(-3)*a*x - a*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^
6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1
/3)*log(-(1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 +
24*a^4*b*c^4 + sqrt(-3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3
 + 24*a^4*b*c^4) + (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c
^3 + 64*a^8*c^4 + sqrt(-3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a
^7*b^2*c^3 + 64*a^8*c^4))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b
^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c
^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 4*(b^4*c^3 - 4*a*b^2*c^4 +
 2*a^2*c^5)*x) - (1/2)^(1/3)*(sqrt(-3)*a*x + a*x)*((b^3 - 2*a*b*c - (a^4*b^
2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^
4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2
- 4*a^5*c))^(1/3)*log(-(1/2)^(2/3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*
a^3*b^3*c^3 + 24*a^4*b*c^4 - sqrt(-3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 -

```
62*a^3*b^3*c^3 + 24*a^4*b*c^4) + (a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 -
 112*a^7*b^2*c^3 + 64*a^8*c^4 - sqrt(-3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b
^4*c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*
c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2
 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c +
 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(2/3) + 4*(b^4*c^3 -
 4*a*b^2*c^4 + 2*a^2*c^5)*x) - 6)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} dx$$
$$= \text{RootSum}\left(t^6 \cdot \left(46656a^7c^3 - 34992a^6b^2c^2 + 8748a^5b^4c - 729a^4b^6\right) + t^3\left(-864a^3bc^3 + 864a^2b^3c^2 - 270ab^5c\right.\right.$$
$$- \frac{1}{ax}$$

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*
c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*
_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c
- 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 +
378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 -
 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} dx = \int \frac{1}{(cx^6 + bx^3 + a)x^2} dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{1}{(cx^6+bx^3+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 2978, normalized size of antiderivative = 4.88

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)),x)
[Out] log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b
^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*a*
c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^
3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*a*c -
 b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/
3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/
3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3
*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c -
 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*((b^4
*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - 1/(a*x) + log(36*a^9*c^6 + 9*a^7*b^4*c
^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*
c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-(4
```

```
*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a
*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^7 + b^4*(-(4*a*c - b^2)
^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^
7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b^4
*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(b^6 - 8*a^
3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i -
1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 3
2*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-
(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*
a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b
^4*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*
b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2
)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i + 1)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*
b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2
*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^
2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i - 1)*((b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(
4*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)^3
)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 6
4*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
```

3.150 $\int \frac{1}{x^3(a+bx^3+cx^6)} dx$

Optimal result

Integrand size = 18, antiderivative size = 612

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx$$

$$= -\frac{1}{2ax^2} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] -1/2/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(
1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2
)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2
))^(1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1
/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))
^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+
```

```
(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+
(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1382, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$$

$$= \frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b -
```

```
 Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])
*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*
a*c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c
])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[
b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 -
 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2ax^2} + \frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6}\,dx}{2a} \\
&= -\frac{1}{2ax^2} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}
\end{aligned}
$$

$$= -\frac{1}{2ax^2} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{1}{2ax^2} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{1}{2ax^2} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$-\frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$-\frac{\left(c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{1}{2ax^2} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+\_Z^3b+a\right)}\frac{\left(-c\_R^3-b\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3a}-\frac{1}{2a\,x^2}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^3a^8-48a^7b^2c^2+12a^6b^4c-a^5b^6\right)\_Z^6+\left(-16a^4c^4+56a^3b^2c^3-41b^4c^2a^2+11a\,b^6c-b^8\right)\_Z^3+c^5\right)}\_R\ln\left(\left(\left((224c\right.\right.\right.\right.}{}$

```
[In] int(1/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)-1/2/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. 2(471) = 942.

Time = 0.42 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.27

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)
*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4
)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4
*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1/2)^(1/3)*
(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (a^5*b^6 - 10*a^6*b^4*
c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt
((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*
c))^(1/3)) + 2*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 -
4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*log(2*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*x + (1/2
)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (a^5*b^6 - 10*
a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
```

^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6
*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*
c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2
- 4*a^6*c))^(1/3)) - (1/2)^(1/3)*(sqrt(-3)*a*x^2 + a*x^2)*(-(b^4 - 3*a*b^2*
c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(4*(b^5*c^2 - 5*a*b^3*c^
3 + 5*a^2*b*c^4)*x - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3
*b^2*c^3 + sqrt(-3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3) - (
a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3 + sqrt(-3)*(a^5*b^6 -
10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3))*sqrt((b^10 - 10*a*b^8*c + 35*a
^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4
*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*
b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^2 - 4*a^6*c))^(1/3)) + (1/2)^(1/3)*(sqrt(-3)*a*x^2 - a*x^2)*(-(b^4 - 3*a*
b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(4*(b^5*c^2 - 5*a*b^
3*c^3 + 5*a^2*b*c^4)*x - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20
*a^3*b^2*c^3 - sqrt(-3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)
 - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3 - sqrt(-3)*(a^5*b^
6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3))*sqrt((b^10 - 10*a*b^8*c +
35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2
 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*
a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^2 - 4*a^6*c))^(1/3)) - (1/2)^(1/3)*(sqrt(-3)*a*x^2 + a*x^2)*(-(b^4 -
3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*
b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(4*(b^5*c^2 - 5*
a*b^3*c^3 + 5*a^2*b*c^4)*x - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2
- 20*a^3*b^2*c^3 + sqrt(-3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*
c^3) + (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3 + sqrt(-3)*(a^
5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3))*sqrt((b^10 - 10*a*b^8*
c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b
^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5
*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 +
 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))/(a^5*b^2 - 4*a^6*c))^(1/3)) + (1/2)^(1/3)*(sqrt(-3)*a*x^2 - a*x^2)*(-(b^
4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*
a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*log(4*(b^5*c^2
- 5*a*b^3*c^3 + 5*a^2*b*c^4)*x - (1/2)^(1/3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*

```
c^2 - 20*a^3*b^2*c^3 - sqrt(-3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*
b^2*c^3) + (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3 - sqrt(-3)
*(a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3))*sqrt((b^10 - 10*a*
b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 -
(a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)) - 3)/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx = \int \frac{1}{(cx^6+bx^3+a)x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx = \int \frac{1}{(cx^6+bx^3+a)x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 4063, normalized size of antiderivative = 6.64

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} dx = \text{Too large to display}$$

[In] int(1/(x^3*(a + b*x^3 + c*x^6)),x)

[Out] log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) + log((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) - 1/(2*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*

a*c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/
3))/36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 -
11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b
^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(
3^(1/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^1
0*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*
c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3)
)/36))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2
)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b
^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)*(3^
(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41
*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/
36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4
c^4 - b^5(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*
a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^
3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3
) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2
)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(3^(1
/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^10*b*
c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^
2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -

```
b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a
^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36
))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c
^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*
b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)
^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3)
```

3.151 $\int \frac{x^{11}}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 715, 646, 31}

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

```
[In] Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-4+x+\frac{12+13x}{3+4x+x^2}\right)\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\text{Subst}\left(\int \frac{12+13x}{3+4x+x^2}\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)+\frac{9}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

```
[In] Integrate[x^11/(3 + 4*x^3 + x^6),x]
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	28
risch	$\frac{x^6}{6}-\frac{4x^3}{3}+\frac{8}{3}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	29
norman	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37
parallelrisch	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37

```
[In] int(x^11/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=\frac{x^6}{6}-\frac{4x^3}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{9\log\left(x^3+3\right)}{2}$$

```
[In] integrate(x**11/(x**6+4*x**3+3),x)
[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\,\log\left(x^3+3\right) - \frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\,\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{9\,\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6} - \frac{4\,x^3}{3} + \frac{x^6}{6}$$

```
[In] int(x^11/(4*x^3 + x^6 + 3),x)
[Out] (9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

3.152 $\int \frac{x^8}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3}+\frac{1}{6}\log \left(1+x^3\right)-\frac{3}{2}\log \left(3+x^3\right)$$

```
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 717, 646, 31}

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3}+\frac{1}{6}\log \left(x^3+1\right)-\frac{3}{2}\log \left(x^3+3\right)$$

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3}+\frac{1}{3}\text{Subst}\left(\int \frac{-3-4x}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\
&= \frac{x^3}{3}+\frac{1}{6}\log\left(1+x^3\right)-\frac{3}{2}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{3+4x^3+x^6}\,dx=\frac{x^3}{3}+\frac{1}{6}\log\left(1+x^3\right)-\frac{3}{2}\log\left(3+x^3\right)$$

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6),x]
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
risch	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
norman	$\frac{x^3}{3}+\frac{\ln(x+1)}{6}-\frac{3\ln(x^3+3)}{2}+\frac{\ln(x^2-x+1)}{6}$	32
parallelrisch	$\frac{x^3}{3}+\frac{\ln(x+1)}{6}-\frac{3\ln(x^3+3)}{2}+\frac{\ln(x^2-x+1)}{6}$	32

```
[In] int(x^8/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx=\frac{1}{3}\,x^3-\frac{3}{2}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx=\frac{x^3}{3}+\frac{\log\left(x^3+1\right)}{6}-\frac{3\log\left(x^3+3\right)}{2}$$

```
[In] integrate(x**8/(x**6+4*x**3+3),x)
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\,x^3 - \frac{3}{2}\,\log\left(x^3+3\right) + \frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}\,x^3 - \frac{3}{2}\,\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{\ln\left(x^3+1\right)}{6} - \frac{3\,\ln\left(x^3+3\right)}{2} + \frac{x^3}{3}$$

```
[In] int(x^8/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

3.153 $\int \frac{x^5}{3+4x^3+x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^5}{3+4x^3+x^6} dx = -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 646, 31}

$$\int \frac{x^5}{3+4x^3+x^6} dx = \frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

```
[In] Int[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{3+4x+x^2}\,dx, x, x^3\right)\\ &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\ &= -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

```
[In] Integrate[x^5/(3 + 4*x^3 + x^6),x]
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
risch	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
norman	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27
parallelrisch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27

```
[In] int(x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{\log\left(x^3+1\right)}{6} + \frac{\log\left(x^3+3\right)}{2}$$

```
[In] integrate(x**5/(x**6+4*x**3+3),x)
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\,\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\,\log\left(\left|x^3+1\right|\right)$$

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6}$$

```
[In] int(x^5/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 3)/2 - log(x^3 + 1)/6
```

3.154 $\int \frac{x^2}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{x^2}{3+4x^3+x^6} \, dx = -\frac{1}{3}\text{arctanh}\left(2+x^3\right)$$

```
[Out] -1/3*arctanh(x^3+2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 630, 31}

$$\int \frac{x^2}{3+4x^3+x^6} \, dx = \frac{1}{6}\log\left(x^3+1\right) - \frac{1}{6}\log\left(x^3+3\right)$$

```
[In] Int[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{3+4x+x^2}\,dx, x, x^3\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\ &= \frac{1}{6}\log\left(1+x^3\right) - \frac{1}{6}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{1}{6}\log\left(1+x^3\right) - \frac{1}{6}\log\left(3+x^3\right)$$

```
[In] Integrate[x^2/(3 + 4*x^3 + x^6),x]
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
default	$-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{3}+1\right)}{6}$	18
risch	$-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{3}+1\right)}{6}$	18
norman	$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	27
parallelrisch	$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x^{3}+3\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	27

```
[In] int(x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x^3+3)+1/6*ln(x^3+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\,\log\left(x^3+3\right) + \frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\log\left(x^3+1\right)}{6} - \frac{\log\left(x^3+3\right)}{6}$$

```
[In] integrate(x**2/(x**6+4*x**3+3),x)
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\,\log\left(x^3+3\right) + \frac{1}{6}\,\log\left(x^3+1\right)$$

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{9}{2\,(8\,x^3+6)}+\frac{5}{4}\right)}{3}$$

```
[In] int(x^2/(4*x^3 + x^6 + 3),x)
[Out] atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(3+4x^3+x^6)} \, dx = \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

```
[Out] 1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 719, 29, 646, 31}

$$\int \frac{1}{x(3+4x^3+x^6)} \, dx = -\frac{1}{6}\log\left(x^3+1\right) + \frac{1}{18}\log\left(x^3+3\right) + \frac{\log(x)}{3}$$

```
[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\,(3+4x+x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{9}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{\log(x)}{3}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)\\
&= \frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{18}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(3+4x^3+x^6)}\,dx = \frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{18}\log\left(3+x^3\right)$$

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\ln(x)}{3}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{18}$	22
default	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31
norman	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31
parallelrisch	$\frac{\ln(x)}{3}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x^2-x+1)}{6}$	31

```
[In] int(1/x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx=\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log\left(x\right)$$

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx=\frac{\log\left(x\right)}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

```
[In] integrate(1/x/(x**6+4*x**3+3),x)
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{1}{18}\,\log\left(x^3+3\right) - \frac{1}{6}\,\log\left(x^3+1\right) + \frac{1}{9}\,\log\left(x^3\right)$$

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{1}{18}\,\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\,\log\left(\left|x^3+1\right|\right) + \frac{1}{3}\,\log\left(\left|x\right|\right)$$

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx = \frac{\ln\left(x^3+3\right)}{18} - \frac{\ln\left(x^3+1\right)}{6} + \frac{\ln\left(x\right)}{3}$$

```
[In] int(1/(x*(4*x^3 + x^6 + 3)),x)
[Out] log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

```
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 723, 814}

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

```
[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(3+4x+x^2)}\,dx, x, x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x(3+4x+x^2)}\,dx, x, x^3\right)\\
&= -\frac{1}{9x^3}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x}+\frac{3}{2(1+x)}-\frac{1}{6(3+x)}\right)dx, x, x^3\right)\\
&= -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx = -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

```
[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x^3+1)}{6}-\frac{\ln(x^3+3)}{54}$	27
default	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln(x^3+3)}{54}+\frac{\ln(x^2-x+1)}{6}$	36
norman	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x+1)}{6}-\frac{\ln(x^3+3)}{54}+\frac{\ln(x^2-x+1)}{6}$	36
parallelrisch	$-\frac{24\ln(x)x^3-9\ln(x+1)x^3+\ln(x^3+3)x^3-9\ln(x^2-x+1)x^3+6}{54x^3}$	48

```
[In] int(1/x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx=-\frac{x^3\log(x^3+3)-9\,x^3\log(x^3+1)+24\,x^3\log(x)+6}{54\,x^3}$$

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx=-\frac{4\log(x)}{9}+\frac{\log(x^3+1)}{6}-\frac{\log(x^3+3)}{54}-\frac{1}{9x^3}$$

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx=-\frac{1}{9\,x^3}-\frac{1}{54}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)-\frac{4}{27}\log\left(x^3\right)$$

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} \, dx = \frac{4\, x^3 - 3}{27\, x^3} - \frac{1}{54} \log \left(\left| x^3 + 3 \right| \right) + \frac{1}{6} \log \left(\left| x^3 + 1 \right| \right) - \frac{4}{9} \log \left(|x| \right)$$

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9
*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} \, dx = \frac{\ln(x^3 + 1)}{6} - \frac{\ln(x^3 + 3)}{54} - \frac{4 \ln(x)}{9} - \frac{1}{9\, x^3}$$

```
[In] int(1/(x^4*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 723, 814}

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

```
[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x^2\left(3+4x+x^2\right)}\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x^2}+\frac{13}{9x}-\frac{3}{2(1+x)}+\frac{1}{18(3+x)}\right)\,dx,x,x^3\right)\\
&= -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

```
[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{162}$	33
default	$-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	41
norman	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	42
parallelrisch	$\frac{78\ln(x)x^6-27\ln(x+1)x^6+\ln(x^3+3)x^6-27\ln(x^2-x+1)x^6-9+24x^3}{162x^6}$	53

```
[In] int(1/x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] (-1/18+4/27*x^3)/x^6+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx=\frac{x^6\log(x^3+3)-27\,x^6\log(x^3+1)+78\,x^6\log(x)+24\,x^3-9}{162\,x^6}$$

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9)
/x^6
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx=\frac{13\log(x)}{27}-\frac{\log(x^3+1)}{6}+\frac{\log(x^3+3)}{162}+\frac{8x^3-3}{54x^6}$$

```
[In] integrate(1/x**7/(x**6+4*x**3+3),x)
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7 (3 + 4x^3 + x^6)} dx = \frac{8 x^3 - 3}{54 x^6} + \frac{1}{162} \log (x^3 + 3) - \frac{1}{6} \log (x^3 + 1) + \frac{13}{81} \log (x^3)$$

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x^
3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7 (3 + 4x^3 + x^6)} dx = -\frac{13 x^6 - 8 x^3 + 3}{54 x^6} + \frac{1}{162} \log (|x^3 + 3|) - \frac{1}{6} \log (|x^3 + 1|) + \frac{13}{27} \log (|x|)$$

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
+ 1)) + 13/27*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (3 + 4x^3 + x^6)} dx = \frac{\ln (x^3 + 3)}{162} - \frac{\ln (x^3 + 1)}{6} + \frac{13 \ln (x)}{27} + \frac{\frac{4 x^3}{27} - \frac{1}{18}}{x^6}$$

```
[In] int(1/(x^7*(4*x^3 + x^6 + 3)),x)
[Out] log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x^
6
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = -2x^2+\frac{x^5}{5}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

```
[Out] -2*x^2+1/5*x^5-9/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-3/
2*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1381, 1516, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)$$

```
[In] Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^5}{5} - \frac{1}{5}\int \frac{x^4(15+20x^3)}{3+4x^3+x^6}\,dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{10}\int \frac{x(120+130x^3)}{3+4x^3+x^6}\,dx \\
&= -2x^2 + \frac{x^5}{5} - \frac{1}{2}\int \frac{x}{1+x^3}\,dx + \frac{27}{2}\int \frac{x}{3+x^3}\,dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{6}\int \frac{1}{1+x}\,dx - \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx \\
&\quad - \frac{1}{2}\left(3\ 3^{2/3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx + \frac{1}{2}\left(3\ 3^{2/3}\right)\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx
\end{aligned}$$

$$= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) - \frac{1}{12}\int\frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int\frac{1}{1-x+x^2}\,dx + \frac{27}{4}\int\frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx+\frac{1}{4}\left(3\ \ 3^{2/3}\right)\int\frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx$$

$$= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right) +\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{2}\left(9\ 3^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x$$

$$= -2x^2+\frac{x^5}{5}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x) - \frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int\frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{60}\Bigg(-120x^2+12x^5 - 270\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+10\log(1+x) - 90\ 3^{2/3}\log\left(3+3^{2/3}x\right)-5\log\left(1-x+x^2\right)+45\ 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\Bigg)$$

```
[In] Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[
3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*
x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.55

method	result
risch	$\frac{x^5}{5}-2x^2-\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}+\frac{\ln(x+1)}{6}$
default	$\frac{x^5}{5}-2x^2+\frac{\ln(x+1)}{6}-\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(x^2-x+1)}{12}-\sqrt{3}$

```
[In] int(x^10/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5-2*x^2-1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+
3/2*sum(_R*ln(_R^2+3*x),_R=RootOf(_Z^3+9))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{x^{10}}{3+4x^3+x^6}\,dx &= \frac{1}{5}\,x^5-2\,x^2+\frac{3}{2}\,\sqrt{3}(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\,\sqrt{3}\left(2\,(-9)^{\frac{1}{3}}\,x+3\right)\right)\\&\quad-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)\\&\quad-\frac{3}{4}\,(-9)^{\frac{1}{3}}\log\left(3\,x^2-(-9)^{\frac{2}{3}}\,x-3\,(-9)^{\frac{1}{3}}\right)\\&\quad+\frac{3}{2}\,(-9)^{\frac{1}{3}}\log\left(3\,x+(-9)^{\frac{2}{3}}\right)-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,\log\left(x+1\right)\end{aligned}$$

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*x
 + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3*x
^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3)) -
1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3188648\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \operatorname{RootSum}\left(8t^3+243,\left(t\mapsto t\log\left(\frac{3872t^5}{3281}+\frac{3188648t^2}{88587}+x\right)\right)\right)$$

```
[In] integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12
 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/1
2 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-
1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(38
72*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*
arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2} \cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{9}{2} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\,\log\left(x^2-x+1\right) + \frac{1}{6}\,\log\left(|x+1|\right)$$

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(
1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{\ln\left(x+1\right)}{6} - \frac{3\,3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - 2\,x^2 + \frac{x^5}{5} - \frac{3\,{\left(-1\right)}^{1/3}\,\ln\left(x - \frac{{\left(-1\right)}^{1/3}\,3^{1/3}}{2} - \frac{{\left(-1\right)}^{1/6}\,3^{5/6}}{2} + \frac{3^{1/3}}{2}\right)\,\left(3^{2/3} + 3^{1/6}\,3\mathrm{i}\right)}{4} + \frac{3\,{\left(-1\right)}^{1/3}\,3^{2/3}\,\ln\left(x + {\left(-1\right)}^{2/3}\,3^{1/3}\right)}{2}$$

```
[In] int(x^10/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 -
((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)^(1/
3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

3.159 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = -4x+\frac{x^4}{4}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

```
[Out] -4*x+1/4*x^4-3/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+3/2*
3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1381, 1516, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)$$

```
[In] Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
```

```
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4}{4} - \frac{1}{4}\int \frac{x^3(12+16x^3)}{3+4x^3+x^6}\,dx \\
&= -4x + \frac{x^4}{4} + \frac{1}{4}\int \frac{48+52x^3}{3+4x^3+x^6}\,dx \\
&= -4x + \frac{x^4}{4} - \frac{1}{2}\int \frac{1}{1+x^3}\,dx + \frac{27}{2}\int \frac{1}{3+x^3}\,dx \\
&= -4x + \frac{x^4}{4} - \frac{1}{6}\int \frac{1}{1+x}\,dx - \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx \\
&\quad + \frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx + \frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx
\end{aligned}$$

$$= -4x + \frac{x^4}{4} - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\left(3\sqrt[3]{3}\right)\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx + \frac{1}{4}\left(9\ 3^{2/3}\right)\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx$$

$$= -4x + \frac{x^4}{4} - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3} - \sqrt[3]{3}x+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1 + 2x\right) + \frac{1}{2}\left(9\sqrt[3]{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)$$

$$= -4x + \frac{x^4}{4} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{12}\left(-48x+3x^4 - 18\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + 18\sqrt[3]{3}\log\left(3+3^{2/3}x\right) + \log\left(1-x+x^2\right)\right.$$

```
[In] Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Lo
g[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result
risch	$\frac{x^4}{4}-4x+\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3-3)}\_R\ln(x+\_R)\right)}{2}$
default	$\frac{x^4}{4}-4x-\frac{\ln(x+1)}{6}+\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln(x^2-x+1)}{12}-\sqrt{3}$

```
[In] int(x^9/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4-4*x+1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/
6*ln(x+1)+3/2*sum(_R*ln(x+_R),_R=RootOf(_Z^3-3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\,\sqrt{3}\right) -\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2} \cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-4\,x+\frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \operatorname{RootSum}\left(8t^3-81, \left(t \mapsto t\log\left(\frac{360t^4}{547} - \frac{9841t}{1641} + x\right)\right)\right)$$

```
[In] integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 98
41*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*I/
12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)*I/
19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_t/16
41 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log(x+1)$$

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - 4\,x + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(|x+1|\right)$$

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(ab
s(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{3\,3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln\left(x+1\right)}{6} - 4\,x + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^4}{4} - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3\,3^{1/3}}{4}+\frac{3^{5/6}\,3\mathrm{i}}{4}\right) + 3^{1/3}\,\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)$$

```
[In] int(x^9/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3))/
4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(1/2)
*3i)/4 - 3/4)
```

3.160 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{x^7}{3+4x^3+x^6}\,dx=\frac{x^2}{2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

```
[Out] 1/2*x^2+3/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/2*3^(2/
3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6
*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1381, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\int \frac{x^7}{3+4x^3+x^6}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)$$

```
[In] Int[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3)
- 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 -
x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
```

```
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{2}-\frac{1}{2}\int \frac{x(6+8x^3)}{3+4x^3+x^6}\,dx \\
&= \frac{x^2}{2}+\frac{1}{2}\int \frac{x}{1+x^3}\,dx-\frac{9}{2}\int \frac{x}{3+x^3}\,dx \\
&= \frac{x^2}{2}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx \\
&\quad +\frac{1}{2}3^{2/3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}3^{2/3}\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx \\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad +\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{9}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx-\frac{1}{4}3^{2/3}\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx \\
&= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right) \\
&\quad -\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\left(3\,3^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right. \\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x) \\
&\quad +\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{12}\left(6x^2 + 18\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + 6\,3^{2/3}\log\left(3+3^{2/3}x\right) + \log\left(1-x+x^2\right) - 3\,3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

```
[In] Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x +
 x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x^2}{2}+\frac{\ln\left(4x^2-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}-\frac{\ln(x+1)}{6}$
default	$\frac{x^2}{2}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan(\ldots}{6}$

```
[In] int(x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*su
m(_R*ln(_R^2+3*x),_R=RootOf(_Z^3-9))-1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x - \frac{1}{3}\,\sqrt{3}\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^2 - 9^{\frac{2}{3}}x + 3\cdot 9^{\frac{1}{3}}\right) + \frac{1}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x + 9^{\frac{2}{3}}\right) + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3)) +
 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2/3
)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1) -
 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{x^2}{2} - \frac{\log\left(x+1\right)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61} + \frac{6562\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{183}\right) + \operatorname{RootSum}\left(8t^3 - 9, \left(t \mapsto t\log\left(-\frac{1872t^5}{61} + \frac{6562t^2}{183} + x\right)\right)\right)$$

```
[In] integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*
I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*lo
g(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183)
 + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 +
 x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(|x+1|\right)$$

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs(
x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arc
tan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x +
1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{3+4x^3+x^6}\,dx$$
$$= \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln(x+1)}{6} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^2}{2} - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4}-\frac{3^{1/6}\,3\mathrm{i}}{4}\right) - \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4}+\frac{3^{1/6}\,3\mathrm{i}}{4}\right)$$

```
[In] int(x^7/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1/6)
*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)/4)
```

3.161 $\int \frac{x^6}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

```
[Out] x+1/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/2*3^(1/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1381, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)$$

```
[In] Int[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)
/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^
2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
```

```
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x-\int \frac{3+4x^3}{3+4x^3+x^6}\,dx \\
&= x+\frac{1}{2}\int \frac{1}{1+x^3}\,dx-\frac{9}{2}\int \frac{1}{3+x^3}\,dx \\
&= x+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx \\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx \\
&\quad +\frac{1}{4}\sqrt[3]{3}\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx-\frac{1}{4}\left(3\ 3^{2/3}\right)\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx \\
&= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x\right. \\
&\qquad \left.+x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1\right. \\
&\qquad \left.+2x\right)-\frac{1}{2}\left(3\sqrt[3]{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{3}}\right) \\
&= x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x) \\
&\quad -\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = \frac{1}{12}\Bigg(12x$$
$$+6\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-6\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)$$

```
[In] Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x
^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.47

method	result
risch	$x+\frac{\ln(x+1)}{6}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+3\right)}\_R\ln\left(x-\_R\right)\right)}{2}$
default	$x+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1}{3}\right.}{6}$

```
[In] int(x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/6*ln(x+1)-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/2*
sum(_R*ln(x-_R),_R=RootOf(_Z^3+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = \frac{1}{2}\sqrt{3}(-3)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\,(-3)^{\frac{2}{3}}\,x-3\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,(-3)^{\frac{1}{3}}\log\left(x^2+(-3)^{\frac{1}{3}}\,x+(-3)^{\frac{2}{3}}\right) + \frac{1}{2}\,(-3)^{\frac{1}{3}}\log\left(x-(-3)^{\frac{1}{3}}\right) + x - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x +
(-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x + \frac{\log(x+1)}{6} + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{41}\right) + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}+\frac{864\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{41}+\frac{121\sqrt{3}i}{246}\right) + \operatorname{RootSum}\left(8t^3+3,\left(t\mapsto t\log\left(\frac{864t^4}{41}+\frac{242t}{41}+x\right)\right)\right)$$

```
[In] integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/2
46 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121
/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t
**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + x-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{3+4x^3+x^6}\,dx$$
$$= x+\frac{\ln(x+1)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2}$$
$$-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$
$$+\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{4}-\frac{3^{5/6}\,1\mathrm{i}}{4}\right)+\frac{(-1)^{1/3}\,3^{1/3}\,\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{2}$$

```
[In] int(x^6/(4*x^3 + x^6 + 3),x)
```

```
[Out] x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*1i)
/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

3.162 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}$$

```
[Out] -1/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/6*3^(2/3)*ln(3
^(1/3)+x)-1/12*ln(x^2-x+1)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1388, 298, 31, 648, 631, 210, 642, 632}

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}} + \frac{1}{6}\log(x+1) - \frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}$$

```
[In] Int[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
```

```
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{x}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{x}{3+x^3}\,dx\\
&= \frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\sqrt[3]{3}}\\
&= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&\quad -\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx+\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{4\sqrt[3]{3}}\\
&= \frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{2}3^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{3}}\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}\\
&\quad -\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{3+4x^3+x^6}\,dx=\frac{1}{12}\left(-6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)\right.$$
$$\left.-2\,3^{2/3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)+3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

```
[In] Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] +
3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3+1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{2}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

```
[In] int(x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)+1
/2*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = -\frac{1}{12}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2(-1)^{\frac{1}{3}}x+3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1/
3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2*(-
1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{5}\right) + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{5}+\frac{2592\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{5}\right) + \operatorname{RootSum}\left(24t^3+1,\left(t\mapsto t\log\left(\frac{2592t^5}{5}+\frac{168t^2}{5}+x\right)\right)\right)$$

```
[In] integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)**
5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 168
*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootSum(
24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
- 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1/6
)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) -\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1/
3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6}-\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) -\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) -\frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}3^{1/3}}{2}-\frac{(-1)^{1/6}3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{12} +\frac{(-1)^{1/3}3^{2/3}\ln\left(x+(-1)^{2/3}3^{1/3}\right)}{6}$$

```
[In] int(x^4/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/6))
/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x + (-
1)^(2/3)*3^(1/3)))/6
```

3.163 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^3}{3+4x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}}$$

```
[Out] -1/6*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/6*3^(1/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1388, 206, 31, 648, 631, 210, 642, 632}

$$\int \frac{x^3}{3+4x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}$$

```
[In] Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
```

```
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int \frac{1}{1+x^3}\,dx\right) + \frac{3}{2}\int \frac{1}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) - \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\ 3^{2/3}} + \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\ 3^{2/3}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{4\ 3^{2/3}} + \frac{1}{4}3^{2/3}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) + \frac{1}{2}\sqrt[3]{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)$$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{3+4x^3+x^6}\,dx$$
$$= \frac{1}{12}\left(-23^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + 2\sqrt[3]{3}\log\left(3+3^{2/3}x\right) + \log\left(1-x+x\right.\right.$$

```
[In] Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] -
3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{3}-1\right)}\_R\ln\left(x+3\_R\right)\right)}{2}-\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$-\frac{\ln\left(x+1\right)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{12}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{}}{3}\right.}{6}$

```
[In] int(x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))-1/6*ln(x+1)+1/12*ln(x^2-x+1)-1/6
*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{3+4x^3+x^6}\,dx=\frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3\,x^{2}-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sq
rt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*lo
g(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2
 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4+\frac{\sqrt{3}i}{4}\right) + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4-\frac{\sqrt{3}i}{4}\right) + \operatorname{RootSum}\left(72t^3-1,\left(t\mapsto t\log\left(648t^4-3t+x\right)\right)\right)$$

```
[In] integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/1
2)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sqrt
(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(648*_
t**4 - 3*_t + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{1}{12}\,\log\left(x^2-x+1\right)-\frac{1}{6}\,\log\left(|x+1|\right)$$

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{6}-\frac{\ln\left(x+1\right)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\,\left(\frac{3^{1/3}}{12}+\frac{3^{5/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\,\left(\frac{3^{1/3}}{12}-\frac{3^{5/6}\,1\mathrm{i}}{12}\right)$$

```
[In] int(x^3/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
- 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)/12
) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}}$$

```
[Out] 1/6*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/18*3^(2/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1389, 298, 31, 648, 632, 210, 642, 631}

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\sqrt[3]{3}} - \frac{1}{6}\log(x+1) + \frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}$$

```
[In] Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

```
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{x}{1+x^3}\,dx - \frac{1}{2}\int \frac{x}{3+x^3}\,dx \\
&= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) + \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\sqrt[3]{3}} \\
&= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx - \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{12\sqrt[3]{3}} \\
&= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}} \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)}{2\sqrt[3]{3}} \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} \\
&\quad - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$
\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{36}\left(6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 6\log(1+x) + 2\,3^{2/3}\log\left(3+3^{2/3}x\right) + 3\log\left(1-x+x^2\right) - 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)
$$

```
[In] Integrate[x/(3 + 4*x^3 + x^6),x]
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] -
 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{6}$
default	$-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

```
[In] int(x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(x+1)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/6*s
um(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5\right) + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5+90\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2\right) + \operatorname{RootSum}\left(648t^3-1,\left(t\mapsto t\log\left(11664t^5+90t^2+x\right)\right)\right)$$

```
[In] integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2 +
 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1/1
2 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3 - 1
, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^(
1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}-\frac{3^{1/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}+\frac{3^{1/6}\,1\mathrm{i}}{12}\right)$$

```
[In] int(x/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i)/1
2) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

3.165 $\int \frac{1}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 112

$$\int \frac{1}{3+4x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}}$$

```
[Out] 1/18*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/18*3^(1/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arct
an(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1361, 206, 31, 648, 632, 210, 642, 631}

$$\int \frac{1}{3+4x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}$$

```
[In] Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
```

```
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1+x^3}\,dx - \frac{1}{2}\int \frac{1}{3+x^3}\,dx \\
&= \frac{1}{6}\int \frac{1}{1+x}\,dx + \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\ 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{6\ 3^{2/3}} \\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{12\ 3^{2/3}} - \frac{\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{4\sqrt[3]{3}} \\
&= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}} \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)}{2\ 3^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} \\
&\quad + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{1}{3+4x^3+x^6}\,dx$$
$$= \frac{1}{36}\left(2\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 6\log(1+x) - 2\sqrt[3]{3}\log\left(3+3^{2/3}x\right) - 3\log\left(1-x+\right.\right.$$

```
[In] Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] +
 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{3}+1\right)}\_R\ln\left(x-3\_R\right)\right)}{6}-\frac{\ln\left(4x^{2}-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{36}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}-\frac{\ln\left(x^{2}-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

```
[In] int(1/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))-1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2)
*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{1}{18}\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{108}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x+3\,x^{2}+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-9^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\,x\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1)
^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1)^
(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9^(2
/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}-\frac{13\sqrt{3}i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{5}\right) + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}+\frac{23328\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{5}+\frac{13\sqrt{3}i}{10}\right) + \operatorname{RootSum}\left(1944t^3+1,\left(t\mapsto t\log\left(\frac{23328t^4}{5}-\frac{78t}{5}+x\right)\right)\right)$$

```
[In] integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 233
28*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23
328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1
, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{18}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{18} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(1/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(1/3
)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^
(5/6)*1i))/36
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{1}{3x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}$$

```
[Out] -1/3/x-1/18*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/54*3^(2
/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+
1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1382, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\,3^{5/6}}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}$$

```
[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/3*1/x + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
- x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3x}+\frac{1}{3}\int \frac{x(-4-x^3)}{3+4x^3+x^6}\,dx \\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{x}{3+x^3}\,dx-\frac{1}{2}\int \frac{x}{1+x^3}\,dx \\
&= -\frac{1}{3x}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{3}} \\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad+\frac{1}{12}\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{36\sqrt[3]{3}} \\
&= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}} \\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{3}}\right)}{6\sqrt[3]{3}} \\
&= -\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}} \\
&\quad+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{36+6\sqrt[6]{3}x\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18\sqrt{3}x\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-18x\log(1+x)+2\,3^{2/3}x\log\left(3+3^{2/3}x\right)+9x}{108x}$$

```
[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x]
+ 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{1}{3x}+\frac{\ln(x+1)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3+1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{18}$
default	$\frac{\ln(x+1)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{108}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$

```
[In] int(1/x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x+1/6*ln(x+1)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
+1/18*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (3+4x^3+x^6)} dx = -\frac{3^{\frac{2}{3}}(-1)^{\frac{1}{3}} x \log \left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}} x + x^2 - 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right) - 2\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}} x \log \left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}} + x\right) + 18\sqrt{3}x \arctan \left(\frac{1}{3}\right.}{1}$$

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)
^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)*x
*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/6)*
(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) + 36)/
x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{1}{x^2 (3+4x^3+x^6)} dx &= \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log \left(x - \frac{8188128\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{41} + \frac{39384\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{41}\right) \\ &+ \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log \left(x + \frac{39384\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{41} - \frac{8188128\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{41}\right) \\ &+ \operatorname{RootSum}\left(17496t^3 + 1, \left(t \mapsto t \log \left(-\frac{8188128t^5}{41} + \frac{39384t^2}{41} + x\right)\right)\right) - \frac{1}{3x}\end{aligned}$$

```
[In] integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/12
)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*log(
x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)**5/
41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39384*
_t**2/41 + x))) - 1/(3*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2 (3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x} -\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3)
) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2 (3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54} \cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) +\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x} -\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^(
1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/
3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
+ 1))
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (3+4x^3+x^6)} \, dx = \frac{\ln(x+1)}{6} - \frac{3^{2/3} \ln\left(x+3^{1/3}\right)}{54} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{3\,x} - \frac{(-1)^{1/3} \ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{108} + \frac{(-1)^{1/3}\,3^{2/3} \ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{54}$$

```
[In] int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/
6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^(2/3
)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = -\frac{1}{6x^2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}$$

```
[Out] -1/6/x^2-1/54*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/54*3^
(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1382, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}$$

```
[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/6*1/x^2 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
```

```
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{-8-2x^3}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{1}{3+x^3}\,dx-\frac{1}{2}\int \frac{1}{1+x^3}\,dx\\
&= -\frac{1}{6x^2}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\ 3^{2/3}}\\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&\quad -\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{36\ 3^{2/3}}+\frac{\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{12\sqrt[3]{3}}\\
&= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{3}}\right)}{6\ 3^{2/3}}\\
&= -\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}\\
&\quad -\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx = \frac{1}{108}\Bigg(-\frac{18}{x^2} - 2\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 18\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 18\log(1+x) + 2\sqrt[3]{3}\log\left(3+3^{2/3}x\right) + 9\log\left(1-x+\right.$$

```
[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1
- x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{1}{6x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^3-1\right)}\_R\ln\left(x+3\_R\right)\right)}{18}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}$
default	$-\frac{1}{6x^2}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{108}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}\arctan\left(\right.}{6}$

```
[In] int(1/x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^2+1/18*sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))+1/12*ln(x^2-x+1)-1/6*3
^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))-1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx = \frac{6\cdot 9^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3\,x\right.}{324\,x^2}$$

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9
^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/
3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) +
27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx &= -\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{61}\right)\\&+\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{61}+\frac{1093\sqrt{3}i}{244}\right)\\&+\operatorname{RootSum}\left(52488t^3-1,\left(t\mapsto t\log\left(\frac{787320t^4}{61}+\frac{3279t}{61}+x\right)\right)\right)-\frac{1}{6x^2}\end{aligned}$$

```
[In] integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/244
 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 1093
/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSum(5
2488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) - 1/(
6*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\,(3+4x^3+x^6)}\,dx = \frac{1}{54}\cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6\,x^2} + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\,(3+4x^3+x^6)}\,dx = \frac{1}{54}\cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{6\,x^2} + \frac{1}{12}\,\log\left(x^2-x+1\right) - \frac{1}{6}\,\log\left(|x+1|\right)$$

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(
x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx = \frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{54}-\frac{\ln\left(x+1\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{6\,x^2}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\,\left(\frac{3^{1/3}}{108}+\frac{3^{5/6}\,1\mathrm{i}}{108}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\,\left(\frac{3^{1/3}}{108}-\frac{3^{5/6}\,1\mathrm{i}}{108}\right)$$

```
[In] int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108 + (
3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (3^(5
/6)*1i)/108)
```

3.168 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$

```
[Out] -1/12/x^4+4/9/x+1/54*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+
1/162*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/
3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1382, 1518, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}$$

```
[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/12*1/x^4 + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{12x^4}+\frac{1}{12}\int\frac{-16-4x^3}{x^2\left(3+4x^3+x^6\right)}\,dx\\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{36}\int\frac{x(-52-16x^3)}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{18}\int\frac{x}{3+x^3}\,dx+\frac{1}{2}\int\frac{x}{1+x^3}\,dx\\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\int\frac{1}{1+x}\,dx+\frac{1}{6}\int\frac{1+x}{1-x+x^2}\,dx+\frac{\int\frac{1}{\sqrt[3]{3}+x}\,dx}{54\sqrt[3]{3}}-\frac{\int\frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{3}}\\
&= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}-\frac{1}{36}\int\frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx\\
&\quad+\frac{1}{12}\int\frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int\frac{1}{1-x+x^2}\,dx-\frac{\int\frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{3}}
\end{aligned}$$

$$= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$

$$-\frac{1}{2}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)}{18\sqrt[3]{3}}$$

$$= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int\frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx = \frac{1}{324}\left(-\frac{27}{x^4} + \frac{144}{x} + 6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 54\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 54\log(1+x) + 2\ 3^{2/3}\log\left(3+3^{2/3}x\right) + 27\log\left(1-x+x^2\right) - 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

```
[In] Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

method	result
risch	$\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{54}$
default	$-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\ln(x+1)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\mathrm{a}}{}$

```
[In] int(1/x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] (4/9*x^3-1/12)/x^4-1/6*ln(x+1)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1
/2)*3^(1/2))+1/54*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx = -\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan\left(\right.}{324\,x^4}$$

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arc
tan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x
+ 1) - 144*x^3 + 27)/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right) + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1028869776\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{547}+\frac{4782978\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{547}\right) + \operatorname{RootSum}\left(472392t^3-1,\left(t\mapsto t\log\left(\frac{1028869776t^5}{547}+\frac{4782978t^2}{547}+x\right)\right)\right) + \frac{16x^3-3}{36x^4}$$

```
[In] integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12)
**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12)*
log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3)*I
/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_t**5
/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)$$

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1/
3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1
/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x + 3
^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1
) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{162}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}-\frac{3^{1/6}\,1\mathrm{i}}{108}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}+\frac{3^{1/6}\,1\mathrm{i}}{108}\right)$$

```
[In] int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3
^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/
3)/324 + (3^(1/6)*1i)/108)
```

3.169 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}}$$

```
[Out] -1/15/x^5+2/9/x^2+1/162*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+
x)-1/162*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/324*3^(1/3)*ln(3^(2/3)-3^
(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1382, 1518, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}$$

```
[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/15*1/x^5 + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{15x^5}+\frac{1}{15}\int \frac{-20-5x^3}{x^3\left(3+4x^3+x^6\right)}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{90}\int \frac{-130-40x^3}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{18}\int \frac{1}{3+x^3}\,dx+\frac{1}{2}\int \frac{1}{1+x^3}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\ 3^{2/3}}-\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{54\ 3^{2/3}}
\end{aligned}
$$

$$= -\frac{1}{15x^5} + \frac{2}{9x^2} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{108\ 3^{2/3}} - \frac{\int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{36\sqrt[3]{3}}$$

$$= -\frac{1}{15x^5} + \frac{2}{9x^2} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{3}}\right)}{18\ 3^{2/3}}$$

$$= -\frac{1}{15x^5} + \frac{2}{9x^2} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = \frac{-\frac{108}{x^5} + \frac{360}{x^2} + 10\ 3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 270\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 270\log(1+x) - 10\sqrt[3]{3}\log\left(3+3^{2/3}x\right)}{1620}$$

```
[In] Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2/3
)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5}+\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^3+1\right)}\_R\ln\left(x-3\_R\right)\right)}{54}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$-\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{\ln(x+1)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{162}-\frac{\ln(x^2-x+1)}{12}+$

```
[In] int(1/x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] (2/9*x^3-1/15)/x^5+1/6*ln(x+1)+1/54*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))-
1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx$$
$$=\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^5\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x+3\,x\right.}{}$$

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*sq
rt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9^(
2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3)*x
^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 324)/x^
5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^6 (3 + 4x^3 + x^6)} dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{88573}{6562} - \frac{88573\sqrt{3}i}{6562} + \frac{119042784\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{3281}\right) + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{88573}{6562} + \frac{119042784\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{3281} + \frac{88573\sqrt{3}i}{6562}\right) + \operatorname{RootSum}\left(1417176t^3 + 1, \left(t \mapsto t\log\left(\frac{119042784t^4}{3281} - \frac{531438t}{3281} + x\right)\right)\right) + \frac{10x^3 - 3}{45x^5}$$

```
[In] integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I/
6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)*l
og(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqrt(3
)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**4/32
81 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6 (3 + 4x^3 + x^6)} dx = -\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) + \frac{10x^3 - 3}{45x^5} - \frac{1}{12}\log\left(x^2 - x + 1\right) + \frac{1}{6}\log(x+1)$$

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6 (3+4x^3+x^6)} dx = -\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{10x^3-3}{45x^5} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x +
1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^6 (3+4x^3+x^6)} dx = \frac{\ln(x+1)}{6} - \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{162} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5} + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{162} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{324}$$

```
[In] int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)
*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3
^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

3.170 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 412

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2
^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/1
8*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+
I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1381, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \frac{(-\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} - \frac{(\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+(1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+(1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} + x + \frac{(3-i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

```
[In] Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
```

```
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
```

```
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x - \int \frac{1-x^3}{1-x^3+x^6}\,dx \\
&= x - \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx + \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= x + \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}
\end{aligned}$$

$$
\begin{aligned}
&= x + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}
\end{aligned}
$$

$$= x + \frac{\left(3 - i\sqrt{3}\right)\log\left(\sqrt[3]{1 - i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1 - i\sqrt{3}\right)^{2/3}} + \frac{\left(3 + i\sqrt{3}\right)\log\left(\sqrt[3]{1 + i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1 + i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3 - i\sqrt{3}\right)\log\left(\left(1 - i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1 - i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1 - i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3 + i\sqrt{3}\right)\log\left(\left(1 + i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1 + i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1 + i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3 - i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1 - i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1 - i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3 + i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1 + i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1 + i\sqrt{3}\right)^{2/3}}$$

$$= x + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[x^6/(1 - x^3 + x^6),x]
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
2*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{3}-1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{3}-1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

```
[In] int(x^6/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)-3\sqrt{-3}+3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)+3\sqrt{-3}+3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)+3\sqrt{-3}+3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)-3\sqrt{-3}+3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)+36\,x\right)+x$$

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(
I*sqrt(-3) - I) - 3*sqrt(-3) + 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^
(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-
3) - I) + 3*sqrt(-3) + 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3)*(-
I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I)
+ 3*sqrt(-3) + 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I*sqrt(
3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) - 3*sq
rt(-3) + 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(3) + 3)^
(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(I*sqrt(3) - 3) + 36*x) + 1/54*18^
(2/3)*(-I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(1/3)*(-I*sqrt(3
) - 3) + 36*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x + \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

```
[In] integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t
 + x)))
```

Maxima [F]

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \int \frac{x^6}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(268) = 536.

Time = 0.33 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.56

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*
cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9
*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2
*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9
*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))
*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*si
n(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/
9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi
) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*
sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*p
i)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*
cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos
(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(x^6/(x^6 - x^3 + 1),x)
```

```
[Out] x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/2
)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*12i
)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3 -
3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^
(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i
)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(1/2)*1i
 + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5/6)*
(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.171 $\int \frac{x^5}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^5}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] 1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 648, 632, 210, 642}

$$\int \frac{x^5}{1-x^3+x^6} \, dx = \frac{1}{6}\log\left(x^6-x^3+1\right) - \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Int[x^5/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1-x^3+x^6}\,dx=\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[In] Integrate[x^5/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	33
risch	$\frac{\ln\left(4x^{6}-4x^{3}+4\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	35

```
[In] int(x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx=\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1-x^3+x^6}\,dx=\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**5/(x**6-x**3+1),x)
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

```
[In] int(x^5/(x^6 - x^3 + 1),x)
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(1/3)/(1+I*3^(1/2))^(1/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1
/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2)
)^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/18
*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/
3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(
3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I
*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1388, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[In] Int[x^4/(1 - x^3 + x^6),x]
```

```
[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
```

```
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - (
(3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
```

```
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int\frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\left(-\frac{\left(-3-i\sqrt{3}\right)\int\frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)$$

$$+\frac{\left(3-i\sqrt{3}\right)\int\frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$-\frac{\left(3-i\sqrt{3}\right)\int\frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+\frac{\left(3+i\sqrt{3}\right)\int\frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$--\frac{(-3-i\sqrt{3})\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$-\frac{1}{12}\left(-3+i\sqrt{3}\right)\int\frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx-\frac{(3-i\sqrt{3})\int\frac{\sqrt[3]{\frac{1}{2}\left(1+\right.}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{}}}{18\ 2^{2/3}\sqrt[3]{1+}}$$

$$= \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$-\frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$-\frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$--\frac{(-3-i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$-\frac{(3-i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$- \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[x^4/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40

```
[In] int(x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{x^4}{1-x^3+x^6}\,dx &= -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+\sqrt{-3}-1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\\ &\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-\sqrt{-3}-1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\\ &\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-\sqrt{-3}-1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{108}\\ &\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+\sqrt{-3}-1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\left(i\sqrt{3}-3\right)^{\frac{2}{3}}+12\,x\right)\\ &+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}+12\,x\right)\end{aligned}$$

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*
(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) - 3)^(2/3) + 24*x) + 1/108*18^(
```

```
2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3
) - I) - sqrt(-3) - 1)*(I*sqrt(3) - 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(-I*s
qrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I) - s
qrt(-3) - 1)*(-I*sqrt(3) - 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt(3) -
3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sqrt(-3)
- 1)*(-I*sqrt(3) - 3)^(2/3) + 24*x) + 1/54*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*l
og(18^(1/3)*(I*sqrt(3) + 1)*(I*sqrt(3) - 3)^(2/3) + 12*x) + 1/54*18^(2/3)*(
-I*sqrt(3) - 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 1)*(-I*sqrt(3) - 3)^(2/3)
+ 12*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)$$

```
[In] integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**
2 + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \int \frac{x^4}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(267) = 534.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.01

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(
```

```
4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)
*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*co
s(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi
)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*c
os(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1
/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(
3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*
cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^
5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*p
i))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)
*sin(1/9*pi))) - 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*co
s(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*
cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(
4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/
9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*
pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*
cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)
^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log
((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos
(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)
*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(
1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
 sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(162\,x+\frac{27\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\left(162\,x+\frac{27\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(x^4/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i - 3
6)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18 - (
2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3
^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/
12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3^(1/2)*1
i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*1i - 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.173 $\int \frac{x^3}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^3}{1-x^3+x^6}\,dx=-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))
^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*
ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3
)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[In] Int[x^3/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/
(2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*
Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sq
rt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(
1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
```

```
- ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
+ 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
```

```
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$=\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+\frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$=\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$-\frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$-\frac{\left(3+i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$-\frac{\left(3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$=\frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}+\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

$$-\frac{(3+i\sqrt{3})\log\left((1-i\sqrt{3})^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}$$

$$-\frac{(3-i\sqrt{3})\log\left((1+i\sqrt{3})^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

$$+\frac{(3-i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

$$+\frac{(3+i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}$$

$$=-\frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}+\frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

$$+\frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}+\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

$$-\frac{(3+i\sqrt{3})\log\left((1-i\sqrt{3})^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}}$$

$$-\frac{(3-i\sqrt{3})\log\left((1+i\sqrt{3})^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[x^3/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}(\_Z^6-\_Z^3+1)}\frac{\_R^3\ln(x-\_R)}{2\_R^5-\_R^2}}{3}$	40
risch	$\frac{\sum_{\_R=\text{RootOf}(\_Z^6-\_Z^3+1)}\frac{\_R^3\ln(x-\_R)}{2\_R^5-\_R^2}}{3}$	40

```
[In] int(x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}+i\right)+36\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}-i\right)+36\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}+i\right)+36\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}-i\right)+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)$$

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*sqrt(3)*(
I*sqrt(3) - 3)^(1/3)*(I*sqrt(-3) + I) + 36*x) + 1/108*18^(2/3)*(-I*sqrt(3)
- 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(1/3)*(I*sq
rt(-3) - I) + 36*x) + 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*l
og(18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) + I) + 36*x) - 1/108
*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*sqrt(3)*(-I*sq
rt(3) - 3)^(1/3)*(-I*sqrt(-3) - I) + 36*x) + 1/54*18^(2/3)*(I*sqrt(3) - 3)^
(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3) + 18*x) + 1/54*18^(2/3)
*(-I*sqrt(3) - 3)^(1/3)*log(I*18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(1/3) + 18*
x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right)$$

```
[In] integrate(x**3/(x**6-x**3+1),x)
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t +
 x)))
```

Maxima [F]

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \int \frac{x^3}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(267) = 534.

Time = 0.33 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9
*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*co
s(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^
4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos
(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1
/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*
sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*p
i))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)
*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(
```

```
4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 -
2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt(3)*cos(4/
9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*p
i) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)
^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log
((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)
^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi)
 - cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(x^3/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i
 - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1/2)*
1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3
)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3))/12)*
```

```
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/
3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3
)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2
/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(
3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36
```

3.174 $\int \frac{x^2}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^2}{1-x^3+x^6}\,dx = -\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 632, 210}

$$\int \frac{x^2}{1-x^3+x^6}\,dx = -\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) \\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\right) \\ &= -\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[x^2/(1 - x^3 + x^6),x]
[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	19
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	19

```
[In] int(x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**2/(x**6-x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1-x^3+x^6}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

```
[In] int(x^2/(x^6 - x^3 + 1),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.175 $\int \frac{x}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 375

$$\int \frac{x}{1-x^3+x^6} \, dx = \frac{i \arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i \arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i \log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}} + \frac{i \log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] 1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1-I*
3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))
*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/
3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/9*I*2^(
1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2)+1/18*I*
```

```
ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(1/3)/(
1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1389, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

```
[In] Int[x/(1 - x^3 + x^6),x]
```

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$
= \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i \int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}
$$

$$
- \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i \int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}
$$

$$
= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}
$$

$$
+ \frac{i \int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{2\sqrt{3}}
$$

$$
- \frac{i \int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{2\sqrt{3}}
$$

$$
- \frac{i \int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}
$$

$$
+ \frac{i \int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}
$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$= \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$+ \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

```
[In] Integrate[x/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	38
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	38

```
[In] int(x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.79

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}-i\right)+\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}-i\right)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}+i\right)-\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}+i\right)+\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+3\right)^{\frac{2}{3}}\left(i\sqrt{3}+1\right)+12\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}\left(-i\sqrt{3}+1\right)+12\,x\right)$$

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*
(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(
```

```
2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3
) - I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(-I*s
qrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I) - s
qrt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt(3) +
3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sqrt(-3)
- 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 1/54*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*l
og(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(I*sqrt(3) + 1) + 12*x) + 1/54*18^(2/3)*(
-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 3)^(2/3)*(-I*sqrt(3) + 1)
+ 12*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(6561t^5-27t^2+x\right)\right)\right)$$

```
[In] integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**
2 + x)))
```

Maxima [F]

$$\int \frac{x}{1-x^3+x^6}\,dx = \int \frac{x}{x^6-x^3+1}\,dx$$

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(241) = 482.

Time = 0.31 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.17

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*
```

```
pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*sin(4
/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9*
pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^
5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*
pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(
2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*s
in(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*si
n(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/
9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(-1/2*((
-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi))) - 1
/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9
*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*p
i)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - co
s(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x +
 x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*
pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi
)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin
(2/9*pi) - cos(2/9*pi)^2 + sin(2/9*pi)^2)*log((-I*sqrt(3)*cos(2/9*pi) - cos
(2/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 +
10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*co
s(1/9*pi)*sin(1/9*pi) - cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1
/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} + \frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(x/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3
)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3
 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (2^(1/3
)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3
^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/
6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^
(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

3.176 $\int \frac{1}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 186

$$\int \frac{1}{1-x^3+x^6} \, dx = -\frac{1}{3}(-1)^{13/18} \arctan\left(\frac{1+2\sqrt[9]{-1}x}{\sqrt{3}}\right) + \frac{1}{3}(-1)^{5/18} \arctan\left(\frac{1-2(-1)^{8/9}x}{\sqrt{3}}\right) - \frac{(-1)^{5/18}\left(\log(2)+3\log\left(\sqrt[9]{-1}-x\right)\right)}{9\sqrt{3}} + \frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}((-1)^{8/}\right.}{3\sqrt{3}}$$

```
[Out] -1/3*(-1)^(13/18)*arctan(1/3*(1+2*(-1)^(1/9)*x)*3^(1/2))+1/3*(-1)^(5/18)*ar
ctan(1/3*(1-2*(-1)^(8/9)*x)*3^(1/2))-1/27*(-1)^(5/18)*(ln(2)+3*ln((-1)^(1/9
)-x))*3^(1/2)+1/9*(-1)^(13/18)*ln(-2^(1/3)*((-1)^(8/9)+x))*3^(1/2)-1/18*(-1
)^(13/18)*ln(-2^(2/3)*((-1)^(7/9)+((-1)^(8/9)-x)*x))*3^(1/2)+1/18*(-1)^(5/1
8)*ln(2^(2/3)*((-1)^(2/9)+x*((-1)^(1/9)+x)))*3^(1/2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

```
[In] Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-1/3*I)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sq
rt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)
*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3)
 - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt
[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(
1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= \frac{i \log \left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x \right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i \log \left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x \right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i \log \left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2 \right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i \log \left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2 \right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} \right)}{\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} \right)}{\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$= -\frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}} \right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}} \right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$+ \frac{i \log \left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x \right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i \log \left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x \right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i \log \left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2 \right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i \log \left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2 \right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.23

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[(1 - x^3 + x^6)^(-1),x]
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	37
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	37

```
[In] int(1/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.53

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{4}{3}}+36\,x\right)$$

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(
I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^
(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-
3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3)*(-
I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I)
- 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I*sqrt(
3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) + 3*sq
rt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(3) + 3)^
(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(4/3) + 36*x) + 1/54*18^(2/3)*(-I*sqrt(3
) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(4/3) + 36*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

```
[In] integrate(1/(x**6-x**3+1),x)
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

Maxima [F]

$$\int \frac{1}{1-x^3+x^6}\,dx = \int \frac{1}{x^6-x^3+1}\,dx$$

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")
[Out] integrate(1/(x^6 - x^3 + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.40

$$\int \frac{1}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9
*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)
^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(
2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) -
 sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(
3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi
)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4
*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(-1/2
*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi)))
- 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9
*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sq
rt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))
```

```
*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(
2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi)
- cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/
9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(1/9*pi))*log((I*sqrt(3)
*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.76

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(1/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3
)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*
(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
```

```
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3
))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x +
 (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)
^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.177 $\int \frac{1}{x(1-x^3+x^6)}\, dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^3+x^6)}\, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$\int \frac{1}{x(1-x^3+x^6)}\, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] Int[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\,(1-x+x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)
\end{aligned}
$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3 + \#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
+ 2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x) - \frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9} + \ln(x)$	35

```
[In] int(1/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{\log(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(1/x/(x**6-x**3+1),x)
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(1-x^3+x^6)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(1-x^3+x^6)}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\,(1-x^3+x^6)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

```
[In] int(1/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

3.178 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 416

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x} + \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1382, 1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = \frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[In] Int[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
```

```
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
```

```
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{x}+\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx\\
&= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\\
&= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\\
&\quad+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\\
&\quad-\frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\\
&\quad+\frac{\left(3+i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}
\end{aligned}$$

$$= -\frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+\frac{1}{12}\left(-3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx-\frac{1}{12}\left(3+i\sqrt{3}\right)\int \frac{}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2}}$$

$$= -\frac{1}{x} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{\left(3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{\left(3+i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3 + \#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#
1 + 2*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.08

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}+9\_Z^{3}+1\right)}\_R\ln\left(-3\_R^{2}+x\right)\right)}{3}$	35
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	50

```
[In] int(1/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/x+1/3*sum(_R*ln(-3*_R^2+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = \frac{18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right)+12\,x\right)+18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log}{}$$

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/108*(18^(2/3)*(sqrt(-3)*x - x)*(I*sqrt(3) - 3)^(1/3)*log(18^(1/3)*(I*sqrt
(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) + 18^(2/3)*(sqrt(-3)*x - x)*(-I*sqrt(
3) - 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) -
18^(2/3)*(sqrt(-3)*x + x)*(I*sqrt(3) - 3)^(1/3)*log(-18^(1/3)*(I*sqrt(3) -
3)^(2/3)*(sqrt(-3) - 1) + 12*x) - 18^(2/3)*(sqrt(-3)*x + x)*(-I*sqrt(3) - 3
)^(1/3)*log(-18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x) + 2*18^
(2/3)*x*(I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(I*sqrt(3) - 3)^(2/3)) + 2
*18^(2/3)*x*(-I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(-I*sqrt(3) - 3)^(2/3
)) - 108)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = \text{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)-\frac{1}{x}$$

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1
/x
```

Maxima [F]

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = \int \frac{1}{(x^6-x^3+1)x^2}\,dx$$

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(272) = 544.

Time = 0.31 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*p
i)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - s
in(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/
9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*p
i)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(
```

```
3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arc
tan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/
9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi
)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^
3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4
/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - c
os(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*s
qrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5
- 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*
cos(2/9*pi)*sin(2/9*pi) + 2*cos(2/9*pi)^2 - 2*sin(2/9*pi)^2)*log((-I*sqrt(3
)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*s
in(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5
 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*p
i)^4 - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^
2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx$$
$$= \frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{1}{x}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(1/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)
^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) -
3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2
^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/
24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i -
 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1
i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)} dx$

Optimal result

Integrand size = 16, antiderivative size = 418

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} - \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I
*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1382, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{(-\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} + \frac{(\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} - \frac{1}{2x^2} + \frac{(3-i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1-i\sqrt{3})}\,x+(1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1+i\sqrt{3})}\,x+(1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

```
[In] Int[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
```

```
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
```

```
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2x^2} + \frac{1}{2}\int \frac{2-2x^3}{1-x^3+x^6}\,dx \\
&= -\frac{1}{2x^2} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{1}{2x^2} - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}
\end{aligned}$$

$$= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3-i\sqrt{3})\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3-i\sqrt{3})\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{(3+i\sqrt{3})\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3+i\sqrt{3})\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3-i\sqrt{3})\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3+i\sqrt{3})\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2} - \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/
(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}+9\_Z^{3}+1\right)}\_R\ln\left(9\_R^{4}+3\_R+x\right)\right)}{3}$	38
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{2x^{2}}$	50

```
[In] int(1/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/3*sum(_R*ln(9*_R^4+3*_R+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx$$
$$= \frac{2\cdot 18^{\frac{2}{3}}x^2\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+36\,x\right)+2\cdot 18^{\frac{2}{3}}x^2\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right.\right.}{}$$

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*x^2*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)*(I
*sqrt(3) - 3)^(1/3) + 36*x) + 2*18^(2/3)*x^2*(-I*sqrt(3) - 3)^(1/3)*log(18^
(2/3)*(-I*sqrt(3) + 3)*(-I*sqrt(3) - 3)^(1/3) + 36*x) + 18^(2/3)*(sqrt(-3)*
x^2 - x^2)*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) - I) + 3
*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2
)*(I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) - I) - 3*sqrt(-3
) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2)*(-I*sq
rt(3) - 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I) - 3*sqrt(-3) - 3)*(
-I*sqrt(3) - 3)^(1/3) + 72*x) + 18^(2/3)*(sqrt(-3)*x^2 - x^2)*(-I*sqrt(3) -
 3)^(1/3)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) + 3*sqrt(-3) - 3)*(-I*sqr
t(3) - 3)^(1/3) + 72*x) - 54)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} dx = \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 + 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

```
[In] integrate(1/x**3/(x**6-x**3+1),x)
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t + x
))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} dx = \int \frac{1}{(x^6 - x^3 + 1)x^3} dx$$

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.

Time = 0.31 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*
pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*c
os(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*
pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos
(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*
pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*
arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*
```

```
pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9
*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
 - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2
/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*s
qrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi
)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(
1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx=\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(1/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2
^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)
```

```
*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
 + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/
108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (
2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i -
3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.180 $\int \frac{1}{x^4(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[In] Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\,(1-x+x^2)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x\,(1-x+x^2)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2}\right) dx, x, x^3\right)$$

$$= -\frac{1}{3x^3} + \log(x) - \frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2} dx, x, x^3\right)$$

$$= -\frac{1}{3x^3} + \log(x) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, x^3\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3\right)$$

$$= -\frac{1}{3x^3} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3\right)$$

$$= -\frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4\left(1-x^3+x^6\right)} dx = -\frac{1}{3x^3} + \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]
[Out] -1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 
2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{3x^3} + \ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	38
default	$-\frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3} + \ln(x)$	40

```
[In] int(1/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = -\frac{2\sqrt{3}x^3 \arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) + 3x^3 \log(x^6 - x^3 + 1) - 18x^3 \log(x) + 6}{18x^3}$$

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
+ 1) - 18*x^3*log(x) + 6)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = \log(x) - \frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

```
[In] integrate(1/x**4/(x**6-x**3+1),x)
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9 - 1/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{3x^3} - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \frac{1}{3}\log\left(x^3\right)$$

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{x^3+1}{3x^3} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(
x^6 - x^3 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{1}{3x^3}$$

```
[In] int(1/(x^4*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9 - 1/(3*x^3)
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 423

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/4/x^4-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I
-3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3
))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*
(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2)
)^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*
3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/
2))^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^
(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/
3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1382, 1518, 12, 1388, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx = -\frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{1}{4x^4} + \frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[In] Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/4*1/x^4 - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
```

```
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4x^4}+\frac{1}{4}\int \frac{4-4x^3}{x^2\left(1-x^3+x^6\right)}\,dx \\
&= -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{4}\int \frac{4x^4}{1-x^3+x^6}\,dx \\
&= -\frac{1}{4x^4}-\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx \\
&= -\frac{1}{4x^4}-\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx
\end{aligned}$$

$$= -\frac{1}{4x^4} - \frac{1}{x} + -\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$-\frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= -\frac{1}{4x^4} - \frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} +$$

$$-\frac{\left(-3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+\frac{1}{12}\left(-3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx+\frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+\right.}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}}}}{18\ 2^{2/3}\sqrt[3]{1+i}}$$

$$= -\frac{1}{4x^4} - \frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ -\frac{\left(-3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{4x^4} - \frac{1}{x} - \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4} - \frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]
[Out] -1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 +
2*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
risch	$\frac{-x^3-\frac{1}{4}}{x^4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-27\_R^5+6\_R^2+x\right)\right)}{3}$	46
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{4x^4}-\frac{1}{x}$	51

```
[In] int(1/x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-x^3-1/4)/x^4+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = \frac{2\cdot 18^{\frac{2}{3}}x^4\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+12x\right)+2\cdot 18^{\frac{2}{3}}x^4\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+\right.\right.}{}$$

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*x^4*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 1)*(
-I*sqrt(3) + 3)^(2/3) + 12*x) + 2*18^(2/3)*x^4*(I*sqrt(3) + 3)^(1/3)*log(18
^(1/3)*(I*sqrt(3) + 3)^(2/3)*(-I*sqrt(3) + 1) + 12*x) - 108*x^3 + 18^(2/3)*
(sqrt(-3)*x^4 - x^4)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3
) + I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(-3)*x
^4 + x^4)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + s
qrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(-3)*x^4 + x^4)*
(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) - I) + sqrt(-3) -
1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 18^(2/3)*(sqrt(-3)*x^4 - x^4)*(-I*sqrt(
3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) - I) - sqrt(-3) - 1)*(-I*s
qrt(3) + 3)^(2/3) + 24*x) - 27)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} dx = \text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 + 54t^2 + x\right)\right)\right) + \frac{-4x^3 - 1}{4x^4}$$

```
[In] integrate(1/x**5/(x**6-x**3+1),x)
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t*
*2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

Maxima [F]

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} dx = \int \frac{1}{(x^6 - x^3 + 1)x^5} dx$$

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(277) = 554.

Time = 0.29 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.98

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*
sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)
^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*co
s(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3
```

```
)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*c
os(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5
 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos
(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*c
os(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*p
i) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*c
os(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^
4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*
sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1
/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/4
*(4*x^3 + 1)/x^4
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5 (1-x^3+x^6)} \, dx$$
$$= \frac{\ln\left(-x+\left(162\,x+\frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(-x-\left(162\,x+\frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{x^3+\frac{1}{4}}{x^4}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(1/(x^5*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - (x^
3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12
 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i +
3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i +
 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3
 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i
+ 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 381

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

```
[Out] -1/21*I*2^(2/3)*ln(2^(1/3)*x+(1-I*7^(1/2))^(1/3))/(1-I*7^(1/2))^(2/3)*7^(1/
2)+1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))
*2^(2/3)/(1-I*7^(1/2))^(2/3)*7^(1/2)+1/21*I*2^(2/3)*ln(2^(1/3)*x+(1+I*7^(1/
2))^(1/3))/(1+I*7^(1/2))^(2/3)*7^(1/2)-1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I
*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3))*2^(2/3)/(1+I*7^(1/2))^(2/3)*7^(1/2)+1/
21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))/(1-I*7
^(1/2))^(2/3)*21^(1/2)-1/21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1+I*7^(1/2
))^(1/3))*3^(1/2))/(1+I*7^(1/2))^(2/3)*21^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

```
[In] Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}} + \frac{i\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}$$

$$+ \frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$+ \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$- \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{7}\sqrt[3]{1-i\sqrt{7}}}$$

$$- \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$+ \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{7}\sqrt[3]{1+i\sqrt{7}}}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$+\frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$-\frac{i\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$-\frac{i\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}$$

$$+\frac{i\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$=\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$-\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

$$+\frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$-\frac{i\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)}{\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[(2 + x^3 + x^6)^(-1),x]
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5+\_R^2}\right)}{3}$	33
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+\_Z^3+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5+\_R^2}\right)}{3}$	33

```
[In] int(1/(x^6+x^3+2),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{1}{588}\cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(i\sqrt{-3}+i)-7\sqrt{-3}-7\right)\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{588}\cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(-i\sqrt{-3}+i)+7\sqrt{-3}-7\right)\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{588}\cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(i\sqrt{-3}-i)+7\sqrt{-3}-7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)-\frac{1}{588}\cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(49^{\frac{2}{3}}\left(\sqrt{7}(-i\sqrt{-3}-i)-7\sqrt{-3}-7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+392\,x\right)+\frac{1}{294}\cdot 49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(49^{\frac{2}{3}}\left(3i\sqrt{7}-7\right)^{\frac{1}{3}}\left(-i\sqrt{7}+7\right)+196\,x\right)+\frac{1}{294}\cdot 49^{\frac{2}{3}}\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(49^{\frac{2}{3}}\left(i\sqrt{7}+7\right)\left(-3i\sqrt{7}-7\right)^{\frac{1}{3}}+196\,x\right)$$

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] -1/588*49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(49^(2/3)*(sqrt(7
)*(I*sqrt(-3) + I) - 7*sqrt(-3) - 7)*(3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1/5
88*49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) - 1)*log(49^(2/3)*(sqrt(7)*(-
I*sqrt(-3) + I) + 7*sqrt(-3) - 7)*(3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1/588*
49^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) - 1)*log(49^(2/3)*(sqrt(7)*(I*s
qrt(-3) - I) + 7*sqrt(-3) - 7)*(-3*I*sqrt(7) - 7)^(1/3) + 392*x) - 1/588*49
^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(49^(2/3)*(sqrt(7)*(-I*sq
rt(-3) - I) - 7*sqrt(-3) - 7)*(-3*I*sqrt(7) - 7)^(1/3) + 392*x) + 1/294*49^
(2/3)*(3*I*sqrt(7) - 7)^(1/3)*log(49^(2/3)*(3*I*sqrt(7) - 7)^(1/3)*(-I*sqrt
(7) + 7) + 196*x) + 1/294*49^(2/3)*(-3*I*sqrt(7) - 7)^(1/3)*log(49^(2/3)*(I
*sqrt(7) + 7)*(-3*I*sqrt(7) - 7)^(1/3) + 196*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{RootSum}\left(1000188t^6+1323t^3+1,\left(t\mapsto t\log\left(-5292t^4+7t+x\right)\right)\right)$$

```
[In] integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_
t + x)))
```

Maxima [F]

$$\int \frac{1}{2+x^3+x^6}\,dx = \int \frac{1}{x^6+x^3+2}\,dx$$

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Invalid _EXT in replace_ext Error: Bad Argument Valueinte
grate(1/(sageVARx^6+sageVARx^3+2),sageVARx)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{\ln\left(x+\frac{7^{1/3}\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{4}+\frac{7^{5/6}\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{28}\right)\,{\left(-49-\sqrt{7}\,21{}\mathrm{i}\right)}^{1/3}}{42}$$

$$+\frac{\ln\left(x+\frac{7^{1/3}\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{4}-\frac{7^{5/6}\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{28}\right)\,{\left(-49+\sqrt{7}\,21{}\mathrm{i}\right)}^{1/3}}{42}$$

$$+\frac{7^{1/3}\,\ln\left(6\,x+\frac{7^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,\left(\frac{7^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{2/3}\,\left(3969\,x+\frac{567\,7^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84}$$

$$+\frac{7^{1/3}\,\ln\left(6\,x+\frac{7^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,\left(\frac{7^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{2/3}\,\left(3969\,x+\frac{567\,7^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84}$$

$$-\frac{7^{1/3}\,\ln\left(6\,x-\frac{7^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,\left(\frac{7^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{2/3}\,\left(3969\,x-\frac{567\,7^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84}$$

$$-\frac{7^{1/3}\,\ln\left(6\,x-\frac{7^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}\,\left(\frac{7^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{2/3}\,\left(3969\,x-\frac{567\,7^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,3{}\mathrm{i}\right)}^{1/3}}{2}\right)}{7056}+63\right)}{84}\right)}{84}$$

```
[In] int(1/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7)
^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2)*
3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i - 4
9)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i -
 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x + (
567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*
(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (7^(1/3)
*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(7^(1
/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^
(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3))/84 - (7
^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3)*((7^(2/
3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1
/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*
(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)
*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2)*3i - 7)^(2/3)
*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/2))/7056 +
 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

3.183 $\int \frac{x^2}{2+x^3+x^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1366, 632, 210}

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2\arctan\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[In] Int[x^2/(2 + x^3 + x^6),x]
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^3\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[In] Integrate[x^2/(2 + x^3 + x^6),x]
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19
risch	$\frac{2\arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19

```
[In] int(x^2/(x^6+x^3+2),x,method=_RETURNVERBOSE)
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

```
[In] integrate(x**2/(x**6+x**3+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

```
[In] int(x^2/(x^3 + x^6 + 2),x)
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

3.184 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 399

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \frac{\left(7+i\sqrt{7}\right)\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{\left(7-i\sqrt{7}\right)\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

```
[Out] 1/42*ln(2^(1/3)*x+(1+I*7^(1/2))^(1/3))*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^
(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3)
)*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^(2/3)+1/42*ln(2^(1/3)*x+(1-I*7^(1/2))
^(1/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^(1/2))^(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/
3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^
(1/2))^(2/3)-1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))
*(1-I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)+1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1+I
*7^(1/2))^(1/3))*3^(1/2))*(1+I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} - \frac{(7+i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{(7-i\sqrt{7})\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{(7+i\sqrt{7})\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{(7-i\sqrt{7})\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

```
[In] Int[x^3/(2 + x^3 + x^6),x]
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = \frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx+\frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx$$

$$=\frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$+\frac{\left(7+i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$=\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$-\frac{\left(7-i\sqrt{7}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$+\frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+x^2}\,dx}{14\ 2^{2/3}\sqrt[3]{1+i\sqrt{7}}}$$

$$-\frac{\left(7+i\sqrt{7}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$+\frac{\left(7+i\sqrt{7}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+x^2}\,dx}{14\ 2^{2/3}\sqrt[3]{1-i\sqrt{7}}}$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$- \frac{\left(7+i\sqrt{7}\right)\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$- \frac{\left(7-i\sqrt{7}\right)\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$+ \frac{\left(7-i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{7\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$+ \frac{\left(7+i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{7\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$= -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}$$

$$+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}$$

$$+\frac{(7+i\sqrt{7})\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{(7-i\sqrt{7})\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

$$-\frac{(7+i\sqrt{7})\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}$$

$$-\frac{(7-i\sqrt{7})\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{1+2\#1^3}\&\right]$$

```
[In] Integrate[x^3/(2 + x^3 + x^6),x]
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^6+\_Z^3+2)}\left(\frac{\_R^3 \ln(x-\_R)}{2\_R^5+\_R^2}\right)}{3}$	36
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^6+\_Z^3+2)}\left(\frac{\_R^3 \ln(x-\_R)}{2\_R^5+\_R^2}\right)}{3}$	36

```
[In] int(x^3/(x^6+x^3+2),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{1}{588} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}(i\sqrt{-3}+i) + 196\,x\right) + \frac{1}{588} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}(i\sqrt{-3}-i) + 196\,x\right) + \frac{1}{588} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}(-i\sqrt{-3}+i) + 196\,x\right) - \frac{1}{588} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}(-i\sqrt{-3}-i) + 196\,x\right) + \frac{1}{294} \cdot 98^{\frac{2}{3}}\left(i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(i \cdot 98^{\frac{2}{3}}\sqrt{7}\left(i\sqrt{7}-7\right)^{\frac{1}{3}} + 98\,x\right) + \frac{1}{294} \cdot 98^{\frac{2}{3}}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}}\log\left(-i \cdot 98^{\frac{2}{3}}\sqrt{7}\left(-i\sqrt{7}-7\right)^{\frac{1}{3}} + 98\,x\right)$$

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] -1/588*98^(2/3)*(-I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(98^(2/3)*sqrt(7)*
(-I*sqrt(7) - 7)^(1/3)*(I*sqrt(-3) + I) + 196*x) + 1/588*98^(2/3)*(I*sqrt(7
) - 7)^(1/3)*(sqrt(-3) - 1)*log(98^(2/3)*sqrt(7)*(I*sqrt(7) - 7)^(1/3)*(I*s
qrt(-3) - I) + 196*x) + 1/588*98^(2/3)*(-I*sqrt(7) - 7)^(1/3)*(sqrt(-3) - 1
)*log(98^(2/3)*sqrt(7)*(-I*sqrt(7) - 7)^(1/3)*(-I*sqrt(-3) + I) + 196*x) -
1/588*98^(2/3)*(I*sqrt(7) - 7)^(1/3)*(sqrt(-3) + 1)*log(98^(2/3)*sqrt(7)*(I
*sqrt(7) - 7)^(1/3)*(-I*sqrt(-3) - I) + 196*x) + 1/294*98^(2/3)*(I*sqrt(7)
- 7)^(1/3)*log(I*98^(2/3)*sqrt(7)*(I*sqrt(7) - 7)^(1/3) + 98*x) + 1/294*98^
(2/3)*(-I*sqrt(7) - 7)^(1/3)*log(-I*98^(2/3)*sqrt(7)*(-I*sqrt(7) - 7)^(1/3)
 + 98*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \operatorname{RootSum}\left(250047t^6+1323t^3+2,\left(t\mapsto t\log\left(7938t^4+21t+x\right)\right)\right)$$

```
[In] integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t
 + x)))
```

Maxima [F]

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \int \frac{x^3}{x^6+x^3+2}\,dx$$

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Invalid _EXT in replace_ext Error: Ba
d Argument ValueDone
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-196-\sqrt{7}\,28\mathrm{i}\right)^{1/3}}{42}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{42}-\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}-\frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}$$

```
[In] int(x^3/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i -
 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7
)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2^(2
/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(- 7
^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (
2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 +
(2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i - 1)*(-
7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/6)*(7^(1
/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))
/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/3)*log(x -
 (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/2)*7^(5/6)*
(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(1/3))/84
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 231

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{(21b^4-56ab^2c+16a^2c^2)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)\,x^3)\,(a+bx^3+cx^6)^{3/2}}{2880c^4} - \frac{(b^2-4ac)\,(21b^4-56ab^2c+16a^2c^2)\,\mathrm{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}$$

```
[Out] -1/20*b*x^6*(c*x^6+b*x^3+a)^(3/2)/c^2+1/18*x^9*(c*x^6+b*x^3+a)^(3/2)/c-1/28
80*(7*b*(-28*a*c+15*b^2)-6*c*(-20*a*c+21*b^2)*x^3)*(c*x^6+b*x^3+a)^(3/2)/c^
4-1/3072*(-4*a*c+b^2)*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*(2*c*x^3+b
)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1536*(16*a^2*c^2-56*a*b^2*c+21*
b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{1371, 756, 846, 793, 626, 635, 212\}$

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = -\frac{(b^2-4ac)\left(16a^2c^2-56ab^2c+21b^4\right)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}} + \frac{\left(16a^2c^2-56ab^2c+21b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{\left(7b(15b^2-28ac)-6cx^3(21b^2-20ac)\right)\left(a+bx^3+cx^6\right)^{3/2}}{2880c^4} - \frac{bx^6\left(a+bx^3+cx^6\right)^{3/2}}{20c^2} + \frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{18c}$$

```
[In] Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^4\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{18c}\\
&= -\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}\\
&\quad+\frac{\text{Subst}\left(\int x\left(9ab+\frac{3}{4}(21b^2-20ac)x\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{90c^2}
\end{aligned}$$

$$= -\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)x^3)(a+bx^3+cx^6)^{3/2}}{2880c^4} + \frac{(21b^4-56ab^2c+16a^2c^2)\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{384c^4}$$

$$= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)x^3)(a+bx^3+cx^6)^{3/2}}{2880c^4} - \frac{((b^2-4ac)(21b^4-56ab^2c+16a^2c^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3072c^5}$$

$$= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)x^3)(a+bx^3+cx^6)^{3/2}}{2880c^4} - \frac{((b^2-4ac)(21b^4-56ab^2c+16a^2c^2))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{1536c^5}$$

$$= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)x^3)(a+bx^3+cx^6)^{3/2}}{2880c^4} - \frac{(b^2-4ac)(21b^4-56ab^2c+16a^2c^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(315b^5-210b^4cx^3+16b^2c^2x^3(56a-9cx^6)+168b^3c(-10a+cx^6)+16bc^2(113a^2-34$$

```
[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^
3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c
*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) + 15*(2
1*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b + 2*c*x^3 - 2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6]])/(46080*c^(11/2))
```

Maple [F]

$$\int x^{14}\sqrt{c\,x^6+b\,x^3+a}dx$$

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.95

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\left[-\frac{15\,(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+\right.}{}\right.$$

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)
*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*
a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 +
 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
```

```
 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*
(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c -
1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*
c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}\,x^{14}\,dx$$

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.35

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\frac{x^9\left(c\,x^6+b\,x^3+a\right)^{3/2}}{18\,c}-\frac{b\left(\frac{x^6\left(c\,x^6+b\,x^3+a\right)^{3/2}}{5\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6\right.\right.}{}\right.}{}\right)}{10\,c}\right.}{}$$

$$+\frac{a\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{c}}{24\,c^2}\right.}{}\right.}{6\,c}$$

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2)
)/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a +
 b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))
/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) -
3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3
+ c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*
c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x
^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4*c) + x^3
/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^
(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*
x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=-\frac{b(7b^2-12ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-32ac-42bcx^3)\,(a+bx^3+cx^6)^{3/2}}{720c^3}+\frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}$$

```
[Out] 1/15*x^6*(c*x^6+b*x^3+a)^(3/2)/c+1/720*(-42*b*c*x^3-32*a*c+35*b^2)*(c*x^6+b
*x^3+a)^(3/2)/c^3+1/768*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3
+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)-1/384*b*(-12*a*c+7*b^2)*(2*c*x^3
+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 756, 793, 626, 635, 212}

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b(7b^2-12ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{(-32ac+35b^2-42bcx^3)\,(a+bx^3+cx^6)^{3/2}}{720c^3}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}$$

```
[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/384*(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^4 + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{15c}\\
&= \frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-32ac-42bcx^3)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}\\
&\quad-\frac{(b(7b^2-12ac))\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{96c^3}\\
&= -\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}\\
&\quad+\frac{(35b^2-32ac-42bcx^3)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}\\
&\quad+\frac{(b(7b^2-12ac)\left(b^2-4ac\right))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{768c^4}\\
&= -\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}\\
&\quad+\frac{(35b^2-32ac-42bcx^3)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}\\
&\quad+\frac{(b(7b^2-12ac)\left(b^2-4ac\right))\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{384c^4}\\
&= -\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}\\
&\quad+\frac{(35b^2-32ac-42bcx^3)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}\\
&\quad+\frac{b(7b^2-12ac)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{\sqrt{a+bx^3+cx^6}(-105b^4+70b^3cx^3+4b^2c(115a-14cx^6)+8bc^2x^3(-29a+6cx^6)+128c^2(-2a^2+acx^6+\ldots}{5760c^4}$$
$$-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{768c^{9/2}}$$

```
[In] Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 70*b^3*c*x^3 + 4*b^2*c*(115*a - 14*c*x
^6) + 8*b*c^2*x^3*(-29*a + 6*c*x^6) + 128*c^2*(-2*a^2 + a*c*x^6 + 3*c^2*x^1
2)))/(5760*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*x^3
 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Maple [F]

$$\int x^{11}\sqrt{c\,x^6+b\,x^3+a}dx$$

```
[In] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.15

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\ldots}{\ldots}\right.$$
$$-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(384\,c^5x^{12}+48\,bc^4x^9-8\,(7\,b^2c^3\ldots}{11520\,c^5}$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
```

```
4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4*c
+ 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*
c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x
^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}\,x^{11}\,dx$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.84

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\frac{x^6\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{15\,c}$$

$$+\frac{7\,b\,\left(\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{4\,c}+\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{\cdots}}{24\,c^2}}{}\right)}{30\,c}$$

$$-\frac{2\,a\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{15\,c}$$

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b
*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^
2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*
x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(15*c)
```

3.187 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{(b^2-4ac)\,(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}$$

```
[Out] -5/72*b*(c*x^6+b*x^3+a)^(3/2)/c^2+1/12*x^3*(c*x^6+b*x^3+a)^(3/2)/c-1/384*(-
4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(
1/2))/c^(7/2)+1/192*(-4*a*c+5*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 756, 654, 626, 635, 212}

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=-\frac{(b^2-4ac)\,(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}$$

```
[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{12c}\\
&= -\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}\\
&\quad+\frac{(5b^2-4ac)\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{48c^2}\\
&= \frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}\\
&\quad+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{((b^2-4ac)\,(5b^2-4ac))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{384c^3}\\
&= \frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}\\
&\quad+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{((b^2-4ac)\,(5b^2-4ac))\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{192c^3}\\
&= \frac{(5b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}\\
&\quad+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int x^8\sqrt{a+bx^3+cx^6}\,dx\\
&= \frac{\sqrt{a+bx^3+cx^6}(15b^3-52abc-10b^2cx^3+24ac^2x^3+8bc^2x^6+48c^3x^9)}{576c^3}\\
&\quad+\frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{384c^{7/2}}
\end{aligned}$$

```
[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3 +
 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))
```

Maple [F]

$$\int x^{8}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

```
[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.98

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,(}{2304\,c^4}\right.$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c
^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \int x^8\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,x^8\,dx$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.26

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}{12\,c} - \frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c} - \frac{5\,b\left(\frac{(8\,c\,(c\,x^6+a)-3\,b^2+2\,b\,c\,x^3)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)(b^3-4\,a\,b\,c)}{16\,c^{5/2}}\right)}{24\,c}$$

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3 +
 c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*
c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*
x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2)
 + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

3.188 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)/c+1/48*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(
1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/24*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2
)/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 654, 626, 635, 212}

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}$$

```
[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/24*(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c} - \frac{b\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{b\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9c}\\
&\quad + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{48c^2}
\end{aligned}
$$

$$= -\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24c^2}$$

$$= -\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c} + \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-3b^2+2bcx^3+8c(a+cx^6))}{72c^2} - \frac{(b^3-4abc)\log\left(c^2\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{48c^{5/2}}$$

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) -
 ((b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(48*c^(5/2))
```

Maple [F]

$$\int x^5\sqrt{c\,x^6+b\,x^3+a}dx$$

```
[In] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^6+2\,b}{288\,c^3}\right.$$

$$-\frac{3\,(b^3-4\,abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(8\,c^3x^6+2\,bc^2x^3-3\,b^2c+8\,ac^2)\sqrt{cx^6+bx^3+a}}{144\,c^3}$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqr
t(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^
2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4
*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/
(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2
)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F]

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{72}\,\sqrt{cx^6+bx^3+a}\left(2\left(4\,x^3+\frac{b}{c}\right)x^3-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{(b^3-4\,abc)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{48\,c^{\frac{5}{2}}}$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
1/48*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt
(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{72\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{48\,c^{5/2}}$$

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
+ (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c)
)/(48*c^(5/2))
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[Out] -1/24*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(3/2)+1/12*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1366, 626, 635, 212}

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c}$$

$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c}$$

$$= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} + \frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{12c^{3/2}}$$

```
[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((-b^2 + 4*a*c)*ArcTanh[(S
qrt[c]*x^3)/(-Sqrt[a] + Sqrt[a + b*x^3 + c*x^6])])/(12*c^(3/2))
```

Maple [F]

$$\int x^{2} \sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.37

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}(}{48\,c^2}\right.$$

[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]

Sympy [F]

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24\,c^{\frac{3}{2}}}$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*(
sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}}{3} + \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{6\,c^{3/2}}$$

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)^
(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))*a^(1/2)+1/6*b*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/3*(c*x^6+b*
x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 748, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = -\frac{1}{3}\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}} + \frac{1}{3}\sqrt{a+bx^3+cx^6}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{6}\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} + \frac{1}{3}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{6}b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)$$
$$= \frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{6}\left(2\sqrt{a+bx^3+cx^6} + 4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - \frac{b\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
[Out] (2*Sqrt[a + b*x^3 + c*x^6] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^
3 + c*x^6])/Sqrt[a]] - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6]])/Sqrt[c])/6
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x}dx$$

[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3}{}\right.}{12\,c}\right.$$
$$-\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-2\,\sqrt{c}}{6\,c}$$

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")

[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/6*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c]

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\frac{\sqrt{c\,x^6+b\,x^3+a}}{3}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3}+\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x,x)
[Out] (a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^
3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3
)/c^(1/2)))/(6*c^(1/2))
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx = -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/3*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/3*(c*x^6+b*
x^3+a)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 746, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx = -\frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right) - \frac{\sqrt{a+bx^3+cx^6}}{3x^3}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{3}c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)$$
$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \frac{1}{3}\left(-\frac{\sqrt{a+bx^3+cx^6}}{x^3} + \frac{b\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] (-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
+ c*x^6])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6]])/3
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{4}}dx$$

[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.37

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \left[\frac{2\,a\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,\cdots}{\cdots}\right)\cdots}{12\,ax^3}\right.$$
$$-\frac{4\,a\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+\cdots}{12\,ax^3}$$

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)]

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^4}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx=\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
[Out] (c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a + b
*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*
x^6)^(1/2))/x^3))/(6*a^(1/2))
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}$$

```
[Out] 1/24*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^
(3/2)-1/12*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a/x^6
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1371, 734, 738, 212}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=\frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -1/12*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcT
anh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}-\frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24a}\\
&= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a}\\
&= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \frac{(-2a-bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{3/2}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*a - b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*ArcTa
nh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(3/2))
```

Maple [F]

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \left[-\frac{(b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{48\,a^2x^6}, -\frac{(b^2-4\,ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{24\,a^2x^6}\right]$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*x^6
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x
^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/a/x^9-1/48*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a
)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/24*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^2/x^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 744, 734, 738, 212}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = -\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}} + \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{b\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} + \frac{(b(b^2-4ac))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48a^2}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{(b(b^2-4ac))\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24a^2}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2-2abx^3+3b^2x^6-8acx^6)}{72a^2x^9} + \frac{(b^3-4abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72*
a^2*x^9) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])
/Sqrt[a]])/(24*a^(5/2))
```

Maple [F]

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{10}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$
$$=\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((3\,ab^2-8\,a^2c)x^6-2\,a^2bx^3}{288\,a^3x^9}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
+ 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b
^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)
, 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*
x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^{10}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{10}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = -\frac{(5b^2-4ac)\,(2a+bx^3)\,\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(b^2-4ac)\,(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}$$

```
[Out] -1/12*(c*x^6+b*x^3+a)^(3/2)/a/x^12+5/72*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^9+1/3
84*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3
+a)^(1/2))/a^(7/2)-1/192*(-4*a*c+5*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a
^3/x^6
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 758, 820, 734, 738, 212}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \frac{(b^2-4ac)\,(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}} - \frac{(5b^2-4ac)\,(2a+bx^3)\,\sqrt{a+bx^3+cx^6}}{192a^3x^6} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -1/192*((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a^3*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72
*a^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]
*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
```

```
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(5b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{48a^2}$$

$$= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} - \frac{((b^2-4ac)(5b^2-4ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{384a^3}$$

$$= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{((b^2-4ac)(5b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{192a^3}$$

$$= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-48a^3-8a^2bx^3+10ab^2x^6-24a^2cx^6-15b^3x^9+52abcx^9)}{576a^3x^{12}} + \frac{(-5b^4+24ab^2c-16a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{7/2}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 - 8*a^2*b*x^3 + 10*a*b^2*x^6 - 24*a^2*c*x
^6 - 15*b^3*x^9 + 52*a*b*c*x^9))/(576*a^3*x^12) + ((-5*b^4 + 24*a*b^2*c - 1
6*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(192*a
^(7/2))
```

Maple [F]

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((15\,ab^3-52\,a^2b}{2304\,a^4x^{12}} - \frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^9+8\,a^3bx^3}{1152\,a^4x^{12}}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
```

```
2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*
a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a
)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2
*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6
 + b*x^3 + a))/(a^4*x^12)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^{13}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} \, dx = \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} - \frac{b(7b^2-12ac)(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}$$

```
[Out] -1/15*(c*x^6+b*x^3+a)^(3/2)/a/x^15+7/120*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1
/720*(-32*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^9-1/768*b*(-12*a*c+7*b^2)
*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2
)+1/384*b*(-12*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^6
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 758, 848, 820, 734, 738, 212}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} \, dx = -\frac{b(7b^2-12ac)(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} + \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)}{15a} \\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{60a^2} \\
&= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} \\
&\quad - \frac{\left(b\left(7b^2-12ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{96a^3}
\end{aligned}
$$

$$= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} + \frac{(b(7b^2-12ac)(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{768a^4}$$

$$= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} - \frac{(b(7b^2-12ac)(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{384a^4}$$

$$= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} - \frac{b(7b^2-12ac)(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

$$= \frac{\sqrt{a+bx^3+cx^6}(-384a^4-48a^3bx^3+56a^2b^2x^6-128a^3cx^6-70ab^3x^9+232a^2bcx^9+105b^4x^{12}-460ab^2cx^{1}}{5760a^4x^{15}} + \frac{(7b^5-40ab^3c+48a^2bc^2)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{384a^{9/2}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a^
3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^12
+ 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2))
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{16}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$
$$= \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((105\,ab^4-4}{23}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2
*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384
*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c
+ 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 2
56*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a
^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^{16}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,-1/2,-1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 358 vs. 2(140) = 280.

Time = 8.91 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.56

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{x\left(8(3b+8cx^3)\left(a+bx^3+cx^6\right)-24ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\right.\right.}{448c\sqrt{a}}$$

```
[In] Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16*a*c
)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/
2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int x^{3}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

```
[In] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^3*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,-1/2,-1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 337 vs. $2(140) = 280$.

Time = 10.03 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.41

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\left(10(a+bx^3+cx^6)+15a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right.}{50\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
[Out] (x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F]

$$\int x\sqrt{c\,x^6+b\,x^3+a}dx$$

```
[In] int(x*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Sympy [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int x\sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int x\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

```
[In] int(x*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \sqrt{a+bx^3+cx^6}\,dx=\frac{x\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*AppellF1(1/3,-1/2,-1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1362, 440}

$$\int \sqrt{a+bx^3+cx^6}\,dx=\frac{x\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^3+cx^6}\int\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(135) = 270$.

Time = 10.24 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.48

$$\int\sqrt{a+bx^3+cx^6}\,dx$$

$$= \frac{x\left(8(a+bx^3+cx^6)+24a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+\right.}{32\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*x^3 + c
*x^6])
```

Maple [F]

$$\int \sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2),x)
[Out] int((c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{a+bx^3+cx^6}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{c\,x^6+b\,x^3+a}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((a + b*x^3 + c*x^6)^(1/2), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -AppellF1(-1/3,-1/2,-1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}} \\ &= -\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. $2(138) = 276$.

Time = 10.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \frac{-20(a+bx^3+cx^6)+15bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{20x\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a
```

```
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{2}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*AppellF1(-2/3,-1/2,-1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 + (
```

```
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. $2(140) = 280$.

Time = 10.21 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \frac{-4(a+bx^3+cx^6)+6bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+}{8x^2\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
```

```
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{3}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^3, x)
```

3.201 $\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\begin{aligned}\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx =\\ &-\frac{(b^2-4ac)\left(33b^4-72ab^2c+16a^2c^2\right)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6}\\ &+\frac{\left(33b^4-72ab^2c+16a^2c^2\right)(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}\\ &-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}\\ &-\frac{\left(3b(77b^2-124ac)-10c(33b^2-28ac)x^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{13440c^4}\\ &+\frac{(b^2-4ac)^2\left(33b^4-72ab^2c+16a^2c^2\right)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}\end{aligned}$$

```
[Out] 1/6144*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^5
-11/336*b*x^6*(c*x^6+b*x^3+a)^(5/2)/c^2+1/24*x^9*(c*x^6+b*x^3+a)^(5/2)/c-1/
13440*(3*b*(-124*a*c+77*b^2)-10*c*(-28*a*c+33*b^2)*x^3)*(c*x^6+b*x^3+a)^(5/
2)/c^4+1/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-72*a*b^2*c+33*b^4)*arctanh(1/2*(2
*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(13/2)-1/16384*(-4*a*c+b^2)*(16*
a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 756, 846, 793, 626, 635, 212}

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}-\frac{\left(b^2-4ac\right)\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{\left(3b(77b^2-124ac)-10cx^3(33b^2-28ac)\right)\left(a+bx^3+cx^6\right)^{5/2}}{13440c^4}-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}$$

```
[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/16384*((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sq
rt[a + b*x^3 + c*x^6])/c^6 + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int x^4\left(a+bx+cx^2\right)^{3/2}\,dx, x, x^3\right)$$

$$=\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{11bx}{2}\right)(a+bx+cx^2)^{3/2}\,dx,x,x^3\right)}{24c}$$

$$=-\frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x\left(11ab+\frac{3}{4}(33b^2-28ac)x\right)(a+bx+cx^2)^{3/2}\,dx,x,x^3\right)}{168c^2}$$

$$=-\frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}-\frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4}+\frac{(33b^4-72ab^2c+16a^2c^2)\,\text{Subst}\left(\int(a+bx+cx^2)^{3/2}\,dx,x,x^3\right)}{768c^4}$$

$$=\frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5}-\frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}-\frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4}-\frac{((b^2-4ac)(33b^4-72ab^2c+16a^2c^2))\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{4096c^5}$$

$$=-\frac{(b^2-4ac)(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5}-\frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}-\frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4}+\frac{\left((b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{32768c^6}$$

$$= -\frac{(b^2-4ac)(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5} - \frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2} + \frac{x^9(a+bx^3+cx^6)^{5/2}}{24c} - \frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4} + \frac{\left((b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{16384c^6}$$

$$= -\frac{(b^2-4ac)(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5} - \frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2} + \frac{x^9(a+bx^3+cx^6)^{5/2}}{24c} - \frac{(3b(77b^2-124ac)-10c(33b^2-28ac)x^3)(a+bx^3+cx^6)^{5/2}}{13440c^4} + \frac{(b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-3465b^7+2310b^6cx^3+84b^5c(365a-22cx^6)+24b^4c^2x^3(-749a+66cx^6$$

```
[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 2310*b^6*c*x^3 + 84*b^5*c*(
365*a - 22*c*x^6) + 24*b^4*c^2*x^3*(-749*a + 66*c*x^6) + 32*b^2*c^3*x^3*(11
81*a^2 - 284*a*c*x^6 + 40*c^2*x^12) - 16*b^3*c^2*(5103*a^2 - 780*a*c*x^6 +
88*c^2*x^12) + 4480*c^4*x^3*(-3*a^3 + 2*a^2*c*x^6 + 24*a*c^2*x^12 + 16*c^3*
x^18) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^6 + 104*a*c^2*x^12 + 1360*c^3*x^18)
) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^3
- 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3440640*c^(13/2))
```

Maple [F]

$$\int x^{14} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.19

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{105\,(33\,b^8 - 336\,ab^6c + 1120\,a^2b^4c^2 - 1280\,a^3b^2c^3 + 256\,a^4c^4)\sqrt{c}\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - \right.}{} \right.$$

$$-\frac{105\,(33\,b^8 - 336\,ab^6c + 1120\,a^2b^4c^2 - 1280\,a^3b^2c^3 + 256\,a^4c^4)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right) - 2\,(7168}{}$$

[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]

Sympy [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{14}(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x^{14}\,dx$$

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{14}\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.202 $\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 223

$$\begin{aligned}\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx &= \frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1024c^5}\\ &-\frac{b(3b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}\\ &+\frac{(21b^2-16ac-30bcx^3)\,(a+bx^3+cx^6)^{5/2}}{840c^3}\\ &-\frac{b(b^2-4ac)^2\,(3b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}\end{aligned}$$

```
[Out] -1/384*b*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^4+1/21*x^6*(c*x
^6+b*x^3+a)^(5/2)/c+1/840*(-30*b*c*x^3-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)
/c^3-1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)
/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*
x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {1371, 756, 793, 626, 635, 212}

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{b(b^2-4ac)^2\,(3b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{384c^4}+\frac{(-16ac+21b^2-30bcx^3)\,(a+bx^3+cx^6)^{5/2}}{840c^3}+\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}$$

```
[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)\\
&= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{21c}\\
&= \frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}+\frac{\left(21b^2-16ac-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}\\
&\quad-\frac{\left(b\left(3b^2-4ac\right)\right)\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{48c^3}\\
&= -\frac{b\left(3b^2-4ac\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{21c}\\
&\quad+\frac{\left(21b^2-16ac-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}\\
&\quad+\frac{\left(b\left(b^2-4ac\right)\left(3b^2-4ac\right)\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{256c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5} \\
&\quad - \frac{b(3b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{384c^4} + \frac{x^6(a+bx^3+cx^6)^{5/2}}{21c} \\
&\quad + \frac{(21b^2-16ac-30bcx^3)\,(a+bx^3+cx^6)^{5/2}}{840c^3} \\
&\quad - \frac{\left(b(b^2-4ac)^2\,(3b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{2048c^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5} \\
&\quad - \frac{b(3b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{384c^4} + \frac{x^6(a+bx^3+cx^6)^{5/2}}{21c} \\
&\quad + \frac{(21b^2-16ac-30bcx^3)\,(a+bx^3+cx^6)^{5/2}}{840c^3} \\
&\quad - \frac{\left(b(b^2-4ac)^2\,(3b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{1024c^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5} \\
&\quad - \frac{b(3b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{384c^4} + \frac{x^6(a+bx^3+cx^6)^{5/2}}{21c} \\
&\quad + \frac{(21b^2-16ac-30bcx^3)\,(a+bx^3+cx^6)^{5/2}}{840c^3} \\
&\quad - \frac{b(b^2-4ac)^2\,(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$
\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \frac{\sqrt{a+bx^3+cx^6}\left(315b^6-210b^5cx^3+16b^3c^2x^3(91a-9cx^6)+168b^4c(-15a+cx^6)+1024c^3(a\right.}{1} + \frac{b(b^2-4ac)^2\,(3b^2-4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{2048c^{11/2}}
$$

```
[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 210*b^5*c*x^3 + 16*b^3*c^2*x^3*(91*a -
9*c*x^6) + 168*b^4*c*(-15*a + c*x^6) + 1024*c^3*(a + c*x^6)^2*(-2*a + 5*c*x
```

```
^6) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^6 + 8*c^2*x^12) + 32*b*c^3*x^3*(-73*a^
2 + 22*a*c*x^6 + 200*c^2*x^12)))/(107520*c^5) + (b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^(11/2
))
```

Maple [F]

$$\int x^{11} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

```
[In] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\right.}{}\right.$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{11}(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x^{11}\,dx$$

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{11}\,(c\,x^6+b\,x^3+a)^{3/2}\,dx$$

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.203 $\int x^8(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 204

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{(7b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{576c^3}-\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2}+\frac{x^3(a+bx^3+cx^6)^{5/2}}{18c}+\frac{(b^2-4ac)^2\,(7b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}$$

```
[Out] 1/576*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^3-7/180*b*(c*x^6+b
*x^3+a)^(5/2)/c^2+1/18*x^3*(c*x^6+b*x^3+a)^(5/2)/c+1/3072*(-4*a*c+b^2)^2*(-
4*a*c+7*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)
-1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 756, 654, 626, 635, 212}

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{(b^2-4ac)^2\,(7b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{(7b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{576c^3}-\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2}+\frac{x^3(a+bx^3+cx^6)^{5/2}}{18c}$$

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/1536*((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6
])/c^4 + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^{3/2}\,dx, x, x^3\right)$$

$$= \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{\text{Subst}\left(\int \left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx, x, x^3\right)}{18c}$$

$$= -\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2}\,dx, x, x^3\right)}{72c^2}$$

$$= \frac{(7b^2-4ac)\,(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} - \frac{((b^2-4ac)\,(7b^2-4ac))\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)\,(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{\left((b^2-4ac)^2\,(7b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3072c^4}$$

$$= -\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)\,(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{\left((b^2-4ac)^2\,(7b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{1536c^4}$$

$$= -\frac{(b^2-4ac)(7b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-105b^5+70b^4cx^3+8b^3c(95a-7cx^6)+48b^2c^2x^3(-9a+cx^6)+160c^3x^3}{}$$

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 70*b^4*c*x^3 + 8*b^3*c*(95*a
 - 7*c*x^6) + 48*b^2*c^2*x^3*(-9*a + c*x^6) + 160*c^3*x^3*(3*a^2 + 14*a*c*x
^6 + 8*c^2*x^12) + 16*b*c^2*(-81*a^2 + 18*a*c*x^6 + 104*c^2*x^12)) - 15*(b^
2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6]])/(46080*c^(9/2))
```

Maple [F]

$$\int x^8\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.21

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+}\right.}{}\right.$$

$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(1280\,c^6x^{15}+1664\,bc^5x}{}$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \int x^8 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^8 \, dx$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int x^8 \left(c\,x^6+b\,x^3+a\right)^{3/2} \, dx$$

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.204 $\int x^5(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{b(b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{48c^2}+\frac{(a+bx^3+cx^6)^{5/2}}{15c}-\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}$$

```
[Out] -1/48*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^2+1/15*(c*x^6+b*x^3+a)^(5/2)/c-
1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/c^(7/2)+1/128*b*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 654, 626, 635, 212}

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b(b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{48c^2}+\frac{(a+bx^3+cx^6)^{5/2}}{15c}$$

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)\\ &= \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b\text{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{6c}\end{aligned}$$

$$= -\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{32c^2}$$

$$= \frac{b(b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} - \frac{\left(b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{256c^3}$$

$$= \frac{b(b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} - \frac{\left(b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{128c^3}$$

$$= \frac{b(b^2-4ac)\,(b+2cx^3)\sqrt{a+bx^3+cx^6}}{128c^3} - \frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2} + \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c} - \frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx = \frac{\sqrt{a+bx^3+cx^6}\,\big(15b^4-10b^3cx^3+128c^2(a+cx^6)^2+4b^2c(-25a+2cx^6)+8bc^2x^3(7a+22cx^6}{1920c^3}$$

$$+\frac{b(b^2-4ac)^2\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}}$$

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 10*b^3*c*x^3 + 128*c^2*(a + c*x^6)^2 + 4
*b^2*c*(-25*a + 2*c*x^6) + 8*b*c^2*x^3*(7*a + 22*c*x^6)))/(1920*c^3) + (b*(
b^2 - 4*a*c)^2*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(256*c
^(7/2))
```

Maple [F]

$$\int x^{5} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}d x$$

```
[In] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int x^5(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-\right.}{}\right.$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*
a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/
2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))
 + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6
+ b*x^3 + a))/c^4]
```

Sympy [F]

$$\int x^5(a+bx^3+cx^6)^{3/2}\,dx = \int x^{5} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 (a + bx^3 + cx^6)^{3/2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int x^5 (a + bx^3 + cx^6)^{3/2} \, dx = \frac{1}{1920} \sqrt{cx^6 + bx^3 + a} \left(2 \left(4 \left(2 \left(8 cx^3 + 11 b\right) x^3 + \frac{b^2 c^3 + 32 ac^4}{c^4}\right) x^3 - \frac{5 b^3 c^2 - 28 abc^3}{c^4}\right) x^3 + \cdots + \frac{(b^5 - 8 ab^3 c + 16 a^2 bc^2) \log \left(\left|2 \left(\sqrt{c} x^3 - \sqrt{cx^6 + bx^3 + a}\right) \sqrt{c} + b\right|\right)}{256 c^{\frac{7}{2}}}$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.49

$$\int x^5 (a + bx^3 + cx^6)^{3/2} \, dx = \frac{(c\,x^6 + b\,x^3 + a)^{5/2}}{15\,c} - \frac{b \left(\frac{3\,a \left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right) \left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\,\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{4} + \frac{x^3\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4} - \frac{3\,b^2 \left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}} \cdots\right.\right.}{\cdots}\right)}{6\,c}$$

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)
*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/4 -
 (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

3.205 $\int x^2(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{(b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{24c}+\frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}$$

```
[Out] 1/24*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c+1/128*(-4*a*c+b^2)^2*arctanh(1/2*(
2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/64*(-4*a*c+b^2)*(2*c*x^
3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1366, 626, 635, 212}

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{(b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{24c}$$

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/64*((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)\\
&= \frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{16c}\\
&= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}\\
&\quad+\frac{\left(b^2-4ac\right)^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{128c^2}\\
&= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}\\
&\quad+\frac{\left(b^2-4ac\right)^2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{64c^2}\\
&= -\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}\\
&\quad+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}(-3b^2+20ac+8bcx^3+8c^2x^6)}{192c^2}+\frac{(-b^2+4ac)^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{64c^{5/2}}$$

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 20*a*c + 8*b*c*x^3 + 8*c^2
*x^6))/(192*c^2) + ((-b^2 + 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3)/(-Sqrt[a] + Sqrt
[a + b*x^3 + c*x^6])])/(64*c^(5/2))
```

Maple [F]

$$\int x^2\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

```
[In] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,a\right.}{768\,c^3}\right.$$

$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(16\,c^4x^9+24\,bc^3x^6-3\,b^3c+20\,abc^2+2\,(}{384\,c^3}$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3
 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^3)*
sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b
*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F]

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx = \int x^2(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int x^2(a+bx^3+cx^6)^{3/2}\,dx = \frac{1}{192}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,cx^3+3\,b\right)x^3+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^3-\frac{3\,b^3c-20\,abc^2}{c^3}\right) - \frac{(b^4-8\,ab^2c+16\,a^2c^2)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(5
/2)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(c\,x^3+\frac{b}{2}\right)\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] ((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((b
/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

3.206 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx = \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}(a+bx^3+cx^6)^{3/2} - \frac{1}{3}a^{3/2}\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)-1/3*a^(3/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^
6+b*x^3+a)^(1/2))-1/48*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/c^(3/2)+1/24*(2*b*c*x^3+8*a*c+b^2)*(c*x^6+b*x^3+a)^(1/2)
/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 748, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx = -\frac{1}{3}a^{3/2}\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}} + \frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}(a+bx^3+cx^6)^{3/2}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx,x,x^3\right)\\
&= \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}-\frac{1}{6}\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)\\
&= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24c}\\
&= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}\\
&\quad+\frac{1}{3}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\
&\quad-\frac{\left(b\left(b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{48c}\\
&= \frac{\left(b^2+8ac+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2}\\
&\quad-\frac{1}{3}\left(2a^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&\quad-\frac{\left(b\left(b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24c}
\end{aligned}
$$

$$= \frac{(b^2 + 8ac + 2bcx^3)\sqrt{a + bx^3 + cx^6}}{24c} + \frac{1}{9}(a + bx^3 + cx^6)^{3/2} - \frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right) - \frac{b(b^2 - 12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x} dx = \frac{1}{144}\left(\frac{2\sqrt{a + bx^3 + cx^6}(3b^2 + 14bcx^3 + 8c(4a + cx^6))}{c} - \frac{3(b^3 - 12abc)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}} + 96a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}}\right)\right)$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c - (
3*(b^3 - 12*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/c^(3/2) + 96*a^(3/2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sq
rt[a]])/144
```

Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^{\frac{3}{2}}}{x} dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.69

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x} dx = \left[\frac{48\,a^{\frac{3}{2}}c^2\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right) - 3\,(b^3 - 12\,abc)\sqrt{c}\,\mathrm{lo}}{}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a
^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^
2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a
)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6
+ a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x
^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 +
14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]
```

Sympy [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x}\,dx = \int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

3.207 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}$$

```
[Out] -1/3*(c*x^6+b*x^3+a)^(3/2)/x^3-1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6
+b*x^3+a)^(1/2))*a^(1/2)+1/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/4*(2*c*x^3+3*b)*(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 746, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{(4ac+b^2)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}} - \frac{1}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} + \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*
x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])
])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6])])/(8*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}+\frac{1}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}-\frac{\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{8c}\\
&= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}\\
&\quad+\frac{1}{2}(ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&\quad-\frac{1}{8}\left(-b^2-4ac\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}\\
&\quad-(ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&\quad-\frac{1}{4}\left(-b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)
\end{aligned}$$

$$= \frac{1}{4}(3b + 2cx^3)\sqrt{a + bx^3 + cx^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right) + \frac{(b^2 + 4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4}\,dx = \frac{\sqrt{a + bx^3 + cx^6}(-4a + 5bx^3 + 2cx^6)}{12x^3} + \sqrt{a}b\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}}\right) - \frac{(b^2 + 4ac)\log\left(b + 2cx^3 - 2\sqrt{c}\sqrt{a + bx^3 + cx^6}\right)}{8\sqrt{c}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + Sqrt[a]*b*A
rcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]] - ((b^2 + 4*a*c)*Lo
g[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(8*Sqrt[c])
```

Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^{\frac{3}{2}}}{x^4}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.75

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4}\,dx = \left[\frac{12\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right) + 3(b^2 + 4ac)\sqrt{cx}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^
6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 + b*x
^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqr
t(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*
x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3)]
```

Sympy [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4} \, dx = \int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^4} \, dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^4}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

3.208 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7} dx = -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{(b^2+4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} + \frac{1}{2}b\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -1/6*(c*x^6+b*x^3+a)^(3/2)/x^6-1/8*(4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1
/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/4*(-2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 746, 826, 857, 635, 212, 738}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7} dx = -\frac{(4ac+b^2)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} + \frac{1}{2}b\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right) - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/4*((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^3 - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
```

```
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{6x^6} + \frac{1}{4}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{1}{8}\text{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6}\\
&\quad + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&\quad - \frac{1}{8}\left(-b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6}\\
&\quad + (bc)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&\quad - \frac{1}{4}\left(b^2+4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6}\\
&\quad - \frac{(b^2+4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} + \frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \frac{1}{12}\left(\frac{\sqrt{a+bx^3+cx^6}(-2a-5bx^3+4cx^6)}{x^6} + \frac{3(b^2+4ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}} - 6b\sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((Sqrt[a + b*x^3 + c*x^6]*(-2*a - 5*b*x^3 + 4*c*x^6))/x^6 + (3*(b^2 + 4*a*c
)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/Sqrt[a] - 6*b*S
qrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/12
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.72

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \left[\frac{12\,ab\sqrt{c}x^6\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\cdots}{48\,ax^6}\right.$$

$$-\frac{24\,ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-3\,(b^2+4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\cdots}{x^6}\cdots\right.}{48\,ax^6}$$

$$-\frac{12\,ab\sqrt{-c}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-3\,(b^2+4\,ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)-2\,(4\,a\cdots}{24\,ax^6}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*lo
```

```
g(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 + b*x
^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sq
rt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6)]
```

Sympy [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7}\,dx = \int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^7}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

3.209 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx=-\frac{\left(2ab+\left(b^2+8ac\right)x^3\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}+\frac{b\left(b^2-12ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}+\frac{1}{3}c^{3/2}\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/x^9+1/48*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)
/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+1/3*c^(3/2)*arctanh(1/2*(2*c*x^3+b)
/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))-1/24*(2*a*b+(8*a*c+b^2)*x^3)*(c*x^6+b*x^3+a
)^(1/2)/a/x^6
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 746, 824, 857, 635, 212, 738}

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{10}}\,dx=\frac{b\left(b^2-12ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}+\frac{1}{3}c^{3/2}\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{\left(x^3\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9x^9}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/24*((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (a +
b*x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*S
```

```
qrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*
x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^4}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{1}{6}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{(2ab+(b^2+8ac)x^3)\sqrt{a+bx^3+cx^6}}{24ax^6}\\
&\quad - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2-12ac)-8ac^2x}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a}\\
&= -\frac{(2ab+(b^2+8ac)x^3)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9}\\
&\quad + \frac{1}{3}c^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&\quad - \frac{(b(b^2-12ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48a}\\
&= -\frac{(2ab+(b^2+8ac)x^3)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9}\\
&\quad + \frac{1}{3}(2c^2)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\
&\quad + \frac{(b(b^2-12ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24a}
\end{aligned}$$

$$= -\frac{(2ab + (b^2 + 8ac)\,x^3)\,\sqrt{a + bx^3 + cx^6}}{24ax^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} + \frac{b(b^2 - 12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}} + \frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}}\right)$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a + bx^3 + cx^6}(-8a^2 - 14abx^3 - 3b^2x^6 - 32acx^6)}{72ax^9} + \frac{(b^3 - 12abc)\,\text{arctanh}\left(\frac{-\sqrt{c}x^3+\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{1}{3}c^{3/2}\log\left(b + 2cx^3 - 2\sqrt{c}\sqrt{a + bx^3 + cx^6}\right)$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(7
2*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c*x
^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a +
 b*x^3 + c*x^6]])/3
```

Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^{\frac{3}{2}}}{x^{10}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.73

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{10}}\,dx = \left[\frac{48\,a^2c^{\frac{3}{2}}x^9\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - 4\,\sqrt{cx^6 + bx^3 + a}(2\,cx^3 + b)\sqrt{c} - 4\,ac\right) -}{}\right.$$

$$-\frac{96\,a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right) + 3\,(b^3 - 12\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+}{x^6}\right.}{288\,a^2x^9}$$

$$-\frac{48\,a^2\sqrt{-c}cx^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right) + 3\,(b^3 - 12\,abc)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right) + 2\,(}{144\,a^2x^9}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8
*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*
b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x
^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt
(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3
*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14
*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{10}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{10}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)
```

3.210 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}} dx$

Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}} dx = \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}$$

```
[Out] -1/24*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a/x^12-1/128*(-4*a*c+b^2)^2*arctanh
(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/64*(-4*a*c+b^2)*(
b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1371, 734, 738, 212}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}} dx = -\frac{(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}} + \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)\\
&= -\frac{(2a+bx^3)\,(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{16a}\\
&= \frac{(b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)\,(a+bx^3+cx^6)^{3/2}}{24ax^{12}}\\
&\quad + \frac{(b^2-4ac)^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{128a^2}
\end{aligned}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{64a^2}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}(8a^2+8abx^3-3b^2x^6+20acx^6)}{192a^2x^{12}} + \frac{(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/192*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 8*a*b*x^3 - 3*b^2*x^
6 + 20*a*c*x^6))/(a^2*x^12) + ((b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[
a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2))
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{13}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.40

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a}{x^6}\right.}{76}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x
^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)
*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 -
24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a)
)/(a^3*x^12)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{13}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{13}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}\,dx=\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{13}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

3.211 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx=-\frac{b(b^2-4ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}}+\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}$$

```
[Out] 1/48*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/15*(c*x^6+b*x^3+a)^(5/2
)/a/x^15+1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^
3+a)^(1/2))/a^(7/2)-1/128*b*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/
a^3/x^6
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 744, 734, 738, 212}

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx=\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/128*(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a^3*x^6) +
(b*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*
x^6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a
]*Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} - \frac{b\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{32a^2}$$

$$= -\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} - \frac{\left(b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{256a^3}$$

$$= -\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} + \frac{\left(b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{128a^3}$$

$$= -\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} + \frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\left(128a^4+15b^4x^{12}-10ab^2x^9(b+10cx^3)+16a^3(11bx^3+16cx^6)+8a^2x^6(b^2+7bcx^3+16c^2x^6)\right)}{x^{15}} -}{1920a^{7/2}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(128*a^4 + 15*b^4*x^12 - 10*a*b^2*x^9*(
b + 10*c*x^3) + 16*a^3*(11*b*x^3 + 16*c*x^6) + 8*a^2*x^6*(b^2 + 7*b*c*x^3 +
 16*c^2*x^6)))/x^15) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
 b*x^3 + c*x^6])/Sqrt[a]])/(1920*a^(7/2))
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{16}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+}{x^6}\right.}{}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^{12}}{3840\,a^4x^{15}}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 -
 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sq
rt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*
x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4
*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{16}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{16}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{16}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

3.212 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} dx = \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{(b^2-4ac)^2(7b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}$$

```
[Out] -1/576*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^12-1/18*(c*x^
6+b*x^3+a)^(5/2)/a/x^18+7/180*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^15-1/3072*(-4*a
*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1
/2))/a^(9/2)+1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^4/x^6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 758, 820, 734, 738, 212}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = -\frac{(b^2-4ac)^2(7b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}} + \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^7}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^3\right)}{18a}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)}{72a^2}$$

$$= -\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{((b^2-4ac)(7b^2-4ac))\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{384a^3}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} + \frac{\left((b^2-4ac)^2(7b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3072a^4}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{\left((b^2-4ac)^2(7b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{1536a^4}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}(1280a^5-105b^5x^{15}+10ab^3x^{12}(7b+76cx^3)+64a^4(26bx^3+35cx^6)+48a^3x^6(b^2+6bcx^3+10c^2x^6}{x^{18}}}{2304\ldots}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(1280*a^5 - 105*b^5*x^15 + 10*a*b^3*x^1
2*(7*b + 76*c*x^3) + 64*a^4*(26*b*x^3 + 35*c*x^6) + 48*a^3*x^6*(b^2 + 6*b*c
*x^3 + 10*c^2*x^6) - 8*a^2*b*x^9*(7*b^2 + 54*b*c*x^3 + 162*c^2*x^6)))/x^18)
 + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/(23040*a^(9/2))
```

Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{19}}dx$$

[In] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{a}x^{18}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6-}}{x^6}\right.}{}\right.$$

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")

[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{19}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{19}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

3.213 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 255

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{b(b^2-4ac)^2(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}$$

```
[Out] 1/384*b*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^4/x^12-1/21*(c*x
^6+b*x^3+a)^(5/2)/a/x^21+1/28*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^18-1/840*(-16*a
*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)/a^3/x^15+1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3
*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(11/2)-1/102
4*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^5/x^6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 758, 848, 820, 734, 738, 212}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \frac{b(b^2-4ac)^2(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}} - \frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -1/1024*(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2)
)/(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 +
c*x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))
/(840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/
(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^8}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{9b}{2}+2cx\right)(a+bx+cx^2)^{3/2}}{x^7}\,dx, x, x^3\right)}{21a}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{4}(21b^2-16ac)+\frac{9bcx}{2}\right)(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^3\right)}{126a^2}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} - \frac{(b(3b^2-4ac))\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)}{48a^3}$$

$$= \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{(b(b^2-4ac)(3b^2-4ac))\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{256a^4}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} - \frac{\left(b(b^2-4ac)^2(3b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{2048a^5}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{\left(b(b^2-4ac)^2(3b^2-4ac)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{1024a^5}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{b(b^2-4ac)^2(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}(5120a^6+315b^6x^{18}-210ab^4x^{15}(b+12cx^3)+256a^5(25bx^3+32cx^6)+64a^4x^6(2b^2+11bcx^3+1}{x^{21}}}{}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(5120*a^6 + 315*b^6*x^18 - 210*a*b^4*x^
15*(b + 12*c*x^3) + 256*a^5*(25*b*x^3 + 32*c*x^6) + 64*a^4*x^6*(2*b^2 + 11*
b*c*x^3 + 16*c^2*x^6) + 56*a^2*b^2*x^12*(3*b^2 + 26*b*c*x^3 + 98*c^2*x^6) -
 16*a^3*x^9*(9*b^3 + 62*b^2*c*x^3 + 146*b*c^2*x^6 + 128*c^3*x^9)))/x^21) -
105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/(107520*a^(11/2))
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{22}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{a}x^{21}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6}}{x^{}}\right.}{}\right.$$

$$-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-a}x^{21}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((315\,ab^6-2520\,a^{}}{}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a
)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3
*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b
*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b
*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c
)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^
5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^
6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5
 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 1
28*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*
a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{22}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{22}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{22}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

3.214 $\int x^3(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int x^3(a+bx^3+cx^6)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/3,-3/2,-3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int x^3(a+bx^3+cx^6)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
```

```
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{ax^4\sqrt{a+bx^3+cx^6}F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 453 vs. $2(141) = 282$.

Time = 10.58 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.21

$$\int x^3(a+bx^3+cx^6)^{3/2}\,dx = \frac{x\Big(8(-297b^4x^3-81b^3cx^6+3464b^2c^2x^9+5488bc^3x^{12}+2240c^4x^{15}+4a^2c(459b+1280cx^3)+$$

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 + 2
240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^3
```

```
+ 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 27*(55
*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a + b*x^3 + c*
x^6])
```

Maple [F]

$$\int x^{3} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}d x$$

[In] `int(x^3*(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(x^3*(c*x^6+b*x^3+a)^(3/2),x)`

Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int { {\left(c x^{6} + b x^{3} + a\right)}^{\frac{3}{2}} x^{3} \,d x }$$

[In] `integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")`

[Out] `integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)`

Sympy [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int x^{3} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}\, dx$$

[In] `integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)`

Maxima [F]

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x^3\,\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^3*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.215 $\int x(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/3,-3/2,-3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
```

```
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ax^2\sqrt{a+bx^3+cx^6}F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(141) = 282.

Time = 10.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.91

$$\int x\left(a+bx^3+cx^6\right)^{3/2}dx = \frac{x^2\left(10(27ab^2+448a^2c+27b^3x^3+698abcx^3+277b^2cx^6+608ac^2x^6+410bc^2x^9+160c^3x^{12})\right.}{}$$

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^6
 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*Sqr
```

```
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int x\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

```
[In] int(x*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.216 $\int \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{ax\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*x*AppellF1(1/3,-3/2,-3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1362, 440}

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{ax\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ax\sqrt{a+bx^3+cx^6}F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 408 vs. 2(136) = 272.

Time = 10.48 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.00

$$\int (a+bx^3+cx^6)^{3/2}\,dx = \frac{x\left(8(27ab^2+364a^2c+27b^3x^3+548abcx^3+211b^2cx^6+476ac^2x^6+296bc^2x^9+112c^3x^{12})-\right.}{}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] -
27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

```
[In] int((c*x^6+b*x^3+a)^(3/2),x)
[Out] int((c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((a + b*x^3 + c*x^6)^(3/2), x)
```

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -a*AppellF1(-1/3,-3/2,-3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
```

```
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(139) = 278.

Time = 10.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \frac{10(-80a^2-61abx^3+19b^2x^6-70acx^6+29bcx^9+10c^2x^{12})+810abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}\cdots}{b-\sqrt{b^2\cdots}}}\cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] (10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*x
^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
```

```
+ Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a + b*x^3 +
c*x^6])
```

Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{2}}d x$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^2}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^2, x)
```

3.218 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*a*AppellF1(-2/3,-3/2,-3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 +
```

```
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= -\frac{a\sqrt{a+bx^3+cx^6}F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(141) = 282.

Time = 10.35 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.69

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \frac{8(-28a^2-11abx^3+17b^2x^6-20acx^6+25bcx^9+8c^2x^{12})+648abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}\cdots}{b-\sqrt{b^2-\cdots}}}\cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] (8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^1
2) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
```

```
 Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]])/(448*x^2*Sqrt[a + b*x^3 +
c*x^6])
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{3}}dx$$

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^3}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)\,x^3)\sqrt{a+bx^3+cx^6}}{576c^4} + \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}$$

```
[Out] 1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^
6+b*x^3+a)^(1/2))/c^(9/2)-7/72*b*x^6*(c*x^6+b*x^3+a)^(1/2)/c^2+1/12*x^9*(c*
x^6+b*x^3+a)^(1/2)/c-1/576*(5*b*(-44*a*c+21*b^2)-2*c*(-36*a*c+35*b^2)*x^3)*
(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 756, 846, 793, 635, 212}

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(48a^2c^2-120ab^2c+35b^4)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}} - \frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4} - \frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

```
[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3a-\frac{7bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12c}\\
&= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x\left(7ab+\frac{1}{4}(35b^2-36ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36c^2}\\
&= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}\\
&\quad - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)x^3)\sqrt{a+bx^3+cx^6}}{576c^4}\\
&\quad + \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{384c^4}\\
&= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}\\
&\quad - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)x^3)\sqrt{a+bx^3+cx^6}}{576c^4}\\
&\quad + \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{192c^4}\\
&= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}\\
&\quad - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)x^3)\sqrt{a+bx^3+cx^6}}{576c^4}\\
&\quad + \frac{(35b^4-120ab^2c+48a^2c^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-105b^3+220abc+70b^2cx^3-72ac^2x^3-56bc^2x^6+48c^3x^9)}{576c^4} + \frac{(-35b^4+120ab^2c-48a^2c^2)\log\left(bc^4+2c^5x^3-2c^{9/2}\sqrt{a+bx^3+cx^6}\right)}{384c^{9/2}}$$

```
[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x^
3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*a^
2*c^2)*Log[b*c^4 + 2*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(
9/2))
```

Maple [F]

$$\int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.77

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+}{2304\,c^5}\right.$$
$$-\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(48\,c^4x^9-56\,bc^3x^6-105\,b^3c+}{1152\,c^5}$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b
*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
```

```
*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*
a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c
 + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^
5]
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}$$

```
[Out] -1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/c^(7/2)+1/9*x^6*(c*x^6+b*x^3+a)^(1/2)/c+1/72*(-10*b*c*x^3-16*a*c+15*b^2
)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 756, 793, 635, 212}

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}} + \frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} + \frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

```
[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2-12ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48c^3}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2-12ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24c^3}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(15b^2-16ac-10bcx^3+8c^2x^6)}{72c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{48c^{7/2}}$$

```
[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c^
3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6
]])/(48*c^(7/2))
```

Maple [F]

$$\int \frac{x^{11}}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^6-\right.}{288\,c^4}$$

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*
sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*
b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*
b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c
 - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{cx^6+bx^3+a}}\,dx$$

[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

[In] int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)

[Out] int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}} \, dx = -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}+\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}$$

```
[Out] 1/24*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
c^(5/2)-1/4*b*(c*x^6+b*x^3+a)^(1/2)/c^2+1/6*x^3*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 756, 654, 635, 212}

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}} \, dx = \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

```
[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/4*(b*Sqrt[a + b*x^3 + c*x^6])/c^2 + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}$$

$$= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c^2}$$

$$= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-3b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c^2} + \frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{24c^{5/2}}$$

```
[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Log
[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))
```

Maple [F]

$$\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+}}{48\,c^3}\right.$$
$$\left.-\frac{(3\,b^2-4\,ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,\sqrt{cx^6+bx^3+a}(2\,c^2x^3-3\,bc)}{24\,c^3}\right]$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c
*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*s
qrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}} \, dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+1/3*(
c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1371, 654, 635, 212}

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}} \, dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6]])])/(6*c^(3/2))
```

Maple [F]

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^6+bx^3+a}c}{12\,c^2}, \frac{b\sqrt{-c}\,\mathrm{arct}}{}\right.$$

[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{6\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/
2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c} - \frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 +
 c*x^3)/c^(1/2)))/(6*c^(3/2))
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1366, 635, 212}

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{3\sqrt{c}}$$

```
[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
[Out] -1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)}{6\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)}{3\,c}\right]$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2x^3+\frac{b}{c}\right) + \frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24c^{\frac{3}{2}}}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*(
sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\sqrt{c}}$$

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1371, 738, 212}

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(3*Sqrt[a])
```

Maple [F]

$$\int \frac{1}{x\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$
$$=\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+ax}}\,dx$$

[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\,\sqrt{a}}$$

[In] int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)

[Out] -log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))

3.225 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}+\frac{b\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

```
[Out] 1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)-1/3*(c
*x^6+b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1371, 744, 738, 212}

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

```
[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$
\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}}
$$

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) - (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(3*a^(3/2))
```

Maple [F]

$$\int \frac{1}{x^{4}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

[In] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}a}{12\,a^2x^3}, -\frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}a}{6\,a^2x^3}\right]$$

[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a^2*x^3)]

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^4}\,dx$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}} - \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
- (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

```
[Out] -1/24*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/a^(5/2)-1/6*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/4*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^
3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 758, 820, 738, 212}

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

```
[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/6*Sqrt[a + b*x^3 + c*x^6]/(a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x
^3) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*
x^6])])/(24*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-2a+3bx^3)\sqrt{a+bx^3+cx^6}}{12a^2x^6} + \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}}$$

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)*
ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))
```

Maple [F]

$$\int \frac{1}{x^7\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right) - 4\,\sqrt{cx^6+bx^3+a}(3\,abx^3-2\,a^2)}{48\,a^3x^6}\right.$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4
*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6
 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^7}\,dx$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^7\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}$$

```
[Out] 1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(7/2)-1/9*(c*x^6+b*x^3+a)^(1/2)/a/x^9+5/36*b*(c*x^6+b*x^3+a)^(1/2)/a^2
/x^6-1/72*(-16*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 758, 848, 820, 738, 212}

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

```
[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/9*Sqrt[a + b*x^3 + c*x^6]/(a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^
2*x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b
^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(4
8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15b^2-16ac)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{18a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}\\
&\quad - \frac{(b(5b^2-12ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48a^3}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}\\
&\quad + \frac{(b(5b^2-12ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24a^3}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}\\
&\quad + \frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2+10abx^3-15b^2x^6+16acx^6)}{72a^3x^9} + \frac{(-5b^3+12abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{7/2}}$$

[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/(72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(7/2))

Maple [F]

$$\int \frac{1}{x^{10}\sqrt{c\,x^6+b\,x^3+a}}dx$$

[In] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\left((15\,ab^2-16\,a^2c)x^6-10\,a\right.}{288\,a^4x^9}\right.$$
$$-\frac{3\,(5\,b^3-12\,abc)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\left((15\,ab^2-16\,a^2c)x^6-10\,a^2bx^3+8\,a^3\right)\sqrt{cx^6}}{144\,a^4x^9}$$

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9)]

Sympy [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^{10}}\,dx$$

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}$$

```
[Out] -1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/a^(9/2)-1/12*(c*x^6+b*x^3+a)^(1/2)/a/x^12+7/72*b*(c*x^6+
b*x^3+a)^(1/2)/a^2/x^9-1/288*(-36*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6
+5/576*b*(-44*a*c+21*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {1371, 758, 848, 820, 738, 212}

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(48a^2c^2-120ab^2c+35b^4)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}} - \frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

```
[In] Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/12*Sqrt[a + b*x^3 + c*x^6]/(a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*
a^2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b
*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*
a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x
^6])])/(384*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12a} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(35b^2-36ac)+7bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{36a^2} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{5}{8}b(21b^2-44ac)+\frac{1}{4}c(35b^2-36ac)x}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{72a^3} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} \\
&\quad - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} \\
&\quad + \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{384a^4}
\end{aligned}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{192a^4}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^4-120ab^2c+48a^2c^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-48a^3+56a^2bx^3-70ab^2x^6+72a^2cx^6+105b^3x^9-220abcx^9)}{576a^4x^{12}} + \frac{(35b^4-120ab^2c+48a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{9/2}}$$

```
[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c*
x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2*c
 + 48*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1
92*a^(9/2))
```

Maple [F]

$$\int \frac{1}{x^{13}\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,(5\,(21\,ab^3-44}{2304\,a^5x^{12}}\right.$$

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*
a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2
 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(
35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3
 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)
*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^{13}}\,dx$$

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2
, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^3/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int\frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int\frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2
, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{x}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

[In] int(x/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x/(c*x^6+b*x^3+a)^(1/2),x)

Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral(x/sqrt(c*x^6 + b*x^3 + a), x)

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$$

[In] integrate(x/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x/sqrt(a + b*x**3 + c*x**6), x)

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(x/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}} \, dx = \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1362, 440}

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}} \, dx = \frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$
$$= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2,
1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Maple [F]

$$\int \frac{1}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

```
[In] int(1/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

[In] `integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/sqrt(c*x^6 + b*x^3 + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

[In] `int(1/(a + b*x^3 + c*x^6)^(1/2),x)`

[Out] `int(1/(a + b*x^3 + c*x^6)^(1/2), x)`

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}\\ &= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(138) = 276.

Time = 10.23 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \frac{-20(a+bx^3+cx^6)+5bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{20ax\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^2\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)),x)

[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)), x)

3.233 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*x^3 + c*x^6]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(140) = 280.

Time = 10.20 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

$$= \frac{-4(a+bx^3+cx^6)-2bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8ax^2\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.234 $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}} dx = \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} + \frac{(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}$$

```
[Out] 1/8*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(7/2)+2/3*x^9*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*x^6*(c*
x^6+b*x^3+a)^(1/2)/c/(-4*a*c+b^2)-1/12*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b
^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 752, 846, 793, 635, 212}

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}} dx = \frac{(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}} - \frac{(b(15b^2-52ac)-2cx^3(5b^2-12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3(b^2-4ac)}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{2\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}(5b^2-12ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c(b^2-4ac)}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} + \frac{(5b^2-4ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{8c^3}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} + \frac{(5b^2-4ac)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{4c^3}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} + \frac{(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{4a^2c(-13b+6cx^3)+b^2x^3(15b^2+5bcx^3-2c^2x^6)+a(15b^3-62b^2cx^3-20bc^2x^6}{12c^3\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{(-5b^2+4ac)\log\left(c^3\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{8c^{7/2}}$$

```
[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^2*x^6) + a*(
15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9))/(12*c^3*(-b^2 + 4*a*c)*S
qrt[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*Log[c^3*(b + 2*c*x^3 - 2*Sqrt[c
]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2))
```

Maple [F]

$$\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.03

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^5-24\,ab^3\right.}{} - \frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^3\right)\sqrt{-c}\arctan\left(\right.}{24\left(ab^2c^4-\right.}\right.$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2
*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8*c
^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*
a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*
```

```
c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6
+ 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c
^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*
b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{x^{14}}{\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.235 $\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} - \frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)+2/3*x
^6*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)+1/3*(-2*b*c*x^3-8*a*c+3*b
^2)*(c*x^6+b*x^3+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 752, 793, 635, 212}

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}} + \frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} + \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3(b^2-4ac)}$$

$$= \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}dx, x, x^3\right)}{2c^2}$$

$$= \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{c^2}$$

$$= \frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-3ab^2+8a^2c-3b^3x^3+10abcx^3-b^2cx^6+4ac^2x^6}{3c^2(-b^2+4ac)\sqrt{a+bx^3+cx^6}} + \frac{b\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{2c^{5/2}}$$

```
[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/(
3*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3 -
2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))
```

Maple [F]

$$\int \frac{x^{11}}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

```
[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.35

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^6+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^3\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-\right.}{12\left((b^2c\right.}\right.$$

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a
^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((
b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
 - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b
^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.236 $\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

```
[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+2/3*x^3*
(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*(c*x^6+b*x^3+a)^(1/2)/
c/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 752, 654, 635, 212}

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}} + \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)}$$

```
[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt
[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^3(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\,(b^2-4ac)}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3c}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-\frac{2\sqrt{c}(b^2x^3+a(b-2cx^3))}{(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] ((-2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c
*x^6]) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c^(
3/2))
```

Maple [F]

$$\int \frac{x^8}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.22

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-\right.}{6\left((b^2c^3-4\,ac^4)x^6+ab^2c\right.}\right.$$

$$-\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}(\ldots}{3\left((b^2c^3-4\,ac^4)x^6+ab^2c^2-4\,a^2c^3+(b^3c^2-4\,abc^3)x^3\right)}$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*
b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c
^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^
2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3
+ a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}}+\frac{\frac{a\,b}{2}-x^3\,\left(a\,c-\frac{b^2}{2}\right)}{3\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b)
/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

3.237 $\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[Out] 2/3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1371, 650}

$$\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
gosper	$-\frac{2(b\,x^3+2a)}{3\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	38
trager	$-\frac{2(b\,x^3+2a)}{3\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	38

```
[In] int(x^5/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\sqrt{cx^6+bx^3+a}(bx^3+2\,a)}{3\,((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4
*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\sqrt{cx^6+bx^3+a}}$$

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\,b\,x^3+4\,a}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] -(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.238 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

```
[Out] -2/3*(2*c*x^3+b)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1366, 627}

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$
$$= -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3\,(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	37
trager	$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	37

```
[In] int(x^2/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\sqrt{cx^6+bx^3+a}(2\,cx^3+b)}{3\,((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{4\,c\,x^3+2\,b}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] (2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.239 $\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+2/3*(b*
c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 754, 12, 738, 212}

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(-2ac+b^2+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}
\end{aligned}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\,(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{a}(b^2-2ac+bcx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}} + \operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)}{3a^{3/2}}$$

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*((Sqrt[a]*(b^2 - 2*a*c + b*c*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6
]) + ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]]))/(3*a^(3/2))
```

Maple [F]

$$\int \frac{1}{x\,(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(78) = 156.

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.23

$$\int \frac{1}{x\,(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx}}{x}\right.}{6\,((a^2b^2c-4\,a^3c^2)x^6+a^3b^2-4\,a^4c}\right.$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
- 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4
*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 -
4*a^3*b*c)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.240 $\int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+2/3*(b
*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^6+b*x^3+a)^(1/2)-1/3*(-8*a*c+3*b^
2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 754, 820, 738, 212}

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2x^3\left(b^2-4ac\right)} + \frac{2(-2ac+b^2+bcx^3)}{3ax^3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)x^3\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(-3b^2+8ac)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a(b^2-4ac)}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)x^3\sqrt{a+bx^3+cx^6}} - \frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{3a^2(b^2-4ac)x^3} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{2a^2}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)x^3\sqrt{a+bx^3+cx^6}} - \frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{3a^2(b^2-4ac)x^3} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{a^2}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)x^3\sqrt{a+bx^3+cx^6}} - \frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{3a^2(b^2-4ac)x^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-4a^2c+3b^2x^3(b+cx^3)+a(b^2-10bcx^3-8c^2x^6)}{3a^2(-b^2+4ac)x^3\sqrt{a+bx^3+cx^6}} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
[Out] (-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x^6))/(3*a^
2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - (b*ArcTanh[(Sqrt[c]*x^3 - S
qrt[a + b*x^3 + c*x^6])/Sqrt[a]])/a^(5/2)
```

Maple [F]

$$\int \frac{1}{x^4(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^6}{}\right.}{12\left((a^3b^2c-4\right.}\right.$$

$$-\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\left((3\,ab^2c\right.}{6\left((a^3b^2c-4\,a^4c^2)x^9+(a^3b^3-4\,a^4bc)x^6+(a^4b^2-\right.}$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6
 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))
/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^
5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^
3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*
a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a
^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)
*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^4} dx$$

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} dx = \int \frac{1}{x^4 (c\, x^6 + b\, x^3 + a)^{3/2}} dx$$

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.241 $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 198

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3} - \frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}$$

```
[Out] -1/8*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
a^(7/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^6/(c*x^6+b*x^3+a)^(1/2)-1/
6*(-12*a*c+5*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^6+1/12*b*(-52*a*
c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 754, 848, 820, 738, 212}

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}} + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3\left(b^2-4ac\right)} - \frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6\left(b^2-4ac\right)} + \frac{2(-2ac+b^2+bcx^3)}{3ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
```

```
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b\left(15b^2-52ac\right)-\frac{1}{2}c\left(5b^2-12ac\right)x}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a^2\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6}\\
&\quad + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{8a^3}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6}\\
&\quad + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3} - \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{4a^3}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6}\\
&\quad + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3} - \frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (a+bx^3+cx^6)^{3/2}}\,dx = \frac{-8a^3c-15b^3x^6(b+cx^3)+2a^2(b^2+10bcx^3-12c^2x^6)+abx^3(-5b^2+62bcx^3+}{12a^3(-b^2+4ac)\,x^6\sqrt{a+bx^3+cx^6}} + \frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 - 12*c^2*x^6)
+ a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6))/(12*a^3*(-b^2 + 4*a*c)*x^6*Sq
rt[a + b*x^3 + c*x^6]) + ((5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(4*a^(7/2))
```

Maple [F]

$$\int \frac{1}{x^7\,(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^7 (a+bx^3+cx^6)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^{12}+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^9+(5\,ab^4-\right.}{}\right.$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)*l
og(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a
)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 -
62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b
*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3
- 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*a*b^2*c
^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4
```

```
- 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 +
a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3*c - 52*
a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c
- 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}}\, dx = \int \frac{1}{x^7 (a + bx^3 + cx^6)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}}\, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7}\, dx$$

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^7\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.242 $\int \frac{1}{x^{10}(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 256

$$
\begin{aligned}
\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} \\
&- \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\left(b^2-4ac\right)x^6} \\
&- \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{a+bx^3+cx^6}}{72a^4\left(b^2-4ac\right)x^3} \\
&+ \frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}
\end{aligned}
$$

```
[Out] 5/48*b*(-12*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(9/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^9/(c*x^6+b*x^3+a)^(1/2)
-1/9*(-16*a*c+7*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^9+1/36*b*(-11
6*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^6-1/72*(256*a^2*c^2-
460*a*b^2*c+105*b^4)*(c*x^6+b*x^3+a)^(1/2)/a^4/(-4*a*c+b^2)/x^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{1371, 754, 848, 820, 738, 212\}$

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)} - \frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)} + \frac{2(-2ac+b^2+bcx^3)}{3ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7b^2+16ac\right)-3bcx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)} \\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}b\left(35b^2-116ac\right)-c\left(7b^2-16ac\right)x}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9a^2\left(b^2-4ac\right)}
\end{aligned}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,x^9\sqrt{a+bx^3+cx^6}} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\,(b^2-4ac)\,x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\,(b^2-4ac)\,x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{8}(-105b^4+460ab^2c-256a^2c^2)-\frac{1}{4}bc(35b^2-116ac)x}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9a^3\,(b^2-4ac)}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,x^9\sqrt{a+bx^3+cx^6}} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\,(b^2-4ac)\,x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\,(b^2-4ac)\,x^6} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{a+bx^3+cx^6}}{72a^4\,(b^2-4ac)\,x^3} - \frac{(5b(7b^2-12ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{48a^4}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,x^9\sqrt{a+bx^3+cx^6}} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\,(b^2-4ac)\,x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\,(b^2-4ac)\,x^6} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{a+bx^3+cx^6}}{72a^4\,(b^2-4ac)\,x^3} + \frac{(5b(7b^2-12ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{24a^4}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,x^9\sqrt{a+bx^3+cx^6}} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\,(b^2-4ac)\,x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\,(b^2-4ac)\,x^6} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{a+bx^3+cx^6}}{72a^4\,(b^2-4ac)\,x^3} + \frac{5b(7b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{10}\,(a+bx^3+cx^6)^{3/2}}\,dx = \frac{-32a^4c+105b^4x^9(b+cx^3)+5ab^2x^6(7b^2-106bcx^3-92c^2x^6)+8a^3(b^2+7bcx}{72a^4\,(-b^2+4ac)\,x^9\sqrt{a+b}} + \frac{5b(-7b^2+12ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}}$$

```
[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 - 9
2*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 86*
```

```
b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9))/(72*a^4*(-b^2 + 4*a*c)*x^9*Sqrt[a
 + b*x^3 + c*x^6]) + (5*b*(-7*b^2 + 12*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
 b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(9/2))
```

Maple [F]

$$\int \frac{1}{x^{10} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^{10} (a+bx^3+cx^6)^{3/2}}\,dx = \left[-\frac{15\left((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^{15}+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^{12}+(7\right.}{}\right.$$

$$-\frac{15\left((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^{15}+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^{12}+(7\,ab^5-40\,a^2b^3c+48\,a^3bc^2)x^9\right)\sqrt{-}}{}$$

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

```
[Out] [-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b
^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b
^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3
- 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 +
(a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*((7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2
)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^
2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x
^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b
^2 - 4*a^7*c)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^{10} (a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^{10}} \, dx$$

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^{10} (c\,x^6 + b\,x^3 + a)^{3/2}} \, dx$$

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.243 $\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,3/2,3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.

Time = 10.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.38

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\left(-2(b+2cx^3)+2b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c}{b+\sqrt{b}}\right.\right.}{3\,(}$$

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F]

$$\int \frac{x^3}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

```
[In] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*
x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^3/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.244 $\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,3/2,3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(143) = 286.

Time = 10.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.53

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^2\left(-20(b^2-2ac+bcx^3)+5(b^2+4ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\text{AppellF}\right.}{}$$

```
[In] Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(
30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{x}{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}dx$$

[In] int(x/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x/(c*x^6+b*x^3+a)^(3/2),x)

Fricas [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)

Maxima [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)

Giac [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{x}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(x/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.245 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,3/2,3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1362, 440}

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(138) = 276.

Time = 10.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\left(-4(b^2-2ac+bcx^3)-2(b^2-8ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\right.}{}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(6*a*(
-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{1}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

[In] int(1/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(1/(c*x^6+b*x^3+a)^(3/2),x)

Fricas [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral((a + b*x**3 + c*x**6)**(-3/2), x)

Maxima [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)

Giac [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(1/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(1/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,3/2,3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(141) = 282.

Time = 10.52 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx =$$

$$-\frac{5b(-5b^2+12ac)\,x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-4\Big(60a}{}$$

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -1/60*(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*x^3)
 + 5*a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*Sqrt[(
```

```
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/
(a^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(c \,x^{6}+b \,x^{3}+a \right)^{\frac{3}{2}}}d x$$

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{{\left(cx^{6} + bx^{3} + a\right)}^{\frac{3}{2}} x^{2}}\,dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^8
 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x^{2} \left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{{\left(cx^{6} + bx^{3} + a\right)}^{\frac{3}{2}} x^{2}}\,dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} \, dx$$

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^2 (c\,x^6 + b\,x^3 + a)^{3/2}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.247 $\int \frac{1}{x^3(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,3/2,3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 -
```

```
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x^3 + c*x^
6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 405 vs. $2(143) = 286$.

Time = 10.44 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.83

$$\int\frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{-48a^2c+28b^2x^3(b+cx^3)+4a(3b^2-24bcx^3-20c^2x^6)+2b(7b^2-36ac)\,x^3\sqrt{}}{}$$

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6)
 + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^
```

```
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*a^2*(-b^
2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}}d x$$

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^9
 + 2*a*b*x^6 + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{1}{x^3 (a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x^3 (c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.248 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)} +\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(4+m)/d^4/(4+m)+(2*a*c+b^2)*(d*x)^(7+m)
/d^7/(7+m)+2*b*c*(d*x)^(10+m)/d^10/(10+m)+c^2*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1367}

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)} +\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2(dx)^m + \frac{2ab(dx)^{3+m}}{d^3} + \frac{(b^2+2ac)(dx)^{6+m}}{d^6} + \frac{2bc(dx)^{9+m}}{d^9} + \frac{c^2(dx)^{12+m}}{d^{12}}\right) dx \\ &= \frac{a^2(dx)^{1+m}}{d(1+m)} + \frac{2ab(dx)^{4+m}}{d^4(4+m)} + \frac{(b^2+2ac)(dx)^{7+m}}{d^7(7+m)} + \frac{2bc(dx)^{10+m}}{d^{10}(10+m)} + \frac{c^2(dx)^{13+m}}{d^{13}(13+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 dx = x(dx)^m \left(\frac{a^2}{1+m} + \frac{2abx^3}{4+m} + \frac{(b^2+2ac)x^6}{7+m} + \frac{2bcx^9}{10+m} + \frac{c^2x^{12}}{13+m}\right)$$

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
+ (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(101) = 202.

Time = 0.19 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

method	result
gosper	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+1070}{}$
risch	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+1070}{}$
parallelrisch	$\frac{2x^{10}(dx)^m bc\,m^4+50x^{10}(dx)^m bc\,m^3+390x^{10}(dx)^m bc\,m^2+520x^7(dx)^m b^2+3640x(dx)^m a^2+1070x^{10}(dx)^m bcm+2x^7(dx)^m ac\,m^4}{}$

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x^
12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+10
70*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+249*
b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*
a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*
a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/(4+m)/
(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(101) = 202.

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{((c^2m^4+22\,c^2m^3+159\,c^2m^2+418\,c^2m+280\,c^2)x^{13}+2\,(bcm^4+25\,bcm^3+195\,bcm^2+535\,bcm+364\,b}{}$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m
^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 7
42*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*
m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*
a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. 2(90) = 180.

Time = 0.82 (sec) , antiderivative size = 1459, normalized size of antiderivative = 14.45

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a*
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*m**4*x*(d*x)**m/(m**5 + 35*m*
*4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*m**3*x*(d*x)**m/(m**5
+ 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*m**2*x*(d*x)**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*m*x*(
d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*a**2
*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*b
*m**4*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
 + 62*a*b*m**3*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*
m + 3640) + 642*a*b*m**2*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
```

```
*2 + 5714*m + 3640) + 2402*a*b*m*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 1820*a*b*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*m**4*x**7*(d*x)**m/(m**5 + 35*m**
4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*m**3*x**7*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*m**2*x**7*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1484*a*c*m*x
**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1040
*a*c*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
+ b**2*m**4*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
 3640) + 28*b**2*m**3*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 249*b**2*m**2*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 742*b**2*m*x**7*(d*x)**m/(m**5 + 35*m**4 + 44
5*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*x**7*(d*x)**m/(m**5 + 35*m**
4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*m**4*x**10*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 50*b*c*m**3*x**10*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 390*b*c*m**2
*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1
070*b*c*m*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 728*b*c*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 571
4*m + 3640) + c**2*m**4*x**13*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 22*c**2*m**3*x**13*(d*x)**m/(m**5 + 35*m**4 + 445*m**
3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*m**2*x**13*(d*x)**m/(m**5 + 35*m*
*4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 418*c**2*m*x**13*(d*x)**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 280*c**2*x**13*(d*x)*
*m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{c^2 d^m x^{13} x^m}{m+13} + \frac{2\, bcd^m x^{10} x^m}{m+10} + \frac{b^2 d^m x^7 x^m}{m+7} + \frac{2\, acd^m x^7 x^m}{m+7} + \frac{2\, abd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1}\, a^2}{d(m+1)}$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/(
m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m +
 1)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(101) = 202$.

Time = 0.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.45

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^2 \, dx = \frac{(dx)^m c^2 m^4 x^{13} + 22 (dx)^m c^2 m^3 x^{13} + 159 (dx)^m c^2 m^2 x^{13} + 2 (dx)^m bcm^4 x^{10} + 418 (dx)^m c^2 m x^{13} + 50 (dx}{}$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
+ 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^10
 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 +
2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^7 +
56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 4
98*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 14
84*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*
(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 2402*(d*x)^
m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d*x)^m*a^2
*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4 + 445*m^3
 + 2485*m^2 + 5714*m + 3640)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^2 \, dx = (d\,x)^m \left(\frac{c^2\,x^{13}\,(m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280)}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640} + \frac{x^7\,(b^2 + 2\,a\,c)\,(m^4 + 28\,m^3 + 249\,m^2 + 742\,m + 520)}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640} + \frac{a^2\,x\,(m^4 + 34\,m^3 + 411\,m^2 + 2074\,m + 3640)}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640} + \frac{2\,a\,b\,x^4\,(m^4 + 31\,m^3 + 321\,m^2 + 1201\,m + 910)}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640} + \frac{2\,b\,c\,x^{10}\,(m^4 + 25\,m^3 + 195\,m^2 + 535\,m + 364)}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640}\right)$$

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*m
^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2 +
 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) +
```

```
(a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 + 445*
m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m^4 +
910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^10*(53
5*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4
+ m^5 + 3640))
```

3.249 $\int (dx)^m \left(a+bx^3+cx^6\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int (dx)^m \left(a+bx^3+cx^6\right) dx=\frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}$$

```
[Out] a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(4+m)/d^4/(4+m)+c*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\int (dx)^m \left(a+bx^3+cx^6\right) dx=\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(dx)^m+\frac{b(dx)^{3+m}}{d^3}+\frac{c(dx)^{6+m}}{d^6}\right) dx \\
&= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx = x(dx)^m \left(\frac{a}{1+m}+\frac{bx^3}{4+m}+\frac{cx^6}{7+m}\right)$$

[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]

[Out] x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result
norman	$\frac{ax\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{b\,x^4\mathrm{e}^{m\ln(dx)}}{4+m}+\frac{c\,x^7\mathrm{e}^{m\ln(dx)}}{7+m}$
gosper	$\frac{x(c\,m^2x^6+5cm\,x^6+4c\,x^6+b\,m^2x^3+8bm\,x^3+7b\,x^3+a\,m^2+11am+28a)(dx)^m}{(7+m)(4+m)(1+m)}$
risch	$\frac{x(c\,m^2x^6+5cm\,x^6+4c\,x^6+b\,m^2x^3+8bm\,x^3+7b\,x^3+a\,m^2+11am+28a)(dx)^m}{(7+m)(4+m)(1+m)}$
parallelrisch	$\frac{x^7(dx)^m c\,m^2+5x^7(dx)^m cm+4x^7(dx)^m c+x^4(dx)^m b\,m^2+8x^4(dx)^m bm+7x^4(dx)^m b+x(dx)^m a\,m^2+11x(dx)^m am+28x(dx)^m a}{(7+m)(4+m)(1+m)}$

[In] int((d*x)^m*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] a/(1+m)*x*exp(m*ln(d*x))+b/(4+m)*x^4*exp(m*ln(d*x))+c/(7+m)*x^7*exp(m*ln(d*
x))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx = \frac{((cm^2+5\,cm+4\,c)x^7+(bm^2+8\,bm+7\,b)x^4+(am^2+11\,am+28\,a)x)(dx)^m}{m^3+12\,m^2+39\,m+28}$$

[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(42) = 84$.

Time = 0.37 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.75

$$\int (dx)^m \left(a + bx^3 + cx^6\right) dx$$
$$= \begin{cases} \frac{-\frac{a}{6x^6} - \frac{b}{3x^3} + c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3} + b\log(x) + \frac{cx^3}{3}}{d^4} \\ \frac{a\log(x) + \frac{bx^3}{3} + \frac{cx^6}{6}}{d} \\ \frac{am^2x(dx)^m}{m^3+12m^2+39m+28} + \frac{11amx(dx)^m}{m^3+12m^2+39m+28} + \frac{28ax(dx)^m}{m^3+12m^2+39m+28} + \frac{bm^2x^4(dx)^m}{m^3+12m^2+39m+28} + \frac{8bmx^4(dx)^m}{m^3+12m^2+39m+28} + \frac{7bx^4(dx}{m^3+12m^2+3} \end{cases}$$

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*
x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x*
*6/6)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 11*a
*m*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*x*(d*x)**m/(m**3 + 12*m**
2 + 39*m + 28) + b*m**2*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 8*b*m*
x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*x**4*(d*x)**m/(m**3 + 12*m
**2 + 39*m + 28) + c*m**2*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 5*c*
m*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*x**7*(d*x)**m/(m**3 + 12
*m**2 + 39*m + 28), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a + bx^3 + cx^6\right) dx = \frac{cd^m x^7 x^m}{m+7} + \frac{bd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1} a}{d(m+1)}$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int (dx)^m \left(a + bx^3 + cx^6\right) \, dx = \frac{(dx)^m cm^2x^7 + 5 (dx)^m cmx^7 + 4 (dx)^m cx^7 + (dx)^m bm^2x^4 + 8 (dx)^m bmx^4 + 7 (dx)^m bx^4 + (dx)^m am^2x +}{m^3 + 12 m^2 + 39 m + 28}$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^
4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*
x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int (dx)^m \left(a + bx^3 + cx^6\right) \, dx = (d\,x)^m \left(\frac{b\,x^4\,(m^2 + 8\,m + 7)}{m^3 + 12\,m^2 + 39\,m + 28} + \frac{c\,x^7\,(m^2 + 5\,m + 4)}{m^3 + 12\,m^2 + 39\,m + 28} + \frac{a\,x\,(m^2 + 11\,m + 28)}{m^3 + 12\,m^2 + 39\,m + 28}\right)$$

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

```
[Out] (d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m +
 m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 12*
m^2 + m^3 + 28))
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(
1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1389, 371}

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\,\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$$

$$= \frac{(dx)^m \text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^2+2c\#1^5}\&\right]}{3m}$$

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m, 1 - m, -
(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ])/(3*m)
```

Maple [F]

$$\int \frac{(dx)^m}{c\,x^6+b\,x^3+a}dx$$

```
[In] int((d*x)^m/(c*x^6+b*x^3+a),x)
[Out] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(d\,x)^m}{c\,x^6+b\,x^3+a}\,dx$$

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6), x)
```

3.251 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)d(a+bx^3+cx^6)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4}}\right.}{3a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4}}\right.}{3a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 1/3*(d*x)^(1+m)*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^6+b*x^3+a)-1/3*c*
(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(
1/2)))*(b^2*(2-m)-4*a*c*(5-m)-b*(2-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3
/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/3*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*
m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(2-m)-4*a*c*(5-m)+b*(2
-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

$= \{1380, 1524, 371\}$

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$$
$$= \frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$- \frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+ \frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 + c
*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1380

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(
p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
```

```
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\int \frac{(dx)^m\left(-b^2(2-m)+2ac(5-m)-bc(2-m)x^3\right)}{a+bx^3+cx^6}\,dx}{3a\left(b^2-4ac\right)}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} - \frac{\left(c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{6a\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)\right)\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{6a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.25

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2}\,dx = \frac{x(dx)^m\,\text{AppellF1}\left(\frac{1+m}{3},2,2,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
[Out] (x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^2}dx$$

[In] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)

[Out] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")

[Out] integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \text{Timed out}$$

[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")

[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")

[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^2}\,dx$$

[In] int((d*x)^m/(a + b*x^3 + c*x^6)^2,x)

[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^2, x)

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{3}{2},-\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1(1/3+1/3*m,-3/2,-3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1
+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{3}{2},-\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
```

```
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int (dx)^m\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}F_1\left(\frac{1+m}{3};-\frac{3}{2},-\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.

Time = 1.85 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int (dx)^m\left(a+bx^3+cx^6\right)^{3/2}dx = \frac{x(dx)^m\sqrt{a+bx^3+cx^6}\Big(a(28+11m+m^2)\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2\cdots}}\right.}{}$$

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3,
 -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
```

```
Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2, -1/
2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/((1
+ m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

[In] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)

[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)

Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)

Sympy [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int (dx)^m \left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)

Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int \left(cx^6+bx^3+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)

Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int (d\,x)^m \left(c\,x^6+b\,x^3+a\right)^{3/2} dx$$

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,-1/2,-1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1+2
*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx = \frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{1}{2},-\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 +
 m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
```

```
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx$$

$$= \frac{x(dx)^m\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]

$$\int (dx)^m \sqrt{c\,x^6 + b\,x^3 + a}dx$$

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\,dx = \int \sqrt{cx^6 + bx^3 + a}(dx)^m \,dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\,dx = \int (dx)^m \sqrt{a + bx^3 + cx^6}\,dx$$

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\,dx = \int \sqrt{cx^6 + bx^3 + a}(dx)^m \,dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \int (d\,x)^m\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}} dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}} dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,1/2,1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}} dx = \frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{m+1}{3},\frac{1}{2},\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int\frac{(dx)^m}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$=\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int\frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}dx$$
$$=\frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^3+cx^6}}$$

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$$

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(d\,x)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.255 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{3}{2},\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,3/2,3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^6+b*x^3+a)^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{3},\frac{3}{2},\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A] (verified)

Time = 6.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^3\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\text{AppellF1}}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^3+cx^6\right)}$$

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

[In] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.256 $\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(c*x^6+b*x^3+a)^p*AppellF1(1/3+1/3*m,-p,-p,4/3+1/3*m,-2*c*x^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^3/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{AppellF1}\left(\frac{m+1}{3},-p,-p,\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3,
 (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int (dx)^m\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$=\frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1+m}{3};-p,-p;\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.}{d(1+m)}$$

Mathematica [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

$$\int (dx)^m\left(a+bx^3+cx^6\right)^p dx$$

$$=\frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{1+m}$$

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^p dx$$

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \,dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \,dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \,dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \,dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int (d\,x)^m \left(c\,x^6+b\,x^3+a\right)^p \, dx$$

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^p,x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^p, x)
```

3.257 $\int x^8(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 224

$$\int x^8(a+bx^3+cx^6)^p\,dx = -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3(a+bx^3+cx^6)^{1+p}}{3c(3+2p)} + \frac{2^p(2ac-b^2(2+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

```
[Out] -1/6*b*(2+p)*(c*x^6+b*x^3+a)^(p+1)/c^2/(2*p^2+5*p+3)+1/3*x^3*(c*x^6+b*x^3+a
)^(p+1)/c/(3+2*p)+1/3*2^p*(2*a*c-b^2*(2+p))*(c*x^6+b*x^3+a)^(p+1)*hypergeom
([-p, p+1],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-
b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(p+1)/(3+2*p)/
(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1371, 756, 654, 638}

$$\int x^8(a+bx^3+cx^6)^p\,dx = \frac{2^p(2ac-b^2(p+2))(a+bx^3+cx^6)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{6c^2(p+1)(2p+3)}+\frac{x^3(a+bx^3+cx^6)^{p+1}}{3c(2p+3)}$$

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -1/6*(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (x^3
*(a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a*c - b^2*(2 + p))*
(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^
3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 +
 2*p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)$$

$$= \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{\text{Subst}\left(\int (-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{3c(3+2p)}$$

$$= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} - \frac{\left(2ac-b^2(2+p)\right)\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{6c^2(3+2p)}$$

$$= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)} + \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)} + \frac{2^p\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2c}{2\sqrt{b^2-4ac}}\right.}{3c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int x^8\left(a+bx^3+cx^6\right)^p\,dx = \frac{1}{9}x^9\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(3,-p,-p,4,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int x^8\left(c\,x^6+b\,x^3+a\right)^p dx$$

```
[In] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^8 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^8 \, dx$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^8 (a + bx^3 + cx^6)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^8 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^8 \, dx$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Giac [F]

$$\int x^8 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^8 \, dx$$

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 (a + bx^3 + cx^6)^p \, dx = \int x^8 \left(c\,x^6 + b\,x^3 + a\right)^p \,dx$$

```
[In] int(x^8*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x^8*(a + b*x^3 + c*x^6)^p, x)
```

3.258 $\int x^5(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int x^5(a+bx^3+cx^6)^p\,dx=\frac{(a+bx^3+cx^6)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}(1+p)}$$

```
[Out] 1/6*(c*x^6+b*x^3+a)^(p+1)/c/(p+1)+1/3*2^p*b*(c*x^6+b*x^3+a)^(p+1)*hypergeom
([-p, p+1],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-
b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(p+1)/(-4*a*c+b^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1371, 654, 638}

$$\int x^5(a+bx^3+cx^6)^p\,dx$$

$$=\frac{b2^p(a+bx^3+cx^6)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{(a+bx^3+cx^6)^{p+1}}{6c(p+1)}$$

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hyper
geometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^
2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)\\ &= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}-\frac{b\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{6c}\\ &= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}\\ &\quad+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}(1+p)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{6}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
[Out] (x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int x^5\left(c\,x^6+b\,x^3+a\right)^p dx$$

```
[In] int(x^5*(c*x^6+b*x^3+a)^p,x)
[Out] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\int\left(cx^6+bx^3+a\right)^p x^5\,dx$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^5(a+bx^3+cx^6)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^5(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x^5\,dx$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x^5\,dx$$

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a+bx^3+cx^6)^p\,dx=\int x^5\,(c\,x^6+b\,x^3+a)^p\,dx$$

```
[In] int(x^5*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x^5*(a + b*x^3 + c*x^6)^p, x)
```

3.259 $\int x^2(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int x^2(a+bx^3+cx^6)^p\,dx = -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}(1+p)}$$

```
[Out] -1/3*2^(p+1)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+2*c*x^3
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(-1-p)/(p+1)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1366, 638}

$$\int x^2(a+bx^3+cx^6)^p\,dx = -\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^3+cx^6)^{p+1}\,\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -1/3*(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-
1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1
 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \left(a+bx+cx^2\right)^p \, dx, x, x^3\right)$$

$$= -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p} {}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}(1+p)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^2\left(a+bx^3+cx^6\right)^p \, dx$$

$$= \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{3c(1+p)}$$

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2
 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a
*c])^p)
```

Maple [F]

$$\int x^{2} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

```
[In] int(x^2*(c*x^6+b*x^3+a)^p,x)
[Out] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^{2}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\int\left(cx^{6}+bx^{3}+a\right)^{p}x^{2}\,dx$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^{2}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\int\left(cx^{6}+bx^{3}+a\right)^{p}x^{2}\,dx$$

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x^2\,dx$$

[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^3+cx^6)^p\,dx=\int x^2\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

[In] int(x^2*(a + b*x^3 + c*x^6)^p,x)

[Out] int(x^2*(a + b*x^3 + c*x^6)^p, x)

3.260 $\int x^4(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] 1/5*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[In] Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p\,dx\\
&= \frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^4\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int x^{4} \left(c\,x^{6}+b\,x^{3}+a\right)^{p} dx$$

```
[In] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^{4}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\int\left(cx^{6}+bx^{3}+a\right)^{p}x^{4}\,dx$$

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{4}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^{4}\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\int\left(cx^{6}+bx^{3}+a\right)^{p}x^{4}\,dx$$

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^4\,dx$$

[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^3+cx^6)^p\,dx = \int x^4\,(c\,x^6+b\,x^3+a)^p\,dx$$

[In] int(x^4*(a + b*x^3 + c*x^6)^p,x)

[Out] int(x^4*(a + b*x^3 + c*x^6)^p, x)

3.261 $\int x^3(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] 1/4*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^3+cx^6)^p\,dx = \frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int x^{3} \left(c\,x^{6}+b\,x^{3}+a \right)^{p}d x$$

```
[In] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx = \int (cx^6+bx^3+a)^p x^3 \, dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx = \int (cx^6+bx^3+a)^p x^3 \, dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\int\left(cx^6+bx^3+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\int x^3\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

```
[In] int(x^3*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x^3*(a + b*x^3 + c*x^6)^p, x)
```

3.262 $\int x(a+bx^3+cx^6)^p \, dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int x\left(a+bx^3+cx^6\right)^p \, dx = \frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] 1/2*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1399, 524}

$$\int x\left(a+bx^3+cx^6\right)^p \, dx = \frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[In] Int[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p dx\\ &= \frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x\left(a+bx^3+cx^6\right)^p dx = \frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int x\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

```
[In] int(x*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x(a+bx^3+cx^6)^p\,dx=\int (cx^6+bx^3+a)^p x\,dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Giac [F]

$$\int x(a+bx^3+cx^6)^p \, dx = \int (cx^6+bx^3+a)^p x \, dx$$

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^p \, dx = \int x\,(c\,x^6+b\,x^3+a)^p \, dx$$

```
[In] int(x*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x*(a + b*x^3 + c*x^6)^p, x)
```

3.263 $\int (a+bx^3+cx^6)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+bx^3+cx^6)^p \, dx = x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1
+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1362, 440}

$$\int (a+bx^3+cx^6)^p \, dx = x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

```
[In] Int[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p dx\\ &= x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int\left(a+bx^3+cx^6\right)^p dx = x\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

```
[In] int((c*x^6+b*x^3+a)^p,x)
[Out] int((c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^{6}+bx^{3}+a\right)^{p}dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^{6}+bx^{3}+a\right)^{p}dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Giac [F]

$$\int (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p \, dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + bx^3 + cx^6)^p \, dx = \int (c\,x^6 + b\,x^3 + a)^p \, dx$$

```
[In] int((a + b*x^3 + c*x^6)^p,x)
[Out] int((a + b*x^3 + c*x^6)^p, x)
```

3.264 $\int \frac{(a+bx^3+cx^6)^p}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

```
[Out] 1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c+
b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((b+2*c*x^3-(-4*a*c
+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1371, 772, 138}

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx = \frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(3*p*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\right)\text{Subs}\right.$$

$$= \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right.}{3p}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(c*x^3))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x}dx$$

```
[In] int((c*x^6+b*x^3+a)^p/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x,x)
[Out] int((a + b*x^3 + c*x^6)^p/x, x)
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{(a+bx^3+cx^6)^p}{x^2} dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^p}{x^2} dx = -\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^2}\,dx$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4a}}\right.}{x}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int\frac{(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4a}}\right.}{x}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Maple [F]

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{x^{2}}d x$$

```
[In] int((c*x^6+b*x^3+a)^p/x^2,x)
[Out] int((c*x^6+b*x^3+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^2}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^2, x)
```

3.266 $\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[Out] -1/2*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(1 + (2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^3}\,dx$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4a}}\right.}{2x^2}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int\frac{(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4a}}\right.}{2x^2}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}dx$$

[In] int((c*x^6+b*x^3+a)^p/x^3,x)

[Out] int((c*x^6+b*x^3+a)^p/x^3,x)

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^p/x^3, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx=\text{Timed out}$$

[In] integrate((c*x**6+b*x**3+a)**p/x**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^3, x)
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{3(1-2p)x^3}$$

```
[Out] -1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^2)^
(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-2*p)/x^3/(((b+2*c*x^3-(-
4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1371, 772, 138}

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = -\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b}}{2c}\right.}{3(1-2p)x^3}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] -1/3*(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/((1
- 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^2}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\right)\text{Subs}$$

$$= -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4}}{2cx^3}\right.}{3(1-2p)x^3}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\right.}{3(-1+2p)x^3}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] (4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqr
t[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^4}dx$$

```
[In] int((c*x^6+b*x^3+a)^p/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^4} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^4} \, dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^4} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^4} \, dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^3 + cx^6)^p}{x^4} \, dx = \int \frac{(c\,x^6 + b\,x^3 + a)^p}{x^4} \, dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^4,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^4, x)
```

3.268 $\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2}}\right.}{4x^4}$$

```
[Out] -1/4*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = -\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2}}\right.}{4x^4}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^4*(1 + (2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^5}\,dx$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-}\right.}{4x^4}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int\frac{(a+bx^3+cx^6)^p}{x^5}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-}\right.}{4x^4}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^4*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}dx$$

[In] `int((c*x^6+b*x^3+a)^p/x^5,x)`

[Out] `int((c*x^6+b*x^3+a)^p/x^5,x)`

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

[In] `integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")`

[Out] `integral((c*x^6 + b*x^3 + a)^p/x^5, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \text{Timed out}$$

[In] `integrate((c*x**6+b*x**3+a)**p/x**5,x)`

[Out] `Timed out`

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

[In] `integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")`

[Out] `integrate((c*x^6 + b*x^3 + a)^p/x^5, x)`

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^5,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^5, x)
```

3.269 $\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

```
[Out] -1/5*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^5*(1 + (2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^p}{x^6}\,dx$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-}}\right.}{5x^5}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int\frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-}}\right.}{5x^5}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^5*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}dx$$

```
[In] int((c*x^6+b*x^3+a)^p/x^6,x)
[Out] int((c*x^6+b*x^3+a)^p/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p/x**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^6,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^6, x)
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 168

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2(1-p),-p,-p,3-2p,-\frac{b-}{}\right.}{3(1-p)x^6}$$

```
[Out] -1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a*
c+b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-p)/x^6/(((b+2*c*x
^3-(-4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1371, 772, 138}

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = -\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2(1-p),-p,-p,3-2p,-\frac{b-}{}\right.}{3(1-p)x^6}$$

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] -1/3*(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*
p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^
3)])/((1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[
b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^3}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\right)\text{Subs}\right.$$

$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-}{}\right.}{3(1-p)x^6}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right.}{3(-1+p)x^6}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)]])/(3*(
-1 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}dx$$

```
[In] int((c*x^6+b*x^3+a)^p/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^7,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^7} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^7} \, dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Giac [F]

$$\int \frac{(a + bx^3 + cx^6)^p}{x^7} \, dx = \int \frac{(cx^6 + bx^3 + a)^p}{x^7} \, dx$$

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^3 + cx^6)^p}{x^7} \, dx = \int \frac{(c\,x^6 + b\,x^3 + a)^p}{x^7} \, dx$$

```
[In] int((a + b*x^3 + c*x^6)^p/x^7,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^7, x)
```

3.271 $\int \frac{x^m}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},\frac{5+m}{4},-x^4\right)}{1+m}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-x^4)/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 371}

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},-x^4\right)}{m+1}$$

```
[In] Int[x^m/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{(1+x^4)^2}\,dx$$

$$= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};-x^4\right)}{1+m}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},1+\frac{1+m}{4},-x^4\right)}{1+m}$$

```
[In] Integrate[x^m/(1 + 2*x^4 + x^8),x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

Maple [F]

$$\int \frac{x^m}{x^8+2x^4+1}dx$$

```
[In] int(x^m/(x^8+2*x^4+1),x)
[Out] int(x^m/(x^8+2*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 + 2*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{(x^4+1)^2}\,dx$$

```
[In] integrate(x**m/(x**8+2*x**4+1),x)
[Out] Integral(x**m/(x**4 + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

```
[In] int(x^m/(2*x^4 + x^8 + 1),x)
[Out] int(x^m/(2*x^4 + x^8 + 1), x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{3x^2}{4} - \frac{x^6}{4(1+x^4)} - \frac{3\arctan(x^2)}{4}$$

```
[Out] 3/4*x^2-1/4*x^6/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 281, 294, 327, 209}

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = -\frac{3\arctan(x^2)}{4} + \frac{3x^2}{4} - \frac{x^6}{4(x^4+1)}$$

```
[In] Int[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^9}{(1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, x^2\right) \\
&= -\frac{x^6}{4\,(1+x^4)} + \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4\,(1+x^4)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4\,(1+x^4)} - \frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx=\frac{1}{4}\left(x^2\left(2+\frac{1}{1+x^4}\right)-3\arctan\left(x^2\right)\right)$$

```
[In] Integrate[x^9/(1 + 2*x^4 + x^8),x]
[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan(x^2)}{4}$	25
risch	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan(x^2)}{4}$	25
parallelrisch	$\frac{3i\ln(x^2-i)x^4-3i\ln(x^2+i)x^4+4x^6+3i\ln(x^2-i)-3i\ln(x^2+i)+6x^2}{8x^4+8}$	67

```
[In] int(x^9/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x^9}{1+2x^4+x^8}\,dx=\frac{2\,x^6+3\,x^2-3\,(x^4+1)\arctan\left(x^2\right)}{4\,(x^4+1)}$$

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

```
[In] integrate(x**9/(x**8+2*x**4+1),x)
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{4\,\left(x^4+1\right)}-\frac{3\,\mathrm{atan}(x^2)}{4}+\frac{x^2}{2}$$

```
[In] int(x^9/(2*x^4 + x^8 + 1),x)
[Out] x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

3.273 $\int \frac{x^7}{1+2x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{x^7}{1+2x^4+x^8} dx = \frac{1}{4(1+x^4)} + \frac{1}{4}\log(1+x^4)$$

```
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 45}

$$\int \frac{x^7}{1+2x^4+x^8} dx = \frac{1}{4(x^4+1)} + \frac{1}{4}\log(x^4+1)$$

```
[In] Int[x^7/(1 + 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^7}{(1+x^4)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)\,dx, x, x^4\right) \\
&= \frac{1}{4(1+x^4)}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4}\left(\frac{1}{1+x^4}+\log\left(1+x^4\right)\right)$$

```
[In] Integrate[x^7/(1 + 2*x^4 + x^8),x]
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
norman	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
risch	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
parallelrisch	$\frac{\ln(x^4+1)x^4+1+\ln(x^4+1)}{4x^4+4}$	28

```
[In] int(x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\log(x^4+1)}{4} + \frac{1}{4x^4+4}$$

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4(x^4+1)} + \frac{1}{4}\log(x^4+1)$$

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8} \, dx = \frac{1}{4\,(x^4+1)} + \frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8} \, dx = \frac{\ln\left(x^4+1\right)}{4} + \frac{1}{4\,\left(x^4+1\right)}$$

```
[In] int(x^7/(2*x^4 + x^8 + 1),x)
[Out] log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.274 $\int \frac{x^5}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^5}{1+2x^4+x^8} \, dx = -\frac{x^2}{4(1+x^4)}+\frac{\arctan(x^2)}{4}$$

```
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 281, 294, 209}

$$\int \frac{x^5}{1+2x^4+x^8} \, dx = \frac{\arctan(x^2)}{4}-\frac{x^2}{4(x^4+1)}$$

```
[In] Int[x^5/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5}{(1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4(1+x^4)} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4(1+x^4)} + \frac{1}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4(1+x^4)} + \frac{\arctan(x^2)}{4}$$

```
[In] Integrate[x^5/(1 + 2*x^4 + x^8),x]
[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x^{2}}{4(x^{4}+1)}+\frac{\arctan(x^{2})}{4}$	20
risch	$-\frac{x^{2}}{4(x^{4}+1)}+\frac{\arctan(x^{2})}{4}$	20
parallelrisch	$-\frac{i\ln(x^{2}-i)x^{4}-i\ln(x^{2}+i)x^{4}+i\ln(x^{2}-i)-i\ln(x^{2}+i)+2x^{2}}{8(x^{4}+1)}$	62

```
[In] int(x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2-(x^4+1)\arctan(x^2)}{4(x^4+1)}$$

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}(x^2)}{4}$$

```
[In] integrate(x**5/(x**8+2*x**4+1),x)
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2}{4\,(x^4+1)}+\frac{1}{4}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=-\frac{x^2}{4\,(x^4+1)}+\frac{1}{4}\,\arctan\left(x^2\right)$$

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{1+2x^4+x^8}\,dx=\frac{\mathrm{atan}(x^2)}{4}-\frac{x^2}{4\,\left(x^4+1\right)}$$

```
[In] int(x^5/(2*x^4 + x^8 + 1),x)
[Out] atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

3.275 $\int \frac{x^3}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4(1+x^4)}$$

```
[Out] -1/4/(x^4+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 267}

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4(x^4+1)}$$

```
[In] Int[x^3/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(1 + x^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^3}{(1+x^4)^2}\,dx \\ &= -\frac{1}{4(1+x^4)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4(1+x^4)}$$

```
[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]
[Out] -1/4*1/(1 + x^4)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{1}{4(x^4+1)}$	10
default	$-\frac{1}{4(x^4+1)}$	10
norman	$-\frac{1}{4(x^4+1)}$	10
risch	$-\frac{1}{4(x^4+1)}$	10
parallelrisch	$-\frac{1}{4(x^4+1)}$	10

```
[In] int(x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^4+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4\left(x^4+1\right)}$$

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4x^4+4}$$

```
[In] integrate(x**3/(x**8+2*x**4+1),x)
[Out] -1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4\left(x^4+1\right)}$$

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8} \, dx = -\frac{1}{4\left(x^4+1\right)}$$

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

```
[In] int(x^3/(2*x^4 + x^8 + 1),x)
```

```
[Out] -1/(4*(x^4 + 1))
```

3.276 $\int \frac{x}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x}{1+2x^4+x^8} \, dx = \frac{x^2}{4(1+x^4)} + \frac{\arctan(x^2)}{4}$$

```
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 281, 205, 209}

$$\int \frac{x}{1+2x^4+x^8} \, dx = \frac{\arctan(x^2)}{4} + \frac{x^2}{4(x^4+1)}$$

```
[In] Int[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{(1+x^4)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{4(1+x^4)} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{4(1+x^4)} + \frac{1}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{1}{4}\left(\frac{x^2}{1+x^4} + \arctan\left(x^2\right)\right)$$

```
[In] Integrate[x/(1 + 2*x^4 + x^8),x]
[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^{2}}{4x^{4}+4}+\frac{\arctan\left(x^{2}\right)}{4}$	20
risch	$\frac{x^{2}}{4x^{4}+4}+\frac{\arctan\left(x^{2}\right)}{4}$	20
parallelrisch	$-\frac{i\ln\left(x^{2}-i\right)x^{4}-i\ln\left(x^{2}+i\right)x^{4}+i\ln\left(x^{2}-i\right)-i\ln\left(x^{2}+i\right)-2x^{2}}{8\left(x^{4}+1\right)}$	62

```
[In] int(x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{x^2+\left(x^4+1\right)\arctan\left(x^2\right)}{4\left(x^4+1\right)}$$

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

```
[In] integrate(x/(x**8+2*x**4+1),x)
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4\left(x^4+1\right)} + \frac{1}{4}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4\left(x^4+1\right)} + \frac{1}{4}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{\mathrm{atan}(x^2)}{4} + \frac{x^2}{4\,\left(x^4+1\right)}$$

```
[In] int(x/(2*x^4 + x^8 + 1),x)
[Out] atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1}{x(1+2x^4+x^8)} \, dx = \frac{1}{4(1+x^4)} + \log(x) - \frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 46}

$$\int \frac{1}{x(1+2x^4+x^8)} \, dx = \frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

```
[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x(1+x^4)^2}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)dx, x, x^4\right)\\ &= \frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+2x^4+x^8)}\,dx = \frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

```
[In] Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
norman	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
risch	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
parallelrisch	$\frac{4\ln(x)x^4-\ln(x^4+1)x^4+1+4\ln(x)-\ln(x^4+1)}{4x^4+4}$	42

```
[In] int(1/x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = -\frac{\left(x^4+1\right)\log\left(x^4+1\right)-4\left(x^4+1\right)\log\left(x\right)-1}{4\left(x^4+1\right)}$$

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{x^4+2}{4\left(x^4+1\right)} - \frac{1}{4}\log\left(x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^4+1\right)}{4} + \frac{1}{4\left(x^4+1\right)}$$

```
[In] int(1/(x*(2*x^4 + x^8 + 1)),x)
[Out] log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{1}{x^3(1+2x^4+x^8)}\,dx = -\frac{3}{4x^2}+\frac{1}{4x^2(1+x^4)}-\frac{3\arctan(x^2)}{4}$$

```
[Out] -3/4/x^2+1/4/x^2/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 209}

$$\int \frac{1}{x^3(1+2x^4+x^8)}\,dx = -\frac{3}{4}\arctan(x^2)-\frac{3}{4x^2}+\frac{1}{4x^2(x^4+1)}$$

```
[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3 (1+x^4)^2} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (1+x^2)^2} \, dx, x, x^2\right) \\
&= \frac{1}{4x^2 (1+x^4)} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{x^2 (1+x^2)} \, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2 (1+x^4)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2 (1+x^4)} - \frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1+2x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{x^2}{4(1+x^4)} + \frac{3}{4}\arctan\left(\frac{1}{x^2}\right)$$

```
[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
[Out] -1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{1}{2x^2}-\frac{x^2}{4(x^4+1)}-\frac{3\arctan(x^2)}{4}$	25
risch	$\frac{-\frac{3x^4}{4}-\frac{1}{2}}{x^2(x^4+1)}-\frac{3\arctan(x^2)}{4}$	26
parallelrisch	$\frac{3i\ln(x^2-i)x^6-3i\ln(x^2+i)x^6-4+3i\ln(x^2-i)x^2-3i\ln(x^2+i)x^2-6x^4}{8x^2(x^4+1)}$	72

```
[In] int(1/x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 (1+2x^4+x^8)}\,dx = -\frac{3\,x^4+3\,(x^6+x^2)\arctan\left(x^2\right)+2}{4\,(x^6+x^2)}$$

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = \frac{-3x^4 - 2}{4x^6 + 4x^2} - \frac{3 \operatorname{atan}(x^2)}{4}$$

```
[In] integrate(1/x**3/(x**8+2*x**4+1),x)
[Out] (-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3 x^4 + 2}{4 (x^6 + x^2)} - \frac{3}{4} \arctan\left(x^2\right)$$

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3 x^4 + 2}{4 (x^6 + x^2)} - \frac{3}{4} \arctan\left(x^2\right)$$

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3 \operatorname{atan}(x^2)}{4} - \frac{\frac{3 x^4}{4} + \frac{1}{2}}{x^6 + x^2}$$

```
[In] int(1/(x^3*(2*x^4 + x^8 + 1)),x)
[Out] - (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 46}

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx=-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

```
[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^5 (1+x^4)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1+x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx, x, x^4\right) \\
&= -\frac{1}{4x^4}-\frac{1}{4(1+x^4)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1+2x^4+x^8)}\,dx = -\frac{1}{4x^4}-\frac{1}{4(1+x^4)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

```
[In] Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4x^4}-\frac{1}{4(x^4+1)}-2\ln\left(x\right)+\frac{\ln(x^4+1)}{2}$	28
norman	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln\left(x\right)+\frac{\ln(x^4+1)}{2}$	32
risch	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln\left(x\right)+\frac{\ln(x^4+1)}{2}$	32
parallelrisch	$-\frac{8\ln(x)x^8-2\ln(x^4+1)x^8+1+8\ln(x)x^4-2\ln(x^4+1)x^4+2x^4}{4x^4(x^4+1)}$	56

```
[In] int(1/x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = -\frac{2 x^4 - 2 (x^8 + x^4) \log (x^4 + 1) + 8 (x^8 + x^4) \log (x) + 1}{4 (x^8 + x^4)}$$

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8 +
 x^4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = \frac{-2x^4 - 1}{4x^8 + 4x^4} - 2 \log (x) + \frac{\log (x^4 + 1)}{2}$$

```
[In] integrate(1/x**5/(x**8+2*x**4+1),x)
[Out] (-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 + 2x^4 + x^8)} dx = -\frac{2 x^4 + 1}{4 (x^8 + x^4)} + \frac{1}{2} \log \left(x^4 + 1\right) - \frac{1}{2} \log \left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx = -\frac{2\,x^4+1}{4\,(x^8+x^4)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx = \frac{\ln\left(x^4+1\right)}{2}-2\,\ln\left(x\right)-\frac{\frac{x^4}{2}+\frac{1}{4}}{x^8+x^4}$$

```
[In] int(1/(x^5*(2*x^4 + x^8 + 1)),x)
[Out] log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

3.280 $\int \frac{1}{x^7(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^7(1+2x^4+x^8)}\,dx = -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6(1+x^4)}+\frac{5\arctan(x^2)}{4}$$

```
[Out] -5/12/x^6+5/4/x^2+1/4/x^6/(x^4+1)+5/4*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 209}

$$\int \frac{1}{x^7(1+2x^4+x^8)}\,dx = \frac{5\arctan(x^2)}{4}-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6(x^4+1)}$$

```
[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^7 (1+x^4)^2} \, dx \\
&= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^4 (1+x^2)^2} \, dx, x, x^2\right) \\
&= \frac{1}{4x^6 (1+x^4)} + \frac{5}{4} \text{Subst}\left(\int \frac{1}{x^4 (1+x^2)} \, dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6 (1+x^4)} - \frac{5}{4} \text{Subst}\left(\int \frac{1}{x^2 (1+x^2)} \, dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6 (1+x^4)} + \frac{5}{4} \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6 (1+x^4)} + \frac{5}{4} \tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7 (1+2x^4+x^8)} \, dx = -\frac{1}{6x^6} + \frac{1}{x^2} + \frac{x^2}{4(1+x^4)} - \frac{5}{4}\arctan\left(\frac{1}{x^2}\right)$$

```
[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
[Out] -1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{6x^6}+\frac{1}{x^2}+\frac{x^2}{4x^4+4}+\frac{5\arctan(x^2)}{4}$	28
risch	$\frac{\frac{5}{4}x^8+\frac{5}{6}x^4-\frac{1}{6}}{x^6(x^4+1)}+\frac{5\arctan(x^2)}{4}$	31
parallelrisch	$-\frac{15i\ln(x^2-i)x^{10}-15i\ln(x^2+i)x^{10}+4+15i\ln(x^2-i)x^6-15i\ln(x^2+i)x^6-30x^8-20x^4}{24x^6(x^4+1)}$	77

```
[In] int(1/x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/6/x^6+1/x^2+1/4*x^2/(x^4+1)+5/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7 (1+2x^4+x^8)} \, dx = \frac{15\,x^8+10\,x^4+15\,(x^{10}+x^6)\arctan(x^2)-2}{12\,(x^{10}+x^6)}$$

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (1 + 2x^4 + x^8)} \, dx = \frac{5 \operatorname{atan}(x^2)}{4} + \frac{15x^8 + 10x^4 - 2}{12x^{10} + 12x^6}$$

```
[In] integrate(1/x**7/(x**8+2*x**4+1),x)
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (1 + 2x^4 + x^8)} \, dx = \frac{15 \, x^8 + 10 \, x^4 - 2}{12 \, (x^{10} + x^6)} + \frac{5}{4} \arctan\left(x^2\right)$$

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (1 + 2x^4 + x^8)} \, dx = \frac{x^2}{4 \, (x^4 + 1)} + \frac{6 \, x^4 - 1}{6 \, x^6} + \frac{5}{4} \arctan\left(x^2\right)$$

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (1 + 2x^4 + x^8)} \, dx = \frac{5 \operatorname{atan}(x^2)}{4} + \frac{\frac{5 \, x^8}{4} + \frac{5 \, x^4}{6} - \frac{1}{6}}{x^6 \, (x^4 + 1)}$$

```
[In] int(1/(x^7*(2*x^4 + x^8 + 1)),x)
[Out] (5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{5x}{4}-\frac{x^5}{4(1+x^4)}+\frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 5/4*x-1/4*x^5/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(
1/2))*2^(1/2)+5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-5/32*ln(1+x^2+x*2^(1/2))*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 294, 327, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}+\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{x^5}{4(x^4+1)}+\frac{5x}{4}$$

```
[In] Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^8}{(1+x^4)^2}\,dx \\
&= -\frac{x^5}{4(1+x^4)} + \frac{5}{4}\int \frac{x^4}{1+x^4}\,dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{4}\int \frac{1}{1+x^4}\,dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx - \frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx \\
&\quad - \frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{5\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad + \frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(32x+\frac{8x}{1+x^4}+10\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 +
 Sqrt[2]*x + x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

method	result	size
risch	$x+\frac{x}{4x^4+4}-\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	34
default	$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	64

```
[In] int(x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/4*x/(x^4+1)-5/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{32\,x^5-5\,\sqrt{2}((i+1)\,x^4+i+1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)-5\,\sqrt{2}(-(i-1)\,x^4-i+1)\log\left(2\,x-(i-1)\,\sqrt{2}\right.}{32}$$

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 - 5*sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2)) -
 5*sqrt(2)*(-(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) - 5*sqrt(2)*((
I - 1)*x^4 + I - 1)*log(2*x + (I - 1)*sqrt(2)) - 5*sqrt(2)*(-(I + 1)*x^4 -
I - 1)*log(2*x - (I + 1)*sqrt(2)) + 40*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x+\frac{x}{4x^4+4}+\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} -\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) +\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4\left(x^4+1\right)}$$

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + x + \frac{x}{4\left(x^4+1\right)}$$

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x + \frac{x}{4\ \left(x^4+1\right)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

```
[In] int(x^8/(2*x^4 + x^8 + 1),x)
```

```
[Out] x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(1
/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

3.282 $\int \frac{x^6}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))
*2^(1/2)+3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 294, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} - \frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}} - \frac{x^3}{4(x^4+1)}$$

```
[In] Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*x^3/(1 + x^4) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^6}{(1+x^4)^2}\,dx \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4(1+x^4)} - \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad + \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{3\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= -\frac{x^3}{4(1+x^4)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad + \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(-\frac{8x^3}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[
2]*x + x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}+1)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}(\_Z^{4}+1)}\frac{\ln(x-\_R)}{\_R}\right)}{16}$	35
default	$-\frac{x^{3}}{4(x^{4}+1)}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^{2}-x\sqrt{2}}{1+x^{2}+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	65

```
[In] int(x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^3/(x^4+1)+3/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{8\,x^3+3\,\sqrt{2}(-(i-1)\ x^4-i+1)\log\left(2\,x+(i+1)\ \sqrt{2}\right)+3\,\sqrt{2}((i+1)\ x^4+i+1)\log\left(2\,x-(i-1)\ \sqrt{}\right.}{32\,(x}$$

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/32*(8*x^3 + 3*sqrt(2)*(-(I - 1)*x^4 - I + 1)*log(2*x + (I + 1)*sqrt(2))
+ 3*sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x - (I - 1)*sqrt(2)) + 3*sqrt(2)*(-
(I + 1)*x^4 - I - 1)*log(2*x + (I - 1)*sqrt(2)) + 3*sqrt(2)*((I - 1)*x^4 +
I - 1)*log(2*x - (I + 1)*sqrt(2)))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)$$

```
[In] int(x^6/(2*x^4 + x^8 + 1),x)
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

3.283 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*2
^(1/2)-1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 294, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}} - \frac{x}{4(x^4+1)} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}$$

```
[In] Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*x/(1 + x^4) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{(1+x^4)^2}\,dx\\
&= -\frac{x}{4(1+x^4)}+\frac{1}{4}\int \frac{1}{1+x^4}\,dx\\
&= -\frac{x}{4(1+x^4)}+\frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{x}{4(1+x^4)}+\frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx\\
&\quad +\frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}\\
&= -\frac{x}{4(1+x^4)}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}\\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{8\sqrt{2}}\\
&= -\frac{x}{4(1+x^4)}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}\\
&\quad -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$
\int \frac{x^4}{1+2x^4+x^8}\,dx=\frac{1}{32}\left(-\frac{8x}{1+x^4}-2\sqrt{2}\arctan\left(1-\sqrt{2}x\right)+2\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)
$$

```
[In] Integrate[x^4/(1 + 2*x^4 + x^8),x]
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
 x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{x}{4(x^4+1)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$-\frac{x}{4(x^4+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	63

```
[In] int(x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x/(x^4+1)+1/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{1+2x^4+x^8}\,dx$$
$$=\frac{\sqrt{2}((i+1)\,x^4+i+1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+\sqrt{2}(-(i-1)\,x^4-i+1)\log\left(2\,x-(i-1)\,\sqrt{2}\right)+\sqrt{2}((i}{32\,(x^4+1)}$$

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/32*(sqrt(2)*((I + 1)*x^4 + I + 1)*log(2*x + (I + 1)*sqrt(2)) + sqrt(2)*(-
(I - 1)*x^4 - I + 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I - 1)*x^4 + I
- 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x -
(I + 1)*sqrt(2)) - 8*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx=-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2)*
x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x+\sqrt{2}\Big)\right)+\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x-\sqrt{2}\Big)\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\,(x^4+1)}$$

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x+\sqrt{2}\Big)\right)+\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,x-\sqrt{2}\Big)\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\,(x^4+1)}$$

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

```
[In] int(x^4/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

3.284 $\int \frac{x^2}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^2}{1+2x^4+x^8} \, dx = \frac{x^3}{4(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 1/4*x^3/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*
2^(1/2)+1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {28, 296, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{1+2x^4+x^8} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} - \frac{\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}} + \frac{x^3}{4(x^4+1)}$$

```
[In] Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{(1+x^4)^2}\,dx \\
&= \frac{x^3}{4(1+x^4)} + \frac{1}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= \frac{x^3}{4(1+x^4)} - \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{x^3}{4(1+x^4)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= \frac{x^3}{4(1+x^4)} + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= \frac{x^3}{4(1+x^4)} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$
\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(\frac{8x^3}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)
$$

```
[In] Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x
+ x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^3}{4x^4+4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	65

```
[In] int(x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^3/(x^4+1)+1/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{8\,x^3+\sqrt{2}((i-1)\,x^4+i-1)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+\sqrt{2}(-(i+1)\,x^4-i-1)\log\left(2\,x-(i-1)\,\sqrt{2}\right)+}{32\,(x^4+1}$$

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/32*(8*x^3 + sqrt(2)*((I - 1)*x^4 + I - 1)*log(2*x + (I + 1)*sqrt(2)) + sq
rt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x - (I - 1)*sqrt(2)) + sqrt(2)*((I + 1)*
x^4 + I + 1)*log(2*x + (I - 1)*sqrt(2)) + sqrt(2)*(-(I - 1)*x^4 - I + 1)*lo
g(2*x - (I + 1)*sqrt(2)))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4x^4+4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(x**2/(x**8+2*x**4+1),x)
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2
 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)$$

```
[In] int(x^2/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

3.285 $\int \frac{1}{1+2x^4+x^8} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{1}{1+2x^4+x^8} \, dx = \frac{x}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 1/4*x/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))*2^
(1/2)-3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {28, 205, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{1+2x^4+x^8} \, dx = -\frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}} + \frac{x}{4(x^4+1)} - \frac{3\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}} + \frac{3\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}$$

```
[In] Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(1+x^4)^2}\,dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{4}\int \frac{1}{1+x^4}\,dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad - \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} - \frac{3\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= \frac{x}{4(1+x^4)} - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= \frac{x}{4(1+x^4)} - \frac{3\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad - \frac{3\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(\frac{8x}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1),x]
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{4x^4+4}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	63

```
[In] int(1/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x/(x^4+1)+3/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int\frac{1}{1+2x^4+x^8}\,dx = -\frac{3\sqrt{2}(-(i+1)\ x^4-i-1)\log\left(2\,x+(i+1)\ \sqrt{2}\right)+3\sqrt{2}((i-1)\ x^4+i-1)\log\left(2\,x-(i-1)\ \sqrt{2}\right)+3}{32\,(x}$$

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] -1/32*(3*sqrt(2)*(-(I + 1)*x^4 - I - 1)*log(2*x + (I + 1)*sqrt(2)) + 3*sqrt
(2)*((I - 1)*x^4 + I - 1)*log(2*x - (I - 1)*sqrt(2)) + 3*sqrt(2)*(-(I - 1)*
x^4 - I + 1)*log(2*x + (I - 1)*sqrt(2)) + 3*sqrt(2)*((I + 1)*x^4 + I + 1)*l
og(2*x - (I + 1)*sqrt(2)) - 8*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int\frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(1/(x**8+2*x**4+1),x)
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan(s
qrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\,(x^4+1)}$$

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4\,(x^4+1)}$$

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4\,(x^4+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

```
[In] int(1/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

3.286 $\int \frac{1}{x^2(1+2x^4+x^8)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^2(1+2x^4+x^8)} \, dx = -\frac{5}{4x}+\frac{1}{4x(1+x^4)}+\frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -5/4/x+1/4/x/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(1
/2))*2^(1/2)-5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+5/32*ln(1+x^2+x*2^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 296, 331, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^2(1+2x^4+x^8)} \, dx = \frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}+\frac{1}{4x(x^4+1)}-\frac{5\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{5}{4x}$$

```
[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1+x^4\right)}+\frac{5}{4}\int \frac{1}{x^2\left(1+x^4\right)}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{4}\int \frac{x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx\\
&\quad -\frac{5}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}-\frac{5\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}-\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}\\
&\quad -\frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{8\sqrt{2}}\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1+x^4\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}}\\
&\quad -\frac{5\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{1}{32}\left(-\frac{32}{x}-\frac{8x^3}{1+x^4}+10\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{-\frac{5x^4}{4}-1}{(x^4+1)x}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(-\_R^3+x\right)\right)}{16}$	41
default	$-\frac{1}{x}-\frac{x^3}{4(x^4+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	70

```
[In] int(1/x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (-5/4*x^4-1)/(x^4+1)/x+5/16*sum(_R*ln(-_R^3+x),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{40\,x^4+5\,\sqrt{2}((i-1)\;x^5+(i-1)\;x)\log\left(2\,x+(i+1)\;\sqrt{2}\right)+5\,\sqrt{2}(-(i+1)\;x^5-(i+1)\;x)\log\left(2\,x-(\right.}{}$$

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 + 5*sqrt(2)*((I - 1)*x^5 + (I - 1)*x)*log(2*x + (I + 1)*sqrt(
2)) + 5*sqrt(2)*(-(I + 1)*x^5 - (I + 1)*x)*log(2*x - (I - 1)*sqrt(2)) + 5*s
qrt(2)*((I + 1)*x^5 + (I + 1)*x)*log(2*x + (I - 1)*sqrt(2)) + 5*sqrt(2)*(-(
I - 1)*x^5 - (I - 1)*x)*log(2*x - (I + 1)*sqrt(2)) + 32)/(x^5 + x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{-5x^4-4}{4x^5+4x} - \frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] (-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{\frac{5x^4}{4}+1}{x^5+x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)$$

```
[In] int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{7}{12x^3}+\frac{1}{4x^3(1+x^4)}+\frac{7\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -7/12/x^3+1/4/x^3/(x^4+1)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+x
*2^(1/2))*2^(1/2)+7/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-7/32*ln(1+x^2+x*2^(1/2))
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 296, 331, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = \frac{7\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3(x^4+1)}$$

```
[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^4 (1+x^4)^2}\,dx \\
&= \frac{1}{4x^3(1+x^4)} + \frac{7}{4}\int \frac{1}{x^4(1+x^4)}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{4}\int \frac{1}{1+x^4}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{8}\int \frac{1-x^2}{1+x^4}\,dx - \frac{7}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx \\
&\quad - \frac{7}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{7\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{7\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{7\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad - \frac{7\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{7\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{7\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad + \frac{7\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{7\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = \frac{1}{96}\left(-\frac{32}{x^3}-\frac{24x}{1+x^4}+42\sqrt{2}\arctan\left(1-\sqrt{2}x\right) -42\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+21\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) -21\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]*L
og[1 + Sqrt[2]*x + x^2])/96
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{-\frac{7x^4}{12}-\frac{1}{3}}{x^3(x^4+1)}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+1)}\_R\ln\left(x-\_R\right)\right)}{16}$	39
default	$-\frac{1}{3x^3}-\frac{x}{4(x^4+1)}-\frac{7\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	68

```
[In] int(1/x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (-7/12*x^4-1/3)/x^3/(x^4+1)+7/16*sum(_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{56\,x^4+21\,\sqrt{2}((i+1)\,x^7+(i+1)\,x^3)\log\left(2\,x+(i+1)\,\sqrt{2}\right)+21\,\sqrt{2}(-(i-1)\,x^7-(i-1)\,x^3)\log\left(2\,x\right.}{}$$

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 + 21*sqrt(2)*((I + 1)*x^7 + (I + 1)*x^3)*log(2*x + (I + 1)*sq
rt(2)) + 21*sqrt(2)*(-(I - 1)*x^7 - (I - 1)*x^3)*log(2*x - (I - 1)*sqrt(2))
 + 21*sqrt(2)*((I - 1)*x^7 + (I - 1)*x^3)*log(2*x + (I - 1)*sqrt(2)) + 21*s
qrt(2)*(-(I + 1)*x^7 - (I + 1)*x^3)*log(2*x - (I + 1)*sqrt(2)) + 32)/(x^7 +
 x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = \frac{-7x^4-4}{12x^7+12x^3} + \frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

```
[In] integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] (-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
- 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16
 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{7x^4+4}{12(x^7+x^3)}$$

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4\left(x^4+1\right)} - \frac{1}{3\,x^3}$$

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{\frac{7\,x^4}{12}+\frac{1}{3}}{x^7+x^3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

```
[In] int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = -\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5(1+x^4)}-\frac{9\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}-\frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -9/20/x^5+9/4/x+1/4/x^5/(x^4+1)+9/16*arctan(-1+x*2^(1/2))*2^(1/2)+9/16*arct
an(1+x*2^(1/2))*2^(1/2)+9/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-9/32*ln(1+x^2+x*2^
(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 296, 331, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = -\frac{9\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{9\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5(x^4+1)}+\frac{9}{4x}$$

```
[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^6(1+x^4)^2}\,dx \\
&= \frac{1}{4x^5(1+x^4)} + \frac{9}{4}\int \frac{1}{x^6(1+x^4)}\,dx \\
&= -\frac{9}{20x^5} + \frac{1}{4x^5(1+x^4)} - \frac{9}{4}\int \frac{1}{x^2(1+x^4)}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9}{4}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} - \frac{9}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{9}{8}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9}{16}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx \\
&\quad + \frac{9}{16}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{9\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} + \frac{9\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{16\sqrt{2}} \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{9\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}}
\end{aligned}$$

$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} - \frac{9\tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{9\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{1}{160}\left(-\frac{32}{x^5} + \frac{320}{x} + \frac{40x^3}{1+x^4} - 90\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 90\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + 45\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 45\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

```
[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45
*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\frac{9}{4}x^8+\frac{9}{5}x^4-\frac{1}{5}}{x^5(x^4+1)} + \frac{9\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+1)} \_R\ln\left(\_R^3+x\right)\right)}{16}$	44
default	$-\frac{1}{5x^5} + \frac{2}{x} + \frac{x^3}{4x^4+4} + \frac{9\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	75

```
[In] int(1/x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (9/4*x^8+9/5*x^4-1/5)/x^5/(x^4+1)+9/16*sum(_R*ln(_R^3+x),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^6 (1+2x^4+x^8)}\,dx = \frac{360\,x^8 + 288\,x^4 - 45\,\sqrt{2}(-(i-1)\;x^9 - (i-1)\;x^5)\log\left(2\,x + (i+1)\;\sqrt{2}\right) - 45\,\sqrt{2}((i+1)\;x^9 + (i+1)\;x^5}{}$$

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 45*sqrt(2)*(-(I - 1)*x^9 - (I - 1)*x^5)*log(2*x
+ (I + 1)*sqrt(2)) - 45*sqrt(2)*((I + 1)*x^9 + (I + 1)*x^5)*log(2*x - (I -
1)*sqrt(2)) - 45*sqrt(2)*(-(I + 1)*x^9 - (I + 1)*x^5)*log(2*x + (I - 1)*sqr
t(2)) - 45*sqrt(2)*((I - 1)*x^9 + (I - 1)*x^5)*log(2*x - (I + 1)*sqrt(2)) -
 32)/(x^9 + x^5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^6 (1+2x^4+x^8)}\,dx = \frac{9\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{9\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16} + \frac{45x^8+36x^4-4}{20x^9+20x^5}$$

```
[In] integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x + 1
)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/16
+ (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6 (1+2x^4+x^8)}\,dx = \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{9}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{9}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{45\,x^8+36\,x^4-4}{20\,(x^9+x^5)}$$

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{x^3}{4(x^4+1)} + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{9}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{9}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{10x^4-1}{5x^5}$$

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^6(1+2x^4+x^8)}\,dx = \frac{\frac{9x^8}{4}+\frac{9x^4}{5}-\frac{1}{5}}{x^9+x^5} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{16}-\frac{9}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{16}+\frac{9}{16}\mathrm{i}\right)$$

```
[In] int(1/(x^6*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(9/16 - 9i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(9/16 + 9i/16) + ((9*x^4)/5 + (9*x^8)/4 - 1/5)/(x^5 + x^9)
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = -\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7(1+x^4)} - \frac{11\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{11\log\left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}}+\frac{11\log\left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -11/28/x^7+11/12/x^3+1/4/x^7/(x^4+1)+11/16*arctan(-1+x*2^(1/2))*2^(1/2)+11/
16*arctan(1+x*2^(1/2))*2^(1/2)-11/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+11/32*ln(1
+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {28, 296, 331, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = -\frac{11\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}+\frac{11\arctan\left(\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{11}{28x^7}+\frac{11}{12x^3} - \frac{11\log\left(x^2-\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7(x^4+1)}$$

```
[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^8 (1+x^4)^2} \, dx \\
&= \frac{1}{4x^7 (1+x^4)} + \frac{11}{4} \int \frac{1}{x^8 (1+x^4)} \, dx \\
&= -\frac{11}{28x^7} + \frac{1}{4x^7 (1+x^4)} - \frac{11}{4} \int \frac{1}{x^4 (1+x^4)} \, dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{4} \int \frac{1}{1+x^4} \, dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{8} \int \frac{1-x^2}{1+x^4} \, dx + \frac{11}{8} \int \frac{1+x^2}{1+x^4} \, dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{16} \int \frac{1}{1-\sqrt{2}x+x^2} \, dx \\
&\quad + \frac{11}{16} \int \frac{1}{1+\sqrt{2}x+x^2} \, dx - \frac{11 \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} \, dx}{16\sqrt{2}} - \frac{11 \int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} \, dx}{16\sqrt{2}} \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} - \frac{11 \log \left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{11 \log \left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}} \\
&\quad + \frac{11\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{11\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} - \frac{11 \tan^{-1}\left(1-\sqrt{2}x\right)}{8\sqrt{2}} \\
&\quad + \frac{11 \tan^{-1}\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{11 \log \left(1-\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{11 \log \left(1+\sqrt{2}x+x^2\right)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^8 (1+2x^4+x^8)} \, dx = \frac{1}{672} \left(-\frac{96}{x^7} + \frac{448}{x^3} + \frac{168x}{1+x^4} - 462\sqrt{2} \arctan \left(1-\sqrt{2}x \right) + 462\sqrt{2} \arctan \left(1+\sqrt{2}x \right) - 231\sqrt{2} \log \left(1-\sqrt{2}x+x^2 \right) + 231\sqrt{2} \log \left(1+\sqrt{2}x+x^2 \right) \right)$$

```
[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
+ 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
+ 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{\frac{11}{12}x^8+\frac{11}{21}x^4-\frac{1}{7}}{x^7(x^4+1)}+\frac{11\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)} \_R\ln\left(x+\_R\right)\right)}{16}$	42
default	$-\frac{1}{7x^7}+\frac{2}{3x^3}+\frac{x}{4x^4+4}+\frac{11\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{32}$	73

```
[In] int(1/x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (11/12*x^8+11/21*x^4-1/7)/x^7/(x^4+1)+11/16*sum(_R*ln(x+_R),_R=RootOf(_Z^4+
1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^8 (1+2x^4+x^8)} \, dx = \frac{616\,x^8+352\,x^4-231\,\sqrt{2}(-(i+1)\,x^{11}-(i+1)\,x^7)\log\left(2\,x+(i+1)\,\sqrt{2}\right)-231\,\sqrt{2}((i-1)\,x^{11}+(i-1)}{}$$

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 231*sqrt(2)*(-(I + 1)*x^11 - (I + 1)*x^7)*log(2*
x + (I + 1)*sqrt(2)) - 231*sqrt(2)*((I - 1)*x^11 + (I - 1)*x^7)*log(2*x - (
I - 1)*sqrt(2)) - 231*sqrt(2)*(-(I - 1)*x^11 - (I - 1)*x^7)*log(2*x + (I -
1)*sqrt(2)) - 231*sqrt(2)*((I + 1)*x^11 + (I + 1)*x^7)*log(2*x - (I + 1)*sq
rt(2)) - 96)/(x^11 + x^7)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = -\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{11\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{11\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16} + \frac{77x^8+44x^4-12}{84x^{11}+84x^7}$$

```
[In] integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
+ 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1
)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^8(1+2x^4+x^8)}\,dx = \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{77x^8+44x^4-12}{84(x^{11}+x^7)}$$

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7
)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^8\left(1+2x^4+x^8\right)}\,dx = \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{4\left(x^4+1\right)} + \frac{14x^4-3}{21x^7}$$

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^8\left(1+2x^4+x^8\right)}\,dx = \frac{\frac{11x^8}{12}+\frac{11x^4}{21}-\frac{1}{7}}{x^{11}+x^7} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}+\frac{11}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}-\frac{11}{16}\mathrm{i}\right)$$

```
[In] int(1/(x^8*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x^7
 + x^11)
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},\frac{5+m}{4},x^4\right)}{1+m}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],x^4)/(1+m)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 371}

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},x^4\right)}{m+1}$$

```
[In] Int[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{(-1+x^4)^2}\,dx$$
$$= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4\right)}{1+m}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{4},1+\frac{1+m}{4},x^4\right)}{1+m}$$

```
[In] Integrate[x^m/(1 - 2*x^4 + x^8),x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

Maple [F]

$$\int \frac{x^m}{x^8-2x^4+1}dx$$

```
[In] int(x^m/(x^8-2*x^4+1),x)
[Out] int(x^m/(x^8-2*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 - 2*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\int \frac{x^m}{(x-1)^2\,(x+1)^2\,(x^2+1)^2}\,dx$$

```
[In] integrate(x**m/(x**8-2*x**4+1),x)
[Out] Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-2x^4+x^8}\,dx=\int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

```
[In] int(x^m/(x^8 - 2*x^4 + 1),x)
[Out] int(x^m/(x^8 - 2*x^4 + 1), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^9}{1-2x^4+x^8} \, dx = \frac{3x^2}{4} + \frac{x^6}{4(1-x^4)} - \frac{3\text{arctanh}(x^2)}{4}$$

```
[Out] 3/4*x^2+1/4*x^6/(-x^4+1)-3/4*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 294, 327, 213}

$$\int \frac{x^9}{1-2x^4+x^8} \, dx = -\frac{3\text{arctanh}(x^2)}{4} + \frac{3x^2}{4} + \frac{x^6}{4(1-x^4)}$$

```
[In] Int[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^9}{(-1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^6}{4(1-x^4)} + \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4(1-x^4)} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4(1-x^4)} - \frac{3}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{1}{8}\left(2x^2\left(2+\frac{1}{1-x^4}\right)+3\log\left(1-x^2\right)-3\log\left(1+x^2\right)\right)$$

```
[In] Integrate[x^9/(1 - 2*x^4 + x^8),x]
[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{x^2}{2}-\frac{x^2}{4(x^4-1)}-\frac{3\ln(x^2+1)}{8}+\frac{3\ln(x^2-1)}{8}$	35
default	$\frac{x^2}{2}-\frac{1}{8(x^2+1)}-\frac{3\ln(x^2+1)}{8}-\frac{1}{8(x^2-1)}+\frac{3\ln(x^2-1)}{8}$	41
norman	$\frac{-\frac{3}{4}x^2+\frac{1}{2}x^6}{x^4-1}+\frac{3\ln(x-1)}{8}+\frac{3\ln(x+1)}{8}-\frac{3\ln(x^2+1)}{8}$	41
parallelrisch	$\frac{4x^6+3\ln(x-1)x^4+3\ln(x+1)x^4-3\ln(x^2+1)x^4-6x^2-3\ln(x-1)-3\ln(x+1)+3\ln(x^2+1)}{8x^4-8}$	70

```
[In] int(x^9/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/4*x^2/(x^4-1)-3/8*ln(x^2+1)+3/8*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{4\,x^6-6\,x^2-3\,(x^4-1)\log\left(x^2+1\right)+3\,(x^4-1)\log\left(x^2-1\right)}{8\,(x^4-1)}$$

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(
x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4x^4-4} + \frac{3\log\left(x^2-1\right)}{8} - \frac{3\log\left(x^2+1\right)}{8}$$

```
[In] integrate(x**9/(x**8-2*x**4+1),x)
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{3}{8}\,\log\left(x^2+1\right) + \frac{3}{8}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{3}{8}\,\log\left(x^2+1\right) + \frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4\,(x^4-1)} - \frac{3\,\mathrm{atanh}(x^2)}{4}$$

```
[In] int(x^9/(x^8 - 2*x^4 + 1),x)
[Out] x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^7}{1-2x^4+x^8} dx = \frac{1}{4(1-x^4)} + \frac{1}{4}\log(1-x^4)$$

```
[Out] 1/4/(-x^4+1)+1/4*ln(-x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 45}

$$\int \frac{x^7}{1-2x^4+x^8} dx = \frac{1}{4(1-x^4)} + \frac{1}{4}\log(1-x^4)$$

```
[In] Int[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^7}{(-1+x^4)^2}\,dx \\ &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(-1+x)^2}\,dx, x, x^4\right) \\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, x^4\right) \\ &= \frac{1}{4(1-x^4)}+\frac{1}{4}\log\left(1-x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4(-1+x^4)}+\frac{1}{4}\log\left(-1+x^4\right)$$

```
[In] Integrate[x^7/(1 - 2*x^4 + x^8),x]
[Out] -1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
risch	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
norman	$-\frac{1}{4(x^4-1)}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\ln(x^2+1)}{4}$	31
parallelrisch	$\frac{\ln(x-1)x^4+\ln(x+1)x^4+\ln(x^2+1)x^4-1-\ln(x-1)-\ln(x+1)-\ln(x^2+1)}{4x^4-4}$	58

```
[In] int(x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{1-2x^4+x^8}\,dx=\frac{(x^4-1)\log(x^4-1)-1}{4(x^4-1)}$$

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{1-2x^4+x^8}\,dx=\frac{\log(x^4-1)}{4}-\frac{1}{4x^4-4}$$

```
[In] integrate(x**7/(x**8-2*x**4+1),x)
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^7}{1-2x^4+x^8}\,dx=-\frac{1}{4(x^4-1)}+\frac{1}{4}\log(x^4-1)$$

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)} + \frac{1}{4}\,\log\left(\left|x^4-1\right|\right)$$

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{\ln\left(x^4-1\right)}{4} - \frac{1}{4\,\left(x^4-1\right)}$$

```
[In] int(x^7/(x^8 - 2*x^4 + 1),x)
[Out] log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.293 $\int \frac{x^5}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^5}{1-2x^4+x^8}\,dx=\frac{x^2}{4(1-x^4)}-\frac{\operatorname{arctanh}(x^2)}{4}$$

```
[Out] 1/4*x^2/(-x^4+1)-1/4*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 281, 294, 213}

$$\int \frac{x^5}{1-2x^4+x^8}\,dx=\frac{x^2}{4(1-x^4)}-\frac{\operatorname{arctanh}(x^2)}{4}$$

```
[In] Int[x^5/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^5}{(-1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4(1-x^4)} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4(1-x^4)} - \frac{1}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = \frac{1}{8}\left(-\frac{2x^2}{-1+x^4} + \log\left(1-x^2\right) - \log\left(1+x^2\right)\right)$$

```
[In] Integrate[x^5/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^{2}}{4(x^{4}-1)}+\frac{\ln(x^{2}-1)}{8}-\frac{\ln(x^{2}+1)}{8}$	30
norman	$-\frac{x^{2}}{4(x^{4}-1)}+\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{\ln(x^{2}+1)}{8}$	34
default	$-\frac{1}{8(x^{2}+1)}-\frac{\ln(x^{2}+1)}{8}-\frac{1}{8(x^{2}-1)}+\frac{\ln(x^{2}-1)}{8}$	36
parallelrisch	$\frac{\ln(x-1)x^{4}+\ln(x+1)x^{4}-\ln(x^{2}+1)x^{4}-2x^{2}-\ln(x-1)-\ln(x+1)+\ln(x^{2}+1)}{8x^{4}-8}$	61

```
[In] int(x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(x^4-1)+1/8*ln(x^2-1)-1/8*ln(x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x^5}{1-2x^4+x^8}\,dx=-\frac{2\,x^{2}+\left(x^{4}-1\right)\log\left(x^{2}+1\right)-\left(x^{4}-1\right)\log\left(x^{2}-1\right)}{8\,\left(x^{4}-1\right)}$$

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1-2x^4+x^8}\,dx=-\frac{x^{2}}{4x^{4}-4}+\frac{\log\left(x^{2}-1\right)}{8}-\frac{\log\left(x^{2}+1\right)}{8}$$

```
[In] integrate(x**5/(x**8-2*x**4+1),x)
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{8}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} - \frac{1}{8}\,\log\left(x^2+1\right) + \frac{1}{8}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\ (x^4-1)}$$

```
[In] int(x^5/(x^8 - 2*x^4 + 1),x)
[Out] - atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.294 $\int \frac{x^3}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = \frac{1}{4(1-x^4)}$$

```
[Out] 1/4/(-x^4+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 267}

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = \frac{1}{4(1-x^4)}$$

```
[In] Int[x^3/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^3}{(-1+x^4)^2}\,dx \\ &= \frac{1}{4(1-x^4)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4(-1+x^4)}$$

```
[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]
[Out] -1/4*1/(-1 + x^4)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{4(x^4-1)}$	10
default	$-\frac{1}{4(x^4-1)}$	10
norman	$-\frac{1}{4(x^4-1)}$	10
risch	$-\frac{1}{4(x^4-1)}$	10
parallelrisch	$-\frac{1}{4(x^4-1)}$	10

```
[In] int(x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^4-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4x^4-4}$$

```
[In] integrate(x**3/(x**8-2*x**4+1),x)
[Out] -1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4/(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8}\,dx=-\frac{1}{4\,\left(x^4-1\right)}$$

```
[In] int(x^3/(x^8 - 2*x^4 + 1),x)
[Out] -1/(4*(x^4 - 1))
```

3.295 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x}{1-2x^4+x^8} \, dx = \frac{x^2}{4(1-x^4)} + \frac{\operatorname{arctanh}(x^2)}{4}$$

```
[Out] 1/4*x^2/(-x^4+1)+1/4*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {28, 281, 205, 213}

$$\int \frac{x}{1-2x^4+x^8} \, dx = \frac{\operatorname{arctanh}(x^2)}{4} + \frac{x^2}{4(1-x^4)}$$

```
[In] Int[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{(-1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4(1-x^4)} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{4(1-x^4)} + \frac{1}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x}{1-2x^4+x^8}\,dx = \frac{1}{8}\left(-\frac{2x^2}{-1+x^4} - \log\left(1-x^2\right) + \log\left(1+x^2\right)\right)$$

```
[In] Integrate[x/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^{2}}{4(x^{4}-1)}-\frac{\ln(x^{2}-1)}{8}+\frac{\ln(x^{2}+1)}{8}$	30
norman	$-\frac{x^{2}}{4(x^{4}-1)}-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}+\frac{\ln(x^{2}+1)}{8}$	34
default	$-\frac{1}{8(x^{2}+1)}+\frac{\ln(x^{2}+1)}{8}-\frac{1}{8(x^{2}-1)}-\frac{\ln(x^{2}-1)}{8}$	36
parallelrisch	$-\frac{\ln(x-1)x^{4}+\ln(x+1)x^{4}-\ln(x^{2}+1)x^{4}+2x^{2}-\ln(x-1)-\ln(x+1)+\ln(x^{2}+1)}{8(x^{4}-1)}$	61

```
[In] int(x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(x^4-1)-1/8*ln(x^2-1)+1/8*ln(x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{2\,x^2-(x^4-1)\log(x^2+1)+(x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4x^4-4}-\frac{\log(x^2-1)}{8}+\frac{\log(x^2+1)}{8}$$

```
[In] integrate(x/(x**8-2*x**4+1),x)
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(x^2-1\right)$$

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(|x^2-1|\right)$$

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x}{1-2x^4+x^8}\,dx = \frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

```
[In] int(x/(x^8 - 2*x^4 + 1),x)
[Out] atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)} dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{x(1-2x^4+x^8)} dx = \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log(1-x^4)$$

```
[Out] 1/4/(-x^4+1)+ln(x)-1/4*ln(-x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 46}

$$\int \frac{1}{x(1-2x^4+x^8)} dx = \frac{1}{4(1-x^4)} - \frac{1}{4}\log(1-x^4) + \log(x)$$

```
[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2x}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{(-1+x)^2}+\frac{1}{x}\right)\,dx,x,x^4\right)\\ &= \frac{1}{4\left(1-x^4\right)}+\log(x)-\frac{1}{4}\log\left(1-x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{4\left(-1+x^4\right)}+\log(x)-\frac{1}{4}\log\left(1-x^4\right)$$

```
[In] Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
[Out] -1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{1}{4(x^4-1)}+\ln\left(x\right)-\frac{\ln(x^4-1)}{4}$	21
norman	$-\frac{1}{4(x^4-1)}-\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}-\frac{\ln(x^2+1)}{4}+\ln\left(x\right)$	33
default	$\ln\left(x\right)+\frac{1}{16x+16}-\frac{\ln(x+1)}{4}-\frac{\ln(x^2+1)}{4}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{4}$	47
parallelrisch	$\frac{4\ln(x)x^4-\ln(x-1)x^4-\ln(x+1)x^4-\ln(x^2+1)x^4-1-4\ln(x)+\ln(x-1)+\ln(x+1)+\ln(x^2+1)}{4x^4-4}$	66

```
[In] int(1/x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^4-1)+ln(x)-1/4*ln(x^4-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = -\frac{\left(x^4-1\right)\log\left(x^4-1\right)-4\left(x^4-1\right)\log\left(x\right)+1}{4\left(x^4-1\right)}$$

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = \log\left(x\right)-\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

```
[In] integrate(1/x/(x**8-2*x**4+1),x)
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = -\frac{1}{4\left(x^4-1\right)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = \frac{x^4-2}{4\left(x^4-1\right)} + \frac{1}{4}\,\log\left(x^4\right) - \frac{1}{4}\,\log\left(\left|x^4-1\right|\right)$$

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^4-1\right)}{4} - \frac{1}{4\,\left(x^4-1\right)}$$

```
[In] int(1/(x*(x^8 - 2*x^4 + 1)),x)
[Out] log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx=-\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}+\frac{3\operatorname{arctanh}(x^2)}{4}$$

```
[Out] -3/4/x^2+1/4/x^2/(-x^4+1)+3/4*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 213}

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx=\frac{3\operatorname{arctanh}(x^2)}{4}-\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}$$

```
[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3 (-1+x^4)^2} \, dx \\
&= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2 (-1+x^2)^2} \, dx, x, x^2\right) \\
&= \frac{1}{4x^2 (1-x^4)} - \frac{3}{4} \text{Subst}\left(\int \frac{1}{x^2 (-1+x^2)} \, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2 (1-x^4)} - \frac{3}{4} \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right) \\
&= -\frac{3}{4x^2} + \frac{1}{4x^2 (1-x^4)} + \frac{3}{4} \tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3 (1-2x^4+x^8)} \, dx = \frac{1}{8}\left(\frac{4-6x^4}{x^2(-1+x^4)} - 3\log\left(1-x^2\right) + 3\log\left(1+x^2\right)\right)$$

```
[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)} - \frac{3\ln(x^2-1)}{8} + \frac{3\ln(x^2+1)}{8}$	36
norman	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)} - \frac{3\ln(x-1)}{8} - \frac{3\ln(x+1)}{8} + \frac{3\ln(x^2+1)}{8}$	40
default	$-\frac{1}{2x^2} + \frac{1}{16x+16} - \frac{3\ln(x+1)}{8} + \frac{3\ln(x^2+1)}{8} - \frac{1}{8(x^2+1)} - \frac{1}{16(x-1)} - \frac{3\ln(x-1)}{8}$	50
parallelrisch	$-\frac{3\ln(x-1)x^6+3\ln(x+1)x^6-3\ln(x^2+1)x^6-4+6x^4-3\ln(x-1)x^2-3\ln(x+1)x^2+3\ln(x^2+1)x^2}{8x^2(x^4-1)}$	78

```
[In] int(1/x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (1/2-3/4*x^4)/x^2/(x^4-1)-3/8*ln(x^2-1)+3/8*ln(x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3 (1-2x^4+x^8)} \, dx = -\frac{6\,x^4 - 3\left(x^6-x^2\right)\log\left(x^2+1\right) + 3\left(x^6-x^2\right)\log\left(x^2-1\right) - 4}{8\left(x^6-x^2\right)}$$

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/
(x^6 - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \frac{2-3x^4}{4x^6-4x^2} - \frac{3\log(x^2-1)}{8} + \frac{3\log(x^2+1)}{8}$$

```
[In] integrate(1/x**3/(x**8-2*x**4+1),x)
[Out] (2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{3\,x^4-2}{4\,(x^6-x^2)} + \frac{3}{8}\,\log\left(x^2+1\right) - \frac{3}{8}\,\log\left(x^2-1\right)$$

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{3\,x^4-2}{4\,(x^6-x^2)} + \frac{3}{8}\,\log\left(x^2+1\right) - \frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \frac{3\,\mathrm{atanh}(x^2)}{4} + \frac{\frac{3\,x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

```
[In] int(1/(x^3*(x^8 - 2*x^4 + 1)),x)
[Out] (3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)
```

3.298 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

```
[Out] -1/4/x^4+1/4/(-x^4+1)+2*ln(x)-1/2*ln(-x^4+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 46}

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx=\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

```
[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^5 (-1+x^4)^2}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4(1-x^4)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = -\frac{1}{4x^4}-\frac{1}{4(-1+x^4)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

```
[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
[Out] -1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x^4-1)}{2}$	32
norman	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}$	44
default	$-\frac{1}{4x^4}+2\ln(x)+\frac{1}{16x+16}-\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{8x^2+8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{2}$	54
parallelrisch	$\frac{8\ln(x)x^8-2\ln(x-1)x^8-2\ln(x+1)x^8-2\ln(x^2+1)x^8+1-8\ln(x)x^4+2\ln(x-1)x^4+2\ln(x+1)x^4+2\ln(x^2+1)x^4-2x^4}{4x^4(x^4-1)}$	92

```
[In] int(1/x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (1/4-1/2*x^4)/x^4/(x^4-1)+2*ln(x)-1/2*ln(x^4-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} dx = -\frac{2 x^4 + 2 (x^8 - x^4) \log (x^4 - 1) - 8 (x^8 - x^4) \log (x) - 1}{4 (x^8 - x^4)}$$

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8 -
 x^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} dx = \frac{1 - 2x^4}{4x^8 - 4x^4} + 2 \log (x) - \frac{\log (x^4 - 1)}{2}$$

```
[In] integrate(1/x**5/(x**8-2*x**4+1),x)
[Out] (1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} dx = -\frac{2 x^4 - 1}{4 (x^8 - x^4)} - \frac{1}{2} \log \left(x^4 - 1\right) + \frac{1}{2} \log \left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} \, dx = -\frac{2\,x^4 - 1}{4\,(x^8 - x^4)} + \frac{1}{2}\,\log\left(x^4\right) - \frac{1}{2}\,\log\left(\left|x^4 - 1\right|\right)$$

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 (1 - 2x^4 + x^8)} \, dx = 2\,\ln\left(x\right) - \frac{\ln\left(x^4 - 1\right)}{2} + \frac{\frac{x^4}{2} - \frac{1}{4}}{x^4 - x^8}$$

```
[In] int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
[Out] 2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

3.299 $\int \frac{1}{x^7(1-2x^4+x^8)} dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{x^7 (1-2x^4+x^8)} dx = -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 (1-x^4)} + \frac{5\operatorname{arctanh}(x^2)}{4}$$

```
[Out] -5/12/x^6-5/4/x^2+1/4/x^6/(-x^4+1)+5/4*arctanh(x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 213}

$$\int \frac{1}{x^7 (1-2x^4+x^8)} dx = \frac{5\operatorname{arctanh}(x^2)}{4} - \frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 (1-x^4)}$$

```
[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^7 \left(-1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 \left(-1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{x^4 \left(-1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{x^2 \left(-1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 \left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6 \left(1-x^4\right)} + \frac{5}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^7 (1-2x^4+x^8)} \, dx = -\frac{1}{6x^6} - \frac{1}{x^2} - \frac{x^2}{4(-1+x^4)} - \frac{5}{8}\log\left(1-x^2\right) + \frac{5}{8}\log\left(1+x^2\right)$$

```
[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
[Out] -1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 + x^2])/8
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)} + \frac{5\ln(x^2+1)}{8} - \frac{5\ln(x^2-1)}{8}$	41
norman	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)} - \frac{5\ln(x-1)}{8} - \frac{5\ln(x+1)}{8} + \frac{5\ln(x^2+1)}{8}$	45
default	$-\frac{1}{6x^6} - \frac{1}{x^2} + \frac{1}{16x+16} - \frac{5\ln(x+1)}{8} + \frac{5\ln(x^2+1)}{8} - \frac{1}{8(x^2+1)} - \frac{1}{16(x-1)} - \frac{5\ln(x-1)}{8}$	55
parallelrisch	$-\frac{15\ln(x-1)x^{10}+15\ln(x+1)x^{10}-15\ln(x^2+1)x^{10}-4+30x^8-15\ln(x-1)x^6-15\ln(x+1)x^6+15\ln(x^2+1)x^6-20x^4}{24x^6(x^4-1)}$	83

```
[In] int(1/x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (1/6+5/6*x^4-5/4*x^8)/x^6/(x^4-1)+5/8*ln(x^2+1)-5/8*ln(x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^7 (1-2x^4+x^8)} \, dx = -\frac{30\,x^8 - 20\,x^4 - 15\,(x^{10}-x^6)\log\left(x^2+1\right) + 15\,(x^{10}-x^6)\log\left(x^2-1\right) - 4}{24\,(x^{10}-x^6)}$$

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log(x^2 - 1) - 4)/(x^10 - x^6)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)}\, dx = -\frac{5\log(x^2 - 1)}{8} + \frac{5\log(x^2 + 1)}{8} + \frac{-15x^8 + 10x^4 + 2}{12x^{10} - 12x^6}$$

```
[In] integrate(1/x**7/(x**8-2*x**4+1),x)
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**10
 - 12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)}\, dx = -\frac{15\,x^8 - 10\,x^4 - 2}{12\,(x^{10} - x^6)} + \frac{5}{8}\,\log\left(x^2 + 1\right) - \frac{5}{8}\,\log\left(x^2 - 1\right)$$

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)}\, dx = -\frac{x^2}{4\,(x^4 - 1)} - \frac{6\,x^4 + 1}{6\,x^6} + \frac{5}{8}\,\log\left(x^2 + 1\right) - \frac{5}{8}\,\log\left(\left|x^2 - 1\right|\right)$$

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(x
^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7 (1-2x^4+x^8)} \, dx = \frac{5 \operatorname{atanh}(x^2)}{4} - \frac{-\frac{5x^8}{4} + \frac{5x^4}{6} + \frac{1}{6}}{x^6 - x^{10}}$$

```
[In] int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
[Out] (5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{x^8}{1-2x^4+x^8}\,dx=\frac{5x}{4}+\frac{x^5}{4(1-x^4)}-\frac{5\arctan(x)}{8}-\frac{5\text{arctanh}(x)}{8}$$

```
[Out] 5/4*x+1/4*x^5/(-x^4+1)-5/8*arctan(x)-5/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 294, 327, 218, 212, 209}

$$\int \frac{x^8}{1-2x^4+x^8}\,dx=-\frac{5\arctan(x)}{8}-\frac{5\text{arctanh}(x)}{8}+\frac{x^5}{4(1-x^4)}+\frac{5x}{4}$$

```
[In] Int[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^8}{(-1+x^4)^2}\,dx\\
&= \frac{x^5}{4(1-x^4)}+\frac{5}{4}\int \frac{x^4}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4(1-x^4)}+\frac{5}{4}\int \frac{1}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4(1-x^4)}-\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4(1-x^4)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4\left(-1+x^4\right)} - \frac{5\arctan(x)}{8} + \frac{5}{16}\log(1-x) - \frac{5}{16}\log(1+x)$$

```
[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])
/16
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
risch	$x-\frac{x}{4(x^4-1)}+\frac{5\ln(x-1)}{16}-\frac{5\arctan(x)}{8}-\frac{5\ln(x+1)}{16}$	29
default	$x-\frac{1}{16(x+1)}-\frac{5\ln(x+1)}{16}+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{5\ln(x-1)}{16}$	43
parallelrisch	$\frac{5i\ln(x-i)x^4-5i\ln(x+i)x^4+5\ln(x-1)x^4-5\ln(x+1)x^4+16x^5-5i\ln(x-i)+5i\ln(x+i)-5\ln(x-1)+5\ln(x+1)-20x}{16x^4-16}$	87

```
[In] int(x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] x-1/4*x/(x^4-1)+5/16*ln(x-1)-5/8*arctan(x)-5/16*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = \frac{16\,x^5 - 10\,(x^4-1)\arctan(x) - 5\,(x^4-1)\log(x+1) + 5\,(x^4-1)\log(x-1) - 20\,x}{16\,(x^4-1)}$$

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1
)*log(x - 1) - 20*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4x^4-4} + \frac{5\log{(x-1)}}{16} - \frac{5\log{(x+1)}}{16} - \frac{5\operatorname{atan}{(x)}}{8}$$

```
[In] integrate(x**8/(x**8-2*x**4+1),x)
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(x+1) + \frac{5}{16}\log(x-1)$$

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(|x+1|) + \frac{5}{16}\log(|x-1|)$$

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x
 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{5\,\mathrm{atan}(x)}{8} - \frac{x}{4\,\left(x^4-1\right)} + \frac{\mathrm{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{8}$$

```
[In] int(x^8/(x^8 - 2*x^4 + 1),x)
[Out] x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

3.301 $\int \frac{x^6}{1-2x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{x^3}{4(1-x^4)} + \frac{3\arctan(x)}{8} - \frac{3\text{arctanh}(x)}{8}$$

```
[Out] 1/4*x^3/(-x^4+1)+3/8*arctan(x)-3/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 294, 304, 209, 212}

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{3\arctan(x)}{8} - \frac{3\text{arctanh}(x)}{8} + \frac{x^3}{4(1-x^4)}$$

```
[In] Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^6}{(-1+x^4)^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}+\frac{3}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= \frac{x^3}{4(1-x^4)}-\frac{3}{8}\int \frac{1}{1-x^2}\,dx+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{1-2x^4+x^8}\,dx=\frac{1}{16}\left(-\frac{4x^3}{-1+x^4}+6\arctan(x)+3\log(1-x)-3\log(1+x)\right)$$

```
[In] Integrate[x^6/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}-1)}+\frac{3\arctan(x)}{8}+\frac{3\ln(x-1)}{16}-\frac{3\ln(x+1)}{16}$	30
default	$-\frac{1}{16(x+1)}-\frac{3\ln(x+1)}{16}-\frac{x}{8(x^{2}+1)}+\frac{3\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{3\ln(x-1)}{16}$	42
parallelrisch	$\frac{-3i\ln(x-i)x^{4}+3i\ln(x+i)x^{4}+3\ln(x-1)x^{4}-3\ln(x+1)x^{4}-4x^{3}+3i\ln(x-i)-3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)}{16x^{4}-16}$	84

```
[In] int(x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^3/(x^4-1)+3/8*arctan(x)+3/16*ln(x-1)-3/16*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{4\,x^{3}-6\,(x^{4}-1)\arctan(x)+3\,(x^{4}-1)\log(x+1)-3\,(x^{4}-1)\log(x-1)}{16\,(x^{4}-1)}$$

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)
*log(x - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^{3}}{4x^{4}-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}\,(x)}{8}$$

```
[In] integrate(x**6/(x**8-2*x**4+1),x)
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) - \frac{3}{16}\,\log\left(x+1\right) + \frac{3}{16}\,\log\left(x-1\right)$$

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) - \frac{3}{16}\,\log\left(|x+1|\right) + \frac{3}{16}\,\log\left(|x-1|\right)$$

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
- 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} - \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x^3}{4\,\left(x^4-1\right)}$$

```
[In] int(x^6/(x^8 - 2*x^4 + 1),x)
[Out] (3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

3.302 $\int \frac{x^4}{1-2x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{x^4}{1-2x^4+x^8} dx = \frac{x}{4(1-x^4)} - \frac{\arctan(x)}{8} - \frac{\operatorname{arctanh}(x)}{8}$$

```
[Out] 1/4*x/(-x^4+1)-1/8*arctan(x)-1/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 294, 218, 212, 209}

$$\int \frac{x^4}{1-2x^4+x^8} dx = -\frac{\arctan(x)}{8} - \frac{\operatorname{arctanh}(x)}{8} + \frac{x}{4(1-x^4)}$$

```
[In] Int[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{\left(-1+x^4\right)^2}\,dx \\
&= \frac{x}{4\left(1-x^4\right)}+\frac{1}{4}\int \frac{1}{-1+x^4}\,dx \\
&= \frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\int \frac{1}{1-x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{4\left(1-x^4\right)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{1-2x^4+x^8}\,dx=\frac{1}{16}\left(-\frac{4x}{-1+x^4}-2\arctan(x)+\log(1-x)-\log(1+x)\right)$$

```
[In] Integrate[x^4/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{\arctan(x)}{8}-\frac{\ln(x+1)}{16}+\frac{\ln(x-1)}{16}$	28
default	$-\frac{1}{16(x+1)}-\frac{\ln(x+1)}{16}+\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16(x-1)}+\frac{\ln(x-1)}{16}$	42
parallelrisch	$-\frac{i\ln(x+i)x^4-i\ln(x-i)x^4-\ln(x-1)x^4+\ln(x+1)x^4-i\ln(x+i)+i\ln(x-i)+\ln(x-1)-\ln(x+1)+4x}{16(x^4-1)}$	79

```
[In] int(x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x/(x^4-1)-1/8*arctan(x)-1/16*ln(x+1)+1/16*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{2\left(x^4-1\right)\arctan\left(x\right)+\left(x^4-1\right)\log\left(x+1\right)-\left(x^4-1\right)\log\left(x-1\right)+4\,x}{16\left(x^4-1\right)}$$

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
+ 4*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4}+\frac{\log\left(x-1\right)}{16}-\frac{\log\left(x+1\right)}{16}-\frac{\operatorname{atan}\left(x\right)}{8}$$

```
[In] integrate(x**4/(x**8-2*x**4+1),x)
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\left(x^4-1\right)} - \frac{1}{8}\arctan\left(x\right) - \frac{1}{16}\log\left(x+1\right) + \frac{1}{16}\log\left(x-1\right)$$

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\left(x^4-1\right)} - \frac{1}{8}\arctan\left(x\right) - \frac{1}{16}\log\left(|x+1|\right) + \frac{1}{16}\log\left(|x-1|\right)$$

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{\mathrm{atan}(x)}{8} - \frac{\mathrm{atanh}(x)}{8} - \frac{x}{4\,\left(x^4-1\right)}$$

```
[In] int(x^4/(x^8 - 2*x^4 + 1),x)
[Out] - atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

3.303 $\int \frac{x^2}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^2}{1-2x^4+x^8}\,dx=\frac{x^3}{4\left(1-x^4\right)}-\frac{\arctan(x)}{8}+\frac{\operatorname{arctanh}(x)}{8}$$

```
[Out] 1/4*x^3/(-x^4+1)-1/8*arctan(x)+1/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 296, 304, 209, 212}

$$\int \frac{x^2}{1-2x^4+x^8}\,dx=-\frac{\arctan(x)}{8}+\frac{\operatorname{arctanh}(x)}{8}+\frac{x^3}{4\left(1-x^4\right)}$$

```
[In] Int[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

<u>Rule 28</u>

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{(-1+x^4)^2}\,dx \\
&= \frac{x^3}{4(1-x^4)} - \frac{1}{4}\int \frac{x^2}{-1+x^4}\,dx \\
&= \frac{x^3}{4(1-x^4)} + \frac{1}{8}\int \frac{1}{1-x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = \frac{1}{16}\left(-\frac{4x^3}{-1+x^4} - 2\arctan(x) - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[x^2/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}-1)}-\frac{\arctan(x)}{8}-\frac{\ln(x-1)}{16}+\frac{\ln(x+1)}{16}$	30
default	$-\frac{1}{16(x+1)}+\frac{\ln(x+1)}{16}-\frac{x}{8(x^{2}+1)}-\frac{\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{16}$	42
parallelrisch	$-\frac{-i\ln(x-i)x^{4}+i\ln(x+i)x^{4}+\ln(x-1)x^{4}-\ln(x+1)x^{4}+4x^{3}+i\ln(x-i)-i\ln(x+i)-\ln(x-1)+\ln(x+1)}{16(x^{4}-1)}$	81

```
[In] int(x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^3/(x^4-1)-1/8*arctan(x)-1/16*ln(x-1)+1/16*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{4\,x^{3}+2\,(x^{4}-1)\arctan(x)-(x^{4}-1)\log(x+1)+(x^{4}-1)\log(x-1)}{16\,(x^{4}-1)}$$

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log
(x - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^{3}}{4x^{4}-4}-\frac{\log(x-1)}{16}+\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

```
[In] integrate(x**2/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) + \frac{1}{16}\,\log\left(x+1\right) - \frac{1}{16}\,\log\left(x-1\right)$$

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) + \frac{1}{16}\,\log\left(|x+1|\right) - \frac{1}{16}\,\log\left(|x-1|\right)$$

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
- 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = \frac{\mathrm{atanh}(x)}{8} - \frac{\mathrm{atan}(x)}{8} - \frac{x^3}{4\,\left(x^4-1\right)}$$

```
[In] int(x^2/(x^8 - 2*x^4 + 1),x)
[Out] atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

3.304 $\int \frac{1}{1-2x^4+x^8}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1-2x^4+x^8}\,dx=\frac{x}{4(1-x^4)}+\frac{3\arctan(x)}{8}+\frac{3\text{arctanh}(x)}{8}$$

```
[Out] 1/4*x/(-x^4+1)+3/8*arctan(x)+3/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {28, 205, 218, 212, 209}

$$\int \frac{1}{1-2x^4+x^8}\,dx=\frac{3\arctan(x)}{8}+\frac{3\text{arctanh}(x)}{8}+\frac{x}{4(1-x^4)}$$

```
[In] Int[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(-1+x^4)^2}\,dx \\ &= \frac{x}{4(1-x^4)} - \frac{3}{4}\int \frac{1}{-1+x^4}\,dx \\ &= \frac{x}{4(1-x^4)} + \frac{3}{8}\int \frac{1}{1-x^2}\,dx + \frac{3}{8}\int \frac{1}{1+x^2}\,dx \\ &= \frac{x}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{1}{16}\left(-\frac{4x}{-1+x^4} + 6\arctan(x) - 3\log(1-x) + 3\log(1+x)\right)$$

```
[In] Integrate[(1 - 2*x^4 + x^8)^(-1),x]
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}+\frac{3\ln(x+1)}{16}-\frac{3\ln(x-1)}{16}+\frac{3\arctan(x)}{8}$	28
default	$-\frac{1}{16(x+1)}+\frac{3\ln(x+1)}{16}+\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{3\ln(x-1)}{16}$	42
parallelrisch	$-\frac{3i\ln(x-i)x^4-3i\ln(x+i)x^4+3\ln(x-1)x^4-3\ln(x+1)x^4-3i\ln(x-i)+3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)+4x}{16(x^4-1)}$	82

```
[In] int(1/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x/(x^4-1)+3/16*ln(x+1)-3/16*ln(x-1)+3/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{6\,(x^4-1)\arctan(x)+3\,(x^4-1)\log(x+1)-3\,(x^4-1)\log(x-1)-4\,x}{16\,(x^4-1)}$$

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
1) - 4*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4}-\frac{3\log(x-1)}{16}+\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}(x)}{8}$$

```
[In] integrate(1/(x**8-2*x**4+1),x)
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) + \frac{3}{16}\,\log\left(x+1\right) - \frac{3}{16}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} + \frac{3}{8}\,\arctan\left(x\right) + \frac{3}{16}\,\log\left(|x+1|\right) - \frac{3}{16}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} + \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x}{4\,\left(x^4-1\right)}$$

```
[In] int(1/(x^8 - 2*x^4 + 1),x)
[Out] (3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

3.305 $\int \frac{1}{x^2(1-2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5\arctan(x)}{8}+\frac{5\text{arctanh}(x)}{8}$$

```
[Out] -5/4/x+1/4/x/(-x^4+1)-5/8*arctan(x)+5/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 296, 331, 304, 209, 212}

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = -\frac{5\arctan(x)}{8}+\frac{5\text{arctanh}(x)}{8}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4x}$$

```
[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}+\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = \frac{1}{16}\left(-\frac{16}{x}-\frac{4x^3}{-1+x^4}-10\arctan(x)-5\log(1-x)+5\log(1+x)\right)$$

```
[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/1
6
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result
risch	$\frac{-\frac{5x^4}{4}+1}{(x^4-1)x}+\frac{5\ln(x+1)}{16}-\frac{5\ln(x-1)}{16}-\frac{5\arctan(x)}{8}$
default	$-\frac{1}{x}-\frac{1}{16(x+1)}+\frac{5\ln(x+1)}{16}-\frac{x}{8(x^2+1)}-\frac{5\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{5\ln(x-1)}{16}$
parallelrisch	$-\frac{-5i\ln(x-i)x^5+5i\ln(x+i)x^5+5\ln(x-1)x^5-5\ln(x+1)x^5-16+20x^4+5i\ln(x-i)x-5i\ln(x+i)x-5\ln(x-1)x+5\ln(x+1)x}{16x(x^4-1)}$

```
[In] int(1/x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (-5/4*x^4+1)/(x^4-1)/x+5/16*ln(x+1)-5/16*ln(x-1)-5/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{20x^4+10(x^5-x)\arctan(x)-5(x^5-x)\log(x+1)+5(x^5-x)\log(x-1)-16}{16(x^5-x)}$$

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
x)*log(x - 1) - 16)/(x^5 - x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (1 - 2x^4 + x^8)} \, dx = \frac{4 - 5x^4}{4x^5 - 4x} - \frac{5 \log (x - 1)}{16} + \frac{5 \log (x + 1)}{16} - \frac{5 \operatorname{atan} (x)}{8}$$

```
[In] integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x)
/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2 (1 - 2x^4 + x^8)} \, dx = -\frac{5 \, x^4 - 4}{4 \, (x^5 - x)} - \frac{5}{8} \, \arctan (x) + \frac{5}{16} \, \log (x + 1) - \frac{5}{16} \, \log (x - 1)$$

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (1 - 2x^4 + x^8)} \, dx = -\frac{5 \, x^4 - 4}{4 \, (x^5 - x)} - \frac{5}{8} \, \arctan (x) + \frac{5}{16} \, \log (|x + 1|) - \frac{5}{16} \, \log (|x - 1|)$$

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*lo
g(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx=\frac{5\,\mathrm{atanh}(x)}{8}-\frac{5\,\mathrm{atan}(x)}{8}+\frac{\frac{5\,x^4}{4}-1}{x-x^5}$$

```
[In] int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
[Out] (5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7\arctan(x)}{8}+\frac{7\text{arctanh}(x)}{8}$$

```
[Out] -7/12/x^3+1/4/x^3/(-x^4+1)+7/8*arctan(x)+7/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 296, 331, 218, 212, 209}

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = \frac{7\arctan(x)}{8}+\frac{7\text{arctanh}(x)}{8}-\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}$$

```
[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^4\left(-1+x^4\right)^2}\,dx \\
&= \frac{1}{4x^3\left(1-x^4\right)} - \frac{7}{4}\int \frac{1}{x^4\left(-1+x^4\right)}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} - \frac{7}{4}\int \frac{1}{-1+x^4}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} + \frac{7}{8}\int \frac{1}{1-x^2}\,dx + \frac{7}{8}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3\left(1-x^4\right)} + \frac{7}{8}\tan^{-1}(x) + \frac{7}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=\frac{1}{48}\left(-\frac{16}{x^3}-\frac{12x}{-1+x^4}+42\arctan(x)-21\log(1-x)+21\log(1+x)\right)$$

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x]
)/48
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result
risch	$\frac{-\frac{7x^4}{12}+\frac{1}{3}}{x^3(x^4-1)}-\frac{7\ln(x-1)}{16}+\frac{7\ln(x+1)}{16}+\frac{7\arctan(x)}{8}$
default	$-\frac{1}{3x^3}-\frac{1}{16(x+1)}+\frac{7\ln(x+1)}{16}+\frac{x}{8x^2+8}+\frac{7\arctan(x)}{8}-\frac{1}{16(x-1)}-\frac{7\ln(x-1)}{16}$
parallelrisch	$-\frac{21i\ln(x-i)x^7-21i\ln(x+i)x^7+21\ln(x-1)x^7-21\ln(x+1)x^7-16-21i\ln(x-i)x^3+21i\ln(x+i)x^3-21\ln(x-1)x^3+21\ln(x+1)x^3}{48x^3(x^4-1)}$

```
[In] int(1/x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (-7/12*x^4+1/3)/x^3/(x^4-1)-7/16*ln(x-1)+7/16*ln(x+1)+7/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx = -\frac{28\,x^4-42\left(x^7-x^3\right)\arctan\left(x\right)-21\left(x^7-x^3\right)\log\left(x+1\right)+21\left(x^7-x^3\right)\log\left(x-1\right)-16}{48\left(x^7-x^3\right)}$$

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(
x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = \frac{4-7x^4}{12x^7-12x^3} - \frac{7\log(x-1)}{16} + \frac{7\log(x+1)}{16} + \frac{7\operatorname{atan}(x)}{8}$$

```
[In] integrate(1/x**4/(x**8-2*x**4+1),x)
[Out] (4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*at
an(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{7\,x^4-4}{12\,(x^7-x^3)} + \frac{7}{8}\arctan(x) + \frac{7}{16}\log(x+1) - \frac{7}{16}\log(x-1)$$

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(
x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{x}{4\,(x^4-1)} - \frac{1}{3\,x^3} + \frac{7}{8}\arctan(x) + \frac{7}{16}\log(|x+1|) - \frac{7}{16}\log(|x-1|)$$

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*lo
g(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4 (1 - 2x^4 + x^8)} \, dx = \frac{7 \operatorname{atan}(x)}{8} + \frac{7 \operatorname{atanh}(x)}{8} + \frac{\frac{7 x^4}{12} - \frac{1}{3}}{x^3 - x^7}$$

```
[In] int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
[Out] (7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{x^6(1-2x^4+x^8)} \, dx = -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5(1-x^4)} - \frac{9\arctan(x)}{8} + \frac{9\text{arctanh}(x)}{8}$$

```
[Out] -9/20/x^5-9/4/x+1/4/x^5/(-x^4+1)-9/8*arctan(x)+9/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 296, 331, 304, 209, 212}

$$\int \frac{1}{x^6(1-2x^4+x^8)} \, dx = -\frac{9\arctan(x)}{8} + \frac{9\text{arctanh}(x)}{8} - \frac{9}{20x^5} + \frac{1}{4x^5(1-x^4)} - \frac{9}{4x}$$

```
[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[
x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^6 \left(-1+x^4\right)^2} \, dx \\
&= \frac{1}{4x^5 \left(1-x^4\right)} - \frac{9}{4} \int \frac{1}{x^6 \left(-1+x^4\right)} \, dx \\
&= -\frac{9}{20x^5} + \frac{1}{4x^5 \left(1-x^4\right)} - \frac{9}{4} \int \frac{1}{x^2 \left(-1+x^4\right)} \, dx \\
&= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5 \left(1-x^4\right)} - \frac{9}{4} \int \frac{x^2}{-1+x^4} \, dx \\
&= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5 \left(1-x^4\right)} + \frac{9}{8} \int \frac{1}{1-x^2} \, dx - \frac{9}{8} \int \frac{1}{1+x^2} \, dx \\
&= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5 \left(1-x^4\right)} - \frac{9}{8} \tan^{-1}(x) + \frac{9}{8} \tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^6 (1-2x^4+x^8)} \, dx = -\frac{1}{5x^5} - \frac{2}{x} - \frac{x^3}{4(-1+x^4)} - \frac{9\arctan(x)}{8} - \frac{9}{16}\log(1-x) + \frac{9}{16}\log(1+x)$$

```
[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
[Out] -1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16
 + (9*Log[1 + x])/16
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result
risch	$\frac{-\frac{9}{4}x^8+\frac{9}{5}x^4+\frac{1}{5}}{x^5(x^4-1)} - \frac{9\ln(x-1)}{16} + \frac{9\ln(x+1)}{16} - \frac{9\arctan(x)}{8}$
default	$-\frac{1}{5x^5} - \frac{2}{x} - \frac{1}{16(x+1)} + \frac{9\ln(x+1)}{16} - \frac{x}{8(x^2+1)} - \frac{9\arctan(x)}{8} - \frac{1}{16(x-1)} - \frac{9\ln(x-1)}{16}$
parallelrisch	$-\frac{-45i\ln(x-i)x^9+45i\ln(x+i)x^9+45\ln(x-1)x^9-45\ln(x+1)x^9-16+180x^8+45i\ln(x-i)x^5-45i\ln(x+i)x^5-45\ln(x-1)x^5+4}{80x^5(x^4-1)}$

```
[In] int(1/x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] (-9/4*x^8+9/5*x^4+1/5)/x^5/(x^4-1)-9/16*ln(x-1)+9/16*ln(x+1)-9/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^6 (1-2x^4+x^8)} \, dx = -\frac{180\,x^8 - 144\,x^4 + 90\,(x^9-x^5)\arctan(x) - 45\,(x^9-x^5)\log(x+1) + 45\,(x^9-x^5)\log(x-1) - 16}{80\,(x^9-x^5)}$$

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
+ 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6 (1-2x^4+x^8)}\,dx = -\frac{9\log(x-1)}{16} + \frac{9\log(x+1)}{16} - \frac{9\,\mathrm{atan}(x)}{8} + \frac{-45x^8+36x^4+4}{20x^9-20x^5}$$

```
[In] integrate(1/x**6/(x**8-2*x**4+1),x)
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 + (-45*x**8 + 36*x**4 + 4)
/(20*x**9 - 20*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^6 (1-2x^4+x^8)}\,dx = -\frac{45\,x^8-36\,x^4-4}{20\,(x^9-x^5)} - \frac{9}{8}\arctan(x) + \frac{9}{16}\log(x+1) - \frac{9}{16}\log(x-1)$$

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1) -
 9/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 (1-2x^4+x^8)}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{10\,x^4+1}{5\,x^5} - \frac{9}{8}\arctan(x) + \frac{9}{16}\log(|x+1|) - \frac{9}{16}\log(|x-1|)$$

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
+ 1)) - 9/16*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^6 (1 - 2x^4 + x^8)} \, dx = \frac{9 \operatorname{atanh}(x)}{8} - \frac{9 \operatorname{atan}(x)}{8} - \frac{-\frac{9 x^8}{4} + \frac{9 x^4}{5} + \frac{1}{5}}{x^5 - x^9}$$

```
[In] int(1/(x^6*(x^8 - 2*x^4 + 1)),x)
[Out] (9*atanh(x))/8 - (9*atan(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)
```

3.308 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx=-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11\arctan(x)}{8}+\frac{11\text{arctanh}(x)}{8}$$

```
[Out] -11/28/x^7-11/12/x^3+1/4/x^7/(-x^4+1)+11/8*arctan(x)+11/8*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 296, 331, 218, 212, 209}

$$\int \frac{1}{x^8\left(1-2x^4+x^8\right)}\,dx=\frac{11\arctan(x)}{8}+\frac{11\text{arctanh}(x)}{8}-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}$$

```
[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^8 \left(-1+x^4\right)^2} \, dx \\
&= \frac{1}{4 x^7 \left(1-x^4\right)}-\frac{11}{4} \int \frac{1}{x^8 \left(-1+x^4\right)} \, dx \\
&= -\frac{11}{28 x^7}+\frac{1}{4 x^7 \left(1-x^4\right)}-\frac{11}{4} \int \frac{1}{x^4 \left(-1+x^4\right)} \, dx \\
&= -\frac{11}{28 x^7}-\frac{11}{12 x^3}+\frac{1}{4 x^7 \left(1-x^4\right)}-\frac{11}{4} \int \frac{1}{-1+x^4} \, dx \\
&= -\frac{11}{28 x^7}-\frac{11}{12 x^3}+\frac{1}{4 x^7 \left(1-x^4\right)}+\frac{11}{8} \int \frac{1}{1-x^2} \, dx+\frac{11}{8} \int \frac{1}{1+x^2} \, dx \\
&= -\frac{11}{28 x^7}-\frac{11}{12 x^3}+\frac{1}{4 x^7 \left(1-x^4\right)}+\frac{11}{8} \tan^{-1}(x)+\frac{11}{8} \tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8 \left(1-2 x^4+x^8\right)} \, dx=\frac{1}{336} \left(-\frac{48}{x^7}-\frac{224}{x^3}-\frac{84 x}{-1+x^4}+462 \arctan (x)-231 \log (1-x)+231 \log (1+x)\right)$$

```
[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 2
31*Log[1 + x])/336
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result
risch	$\frac{-\frac{11}{12} x^{8}+\frac{11}{21} x^{4}+\frac{1}{7}}{x^{7} \left(x^{4}-1\right)}+\frac{11 \arctan \left(x \right)}{8}-\frac{11 \ln \left(x -1\right)}{16}+\frac{11 \ln \left(x +1\right)}{16}$
default	$-\frac{1}{7 x^{7}}-\frac{2}{3 x^{3}}-\frac{1}{16 \left(x +1\right)}+\frac{11 \ln \left(x +1\right)}{16}+\frac{x}{8 x^{2}+8}+\frac{11 \arctan \left(x \right)}{8}-\frac{1}{16 \left(x -1\right)}-\frac{11 \ln \left(x -1\right)}{16}$
parallelrisch	$-\frac{231 i \ln \left(x -i\right) x^{11}-231 i \ln \left(x +i\right) x^{11}+231 \ln \left(x -1\right) x^{11}-231 \ln \left(x +1\right) x^{11}-48-231 i \ln \left(x -i\right) x^{7}+231 i \ln \left(x +i\right) x^{7}-231 \ln \left(x -1\right) x^{7}+}{336 x^{7} \left(x^{4}-1\right)}$

```
[In] int(1/x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-11/12*x^8+11/21*x^4+1/7)/x^7/(x^4-1)+11/8*arctan(x)-11/16*ln(x-1)+11/16*l
n(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^8 \left(1-2 x^4+x^8\right)} \, dx=-\frac{308 \, x^{8} - 176 \, x^{4} - 462 \, {\left(x^{11} - x^{7}\right)} \arctan \left(x\right) - 231 \, {\left(x^{11} - x^{7}\right)} \log \left(x + 1\right) + 231 \, {\left(x^{11} - x^{7}\right)} \log \left(x - 1\right) - 48}{336 \, {\left(x^{11} - x^{7}\right)}}$$

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*l
og(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^8 (1-2x^4+x^8)} \, dx = -\frac{11 \log(x-1)}{16} + \frac{11 \log(x+1)}{16} + \frac{11 \operatorname{atan}(x)}{8} + \frac{-77x^8+44x^4+12}{84x^{11}-84x^7}$$

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 + (-77*x**8 + 44*x**4 +
 12)/(84*x**11 - 84*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^8 (1-2x^4+x^8)} \, dx = -\frac{77 x^8 - 44 x^4 - 12}{84 (x^{11} - x^7)} + \frac{11}{8} \arctan(x) + \frac{11}{16} \log(x+1) - \frac{11}{16} \log(x-1)$$

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
1) - 11/16*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^8 (1-2x^4+x^8)} \, dx = -\frac{x}{4 (x^4-1)} - \frac{14 x^4 + 3}{21 x^7} + \frac{11}{8} \arctan(x) + \frac{11}{16} \log(|x+1|) - \frac{11}{16} \log(|x-1|)$$

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x
 + 1)) - 11/16*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^8 (1-2x^4+x^8)} \, dx = \frac{11 \operatorname{atan}(x)}{8} + \frac{11 \operatorname{atanh}(x)}{8} - \frac{-\frac{11 x^8}{12} + \frac{11 x^4}{21} + \frac{1}{7}}{x^7 - x^{11}}$$

```
[In] int(1/(x^8*(x^8 - 2*x^4 + 1)),x)
[Out] (11*atan(x))/8 + (11*atanh(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7 -
 x^11)
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

```
[Out] 2*c*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b-(-4*a*c+b^2)^(
1/2)))/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^(1+m)*hypergeo
m([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1389, 371}

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{2cx^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2cx^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) - (
2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b + Sq
rt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c\int \frac{x^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}\right)(1+m)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.50

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{x^m \text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^3+2c\#1^7}\&\right]}{4m}$$

```
[In] Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4*m)
```

Maple [F]

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}dx$$

```
[In] int(x^m/(c*x^8+b*x^4+a),x)
[Out] int(x^m/(c*x^8+b*x^4+a),x)
```

Fricas [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] integral(x^m/(c*x^8 + b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate(x**m/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^m}{a + bx^4 + cx^8}\,dx = \int \frac{x^m}{cx^8 + bx^4 + a}\,dx$$

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a + bx^4 + cx^8}\,dx = \int \frac{x^m}{c\,x^8 + b\,x^4 + a}\,dx$$

```
[In] int(x^m/(a + b*x^4 + c*x^8),x)
[Out] int(x^m/(a + b*x^4 + c*x^8), x)
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^{11}}{a+bx^4+cx^8} \, dx = \frac{x^4}{4c} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1371, 717, 648, 632, 212, 642}

$$\int \frac{x^{11}}{a+bx^4+cx^8} \, dx = -\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^4+cx^8\right)}{8c^2} + \frac{x^4}{4c}$$

```
[In] Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^4\right)}{4c}\\
&= \frac{x^4}{4c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}\\
&= \frac{x^4}{4c}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c^2}\\
&= \frac{x^4}{4c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \frac{2cx^4+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

[In] Integrate[x^11/(a + b*x^4 + c*x^8),x]

[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^4}{4c}+\frac{-\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$
risch	$\frac{x^4}{4c}-\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^4+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{2c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(\right.\right.}{}$

[In] int(x^11/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4*x^4/c+1/4/c*(-1/2*b/c*ln(c*x^8+b*x^4+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)x^4-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^3-4\,abc)\log\left(cx^8\right.}{8\,(b^2c^2-4\,ac^3)}\right.$$

[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] [1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1

```
/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(73) = 146$.

Time = 2.70 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{8c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^4}{4c}$$

```
[In] integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*
log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b
 - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a
*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \frac{x^4}{4\,c} - \frac{b\log\left(cx^8+bx^4+a\right)}{8\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2*
c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 3916, normalized size of antiderivative = 48.35

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^11/(a + b*x^4 + c*x^8),x)
```

```
[Out] x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*b
^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c^6
 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2
*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16
*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c
^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*
c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2
)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c
^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c
- b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c
 - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*
b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^
4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(
4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/
(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((8*a^3*c^5 -
20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144
*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^
```

4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 1
6*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*
a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^3 - 16*a*b*c)*(2*a*c -
b^2)^3)/(2*c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*a^3*c^2) -
((b^3 - 3*a*b*c)*(((4*b^3 - 16*a*b*c)*((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*
c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c
^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)
/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*
c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^
2)) - (b^7 - a^3*b*c^3 + 3*a^2*b^3*c^2 - 3*a*b^5*c)/c^4 + ((4*b^3 - 16*a*b*
c)*(((2*a*c - b^2)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b
^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2
)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/
2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 -
16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(6
4*a*c^3 - 16*b^2*c^2)) + ((((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*
c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c
- b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*
c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16
*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c
^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*
(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))
/(8*c^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) -
(b^3*(2*a*c - b^2)^4)/(8*c^4*(4*a*c - b^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1
/2)))*(4*a*c - b^2)^2)/(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3
- 8*a*b^6*c) - (c^2*(a*c - b^2)*(4*a*c - b^2)^2*(((4*b^3 - 16*a*b*c)*(((4*b
^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*
a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2
)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(
1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6
+ 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c
^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16
*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2
)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((24*a*b^5*c^2 + 16
*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a
*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512
*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2
)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a
*c - b^2)^(1/2)) - ((2*a*c - b^2)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a
^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))
)/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b
^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(
4*a*c - b^2)^(1/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c
- b^2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*b^2*(4*b
^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2

```
*c^2))))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6
*c)) + (c^2*(4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*((a*b^6 - 2*a^2*b^4*c + a^3
*b^2*c^2)/c^4 + ((4*b^3 - 16*a*b*c)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*
b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b
^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (5
12*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 1
6*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - (
(4*b^3 - 16*a*b*c)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 +
 (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c
 - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c -
b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1
/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^
3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - ((2*a*c - b^2)*(((4*b^3 -
16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2
*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/
2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*
(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 20
8*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 -
512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*
c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/
2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*b^2*(2*a*c - b^2)^4)/(4*c^4*(4*a*c -
 b^2)^2)))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b
^6*c)))*(2*a*c - b^2))/(4*c^2*(4*a*c - b^2)^(1/2))
```

3.311 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 192

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x^2/c-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1373, 1136, 1180, 211}

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

```
[In] Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}-\frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4c}\\
&\quad-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4c}
\end{aligned}$$

$$= \frac{x^2}{2c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \frac{2\sqrt{c}x^2 - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x^2}{2c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8b^2a\,c^2+b^4c\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+a^3c^2\right)} \_R\ln\left(\left(-a^2c^2+a\,b^2c\right)x^2+\left(-4ab\,c^2+b^3c\right)\_R^3+(2a\right.}{4c}$
default	$\frac{x^2}{2c} + \frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

```
[In] int(x^9/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c+1/4/c*sum(_R*ln((-a^2*c^2+a*b^2*c)*x^2+(-4*a*b*c^2+b^3*c)*_R^3+(2
*a^2*c^2-4*a*b^2*c+b^4)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4+(
12*a^2*b*c^2-7*a*b^3*c+b^5)*_Z^2+a^3*c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(150) = 300$.

Time = 0.26 (sec) , antiderivative size = 1071, normalized size of antiderivative = 5.58

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-(ab^2-a^2c)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\right.\right.}{}$$

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 -
 a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
- 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a
*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 -
 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b
^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(4096a^2c^5-2048ab^2c^4+256b^4c^3\right)+t^2\cdot\left(192a^2bc^2-112ab^3c+16b^5\right)+a^3,\left(t\mapsto t\log\left(x^2\right.\right.\right.$$
$$+\frac{x^2}{2c}$$

[In] integrate(x**9/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 + (256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c - 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)

Maxima [F]

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \int \frac{x^9}{cx^8+bx^4+a}\,dx$$

[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2043 vs. 2(150) = 300.

Time = 1.92 (sec) , antiderivative size = 2043, normalized size of antiderivative = 10.64

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] 1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b
^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 +
 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 -
 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 -
4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^
2*c - 8*(b^2 - 4*a*c)*a^2*c^2)*abs(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + s
qrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*
a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2) + 1/8*(2*a*b^3*c^3 - 8*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 -
 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^
4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*ab
s(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3
 - 4*a^2*c^4)*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 5659, normalized size of antiderivative = 29.47

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^9/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*
c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3
*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^
5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^
4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5
*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c +
 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^
6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^
4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^
2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4
*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^
8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16
*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 -
((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*
```

b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^
4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2) + (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16
*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4
+ 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2
c^8)(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
c - b^2)^3)^(1/2))/(32(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x
^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^
5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i + atan(((((16*(a*b^8 + 4*a^5*c^4 -
8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 -
256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6
- 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3
*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*
b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)
(-(b^5 - b^2(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3)
)/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3
*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^
(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*

c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2
*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3
*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*
a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 -
256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^
7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8
*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(
-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*
a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2
*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*(a*b^8 + 4*a
^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*
b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a
*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c
^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3
 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c
^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
+ a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^
(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*2i + x^2/(2*c)

3.312 $\int \frac{x^7}{a+bx^4+cx^8} dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^7}{a+bx^4+cx^8} dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log(a+bx^4+cx^8)}{8c}$$

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1371, 648, 632, 212, 642}

$$\int \frac{x^7}{a+bx^4+cx^8} dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log(a+bx^4+cx^8)}{8c}$$

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c}\\
&= \frac{\log\left(a+bx^4+cx^8\right)}{8c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^4 + c*x^8])/(8*c)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^8+b\,x^4+a\right)}{8c}-\frac{b\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^4+2\sqrt{-b^2(4ac-b^2)}\,a\right)a}{8ac-2b^2}-\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^4+2\sqrt{-b^2(4ac-b^2)}\,a\right)b^2}{8c(4ac-b^2)}+$

```
[In] int(x^7/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c
-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)}{8\,(b^2c-4\,ac^2)},\frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{ar}\right.$$

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 1.50 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)} + \frac{1}{8c}\right)\log\left(x^4 + \frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)} + \frac{1}{8c}\right)\log\left(x^4 + \frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

```
[In] integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c
*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(-b
*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2
)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = -\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
8*log(c*x^8 + b*x^4 + a)/c
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 2654, normalized size of antiderivative = 42.13

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^7/(a + b*x^4 + c*x^8),x)
```

```
[Out] (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*at
an((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^
4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32
*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(1
6*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^
2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((6
4*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b
^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*
c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b
^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)*(4*a*c - b^2
)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c - b^2)^2) + b^3 -
((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))
/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*
c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4
*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*
a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b
```

```
^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b
^2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(448*b^3*c^3 -
 (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)
^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (25
6*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64
*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2))
))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/b^4 + ((4*a*c - b^2)^(
3/2)*(b^3 - 3*a*b*c)*(a*b^2 + (((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*
a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3
*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(
4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)
*(4*a*c - b^2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) + (a*b^6)/(4*(
4*a*c - b^2)^2) - ((16*a*c - 4*b^2)*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (
512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2
*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*
b^2*c))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(768*a*b^2*c^
3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c -
b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((768*a*b^2*c^3 - (512*a*b
^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*
c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c
- b^2)^(1/2))))/(a^3*b^4*c^2) + ((a*c - b^2)*(4*a*c - b^2)^2*(((((16*a*c -
4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16
*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*
a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((
768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16
*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^
2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(768*a*
b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*
a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*
(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4
*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c - b^2)^(1/2)) +
(b*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(
64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b
^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c))/(8*c*(4*a*c - b^2)^(1/2))
 + (a*b^5*c*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2))))
/(a^3*b^4*c^2)))/(4*c*(4*a*c - b^2)^(1/2))
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8} dx$

Optimal result

Integrand size = 18, antiderivative size = 159

$$\int \frac{x^5}{a+bx^4+cx^8} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/4*arctan(x^2*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c
^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1373, 1144, 211}

$$\int \frac{x^5}{a+bx^4+cx^8} dx = \frac{\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2
```

```
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)\\
&\quad+\frac{1}{4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$=\frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[In] Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8b^2a\,c^2+b^4c\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+a\right)}\_R\ln\left(\left(\left(-4a\,c^2+b^2c\right)\_R^2+b\right)x^2+\left(4ab\,c^2-b^3c\right)\_R^3+\left(2ac-b^2\right)\_\right.\right.}{4}$
default	$2c\left(\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$

```
[In] int(x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(((-4*a*c^2+b^2*c)*_R^2+b)*x^2+(4*a*b*c^2-b^3*c)*_R^3+(2*a*c-b
^2)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4+(-4*a*b*c+b^3)*_Z^2+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(119) = 238.

Time = 0.26 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.57

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)$$

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
```

```
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b^2\right.\right.\right.$$

```
[In] integrate(x**5/(c*x**8+b*x**4+a),x)
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64
*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*
c - 4*_t*b + x**2)))
```

Maxima [F]

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx=\int \frac{x^5}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(119) = 238.

Time = 2.04 (sec) , antiderivative size = 1034, normalized size of antiderivative = 6.50

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$=\frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{\right.}{}$$
$$+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c+2\,b^4c+16\,\sqrt{2}\right.}{}$$

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^
3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(
b^2 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))
/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c
^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 1220, normalized size of antiderivative = 7.67

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$= \operatorname{atan}\left(\frac{}{8\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}}+128\,b^5\,c\,\left(\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{512\,a^2\,c^3-256\,a\,b^2\,c^2+32\,b^4\,c}\right)^{3/2}+64\right.$$

```
[In] int(x^5/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*x
^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)
 + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3
 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*((
```

```
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32
*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 51
2*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 -
 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2)
)^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i - atan((x^2*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3*x^2*1i + a*b*c
*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) + 128*b^5*c
*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)
/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*(-(b^3 + (b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 5
12*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 48*a*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 2
56*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c
^2))^(3/2) + 2048*a^2*b*c^3*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2)
))*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1366, 632, 212}

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

```
[In] Int[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \frac{\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}$$

```
[In] Integrate[x^3/(a + b*x^4 + c*x^8),x]
[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^4-2a\right)}{4\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^4+2a\right)}{4\sqrt{-4ac+b^2}}$	70

```
[In] int(x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}},\ -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(b^2-4\,ac)}\right]$$

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] [1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72.

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

```
[In] integrate(x**3/(c*x**8+b*x**4+a),x)
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.84

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = -\frac{\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^2\left(\frac{\left(\frac{4\,a\,c^4}{4\,a\,c-b^2}-\frac{4\,a\,b^2\,c^4}{(4\,a\,c-b^2)^2}\right)(b^3-3\,a\,b\,c)}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}-x^4\left(\frac{\left(\frac{2\,c^4}{\sqrt{4\,a\,c-b^2}}-\frac{6\,b^2\,c^4}{(4\,a\,c-b^2)^{3/2}}\right)(a\,c-b^2)}{8\,a^3\,c^2}-\frac{(b^3-3\,a\,b\,c)\left(\frac{6\,b\,c^4}{4\,a\,c-b^2}-\frac{2\,b^3\,c^4}{(4\,a\,c-b^2)^2}\right.}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}\right.\right.}{2\,c^4}\right.}{2\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^3/(a + b*x^4 + c*x^8),x)
```

```
[Out] -atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c -
b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)/(
```

```
4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^3*c
^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c - b^2)
^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*c - b
^2)^(3/2))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))
```

3.315 $\int \frac{x}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2
)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x^2*2^(1/2)*c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1373, 1107, 211}

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^2\right) \\
&= \frac{c\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}} \\
&= \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$
\int \frac{x}{a+bx^4+cx^8}\,dx = \frac{\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}
$$

```
[In] Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
- Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^3c^2-8a^2b^2c+b^4a\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+c\right)}\_R\ln\left(\left(\left(4abc-b^3\right)\_R^2-c\right)x^2+\left(4cb\,a^2-a\,b^3\right)\_R^3-2ac\_R\right)\right)}{4}$
default	$2c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$

```
[In] int(x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(((4*a*b*c-b^3)*_R^2-c)*x^2+(4*a^2*b*c-a*b^3)*_R^3-2*a*c*_R),_
R=RootOf((16*a^3*c^2-8*a^2*b^2*c+a*b^4)*_Z^4+(-4*a*b*c+b^3)*_Z^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(119) = 238.

Time = 0.28 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.02

$$\int \frac{x}{a+bx^4+cx^8}\,dx = -\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c
 - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/
2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqr
t(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/
4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.57

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-\right.\right.\right.$$

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64
*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t
**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\int \frac{x}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(119) = 238.

Time = 1.89 (sec) , antiderivative size = 1028, normalized size of antiderivative = 6.68

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$
$$=\frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{}\right.}{}$$
$$+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c+2\,b^4c+16\,\sqrt{2}\right.}{}$$

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
```

```
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*
a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*a
bs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
- 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 1105, normalized size of antiderivative = 7.18

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$
$$=\operatorname{atan}\left(\frac{b^4\,x^2}{128\,a^2\,b^5\left(-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}\right)^{3/2}-64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4}{512\,a^3\,c^2-256\,a^2\,b^2\,c+32\,a\,b^4}}}\right.$$

```
[In] int(x/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3
*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2
*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/
2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*
```

```
a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b^3 + (b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512
*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^
2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*
c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(((b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4
+ 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*(((b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c
^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^
2*b^2*c))^(3/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
- b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log(a+bx^4+cx^8)}{8a}$$

```
[Out] ln(x)/a-1/8*ln(c*x^8+b*x^4+a)/a+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{\log(a+bx^4+cx^8)}{8a}+\frac{\log(x)}{a}$$

```
[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}
\end{aligned}$$

$$= \frac{\log(x)}{a} - \frac{\log(a + bx^4 + cx^8)}{8a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx^4\right)}{4a}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx^4 + cx^8)}{8a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a + bx^4 + cx^8)}\,dx$$

$$= \frac{\log(x)}{a} - \frac{\text{RootSum}\left[a + b\#1^4 + c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

```
[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\frac{\ln(c\,x^8+b\,x^4+a)}{4} + \frac{b\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}}{2a}$	66
risch	$\frac{\ln(x)}{a} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)} \_R\ln\left(\left(\left(18ac-5b^2\right)\_R+9c\right)x^4-ab\_R+4b\right)\right)}{4}$	77

```
[In] int(1/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-1/2/a*(1/4*ln(c*x^8+b*x^4+a)+1/2*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)+8\,(b^2-4\,ac)\log}{8\,(ab^2-4\,a^2c)}\right.$$

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.

Time = 99.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{8a\,(4ac-b^2)} - \frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{8a\,(4ac-b^2)} - \frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right) + \frac{\log(x)}{a}$$

```
[In] integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-b*
```

```
sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*s
qrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx=-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{\log\left(x^4\right)}{4\,a}$$

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 1690, normalized size of antiderivative = 24.49

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*
c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*
```

a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c - 4*b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*b^2)^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4*b^2)*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a*c - 4*b^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*c^8*(81*a*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*(((576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)) - (3*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5*c^4*(81*a*c - 20*b^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*b^4*c)*((b^5*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(32768*a^5*(4*a*c - b^2)^(5/2)) - (3*b^3*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^2)/(1024*a^3*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2)) + (b*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^4)/(128*a*(16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^3)/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(81*a*c - 20*b^2)))*(4*a*c - b^2)^(5/2))/(b^4*c^4) + (4*(4*a*c - b^2)^(5/2)*(5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*(((16*a*c - 4*b^2)^4*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(4096*a^4*(4*a*c - b^2)^2) - (b^8*c^4*(16*a*c - 4*b^2))/(8*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2) + (2*b^6*c^4*(16*a*c - 4*b^2)^3)/(a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)) - (3*b^2*(16*a*c - 4*b^2)^2*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(128*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2))))/(b^4*c^8*(81*a*c - 20*b^2))))/(4*a*(4*a*c - b^2)^(1/2))

3.317 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 184

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx=-\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1373, 1137, 1180, 211}

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx=-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*1/(a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}\\
&\quad -\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}
\end{aligned}
$$

$$=-\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx=-\frac{1}{2ax^2}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

```
[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2c\left(\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{1}{2a\,x^2}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(-72a^5c^2+38a^4b^2c-5b^4a^3\right)\_R^4+(-\right.\right.\right.}{4}$

```
[In] int(1/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*c*(1/8*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
-1/8*(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/2
/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.16

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)}\,dx = -\frac{\sqrt{\frac{1}{2}}ax^2\sqrt{-\frac{b^3-3\,abc+(a^3b^2-4\,a^4c)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\log\left(-(b^2c^2-ac^3)x^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-(a^3b^4\right.\right.}{}$$

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2
*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a
^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sq
rt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c +
 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
 - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^
2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 149.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot (4096a^5c^2 - 2048a^4b^2c + 256a^3b^4) + t^2 \cdot (192a^2bc^2 - 112ab^3c + 16b^5) + c^3, \left(t \mapsto t \log\left(x^2\right.\right.\right.$$
$$- \frac{1}{2ax^2}$$

```
[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 +
(-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*
a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*
x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(141) = 282.

Time = 1.87 (sec) , antiderivative size = 2055, normalized size of antiderivative = 11.17

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqr
```

t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 -
 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*
b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2
- 4*a*c)*a*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 -
 4*a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a
^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) + 1/8*(2*a*b^4*c^2 - 8*a
^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 -
4*a*c)*a*b^2*c^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^
2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 - (sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5
*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a
))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^
2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2
- 4*a^3*c^3)*abs(a)*abs(c)) - 1/2/(a*x^2)

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 5451, normalized size of antiderivative = 29.62

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2
*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 245
76*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x
^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a
^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 +
65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11
*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^1
1*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^1
0*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a
^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c
^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
```

3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*
c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4
*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3
)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^
4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024
*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 51
2*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 +
448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan((((64*a^10*
c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
c(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384
*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*
(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 40
96*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 -
 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^
9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
 a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
) - ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^
4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6
) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a
^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c

```
)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(
512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
 b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b
^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^
5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 6553
6*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5
*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^
3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^
2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1
/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4
*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 92
16*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3
072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 -
896*a^10*b^2*c^7))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(32*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(2*a*x^2)
```

3.318 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx=-\frac{1}{4ax^4}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}$$

```
[Out] -1/4/a/x^4-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/4*(-2*a*c+b^2)*arctanh
((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx=-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

```
[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/4*1/(a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(
4*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, x^4\right)$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^4\right)}{4a}$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)dx, x, x^4\right)}{4a}$$
$$= -\frac{1}{4ax^4} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a^2}$$
$$= -\frac{1}{4ax^4} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a^2}$$
$$= -\frac{1}{4ax^4} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^4+cx^8)}{8a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a^2}$$
$$= -\frac{1}{4ax^4} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^4+cx^8)}{8a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx = -\frac{1}{4ax^4} - \frac{b\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a^2}$$

```
[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
[Out] -1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2
)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(18a^{3}c-5a^{2}b^{2}\right)\_R^{2}-8\_Rabc+4c^{2}\right)x^{4}-a^{3}b\right.\right.}{4}$

```
[In] int(1/x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4-b*ln(x)/a^2-1/2/a^2*(-1/4*b*ln(c*x^8+b*x^4+a)+(a*c-1/2*b^2)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)+}{8\,(a^2b^2-4\,a^3c)x^4}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^4\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)+8\,(b^3-4}{8\,(a^2b^2-4\,a^3c)x^4}$$

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2
 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \frac{b \log(cx^8 + bx^4 + a)}{8 a^2} - \frac{b \log(x^4)}{4 a^2} + \frac{(b^2 - 2 ac) \arctan\left(\frac{2 cx^4 + b}{\sqrt{-b^2 + 4 ac}}\right)}{4 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^4 - a}{4 a^2 x^4}$$

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x
^4 - a)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 8817, normalized size of antiderivative = 99.07

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

[In] int(1/(x^5*(a + b*x^4 + c*x^8)),x)

[Out] (atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a*b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + (((a

^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*c - b^2)^5)/(128*a^9*(4*a*c - b^2)^(5/2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4 - 8*a*b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)) - (128*a^10*x^4*(((5*b^6 - a^3*c^3 + 26*a^2*b^2*c^2 - 25*a*b^4*c)*(c^9/a^5 + ((4*b^3 - 16*a*b*c)*((20*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c -

b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((((2*a*c - b^2)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024*a^11*(4*a*c - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^4*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(8192*a^13*(4*a*c - b^2)^2*(64*a^3*c - 16*a^2*b^2))))/(32*a^5*c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)) + ((5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a*b^5*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024*a^11*(4*a*c - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((20*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^

3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2
)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b
^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a
*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 -
4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64
*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4
*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608
*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*
c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 460
8*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3
 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c
)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/
(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(
8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - ((
2*a*c - b^2)^5*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(32768*a^15*(4*a*c -
b^2)^(5/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 20*b^4 + 80*a*b^2*
c)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^8 + b^8*c^4 - 8*a*b^6*c^5 + 24*a^2*b^4*
c^6 - 32*a^3*b^2*c^7) + (4*a^5*(4*a*c - b^2)^(5/2)*(5*b^6 - a^3*c^3 + 26*a^
2*b^2*c^2 - 25*a*b^4*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
c^4(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3
 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2))))
/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 -
256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^
2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 1
6*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(
2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4
*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*
(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (b*c^8)/a^5 + ((4*b^3 - 16*a*b*
c)*((a^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*
a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/
a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a
*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^
2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(6
4*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256
*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(6
4*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4
c^4(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 1
6*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2
*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 -

```
(128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c -
 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c -
16*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*
((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5
*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c
- 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*
(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*a^2*(4*a*c -
b^2)^(1/2)) - ((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*
c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a
*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
- b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)*
(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (b^4
*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^(3/2)*(64*a^3
*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4
*b^3 - 16*a*b*c)*(2*a*c - b^2)^4)/(32*a^7*(4*a*c - b^2)^2*(64*a^3*c - 16*a^
2*b^2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4 - 8*a*
b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)))*(2*a*c - b^2))/(4*a^2*(4*a*c -
 b^2)^(1/2)) - (b*log(x))/a^2 - (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))
/(2*(64*a^3*c - 16*a^2*b^2)) - 1/(4*a*x^4)
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 381

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/3*x^3/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-
2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4
)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*a
rctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a
*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctanh(2^
```

```
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1381, 1524, 304, 211, 214}

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = -\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{x^3}{3c}$$

```
[In] Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx}{3c} \\
&= \frac{x^3}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} \\
&= \frac{x^3}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} \\
&\quad + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}c^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^3}{3c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\
&- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} \\
&+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\
&+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$
\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{4x^3 - 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{12c}
$$

```
[In] Integrate[x^10/(a + b*x^4 + c*x^8),x]
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{x^3}{3c} - \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)} \frac{(\_R^6b+\_R^2a)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{4c}$	63
risch	$\frac{x^3}{3c} + \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)} \frac{(-\_R^6b-\_R^2a)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{4c}$	65

```
[In] int(x^10/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7003 vs. $2(299) = 598$.

Time = 1.02 (sec) , antiderivative size = 7003, normalized size of antiderivative = 18.38

$$\int \frac{x^{10}}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{a + bx^4 + cx^8} \, dx = \text{Timed out}$$

```
[In] integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] 1/3*x^3/c - integrate((b*x^6 + a*x^2)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 12709, normalized size of antiderivative = 33.36

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^10/(a + b*x^4 + c*x^8),x)
[Out] atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3
*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96
*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 +
2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((
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8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((8192*a^6*
b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^
3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((8192*a^6*b*c^6 - 256*
a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^1
0)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^
2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*
b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^

(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5

+ 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*2i + 2*atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*

(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*
a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((8192*a^6*b*c^6 - 256*a^3*b^7*c
^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*
(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/
c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^
2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2
))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 256
0*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^
6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(
b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a
^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*
(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*
b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 25
6*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4
*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^
7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8
 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^
2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^

11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + 2*atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4))/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b

^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9
- 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*
c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 1
6*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i - (4*x*(a^5*b^5
 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2)
- 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7
- 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (((8192*a^
6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(
-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^
5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4
*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
+ 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b
^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4)*1i + (4*x*(a^5*b^5 - 5*
a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*
a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i + (2*(a^8*c - a^7
b^2))/c^3))(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + x^3/(3*c)

3.320 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 376

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1381, 1436, 218, 214, 211}

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{x}{c}$$

```
[In] Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} \\
&= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}}
\end{aligned}$$

$$= \frac{x}{c} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

$$+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

$$+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

```
[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59

```
[In] int(x^8/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(296) = 592$.

Time = 0.41 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*
x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqr
t(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 +
 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^
4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^
3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(-sqrt(1/2
```

)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c
^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b
^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)))) - c*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c +
a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 -
 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*
c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt
(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^
6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^
2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))) + c*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*

```
b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b
^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c
^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(-sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13
*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
) + 4*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^8/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
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^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/

(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^
9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*
a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(
4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
c^2(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a
^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*
a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 +
 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b
^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c +
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6
 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
c^8)))^(3/4)(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120

*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2 *c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1

```
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

3.321 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 325

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}} - \frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^
2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/
c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2
)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+
b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1388, 304, 211, 214}

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} - \frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[In] Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2
 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]
)/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3
/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)\\
&\quad+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\\
&= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}\\
&\quad-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}\\
&= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\
&\quad+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\
&\quad+\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\\
&\quad-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1^3}{b+2c\#1^4}\&\right]$$

```
[In] Integrate[x^6/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4}$	43
risch	$\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4}$	43

```
[In] int(x^6/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4433 vs. 2(245) = 490.

Time = 0.33 (sec) , antiderivative size = 4433, normalized size of antiderivative = 13.64

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c
^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)
*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4
 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c +
```

a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt
(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 1
2*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c +
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4
+ 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a
^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128
*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2
*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b
^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5
)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt
(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 -
 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a
^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-sqrt(1/2)*
sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*
c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*
c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*
c^8 - 64*a^3*c^9)))*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*
sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqr
t(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*
b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^

4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{a + bx^4 + cx^8} dx = \text{Timed out}$$

```
[In] integrate(x**6/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^6}{a + bx^4 + cx^8} dx = \int \frac{x^6}{cx^8 + bx^4 + a} dx$$

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6}{a + bx^4 + cx^8} dx = \int \frac{x^6}{cx^8 + bx^4 + a} dx$$

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 8033, normalized size of antiderivative = 24.72

$$\int \frac{x^6}{a + bx^4 + cx^8} dx = \text{Too large to display}$$

```
[In] int(x^6/(a + b*x^4 + c*x^8),x)
[Out] atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 +
256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b
```

^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x
*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))
)^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5
*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 25
6*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^
2*c^6)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4
096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4
*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4
)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384
*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4)*2i + atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^
(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(
-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 1
6384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c

- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*
c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*
(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*
a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*
b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a
^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
+ 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2)
^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 2
56*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4
- x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 1
6*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 20
48*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b
^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^
5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*
c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 - b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^
2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c
^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^
6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*
b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3

- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 + b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2
048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32
768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c
- 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40
*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7
+ b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-
(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*
b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^
6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b
^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))
^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4
*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(
1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^
5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*
1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4)*1i + 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) -
48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c
^6)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3
+ 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*
(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*
c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a
c - b^2)^5)^(1/2))/(512(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4
*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a
^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48

*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + 2*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)

3.322 $\int \frac{x^4}{a+bx^4+cx^8} dx$

Optimal result

Integrand size = 18, antiderivative size = 325

$$\int \frac{x^4}{a+bx^4+cx^8} dx = \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2
)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1/
4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c
^(1/4)/(-4*a*c+b^2)^(1/2)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b
^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1388, 218, 214, 211}

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[In] Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right) \\
&\quad+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx \\
&= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}} \\
&\quad+\frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}} \\
&\quad-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}} \\
&\quad-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{b^2-4ac}}
\end{aligned}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1}{b+2c\#1^4}\&\right]$$

```
[In] Integrate[x^4/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

```
[In] int(x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2141 vs. 2(245) = 490.

Time = 0.30 (sec) , antiderivative size = 2141, normalized size of antiderivative = 6.59

$$\int \frac{x^4}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(
-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-
(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
- 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)*s
qrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sq
rt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^
5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*
```

```
a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)
*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(-sqrt(1/2)
*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x
- (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)*sqrt(-(b - (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5))
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx$$
$$= \mathrm{RootSum}\left(t^8\cdot\left(16777216a^4c^5-16777216a^3b^2c^4+6291456a^2b^4c^3-1048576ab^6c^2+65536b^8c\right)+t^4\cdot\left(4096\right.\right.$$

```
[In] integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*
b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 +
16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

Maxima [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \int \frac{x^4}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \int \frac{x^4}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 8169, normalized size of antiderivative = 25.14

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int(x^4/(a + b*x^4 + c*x^8),x)
[Out] - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^
4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b
*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^5 + (-(4*a*
c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 40
96*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c
^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) +
x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^
```

3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5

- 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - atan(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i)/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51

2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))
^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + ((((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26214
4*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*
(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c
- b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*
a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a
^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a
^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((((-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a
^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^
6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*
a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i
+ x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4) - ((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4
*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*
a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^
4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a
^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c -
b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*
a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))^(1/4))/(((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*
c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 19660
8*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^
5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8

*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)
*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16
*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^
5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (((
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(26
2144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1
i - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c
^4 + 16*a^2*b^3*c^3)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))^(1/4)*1i - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 - (-(4*a*c - b
^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b
^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i))*(-(b^5 - (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)

3.323 $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = -\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4
)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctanh(2^(1
/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1/2)
)^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4
)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1
/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1389, 304, 211, 214}

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = -\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[In] Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{c\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}}\\
&\quad - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} + \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}}\\
&= -\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\
&\quad + \frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

$$
\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1+2c\#1^5}\&\right]
$$

```
[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

```
[In] int(x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3193 vs. 2(245) = 490.

Time = 0.29 (sec) , antiderivative size = 3193, normalized size of antiderivative = 10.14

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] -1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*
a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 -
8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(
1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sq
rt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1
```

/4*sqrt(-sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a
^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 -
 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b
*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-sqr
t(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*
sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) -
 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*
a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 -
8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(
1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sq
rt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1
/4*sqrt(-sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a
^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b - (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 -

```
 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b
*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-sqr
t(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*
sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x)
```

Sympy [A] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(16777216a^5c^4-16777216a^4b^2c^3+6291456a^3b^4c^2-1048576a^2b^6c+65536ab^8\right)+t^4\cdot(40\right.$$

```
[In] integrate(x**2/(c*x**8+b*x**4+a),x)
[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*
b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b
*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**
7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

Maxima [F]

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \int \frac{x^2}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \int \frac{x^2}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 6067, normalized size of antiderivative = 19.26

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

[In] int(x^2/(a + b*x^4 + c*x^8),x)

[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 2*a*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/

4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + 2*a*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*2i - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6

- 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a
^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
- 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)
- 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(1/4) + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*
a^4*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a
^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4
*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2
*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^
2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c
^5 - 32768*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*
c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4))/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a
^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 3
2768*a^3*b^2*c^6)*1i - 256*a*b^5*c^4 - 4096*a^3*b*c^6 + 2048*a^2*b^3*c^5)*1
i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
c)/(512(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2
*c^3)))^(1/4)*1i - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*

b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(x*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i + 256*a*b^5*c^4 + 4096*a^3*b*c^6 - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + 2*a*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)

3.324 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 315

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] 1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)
/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(3/4)*arctanh(2^(1/
4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))
^(3/4)/(-4*a*c+b^2)^(1/2)-1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
-1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/
4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1361, 218, 214, 211}

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[In] Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}}\\
&\quad - \frac{c\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{c\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}}\\
&= \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\\
&\quad + \frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1^3+2c\#1^7}\&\right]$$

```
[In] Integrate[(a + b*x^4 + c*x^8)^(-1),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40

```
[In] int(1/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. 2(245) = 490.

Time = 0.33 (sec) , antiderivative size = 3125, normalized size of antiderivative = 9.92

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c
^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^
2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))
) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b
^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c -
 a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2))) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c +
```

16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^
2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c
+ 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*lo
g(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c +
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8
*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b
*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3
*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)))) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^
3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-sqrt(1/2)*sqrt(-(b
^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-sqrt(1/2)*sq

```
rt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\text{Timed out}$$

```
[In] integrate(1/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\int \frac{1}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\int \frac{1}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 10337, normalized size of antiderivative = 32.82

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

```
[In] int(1/(a + b*x^4 + c*x^8),x)
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
```

4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144
*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11
264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b
^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b
^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6)
 - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7
 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49
152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 -
 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(6
4*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*
c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*

b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c +

a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*2i - 2*atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*

c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 + b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*
a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 +
196608*a^3*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^
7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^
3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)
*1i - 64*a*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))^(1/4)*1i + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
- 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7
*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96
*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*
c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 + b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a
c(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) - 2*atan((((-(b^7 - b^2*(-(4*a*c -
b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^
2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))
)^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3
*b^3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*
a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a
^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a
*c^7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48
*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(5

12*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3
)))^(1/4) - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
+ 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*
c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*
a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
- 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4))/(((-(b^7 - b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*
b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(
1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^
3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2
*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^
7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^
3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))
^(1/4)*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*
b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(((-(b^7 -
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
+ a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
+ 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*
c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264*
a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b
^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
- 256*a^6*b^2*c^3)))^(3/4)*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7
 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i))*(-(b^7 - b^2*(-(4*a*c -
b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2
 - 256*a^6*b^2*c^3)))^(1/4)

3.325 $\int \frac{1}{x^2(a+bx^4+cx^8)} dx$

Optimal result

Integrand size = 18, antiderivative size = 363

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = &-\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\ &-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\ &+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\ &+\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] -1/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4
)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c+b^2
)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arctan(2^(1/4)
*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/
a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(
```

```
-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1382, 1524, 304, 211, 214}

$$\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx = -\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}} - \frac{1}{ax}$$

```
[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{ax}+\frac{\int \frac{x^2(-b-cx^4)}{a+bx^4+cx^8}\,dx}{a}\\
&= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}
\end{aligned}
$$

$$= -\frac{1}{ax} + \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} + \frac{\left(\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a}$$

$$= -\frac{1}{ax} - \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

```
[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6c+b\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a^9c^4-256b^2c^3a^8+96b^4c^2a^7-16b^6ca^6+b^8a^5\right)\_Z^8+\left(80a^4bc^4-120a^3b^3c^3+61a^2b^5c^2-13cb^7a+b^9\right)\_Z^4+c^5\right)}\right.}{}$

```
[In] int(1/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5758 vs. 2(281) = 562.

Time = 0.53 (sec) , antiderivative size = 5758, normalized size of antiderivative = 15.86

$$\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx = \int \frac{1}{(cx^8 + bx^4 + a)x^2} dx$$

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] -integrate((c*x^6 + b*x^2)/(c*x^8 + b*x^4 + a), x)/a - 1/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx = \int \frac{1}{(cx^8 + bx^4 + a)x^2} dx$$

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 10509, normalized size of antiderivative = 28.95

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + b*x^4 + c*x^8)),x)
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b
^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2
816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c
^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
```

a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*
c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816
*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*
x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2

```
*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 25
6*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768
*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 8
1920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*
c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6
*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
- 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13
*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 28
16*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x
)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a
^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)
^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8
 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15
*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 143
36*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) +
80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5
)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 +
256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((
-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*
a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*
b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 - b^4*(-(4*a*c
 - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^
2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 5
```

1200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*
c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 - b
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c
^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - atan((
((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^
7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 +
51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^
6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 +
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
- 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 8
0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096
*a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^
12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)
+ 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^
2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i)/(((-(b^9 +
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2
- 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)
^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*
a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^
14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 1

0496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(

4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 1/(a*x)

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 365

$$\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx = -\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

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[Out] -1/3/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c
+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctan(2^(
1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3
/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*
c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1382, 1436, 218, 214, 211}

$$\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx = \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{1}{3ax^3}$$

```
[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]

[Out] -1/3*1/(a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^
(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) +
 (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/
4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx}{3a}\\
&= -\frac{1}{3ax^3}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}\\
&= -\frac{1}{3ax^3}+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b+\sqrt{b^2-4ac}}}\\
&\quad+\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b+\sqrt{b^2-4ac}}}
\end{aligned}$$

$$= -\frac{1}{3ax^3} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{3ax^3} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4a}$$

```
[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
[Out] -1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.17

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{1}{3a\,x^{3}}$
risch	$-\frac{1}{3a\,x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256c^{4}a^{11}-256a^{10}b^{2}c^{3}+96a^{9}b^{4}c^{2}-16a^{8}b^{6}c+a^{7}b^{8}\right)\_Z^{8}+\left(-112b\,c^{5}a^{5}+280b^{3}c^{4}a^{4}-231c^{3}b^{5}a^{3}+86b^{7}c^{2}a^{2}-15b^{9}ca+\right.\right.}\right.}{}$

```
[In] int(1/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a)
)-1/3/a/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5030 vs. $2(281) = 562$.

Time = 0.62 (sec) , antiderivative size = 5030, normalized size of antiderivative = 13.78

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} \, dx$$

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)/(c*x^8 + b*x^4 + a), x)/a - 1/3/(a*x^3)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} \, dx$$

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 16497, normalized size of antiderivative = 45.20

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4
 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11
 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*
b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5
+ 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i
```

- 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b

^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(512(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1
/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 -
a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^
2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*
c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*
a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 -
122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c
^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c
- b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a
*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*
c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13
*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*
1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
- 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*
b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2
)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3
*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 25
6*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))
(-(b^11 + b^6(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 -
231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a
*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 2
56*a^10*b^2*c^3)))^(1/4) - atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) -
 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c
^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 -
 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*
b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 12288
0*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2
)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*
c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b
^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a
^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*
c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^

10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a

^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 +
a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11
*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920
*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 -
122880*a^14*b^3*c^7) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c
- b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*
a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^1
3*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)
)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 -
231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*
a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c
^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a
^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c
^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 -
b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5
*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*
a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(1/4)*2i - atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 +
1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b
^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(512(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53
248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^
6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c
^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^
2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c

^3)))^(3/4) - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^1
0*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*
a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*
c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752
*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 -
96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a
^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^
8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4
)*1i)/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
- 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*
c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 -
13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 +
b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^
5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6
*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^
2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 +
245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c
- b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a
^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 128*a
^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*
c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c

^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^
2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(
1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 1
6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*
c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*
a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86
*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^
5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*
a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^
4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^1
1 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3
*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c
 + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10
*b^2*c^3)))^(3/4) + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*
(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112
*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11 + b^6*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 2
80*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(1/4)*2i + 2*atan(-(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5
 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c -
b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
 + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a
^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7
) - (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 1
5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^
14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(
-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)

))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*
a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^
5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4
*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
+ 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8
*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 - b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*
b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*
a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*
c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752
*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
c(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5
*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4))/(((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a
^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)
^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9
*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(
b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*
a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9
*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*
c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a
^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^
8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4
)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^
10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5

+ 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + (-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - 1/(3*a*x^3)

3.327 $\int \frac{x^m}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(1+I*3^(1/2)))/(1+
m)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*
x^4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1389, 371}

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[In] Int[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 - I*Sqr
t[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
```

```
(1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1
 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\int \frac{x^m}{\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}} + \frac{i\int \frac{x^m}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}$$

$$= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.06 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.84

$$\int \frac{x^m}{1+x^4+x^8}\,dx$$

$$= \frac{x^m\left(-\frac{i\left(\left(\frac{x}{-\sqrt[3]{-1}+x}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{\sqrt[3]{-1}}{\sqrt[3]{-1}-x}\right)+\left(\frac{x}{-(-1)^{2/3}+x}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{(-}{(-1}\right.\right.}{}\right.}{}$$

```
[In] Integrate[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) - x)]/
(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-1)
```

```
^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m, (-1
)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^(1/3) + x))^m - Hypergeometric2F1[-m, -m
, 1 - m, (-1)^(2/3)/((-1)^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3] + Ro
otSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/
((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m*x^2 +
m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#
1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*#1^3) & ]/(
2 + 3*m + m^2)))/(4*m)
```

Maple [F]

$$\int \frac{x^m}{x^8+x^4+1}dx$$

[In] `int(x^m/(x^8+x^4+1),x)`

[Out] `int(x^m/(x^8+x^4+1),x)`

Fricas [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\int \frac{x^m}{x^8+x^4+1}\,dx$$

[In] `integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")`

[Out] `integral(x^m/(x^8 + x^4 + 1), x)`

Sympy [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx=\int \frac{x^m}{(x^2-x+1)\,(x^2+x+1)\,(x^4-x^2+1)}\,dx$$

[In] `integrate(x**m/(x**8+x**4+1),x)`

[Out] `Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)`

Maxima [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

```
[In] int(x^m/(x^4 + x^8 + 1),x)
[Out] int(x^m/(x^4 + x^8 + 1), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx=\frac{x^4}{4}-\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1371, 717, 648, 632, 210, 642}

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx=-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

```
[In] Int[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} - \frac{1}{8}\log\left(1+x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= \frac{x^4}{4} - \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+x^4+x^8} \, dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[In] Integrate[x^11/(1 + x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^4}{4} - \frac{\ln(x^8+x^4+1)}{8} - \frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	36
risch	$\frac{x^4}{4} - \frac{\ln(4x^8+4x^4+4)}{8} - \frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	40

```
[In] int(x^11/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8} \, dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

```
[In] int(x^11/(x^4 + x^8 + 1),x)
```

```
[Out] x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
1)/8
```

3.329 $\int \frac{x^9}{1+x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{x^2}{2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1373, 1136, 1175, 632, 210}

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{x^2}{2}$$

```
[In] Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\
&= \frac{x^2}{2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \frac{x^9}{1+x^4+x^8} dx = \frac{x^2}{2} - \frac{\left(i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}} - \frac{\left(-i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

```
[In] Integrate[x^9/(1 + x^4 + x^8),x]
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*Sq
rt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2} - \frac{\sqrt{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6} - \frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	43
risch	$\frac{x^2}{2} - \frac{\sqrt{3}\arctan\left(\frac{x^2\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\arctan\left(\frac{x^6\sqrt{3}}{3}+\frac{2x^2\sqrt{3}}{3}\right)}{6}$	44

```
[In] int(x^9/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*arctan(1/3*(2*x^2+1)*
3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{1+x^4+x^8} dx = \frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x^2\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^6+2x^2)\right)$$

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(x^6 + 2*x^2))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{x^2}{2} + \frac{\sqrt{3}\left(-2\,\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2\,\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right)\right)}{12}$$

```
[In] integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(
3)*x**2/3))/12
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

```
[In] int(x^9/(x^4 + x^8 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(
1/2)*x^2)/3)))/12
```

3.330 $\int \frac{x^7}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1371, 648, 632, 210, 642}

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[In] Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[In] Integrate[x^7/(1 + x^4 + x^8),x]
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	31
risch	$\frac{\ln\left(4x^{8}+4x^{4}+4\right)}{8}-\frac{\arctan\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	35

```
[In] int(x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**7/(x**8+x**4+1),x)
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] int(x^7/(x^4 + x^8 + 1),x)
[Out] log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

3.331 $\int \frac{x^5}{1+x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x^5}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^2+x^4\right) - \frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] 1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(
1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1373, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{x^5}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(x^4-x^2+1\right) - \frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[In] Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$
\int \frac{x^5}{1+x^4+x^8}\,dx
= \frac{\sqrt{1-i\sqrt{3}}\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{4\sqrt{6}}
$$

```
[In] Integrate[x^5/(1 + x^4 + x^8),x]
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1
 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^4+x^2+1)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$-\frac{\ln(4x^4+4x^2+4)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(4x^4-4x^2+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}$	68

```
[In] int(x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^4+x^2
+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)+\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) -\frac{1}{8}\,\log\left(x^4+x^2+1\right)+\frac{1}{8}\,\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8} +\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) + \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{8}\,\log\left(x^4+x^2+1\right) + \frac{1}{8}\,\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) + \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{8}\,\log\left(x^4+x^2+1\right) + \frac{1}{8}\,\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \mathrm{atanh}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \mathrm{atanh}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

```
[In] int(x^5/(x^4 + x^8 + 1),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.332 $\int \frac{x^3}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1366, 632, 210}

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Integrate[x^3/(1 + x^4 + x^8),x]
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19
risch	$\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19

```
[In] int(x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

```
[In] integrate(x**3/(x**8+x**4+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

```
[In] int(x^3/(x^4 + x^8 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

3.333 $\int \frac{x}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{x}{1+x^4+x^8}\,dx=-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] -1/8*ln(x^4-x^2+1)+1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^
(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1373, 1108, 648, 632, 210, 642}

$$\int \frac{x}{1+x^4+x^8}\,dx=-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[In] Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)\\
&\quad +\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^2\right)
\end{aligned}
$$

$$= -\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)-\sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

```
[In] Integrate[x/(1 + x^4 + x^8),x]
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqr
t[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$-\frac{\ln\left(4x^4-4x^2+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(4x^4+4x^2+4\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	68

```
[In] int(x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^4+x^
2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right) - \frac{1}{8}\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x}{1+x^4+x^8}\,dx = -\frac{\log\left(x^4-x^2+1\right)}{8} + \frac{\log\left(x^4+x^2+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right) - \frac{1}{8}\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right) - \frac{1}{8}\log\left(x^4-x^2+1\right)$$

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{x}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2} - \frac{x^2\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12} + \frac{1}{4}\mathrm{i}\right) + \operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2} + \frac{x^2\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{12} - \frac{1}{4}\mathrm{i}\right)$$

```
[In] int(x/(x^4 + x^8 + 1),x)
```

```
[Out] atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2)
/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

3.334 $\int \frac{1}{x(1+x^4+x^8)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^4+x^8)} \, dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$\int \frac{1}{x(1+x^4+x^8)} \, dx = -\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8+x^4+1\right) + \log(x)$$

```
[In] Int[1/(x*(1 + x^4 + x^8)),x]
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right) \\
&= \log(x) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right) \\
&= \log(x) - \frac{1}{8}\log\left(1+x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)
\end{aligned}
$$

$$= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = \frac{1}{24}\Bigg(2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 24\log(x) - \sqrt{3}\left(i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right) - \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right) - 3\log\left(1-x+x^2\right) - 3\log\left(1+x+x^2\right)\Bigg)$$

```
[In] Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 + 24*Log[x] - Sqrt[3]*(I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^2] - Sqrt[3]
*(-I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] - 3*Log[1 - x + x^2] - 3*Log[
1 + x + x^2])/24
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln(x) - \frac{\ln(x^8+x^4+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$\ln(x) - \frac{\ln(x^2-x+1)}{8} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\ln(x^4-x^2+1)}{8} - \frac{\sqrt{3}\arctan\left(\right.}{12}$

```
[In] int(1/x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(1+x^4+x^8)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right) + \log\left(x\right)$$

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(1+x^4+x^8)}\,dx = \log\left(x\right) - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(1+x^4+x^8)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(1+x^4+x^8)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\,(1+x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^8+x^4+1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] int(1/(x*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx=-\frac{1}{2x^2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1373, 1137, 1175, 632, 210}

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx=\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2x^2}$$

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{-1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= -\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} dx = \frac{1}{12}\left(-\frac{6}{x^2} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - i\sqrt{3}\log\left(i+\sqrt{3}-2ix^2\right) + i\sqrt{3}\log\left(-i+\sqrt{3}+2ix^2\right)\right)$$

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - I*Sqrt[3]*Log[I + Sqrt[3] - (2*I)*x^2] + I*Sqrt[3]*Log[-I + Sqr
t[3] + (2*I)*x^2])/12
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{1}{2x^2} - \frac{\sqrt{3}\arctan\left(\frac{x^6\sqrt{3}}{3}+\frac{2x^2\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\arctan\left(\frac{x^2\sqrt{3}}{3}\right)}{6}$	44
default	$-\frac{1}{2x^2} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} - \frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}$	57

```
[In] int(1/x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-1/6*3^(1/2)*arctan(1/3*x^6*3^(1/2)+2/3*x^2*3^(1/2))-1/6*3^(1/2)*ar
ctan(1/3*x^2*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} dx = -\frac{\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}x^2\right) + \sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}(x^6 + 2x^2)\right) + 3}{6x^2}$$

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*
(x^6 + 2*x^2)) + 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} dx = \frac{\sqrt{3} \left(-2 \operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2 \operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right)\right)}{12} - \frac{1}{2x^2}$$

```
[In] integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3
))/12 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} dx = -\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^2 + 1)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^2 - 1)\right) - \frac{1}{2 x^2}$$

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} dx = -\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^2 + 1)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^2 - 1)\right) - \frac{1}{2 x^2}$$

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3(1+x^4+x^8)}\,dx = -\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}-\frac{1}{2\,x^2}$$

```
[In] int(1/(x^3*(x^4 + x^8 + 1)),x)
[Out] - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x
^2)/3)))/12 - 1/(2*x^2)
```

3.336 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\frac{1}{4x^4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \log(x) + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] -1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4x^4} + \frac{1}{8}\log\left(x^8+x^4+1\right) - \log(x)$$

```
[In] Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\,(1+x+x^2)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{x\,(1+x+x^2)}\,dx, x, x^4\right)\end{aligned}$$

$$= -\frac{1}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{x} + \frac{x}{1+x+x^2}\right) dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} - \log(x) + \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2} dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} - \log(x) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2} dx, x, x^4\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2} dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} - \log(x) + \frac{1}{8}\log\left(1+x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4\right)$$
$$= -\frac{1}{4x^4} - \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \log(x) + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^5(1+x^4+x^8)} dx = \frac{1}{24}\left(-\frac{6}{x^4} + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 24\log(x) + \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right) + \sqrt{3}\left(i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right) + 3\log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(-I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] + 3*Log[1 - x + x^2]
 + 3*Log[1 + x + x^2])/24
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{4x^4}-\ln(x)+\frac{\ln(x^8+x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{1}{4x^4}-\ln(x)+\frac{\ln(x^2-x+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(x^4-x^2+1)}{8}-$

```
[In] int(1/x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)-3x^4\log\left(x^8+x^4+1\right)+24x^4\log\left(x\right)+6}{24x^4}$$

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
+ 1) + 24*x^4*log(x) + 6)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\log(x)+\frac{\log(x^8+x^4+1)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

```
[In] integrate(1/x**5/(x**8+x**4+1),x)
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12 - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{4x^4} + \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^4
 + 1) - 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5 (1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) + \frac{x^4-1}{4x^4} + \frac{1}{8}\log\left(x^8+x^4+1\right) - \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+x^4+x^8)}\,dx = \frac{\ln\left(x^8+x^4+1\right)}{8} - \ln(x) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

```
[In] int(1/(x^5*(x^4 + x^8 + 1)),x)
[Out] log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

3.337 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx=-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] -1/6/x^6+1/2/x^2+1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^
2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1373, 1137, 1295, 12, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx=-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

```
[In] Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{-3-3x^2}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{6}\text{Subst}\left(\int -\frac{3x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{8}\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)\\
&\quad+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{1}{6x^6} + \frac{1}{2x^2} + \frac{1}{8}\log\left(1 - x^2 + x^4\right) - \frac{1}{8}\log\left(1 + x^2 + x^4\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, -1 + 2x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 + 2x^2\right)$$

$$= -\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1 - x^2 + x^4\right) - \frac{1}{8}\log\left(1 + x^2 + x^4\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^7 (1 + x^4 + x^8)}\,dx = \frac{1}{24}\left(-\frac{4}{x^6} + \frac{12}{x^2} - 2\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right) - 3\log\left(1 - x + x^2\right) - 3\log\left(1 + x + x^2\right) + 3\log\left(1 - x^2 + x^4\right)\right)$$

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 - 2*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(
1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]] - 3*Log[1 - x + x
^2] - 3*Log[1 + x + x^2] + 3*Log[1 - x^2 + x^4])/24
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

method	result
risch	$\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6} - \frac{\ln(x^4+x^2+1)}{8} + \frac{\sqrt{3}\arctan\left(\frac{2\left(x^2+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^4-x^2+1)}{8} + \frac{\sqrt{3}\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{\ln(x^2-x+1)}{8} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} + \frac{\ln(x^4-x^2+1)}{8} + \frac{\sqrt{3}\,\text{arc}}{}$

```
[In] int(1/x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*x^4-1/6)/x^6-1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(2/3*(x^2+1/2)*3^(1/
2))+1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+x^4+x^8)} \, dx = \frac{2 \sqrt{3} x^6 \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2+1)\right) + 2 \sqrt{3} x^6 \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2-1)\right) - 3 x^6 \log\left(x^4+x^2+1\right) + 3 x^6 \log\left(x^4 -\right.}{24 x^6}$$

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(
1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 +
 1) + 12*x^4 - 4)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7 (1+x^4+x^8)} \, dx = \frac{\log\left(x^4-x^2+1\right)}{8} - \frac{\log\left(x^4+x^2+1\right)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12} + \frac{3x^4-1}{6x^6}$$

```
[In] integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x
**4 - 1)/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7 (1+x^4+x^8)} \, dx = \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2+1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2-1)\right) + \frac{3 x^4-1}{6 x^6} - \frac{1}{8} \log\left(x^4+x^2+1\right) + \frac{1}{8} \log\left(x^4-x^2+1\right)$$

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^7 (1+x^4+x^8)} \, dx = \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2 - 1)\right) + \frac{3 x^4 - 1}{6 x^6} - \frac{1}{8} \log\left(x^4 + x^2 + 1\right) + \frac{1}{8} \log\left(x^4 - x^2 + 1\right)$$

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7 (1+x^4+x^8)} \, dx = \operatorname{atanh}\left(\frac{2 x^2}{-1 + \sqrt{3}\, 1\mathrm{i}}\right) \left(\frac{1}{4} + \frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) + \operatorname{atanh}\left(\frac{2 x^2}{1 + \sqrt{3}\, 1\mathrm{i}}\right) \left(-\frac{1}{4} + \frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) + \frac{\frac{x^4}{2} - \frac{1}{6}}{x^6}$$

```
[In] int(1/(x^7*(x^4 + x^8 + 1)),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

3.338 $\int \frac{x^8}{1+x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^
2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1381, 1433, 1108, 648, 632, 210, 642}

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x$$

```
[In] Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x-\int \frac{1+x^4}{1+x^4+x^8}\,dx \\
&= x-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx \\
&= x-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} \\
&= x-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx \\
&\quad -\frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx+\frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{8\sqrt{3}} \\
&= x+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} \\
&\quad +\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&\quad +\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right) \\
&= x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) \\
&\quad +\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{i\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}+\frac{1}{24}\left(24x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(
(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[
(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x
^2] - 3*Log[1 + x + x^2])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

method	result
risch	$x-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\operatorname{RootOf}(9\_Z^4+3\_Z^2+1)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{4}+\frac{\ln(4x^2-4x+4)}{8}$
default	$x+\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-\frac{\arctan\left(2x-\sqrt{\right.}}{4}$

```
[In] int(x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*sum(_R*l
n(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(4*x^2-4*x+4)-1/12*3^(1/2)
*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.49

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)+12\,x\right) - \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)+12\,x\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}+12\,x\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)+12\,x\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + x - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)$$

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(3)
 - 3) + 12*x) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3)
 - 1)*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(sqrt
(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) + 1/24*sqrt(6)*sqrt(-I*sqr
t(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) - 1/12*
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)$$

```
[In] integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24
)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sq
rt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*lo
g(-9216*_t**5 - 8*_t + x)))
```

Maxima [F]

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \int \frac{x^8}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x +
 1) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + x - \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) - \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)$$

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(
3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x - \mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(x^8/(x^4 + x^8 + 1),x)
```

```
[Out] x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.339 $\int \frac{x^6}{1+x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{x^6}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1386, 1178, 642, 1175, 632, 210}

$$\int \frac{x^6}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}$$

```
[In] Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx + \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}$$

$$= \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + \log\left(-1+\sqrt{3}x-x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

```
[In] Integrate[x^6/(1 + x^4 + x^8),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt
[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	67
risch	$\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	68

```
[In] int(x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2-2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

```
[In] integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Maxima [F]

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \int \frac{x^6}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

$$\int \frac{x^6}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) + \operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

```
[In] int(x^6/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

3.340 $\int \frac{x^4}{1+x^4+x^8} dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{x^4}{1+x^4+x^8} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2+
x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1387, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{x^4}{1+x^4+x^8} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) - \frac{1}{8}\log\left(x^2-x+1\right) + \frac{1}{8}\log\left(x^2+x+1\right) + \frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}} - \frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

```
[In] Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx\\
&= -\left(\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\left(\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx\right)-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx\\
&\quad+\frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{8\sqrt{3}}\\
&= -\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}\\
&\quad+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)\\
&\quad-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{1}{24}\Bigg(&-2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\\
&+2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)\\
&-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\Bigg)
\end{aligned}$$

```
[In] Integrate[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt
[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] +
 3*Log[1 + x + x^2])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}+\frac{\ln(4x^2+4x+4)}{8}-$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x-\right.\right.}{12}$

```
[In] int(x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(4*x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*ln
(6*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(4*x^2+4*x+4)-1/12*arctan(1
/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$
\begin{aligned}
\int \frac{x^4}{1+x^4+x^8}\,dx =& \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)\\
&-\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)\\
&-\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}+6x\right)\\
&+\frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}+6x\right)\\
&-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\
&+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)
\end{aligned}
$$

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3) - 1)
+ 6*x) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(I*sqr
t(3) - 1) + 6*x) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*
sqrt(-I*sqrt(3) - 1) + 6*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(-I*sqrt
(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x +
1) - 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{x^4}{1+x^4+x^8}\,dx =& \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)\\ &+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)\\ &+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)\\ &+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)\\ &+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)\end{aligned}$$

```
[In] integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \int \frac{x^4}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1
) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{x^4}{1+x^4+x^8}\,dx =& -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ &- \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) \\ &+ \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) \\ &+ \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
+ 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{x^4}{1+x^4+x^8}\,dx =& -\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) \\ &- \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)\end{aligned}$$

```
[In] int(x^4/(x^4 + x^8 + 1),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

3.341 $\int \frac{x^2}{1+x^4+x^8} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{x^2}{1+x^4+x^8} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) + \frac{1}{8}\log\left(1-x+x^2\right) - \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2+
x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1387, 1108, 648, 632, 210, 642}

$$\int \frac{x^2}{1+x^4+x^8} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{8}\log\left(x^2-x+1\right) - \frac{1}{8}\log\left(x^2+x+1\right) - \frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}} + \frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

```
[In] Int[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8
 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&=-\left(\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx\right)-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&=-\left(\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx\\
&\quad+\frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx-\frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}\\
&=\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}\\
&\quad+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&=\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)\\
&\quad+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{x^2}{1+x^4+x^8}\,dx &= \frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\right.\\
&\quad-4i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-4\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)\\
&\quad\left.-4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+6\log\left(1-x+x^2\right)-6\log\left(1+x+x^2\right)\right)
\end{aligned}
$$

```
[In] Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] -
6*Log[1 + x + x^2])/48
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+1)}\_R\ln(\right.}{4}$
default	$\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+$

```
[In] int(x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(4*x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(4*x^2+4
*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*sum(_R*ln(-3*_R^3+_R+x),
_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{x^2}{1+x^4+x^8}\,dx = & -\frac{1}{24}\,\sqrt{6}\sqrt{i\,\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\,\sqrt{3}-1}\left(i\,\sqrt{3}-3\right)+12\,x\right)\\ & +\frac{1}{24}\,\sqrt{6}\sqrt{i\,\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\,\sqrt{3}-1}\left(-i\,\sqrt{3}+3\right)+12\,x\right)\\ & +\frac{1}{24}\,\sqrt{6}\sqrt{-i\,\sqrt{3}-1}\log\left(\sqrt{6}\left(i\,\sqrt{3}+3\right)\sqrt{-i\,\sqrt{3}-1}+12\,x\right)\\ & -\frac{1}{24}\,\sqrt{6}\sqrt{-i\,\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{-i\,\sqrt{3}-1}\left(-i\,\sqrt{3}-3\right)+12\,x\right)\\ & -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)\\ & -\frac{1}{8}\,\log\left(x^2+x+1\right)+\frac{1}{8}\,\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sqrt(3
) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3
) - 1)*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(sqr
t(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) - 1/24*sqrt(6)*sqrt(-I*sq
rt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) - 1/12
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2
*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(442368t^7-192t^3+x\right)\right)\right)$$

```
[In] integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*
_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \int \frac{x^2}{x^8+x^4+1}\,dx$$

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1)
+ 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2\,x-\sqrt{3}\right) - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)$$

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
- 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(x^2/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.342 $\int \frac{1}{1+x^4+x^8} dx$

Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{1}{1+x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1360, 1178, 642, 1175, 632, 210}

$$\int \frac{1}{1+x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}} + \frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}$$

```
[In] Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx - \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}$$

$$= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \log\left(-1+\sqrt{3}x-x^2\right) + \log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

```
[In] Integrate[(1 + x^4 + x^8)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt
[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} - \frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12} + \frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	67
risch	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12} + \frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12} + \frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	68

```
[In] int(1/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2+2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12} - \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

```
[In] integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 -
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Maxima [F]

$$\int \frac{1}{1+x^4+x^8}\,dx = \int \frac{1}{x^8+x^4+1}\,dx$$

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right) - \mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

```
[In] int(1/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = -\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right) -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] -1/x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln
(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*
3^(1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1382, 1520, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right) -\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}$$

```
[In] Int[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
```

```
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{x}+\int \frac{x^2(-1-x^4)}{1+x^4+x^8}\,dx \\
&= -\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx \\
&= -\frac{1}{x}+\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx \\
&\quad +\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx \\
&= -\frac{1}{x}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx \\
&\quad -\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx \\
&\quad -\frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx-\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{8\sqrt{3}}
\end{aligned}$$

$$= -\frac{1}{x} - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right)$$

$$= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx = \frac{1}{24}\left(-\frac{24}{x} + 2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 3\log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2*
I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x +
 x^2] + 3*Log[1 + x + x^2])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{1}{x}+\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+1)}\right.}{4}$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{x}+\frac{\sqrt{3}\left(-\frac{\ln(1+x^2-x\sqrt{3})}{2}-\sqrt{3}\arctan\left(\right.\right.}{12}$

```
[In] int(1/x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/8*ln(4*
x^2-4*x+4)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*sum(_R*ln(-6*_R^3-_
R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx =$$

$$-\frac{\sqrt{6}x\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-\sqrt{6}x\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-\sqrt{6}}{}$$

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(6)*x*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3) -
 1) + 6*x) - sqrt(6)*x*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(I*sq
rt(3) - 1) + 6*x) - sqrt(6)*x*sqrt(-I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sq
rt(-I*sqrt(3) - 1) + 6*x) + sqrt(6)*x*sqrt(-I*sqrt(3) - 1)*log(-I*sqrt(6)*s
qrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) + 2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x +
1)) + 2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - 3*x*log(x^2 + x + 1) + 3*
x*log(x^2 - x + 1) + 24)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-442368t^7-384t^3+x\right)\right)\right)-\frac{1}{x}$$

```
[In] integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/24
)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4423
68*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384
*_t**3 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \int \frac{1}{(x^8+x^4+1)x^2}\,dx$$

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 +
 x + 1) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \frac{1}{x} - \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) - \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqr
t(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1i}{12}\right) + \operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1i}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{12}\right) - \operatorname{atan}\left(\frac{x\,2i}{-1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right) - \operatorname{atan}\left(\frac{x\,2i}{1+\sqrt{3}\,1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right) - \frac{1}{x}$$

```
[In] int(1/(x^2*(x^4 + x^8 + 1)),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x
```

3.344 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 147

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] -1/3/x^3-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/
8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2
*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/
2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1382, 1433, 1108, 648, 632, 210, 642}

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

```
[In] Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3x^3}+\frac{1}{3}\int\frac{-3-3x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{2}\int\frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int\frac{1}{1+x^2+x^4}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{4}\int\frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int\frac{1+x}{1+x+x^2}\,dx-\frac{\int\frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int\frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{1}{3x^3}-\frac{1}{8}\int\frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int\frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int\frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int\frac{1+2x}{1+x+x^2}\,dx\\
&\quad-\frac{1}{8}\int\frac{1}{1-\sqrt{3}x+x^2}\,dx-\frac{1}{8}\int\frac{1}{1+\sqrt{3}x+x^2}\,dx+\frac{\int\frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int\frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{8\sqrt{3}}\\
&= -\frac{1}{3x^3}+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}\\
&\quad-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}+\frac{1}{4}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{4}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&\quad+\frac{1}{4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)+\frac{1}{4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)\\
&\quad+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{8}{x^3} - \frac{4i\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(i+\sqrt{3}\right)}} + \frac{4i\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(-i+\sqrt{3}\right)}} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 3\log\left(1-x+x^2\right) - 3\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] +
 ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*Sq
rt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*
Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{3x^3} - \frac{\ln\left(4x^2+4x+4\right)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} + \frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{4} + \frac{\ln\left(x^2-x+1\right.}{8}$
default	$\frac{\ln\left(x^2-x+1\right)}{8} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^2+x+1\right)}{8} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{1}{3x^3} + \frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24} - \frac{\arctan\left(2x-\right.}{4}$

```
[In] int(1/x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3-1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*s
um(_R*ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/8*ln(x^2-x+1)-1/12*3^(1
/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^4(1+x^4+x^8)} dx = \frac{\sqrt{6}x^3\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}(i\sqrt{3}-3)+12x\right)-\sqrt{6}x^3\sqrt{i\sqrt{3}-1}\log\left(\sqrt{6}\sqrt{i\sqrt{3}-1}(-i\sqrt{3}+3\right.}{}$$

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(6)*x^3*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqrt(3) - 1)*(I*sq
rt(3) - 3) + 12*x) - sqrt(6)*x^3*sqrt(I*sqrt(3) - 1)*log(sqrt(6)*sqrt(I*sqr
t(3) - 1)*(-I*sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sqrt(-I*sqrt(3) - 1)*log(s
qrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1) + 12*x) + sqrt(6)*x^3*sqrt(-I*s
qrt(3) - 1)*log(sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3) + 12*x) - 2*s
qrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*x^3*arctan(1/3*sqrt(3)
*(2*x - 1)) - 3*x^3*log(x^2 + x + 1) + 3*x^3*log(x^2 - x + 1) - 8)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{1}{x^4(1+x^4+x^8)} dx =& \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) \\ &+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) \\ &+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) \\ &+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) \\ &+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right) \\ &-\frac{1}{3x^3}\end{aligned}$$

```
[In] integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5
) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)
*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3
)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9
216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = \int \frac{1}{(x^8 + x^4 + 1)x^4} \, dx$$

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\begin{aligned} \int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = & -\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x + 1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) \\ & - \frac{1}{24} \sqrt{3} \log\left(x^2 + \sqrt{3}x + 1\right) + \frac{1}{24} \sqrt{3} \log\left(x^2 - \sqrt{3}x + 1\right) \\ & - \frac{1}{3x^3} - \frac{1}{4} \arctan\left(2x + \sqrt{3}\right) - \frac{1}{4} \arctan\left(2x - \sqrt{3}\right) \\ & - \frac{1}{8} \log\left(x^2 + x + 1\right) + \frac{1}{8} \log\left(x^2 - x + 1\right) \end{aligned}$$

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
 sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = -\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \frac{1}{3x^3}$$

```
[In] int(1/(x^4*(x^4 + x^8 + 1)),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x^3)
```

3.345 $\int \frac{1}{x^6(1+x^4+x^8)} dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{1}{x^6(1+x^4+x^8)} dx = -\frac{1}{5x^5} + \frac{1}{x} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/5/x^5+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1382, 1518, 12, 1386, 1178, 642, 1175, 632, 210}

$$\int \frac{1}{x^6(1+x^4+x^8)} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{5x^5} + \frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}} - \frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}} + \frac{1}{x}$$

```
[In] Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
```

```
tQ[m, -1] && IntegerQ[p]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{-5-5x^4}{x^2\left(1+x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{5}\int -\frac{5x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\int \frac{x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^6(1+x^4+x^8)}\,dx = \frac{1}{60}\left(-\frac{12}{x^5}+\frac{60}{x}+10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+10\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+5\sqrt{3}\log\left(-1+\sqrt{3}x-x^2\right)-5\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)$$

```
[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
 + Sqrt[3]*x + x^2])/60
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{5x^5}+\frac{1}{x}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	75
risch	$\frac{x^4-\frac{1}{5}}{x^5}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\arctan\left(\frac{x\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}$	77

```
[In] int(1/x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))
*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6(1+x^4+x^8)}\,dx = \frac{10\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+10\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}x\right)+5\sqrt{3}x^5\log\left(\frac{x^4+5x^2-2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)+60x^4 -}{60x^5}$$

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arcta
n(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^6 (1 + x^4 + x^8)} \, dx = \frac{\sqrt{3} \cdot \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right)\right)}{12} + \frac{\sqrt{3} \log\left(x^2 - \sqrt{3}x + 1\right)}{12} - \frac{\sqrt{3} \log\left(x^2 + \sqrt{3}x + 1\right)}{12} + \frac{5x^4 - 1}{5x^5}$$

```
[In] integrate(1/x**6/(x**8+x**4+1),x)
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + (5*x**4 - 1)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (1 + x^4 + x^8)} \, dx = \int \frac{1}{(x^8 + x^4 + 1)x^6} \, dx$$

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1),
 x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6 (1 + x^4 + x^8)} \, dx = \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x + 1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{24} \sqrt{3} \log\left(x^2 + \sqrt{3}x + 1\right) + \frac{1}{24} \sqrt{3} \log\left(x^2 - \sqrt{3}x + 1\right) + \frac{5x^4 - 1}{5x^5} + \frac{1}{4} \arctan\left(2x + \sqrt{3}\right) + \frac{1}{4} \arctan\left(2x - \sqrt{3}\right)$$

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*arc
tan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^6 (1 + x^4 + x^8)} \, dx = \frac{x^4 - \frac{1}{5}}{x^5} - \frac{\sqrt{3}\, \text{atanh}\left(\frac{2\sqrt{3}\, x}{3\left(\frac{2x^2}{3} + \frac{2}{3}\right)}\right)}{6} - \frac{\sqrt{3}\, \text{atan}\left(\frac{2\sqrt{3}\, x}{3\left(\frac{2x^2}{3} - \frac{2}{3}\right)}\right)}{6}$$

```
[In] int(1/(x^6*(x^4 + x^8 + 1)),x)
[Out] (x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
(3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 154

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

```
[Out] -1/7/x^7+1/3/x^3+1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2
-x+1)+1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(
1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2
+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1382, 1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

```
[In] Int[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{-7-7x^4}{x^4(1+x^4+x^8)}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{21}\int -\frac{21x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}+\int \frac{x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx
\end{aligned}
$$

$$= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

$$= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx - \frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx - \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx + \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{8\sqrt{3}}$$

$$= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right)$$

$$= -\frac{1}{7x^7} + \frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{8\sqrt{3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{7x^7} + \frac{1}{3x^3} + \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}} + \frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{-6-6i\sqrt{3}}} - \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right)$$

```
[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

[Out] -1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*Sqrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+1)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}-\frac{\ln(x^2-x+\cdots}{8}$
default	$-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(-\frac{\ln(1+x^2-x\sqrt{3})}{2}-\cdots\right.}{12}$

[In] int(1/x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)

[Out] (1/3*x^4-1/7)/x^7+1/8*ln(4*x^2+4*x+4)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*sum(_R*ln(6*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx$$
$$=\frac{7\sqrt{6}x^7\sqrt{i\sqrt{3}-1}\log\left(i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-7\sqrt{6}x^7\sqrt{i\sqrt{3}-1}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}+6x\right)-\cdots}{\cdots}$$

[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/168*(7*sqrt(6)*x^7*sqrt(I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3) - 1) + 6*x) - 7*sqrt(6)*x^7*sqrt(I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(3) - 1) + 6*x) - 7*sqrt(6)*x^7*sqrt(-I*sqrt(3) - 1)*log(I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) + 7*sqrt(6)*x^7*sqrt(-I*sqrt(3) - 1)*log(-I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1) + 6*x) - 14*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1)) - 14*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) + 21*x^7*log(x^2 + x + 1) - 21*x^7*log(x^2 - x + 1) + 56*x^4 - 24)/x^7

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right) + \frac{7x^4-3}{21x^7}$$

```
[In] integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \int \frac{1}{(x^8+x^4+1)x^8}\,dx$$

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1), x
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) -\frac{1}{24}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) +\frac{7\,x^4-3}{21\,x^7}+\frac{1}{4}\,\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{4}\,\arctan\left(2\,x-\sqrt{3}\right) +\frac{1}{8}\,\log\left(x^2+x+1\right)-\frac{1}{8}\,\log\left(x^2-x+1\right)$$

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) + 1/4
*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = \frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) -\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) -\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) -\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

```
[In] int(1/(x^8*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i
 + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12
+ 1/4)
```

3.347 $\int \frac{x^m}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{x^m}{1-x^4+x^8} \, dx = \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(1+I*3^(1/2)))/(1+m
)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^
4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1389, 371}

$$\int \frac{x^m}{1-x^4+x^8} \, dx = \frac{2x^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)} - \frac{2x^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

```
[In] Int[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 - I*Sqrt
[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
```

```
(1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1 +
 m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\int \frac{x^m}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}+\frac{i\int \frac{x^m}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \frac{x^m \text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1^3+2\#1^7}\&\right]}{4m}$$

```
[In] Integrate[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x -
 #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F]

$$\int \frac{x^m}{x^8 - x^4 + 1} dx$$

[In] int(x^m/(x^8-x^4+1),x)

[Out] int(x^m/(x^8-x^4+1),x)

Fricas [F]

$$\int \frac{x^m}{1 - x^4 + x^8} \, dx = \int \frac{x^m}{x^8 - x^4 + 1} \, dx$$

[In] integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")

[Out] integral(x^m/(x^8 - x^4 + 1), x)

Sympy [F]

$$\int \frac{x^m}{1 - x^4 + x^8} \, dx = \int \frac{x^m}{x^8 - x^4 + 1} \, dx$$

[In] integrate(x**m/(x**8-x**4+1),x)

[Out] Integral(x**m/(x**8 - x**4 + 1), x)

Maxima [F]

$$\int \frac{x^m}{1 - x^4 + x^8} \, dx = \int \frac{x^m}{x^8 - x^4 + 1} \, dx$$

[In] integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(x^m/(x^8 - x^4 + 1), x)

Giac [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx=\int \frac{x^m}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-x^4+x^8}\,dx=\int \frac{x^m}{x^8-x^4+1}\,dx$$

```
[In] int(x^m/(x^8 - x^4 + 1),x)
[Out] int(x^m/(x^8 - x^4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^{11}}{1-x^4+x^8} dx = \frac{x^4}{4} + \frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1371, 717, 648, 632, 210, 642}

$$\int \frac{x^{11}}{1-x^4+x^8} dx = \frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{x^4}{4} + \frac{1}{8}\log\left(x^8-x^4+1\right)$$

```
[In] Int[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{8}\log\left(1-x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= \frac{x^4}{4} + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[In] Integrate[x^11/(1 - x^4 + x^8),x]
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	38
risch	$\frac{x^4}{4}+\frac{\ln(4x^8-4x^4+4)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

```
[In] int(x^11/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{\ln\left(x^8-x^4+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12} + \frac{x^4}{4}$$

```
[In] int(x^11/(x^8 - x^4 + 1),x)
[Out] log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 + x
^4/4
```

3.349 $\int \frac{x^9}{1-x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{x^9}{1-x^4+x^8} dx = \frac{x^2}{2}+\frac{\log \left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}-\frac{\log \left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1136, 1178, 642}

$$\int \frac{x^9}{1-x^4+x^8} dx = \frac{x^2}{2}+\frac{\log \left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log \left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[In] Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\
&= \frac{x^2}{2} + \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)-\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

```
[In] Integrate[x^9/(1 - x^4 + x^8),x]
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
+ x^4])/12
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44
risch	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

```
[In] int(x^9/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{1}{2}\,x^2+\frac{1}{12}\,\sqrt{3}\log\left(\frac{x^8+5\,x^4-2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 -
 x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{x^2}{2}+\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

```
[In] integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt(
3)*x**2 + 1)/12
```

Maxima [F]

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \int \frac{x^9}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right) + \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)$$

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x^
2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) - 1/
24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x^2}{9\,\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

```
[In] int(x^9/(x^8 - x^4 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

3.350 $\int \frac{x^7}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^7}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] 1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 648, 632, 210, 642}

$$\int \frac{x^7}{1-x^4+x^8} \, dx = \frac{1}{8}\log\left(x^8-x^4+1\right) - \frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[In] Int[x^7/(1 - x^4 + x^8),x]
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right) \\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right) \\
&= \frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[In] Integrate[x^7/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	33
risch	$\frac{\ln\left(4x^{8}-4x^{4}+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	35

```
[In] int(x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**7/(x**8-x**4+1),x)
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

```
[In] int(x^7/(x^8 - x^4 + 1),x)
[Out] log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.351 $\int \frac{x^5}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{x^5}{1-x^4+x^8} \, dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4-x^2*3^(1/
2))*3^(1/2)-1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1373, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{x^5}{1-x^4+x^8} \, dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[In] Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*
x^2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx,x,x^2\right)\\
&\quad+\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{8\sqrt{3}}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{8\sqrt{3}}\\
&= \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}\\
&\quad-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x^2\right)\\
&= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{\sqrt{-1-i\sqrt{3}}\,(i+\sqrt{3})\arctan\left(\frac{1}{2}(1-i\sqrt{3})\,x^2\right)+\sqrt{-1+i\sqrt{3}}\,(-i+\sqrt{3})\arctan\left(\frac{1}{2}(1+i\sqrt{3})\,x^2\right)}{4\sqrt{6}}$$

```
[In] Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[
-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R\ln\left(6\_R^{3}+x^{2}+\_R\right)\right)}{4}$	32
default	$-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^{4}-x^{2}\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x^{2}-\sqrt{3}\right)\right)}{12}-\frac{\sqrt{3}\left(\frac{\ln\left(1+x^{4}+x^{2}\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x^{2}+\sqrt{3}\right)\right)}{12}$	77

```
[In] int(x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(6*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(6\,x^2+i\,\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right) - \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(6\,x^2-i\,\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(6\,x^2+i\,\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(6\,x^2-i\,\sqrt{6}\sqrt{3}\sqrt{-i\sqrt{3}-1}\right)$$

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(I*sqrt(
3) - 1)) - 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sqrt(3)*s
qrt(I*sqrt(3) - 1)) - 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(
6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log(6*
x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

```
[In] integrate(x**5/(x**8-x**4+1),x)
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2 +
 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \int \frac{x^5}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}x^4\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}x^4\arctan\left(2\,x^2-\sqrt{3}\right)$$

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - sq
rt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 - s
qrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

```
[In] int(x^5/(x^8 - x^4 + 1),x)
[Out] - atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.352 $\int \frac{x^3}{1-x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 632, 210}

$$\int \frac{x^3}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[x^3/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Integrate[x^3/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19

```
[In] int(x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

```
[In] integrate(x**3/(x**8-x**4+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1-x^4+x^8}\,dx=\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

```
[In] int(x^3/(x^8 - x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

3.353 $\int \frac{x}{1-x^4+x^8} dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{x}{1-x^4+x^8} dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-x^2*3^(1/
2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1373, 1108, 648, 632, 210, 642}

$$\int \frac{x}{1-x^4+x^8} dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right) - \frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[In] Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*
x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)}{8\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)}{8\sqrt{3}}
\end{aligned}$$

$$= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x^2\right)$$

$$= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{i\left(\sqrt{-1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right) - \sqrt{-1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

```
[In] Integrate[x/(1 - x^4 + x^8),x]
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*
Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+3\_Z^2+1\right)} \_R\ln\left(-3\_R^3+x^2+\_R\right)\right)}{4}$	32
default	$\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4} + \frac{\arctan\left(2x^2+\sqrt{3}\right)}{4} - \frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24} + \frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$	65

```
[In] int(x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(-3*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int \frac{x}{1-x^4+x^8}\,dx = -\frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(12\,x^2+\sqrt{6}\sqrt{i\sqrt{3}-1}\left(i\sqrt{3}-3\right)\right) + \frac{1}{24}\sqrt{6}\sqrt{i\sqrt{3}-1}\log\left(12\,x^2+\sqrt{6}\sqrt{i\sqrt{3}-1}\left(-i\sqrt{3}+3\right)\right) + \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(12\,x^2+\sqrt{6}\left(i\sqrt{3}+3\right)\sqrt{-i\sqrt{3}-1}\right) - \frac{1}{24}\sqrt{6}\sqrt{-i\sqrt{3}-1}\log\left(12\,x^2+\sqrt{6}\sqrt{-i\sqrt{3}-1}\left(-i\sqrt{3}-3\right)\right)$$

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*sqrt(I*sqrt(3) - 1)*
(I*sqrt(3) - 3)) + 1/24*sqrt(6)*sqrt(I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*sq
rt(I*sqrt(3) - 1)*(-I*sqrt(3) + 3)) + 1/24*sqrt(6)*sqrt(-I*sqrt(3) - 1)*log
(12*x^2 + sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3) - 1)) - 1/24*sqrt(6)*sqrt
(-I*sqrt(3) - 1)*log(12*x^2 + sqrt(6)*sqrt(-I*sqrt(3) - 1)*(-I*sqrt(3) - 3)
)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{x}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

```
[In] integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x}{1-x^4+x^8}\,dx = \int \frac{x}{x^8-x^4+1}\,dx$$

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2\,x^2-\sqrt{3}\right)$$

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x^
2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{x}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

```
[In] int(x/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)*
x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^4+x^8)}\,dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$\int \frac{1}{x(1-x^4+x^8)}\,dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8-x^4+1\right) + \log(x)$$

```
[In] Int[1/(x*(1 - x^4 + x^8)),x]
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\,(1-x+x^2)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)
\end{aligned}
$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1 - x^4 + x^8\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \log(x) - \frac{1}{4}\text{RootSum}\left[1 - \#1^4 + \#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x*(1 - x^4 + x^8)),x]
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
+ 2*#1^4) & ]/4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln\left(x\right)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}$	35

```
[In] int(1/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\big(2\,x^4-1\big)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx=\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(1/x/(x**8-x**4+1),x)
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\big(2\,x^4-1\big)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)+\frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

```
[In] int(1/(x*(x^8 - x^4 + 1)),x)
[Out] log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] -1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1137, 1178, 642}

$$\int \frac{1}{x^3(1-x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}} + \frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[In] Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\
&= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)+\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44
risch	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

```
[In] int(1/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\frac{\sqrt{3}x^2\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)-6}{12\,x^2}$$

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
+ 1)) - 6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}-\frac{1}{2x^2}$$

```
[In] integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12 - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\int \frac{1}{(x^8-x^4+1)x^3}\,dx$$

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43)=86$.

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=&-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right)\\&-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)\\&+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{2\,x^2}\end{aligned}$$

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(
3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqrt(
3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6}-\frac{1}{2\,x^2}$$

```
[In] int(1/(x^3*(x^8 - x^4 + 1)),x)
[Out] (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)
```

3.356 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx = -\frac{1}{4x^4}+\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\int \frac{1}{x^5(1-x^4+x^8)}\,dx = \frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[In] Int[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1-x+x^2)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x(1-x+x^2)}\,dx, x, x^4\right)\end{aligned}$$

$$= -\frac{1}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2}\right) dx, x, x^4\right)$$

$$= -\frac{1}{4x^4} + \log(x) - \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2} dx, x, x^4\right)$$

$$= -\frac{1}{4x^4} + \log(x) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, x^4\right) - \frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4\right)$$

$$= -\frac{1}{4x^4} + \log(x) - \frac{1}{8}\log\left(1-x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4\right)$$

$$= -\frac{1}{4x^4} + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5(1-x^4+x^8)} dx = -\frac{1}{4x^4} + \log(x) - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]
[Out] -1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
2*#1^4) & ]/4
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{4x^4} + \ln(x) - \frac{\ln(x^8-x^4+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	38
default	$-\frac{1}{4x^4} + \ln(x) - \frac{\ln(x^8-x^4+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

```
[In] int(1/x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx = -\frac{2 \sqrt{3} x^4 \arctan\left(\frac{1}{3} \sqrt{3} (2 x^4 - 1)\right) + 3 x^4 \log\left(x^8 - x^4 + 1\right) - 24 x^4 \log\left(x\right) + 6}{24 x^4}$$

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
+ 1) - 24*x^4*log(x) + 6)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx = \log\left(x\right) - \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

```
[In] integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12 - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx = -\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^4 - 1)\right) - \frac{1}{4 x^4} - \frac{1}{8} \log\left(x^8 - x^4 + 1\right) + \frac{1}{4} \log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx = -\frac{1}{12} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^4 - 1) \right) - \frac{x^4 + 1}{4 x^4} - \frac{1}{8} \log \left(x^8 - x^4 + 1 \right) + \frac{1}{4} \log \left(x^4 \right)$$

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx = \ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}\, \mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\, x^4}{3} \right)}{12} - \frac{1}{4 x^4}$$

```
[In] int(1/(x^5*(x^8 - x^4 + 1)),x)
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

3.357 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*l
n(1+x^4-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1373, 1137, 1295, 12, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx = \frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[In] Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(1-x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{3-3x^2}{x^2(1-x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{3x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right)\\
&\quad-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right)\\
&\quad-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{8\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{8\sqrt{3}}
\end{aligned}
$$

$$= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x^2\right)$$

$$= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx = -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x^7*(1 - x^4 + x^8)),x]
[Out] -1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1
 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{-\frac{x^4}{2}-\frac{1}{6}}{x^6} + \frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+3\_Z^2+1\right)} \_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	46
default	$-\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2} - \sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12} + \frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2} - \sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	87

```
[In] int(1/x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] (-1/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = -\frac{\sqrt{6}x^6\sqrt{i\sqrt{3}-1}\log\left(6x^2+i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-\sqrt{6}x^6\sqrt{i\sqrt{3}-1}\log\left(6x^2-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-}{}$$

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(6)*x^6*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(I
*sqrt(3) - 1)) - sqrt(6)*x^6*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sqrt
(3)*sqrt(I*sqrt(3) - 1)) - sqrt(6)*x^6*sqrt(-I*sqrt(3) - 1)*log(6*x^2 + I*s
qrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + sqrt(6)*x^6*sqrt(-I*sqrt(3) - 1)*log
(6*x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 12*x^4 + 4)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} - \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}+\frac{-3x^4-1}{6x^6}$$

```
[In] integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4 -
1)/(6*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = \int \frac{1}{(x^8-x^4+1)x^7} \, dx$$

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = -\frac{1}{12} \sqrt{3} x^4 \log \left(x^4+\sqrt{3} x^2+1\right) + \frac{1}{12} \sqrt{3} x^4 \log \left(x^4-\sqrt{3} x^2+1\right) - \frac{3 x^4+1}{6 x^6}$$

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = \mathrm{atan}\left(\frac{2 x^2}{-1+\sqrt{3}\, 1\mathrm{i}}\right) \left(\frac{1}{4}+\frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) + \mathrm{atan}\left(\frac{2 x^2}{1+\sqrt{3}\, 1\mathrm{i}}\right) \left(-\frac{1}{4}+\frac{\sqrt{3}\, 1\mathrm{i}}{12}\right) - \frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

```
[In] int(1/(x^7*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 356

$$\begin{aligned}\int \frac{x^8}{1-x^4+x^8}\,dx = {} & x+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} \\ & +\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) \\ & +\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\ & +\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) \\ & -\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

```
[Out] x-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*l
n(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6
^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))
/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+
1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(
1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1381, 1435, 1183, 648, 632, 210, 642}

$$\int \frac{x^8}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right) + x$$

```
[In] Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x-\int \frac{1-x^4}{1-x^4+x^8}\,dx\\ &= x+\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}\end{aligned}$$

$$= x - \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$- \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(2+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= x + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx - \frac{\left(-2+\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$= x - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,-\sqrt{2-\sqrt{3}}+2x\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx,x,\sqrt{2-\sqrt{3}}+2x\right)$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.17

$$\int\frac{x^8}{1-x^4+x^8}\,dx = x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[x^8/(1 - x^4 + x^8),x]
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 
2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

```
[In] int(x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

$$\int \frac{x^8}{1-x^4+x^8}\,dx = -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12x\right) - \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12x\right) + \frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)+12x\right) + x$$

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*s
qrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*s
```

```
qrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*sqr
t(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt
(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6
)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(
3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3)
 + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*
x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt
(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(
2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I
*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*lo
g(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3) + 12*x) + x
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x + \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

```
[In] integrate(x**8/(x**8-x**4+1),x)
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
8*_t + x)))
```

Maxima [F]

$$\int \frac{x^8}{1-x^4+x^8}\,dx = \int \frac{x^8}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{1-x^4+x^8}\,dx = -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + x$$

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) + x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{12}$$

```
[In] int(x^8/(x^8 - x^4 + 1),x)
```

```
[Out] x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i
)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*
8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3
^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2
^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^
(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.359 $\int \frac{x^6}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 275

$$\int \frac{x^6}{1-x^4+x^8}\,dx=-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)-1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1386, 1183, 648, 632, 210, 642}

$$\int \frac{x^6}{1-x^4+x^8}\,dx=-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}$$

```
[In] Int[x^6/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}\\
&\quad+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}}+\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}+\frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}
\end{aligned}
$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[x^6/(1 - x^4 + x^8),x]
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32

```
[In] int(x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right) - \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) + \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right) - \left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)$$

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
[Out] (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 + 6
*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*
x^2 - 6*I) + (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(2)*
x + 6*x^2 - 6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(3)*s
qrt(2)*x + 6*x^2 + 6*I)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} + \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} - \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

```
[In] integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Maxima [F]

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \int \frac{x^6}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

```
[In] int(x^6/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
```

3.360 $\int \frac{x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 347

$$\begin{aligned}\int \frac{x^4}{1-x^4+x^8}\,dx=&\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}\\&-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}\\&-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}\\&+\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

```
[Out] -1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*
2^(1/2)-1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/
(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^
(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1
/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x
^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/8*ln(1+x^2+x*(1
/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1387, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[In] Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\ &= -\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}\end{aligned}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}$$
$$- \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{-1-\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{-1+\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$
$$+ \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$
$$+ \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[x^4/(1 - x^4 + x^8),x]
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40

```
[In] int(x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{x^4}{1-x^4+x^8}\,dx &= -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
&\quad+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\\
&\quad-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}+6x\right)\end{aligned}$$

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*sqrt(
sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt
(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) -
 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*sqrt
(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*s
qrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*
x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*
sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(
-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))
+ 6*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*sqr
t(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) - 1/24*sqrt(6)*sqrt(-sqrt(2
)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3
) + 1)) + 6*x)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)$$

```
[In] integrate(x**4/(x**8-x**4+1),x)
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*
_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \int \frac{x^4}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{x^4}{1-x^4+x^8}\,dx &= \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\
&\quad+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\
&\quad+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\
&\quad+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\
&\quad+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\
&\quad-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\
&\quad+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\
&\quad-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.37

$$
\begin{aligned}
\int \frac{x^4}{1-x^4+x^8}\,dx
&= -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} \\
&\quad -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} \\
&\quad +\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}-\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} \\
&\quad +\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}+\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}
\end{aligned}
$$

```
[In] int(x^4/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1
/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2
^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8
 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*
8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8
i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2
)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)
```

```
^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 +
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3
^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) +
 (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4))/12
```

3.361 $\int \frac{x^2}{1-x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 355

$$\begin{aligned}\int \frac{x^2}{1-x^4+x^8} dx = &\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}\\
&-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

```
[Out] 1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1
/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1
/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(
1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/
2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^
2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/
2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1387, 1108, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{x^2}{1-x^4+x^8}\,dx=&\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)\\&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\\&+\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}\\&-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}\end{aligned}$$

```
[In] Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} \\
&= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} \\
&\quad + \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}}
\end{aligned}
$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)}{-\#1+2\#1^5}\&\right]$$

```
[In] Integrate[x^2/(1 - x^4 + x^8),x]
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40

```
[In] int(x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{x^2}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt
(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1) + 24
*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3
)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1
) + 24*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*
sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt
(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt
(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqr
t(I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*
log(sqrt(6)*(I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*sqrt(3) +
1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3
) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*s
qrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt
(-I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)
*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(-sq
rt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sq
rt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-442368t^7-192t^3+x\right)\right)\right)$$

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1
92*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \int \frac{x^2}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

```
[In] int(x^2/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4)*
1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/
2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i
- 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^
(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*
1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.362 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 275

$$\int \frac{1}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1360, 1183, 648, 632, 210, 642}

$$\int \frac{1}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}} + \frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}} - \frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}} + \frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}$$

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx,x,-\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx,x,\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

```
[In] int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\begin{aligned}\int \frac{1}{1-x^4+x^8}\,dx = {} & \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6\,x^2+6i\right) \\ & -\left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6\,x^2-6i\right) \\ & +\left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6\,x^2-6i\right) \\ & -\left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6\,x^2+6i\right)\end{aligned}$$

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
[Out] (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 + 6
*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*
x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(2)*
x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(3)*s
qrt(2)*x + 6*x^2 + 6*I)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\,\text{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\text{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\,\text{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\text{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Maxima [F]

$$\int \frac{1}{1-x^4+x^8}\,dx = \int \frac{1}{x^8-x^4+1}\,dx$$

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx = &-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

```
[Out] -1/x+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/
8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arcta
n((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/
2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/
2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*
2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2)
)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1520, 1293, 1183, 648, 632, 210, 642}

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right) - \frac{1}{x}$$

```
[In] Int[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{x} + \int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx \\
&= -\frac{1}{x} + \frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{x} - \frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{x} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} \\
&\quad - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+\left(-2-\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} \\
&= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx - \frac{\left(-2+\sqrt{3}\right)\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} \\
&\quad - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx \\
&\quad - \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx \\
&\quad - \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx
\end{aligned}
$$

$$= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{2}{3} - \frac{1}{\sqrt{3}}}\log\left(1 - \sqrt{2-\sqrt{3}}x + x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1 + \sqrt{2-\sqrt{3}}x + x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 - \sqrt{2+\sqrt{3}}x + x^2\right)$$
$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 + \sqrt{2+\sqrt{3}}x + x^2\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)$$

$$= -\frac{1}{x} + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$+ \frac{1}{8}\sqrt{\frac{2}{3} - \frac{1}{\sqrt{3}}}\log\left(1 - \sqrt{2-\sqrt{3}}x + x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1 + \sqrt{2-\sqrt{3}}x + x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 - \sqrt{2+\sqrt{3}}x + x^2\right)$$
$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 + \sqrt{2+\sqrt{3}}x + x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]

[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2*#1^5) &]/4

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-27\_R^7+6\_R^3+x\right)\right)}{4}$	40
default	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^6-\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{x}$	52

[In] int(1/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)

[Out] -1/x+1/4*sum(_R*ln(-27*_R^7+6*_R^3+x),_R=RootOf(81*_Z^8-9*_Z^4+1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{\sqrt{6}x\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}(i\sqrt{3}\sqrt{2}-3\sqrt{2})\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)-\sqrt{6}x\sqrt{\sqrt{2}\sqrt{i\sqrt{3}}}}{}$$

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(6)*x*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*s
qrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1) +
 24*x) - sqrt(6)*x*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3
)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1
) + 24*x) - sqrt(6)*x*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqr
t(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3)
 + 1) + 24*x) + sqrt(6)*x*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-
I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sq
rt(3) + 1) + 24*x) - sqrt(6)*x*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(
6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(
-I*sqrt(3) + 1) + 24*x) + sqrt(6)*x*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(
sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))
*sqrt(-I*sqrt(3) + 1) + 24*x) + sqrt(6)*x*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1
))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(-I*sqrt(3
) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - sqrt(6)*x*sqrt(-sqrt(2)*sqrt(-I*sqrt
(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(-
I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 24)/x
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)} dx$$
$$= \text{RootSum}\left(5308416t^8-2304t^4+1, \left(t\mapsto t\log\left(-442368t^7+384t^3+x\right)\right)\right)-\frac{1}{x}$$

```
[In] integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 3
84*_t**3 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)} dx = \int \frac{1}{(x^8-x^4+1)x^2} dx$$

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{x}$$

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx=-\frac{1}{x}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}-\frac{2^{3/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}+\frac{2^{3/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

```
[In] int(1/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)
) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8
i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
 - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
+ (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/1
2
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 370

$$\begin{aligned}\int \frac{1}{x^4(1-x^4+x^8)}\,dx = &-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

```
[Out] -1/3/x^3+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*
6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)
-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))
/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6
```

```
^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/
8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+
x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1382, 1435, 1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{1}{x^4(1-x^4+x^8)}\,dx = &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)\\&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{3x^3}\\&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)\\&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)\\&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)\\&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)\end{aligned}$$

```
[In] Int[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3x^3}+\frac{1}{3}\int\frac{3-3x^4}{1-x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}-\frac{\int\frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}-\frac{\int\frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}\\
&= -\frac{1}{3x^3}+\frac{\int\frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int\frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}\\
&\quad+\frac{\int\frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int\frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(2+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}\\
&= -\frac{1}{3x^3}-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int\frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx\\
&\quad-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int\frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int\frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx+\frac{\left(-2+\sqrt{3}\right)\int\frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}}\\
&\quad-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int\frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int\frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int\frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx\\
&\quad+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int\frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx
\end{aligned}
$$

$$= -\frac{1}{3x^3} + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\mathrm{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\mathrm{Subst}\left(\int\frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\mathrm{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\mathrm{Subst}\left(\int\frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)$$

$$= -\frac{1}{3x^3} - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)$$
$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-9\_R^5+2\_R+x\right)\right)}{4}$	38
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{3x^3}$	50

```
[In] int(1/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+1/4*sum(_R*ln(-9*_R^5+2*_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx$$
$$= \frac{\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}(i\sqrt{3}+3)+12x\right)+\sqrt{6}x^3\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\right.}{}$$

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(
2)*sqrt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + sqrt(6)*x^3*sqrt(-sqrt(2
)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*
sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sq
rt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - sqrt(6)*x
^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt
(3) + 1))*(I*sqrt(3) - 3) + 12*x) + sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*sqrt(3)
 + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*
x) + sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(
2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - sqrt(6)*x^3*sqrt(sqrt(2)
*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*s
qrt(3) - 3) + 12*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(s
qrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3) + 12*x) - 8)/x^
3
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-9216t^5+8t+x\right)\right)\right) - \frac{1}{3x^3}$$

```
[In] integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_
t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^4}\,dx$$

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4 (1 - x^4 + x^8)} \, dx = \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2} x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) - \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2} x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) + \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2} x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2} x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{1}{3x^3}$$

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4 (1-x^4+x^8)} \, dx = -\frac{1}{3 x^3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{x}{\left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4}} + \frac{\sqrt{3}\, x\, 1\mathrm{i}}{\left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4}}\right) \left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4} 1\mathrm{i}}{12} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{x\, 1\mathrm{i}}{\left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4}} - \frac{\sqrt{3}\, x}{\left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4}}\right) \left(8-\sqrt{3}\, 8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4} \sqrt{3} \operatorname{atan}\left(\frac{2^{1/4}\, x}{2 \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4}} - \frac{2^{1/4} \sqrt{3}\, x\, 1\mathrm{i}}{2 \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4}}\right) \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4} 1\mathrm{i}}{12} + \frac{2^{3/4} \sqrt{3} \operatorname{atan}\left(\frac{2^{1/4}\, x\, 1\mathrm{i}}{2 \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4}} + \frac{2^{1/4} \sqrt{3}\, x}{2 \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4}}\right) \left(1+\sqrt{3}\, 1\mathrm{i}\right)^{1/4}}{12}$$

```
[In] int(1/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))
*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4
) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1/(3*x
^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^
(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 287

$$\begin{aligned}\int \frac{1}{x^6(1-x^4+x^8)}\,dx = & -\frac{1}{5x^5}-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ & -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ & -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\ & -\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\end{aligned}$$

```
[Out] -1/5/x^5-1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*
2^(1/2)))*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1
/2*2^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)
-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1
/24*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1
/2)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1382, 1518, 12, 1386, 1183, 648, 632, 210, 642}

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{1}{5x^5} - \frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}} + \frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}} - \frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}} + \frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}} - \frac{1}{x}$$

```
[In] Int[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]
) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTa
n[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2
 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sq
rt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
```

`gerQ[p]`

Rubi steps

$$\text{integral} = -\frac{1}{5x^5} + \frac{1}{5}\int \frac{5-5x^4}{x^2(1-x^4+x^8)}\,dx$$

$$= -\frac{1}{5x^5} - \frac{1}{x} - \frac{1}{5}\int \frac{5x^6}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{5x^5} - \frac{1}{x} - \int \frac{x^6}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{5x^5} - \frac{1}{x} + \frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{5x^5} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx = -\frac{1}{5x^5} - \frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.15

method	result	size
default	$-\frac{1}{5x^5}-\frac{1}{x}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	43
risch	$\frac{-x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	44

```
[In] int(1/x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx = \frac{-(5i-5)\,\sqrt{3}\sqrt{2}x^5\log\left((3i+3)\,\sqrt{3}\sqrt{2}x+6\,x^2+6i\right)+(5i+5)\,\sqrt{3}\sqrt{2}x^5\log\left(-(3i-3)\,\sqrt{3}\sqrt{2}x+6\,x^2-\right.}{}$$

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/120*(-(5*I - 5)*sqrt(3)*sqrt(2)*x^5*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x
^2 + 6*I) + (5*I + 5)*sqrt(3)*sqrt(2)*x^5*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x
+ 6*x^2 - 6*I) - (5*I + 5)*sqrt(3)*sqrt(2)*x^5*log((3*I - 3)*sqrt(3)*sqrt(2
)*x + 6*x^2 - 6*I) + (5*I - 5)*sqrt(3)*sqrt(2)*x^5*log(-(3*I + 3)*sqrt(3)*s
qrt(2)*x + 6*x^2 + 6*I) - 120*x^4 - 24)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = \frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24} + \frac{-5x^4-1}{5x^5}$$

```
[In] integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
- sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x
+ 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^6}\,dx$$

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{5x^4+1}{5x^5}$$

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = -\frac{x^4+\frac{1}{5}}{x^5} + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

```
[In] int(1/(x^6*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
(x^4 + 1/5)/x^5
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 377

$$\begin{aligned}\int \frac{1}{x^8(1-x^4+x^8)}\,dx &= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)\end{aligned}$$

```
[Out] -1/7/x^7-1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2
*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2
))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2
*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*
2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2
```

```
*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(
1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1
/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1382, 1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$\begin{aligned}\int \frac{1}{x^8(1-x^4+x^8)}\,dx = &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{7x^7}\\ &-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)\\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)\end{aligned}$$

```
[In] Int[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] -
 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[
3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 -
 Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x
^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
```

```
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{7-7x^4}{x^4\left(1-x^4+x^8\right)}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{21}\int \frac{21x^4}{1-x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}
\end{aligned}
$$

$$= -\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}$$

$$- \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$+ \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{-1-\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{-1+\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$= -\frac{1}{7x^7} - \frac{1}{3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$- \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]
[Out] -1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 +
 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
risch	$\frac{-\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\text{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	44
default	$-\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	51

```
[In] int(1/x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] (-1/3*x^4-1/7)/x^7+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = \frac{7\sqrt{6}x^7\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)-7\sqrt{6}x^7\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\right.}$$

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/168*(7*sqrt(6)*x^7*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3
)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(sqrt(2)*sqr
t(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))
+ 6*x) + 7*sqrt(6)*x^7*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqr
t(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(-sqrt(2
)*sqrt(I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3)
+ 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(
6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + 7*sqrt(6)*x^7*sqrt(s
qrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sq
rt(3) + 1)) + 6*x) - 7*sqrt(6)*x^7*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(
I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + 7*sqrt(6)*x^
7*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*
sqrt(-I*sqrt(3) + 1)) + 6*x) - 56*x^4 - 24)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} dx = \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(18432t^5 - 4t + x\right)\right)\right) + \frac{-7x^4 - 3}{21x^7}$$

```
[In] integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_
t + x))) + (-7*x**4 - 3)/(21*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} dx = \int \frac{1}{(x^8 - x^4 + 1)x^8} dx$$

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{7x^4+3}{21x^7}$$

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^8 (1-x^4+x^8)}\,dx$$
$$= -\frac{\frac{x^4}{3}+\frac{1}{7}}{x^7} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,(8-\sqrt{3}\,8i)^{1/4}}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)} + \frac{\sqrt{3}\,x\,(8-\sqrt{3}\,8i)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)(8-\sqrt{3}\,8i)^{1/4}\,1i}{12}$$
$$+ \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,(8-\sqrt{3}\,8i)^{1/4}\,1i}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)} - \frac{\sqrt{3}\,x\,(8-\sqrt{3}\,8i)^{1/4}}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)(8-\sqrt{3}\,8i)^{1/4}}{12}$$
$$- \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(1+\sqrt{3}\,1i)^{1/4}}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)} - \frac{2^{3/4}\sqrt{3}\,x\,(1+\sqrt{3}\,1i)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)(1+\sqrt{3}\,1i)^{1/4}\,1i}{12}$$
$$- \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,(1+\sqrt{3}\,1i)^{1/4}\,1i}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)} + \frac{2^{3/4}\sqrt{3}\,x\,(1+\sqrt{3}\,1i)^{1/4}}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)(1+\sqrt{3}\,1i)^{1/4}}{12}$$

```
[In] int(1/(x^8*(x^8 - x^4 + 1)),x)
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8 - 3^
(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2
)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1
/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/1
2 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
- (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i +
1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)
*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i
+ 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2
)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2
)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.367 $\int \frac{x^m}{1+3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1+3x^4+x^8} \, dx = \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(3-5^(1/2)))/(1+m)/
(3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4
/(3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1389, 371}

$$\int \frac{x^m}{1+3x^4+x^8} \, dx = \frac{2x^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)} - \frac{2x^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[In] Int[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (
```

```
1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m
))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{x^m}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}-\frac{\int \frac{x^m}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \frac{x^m\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{3\#1^3+2\#1^7}\&\right]}{4m}$$

```
[In] Integrate[x^m/(1 + 3*x^4 + x^8),x]
[Out] (x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F]

$$\int \frac{x^m}{x^8+3x^4+1}dx$$

[In] int(x^m/(x^8+3*x^4+1),x)

[Out] int(x^m/(x^8+3*x^4+1),x)

Fricas [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] integral(x^m/(x^8 + 3*x^4 + 1), x)

Sympy [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3x^4+1}\,dx$$

[In] integrate(x**m/(x**8+3*x**4+1),x)

[Out] Integral(x**m/(x**8 + 3*x**4 + 1), x)

Maxima [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)

Giac [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

```
[In] int(x^m/(3*x^4 + x^8 + 1),x)
[Out] int(x^m/(3*x^4 + x^8 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] 1/4*x^4-1/40*ln(2*x^4-5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(2*x^4+5^(1/2)+3)*(1
5+7*5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 717, 646, 31}

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{x^4}{4} - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[In] Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1+3x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-3x}{1+3x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad -\frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{1}{40}\Big(10x^4 + \left(-15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) - \left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\Big)$$

[In] Integrate[x^11/(1 + 3*x^4 + x^8),x]

[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x^4])/40

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}-\frac{3\ln(x^8+3x^4+1)}{8}-\frac{7\,\mathrm{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	38
risch	$\frac{x^4}{4}-\frac{3\ln\left(2x^4-\sqrt{5}+3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{40}-\frac{3\ln\left(2x^4+\sqrt{5}+3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{40}$	69

[In] int(x^11/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)

[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+3x^4+x^8} dx = \frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right) - \frac{3}{8}\log\left(x^8+3x^4+1\right)$$

[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 +
 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{x^4}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

```
[In] integrate(x**11/(x**8+3*x**4+1),x)
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/40
 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{7\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}-\frac{3\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{7\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

```
[In] int(x^11/(3*x^4 + x^8 + 1),x)
```

```
[Out] (7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))/
8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2
))/40 + x^4/4
```

3.369 $\int \frac{x^9}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{x^9}{1+3x^4+x^8}\,dx=\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctan(x^2*2^
(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1136, 1180, 209}

$$\int \frac{x^9}{1+3x^4+x^8}\,dx=-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)+\frac{x^2}{2}$$

```
[In] Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1+3x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&\quad - \frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{20}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{40}\left(20x^2-\sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\sqrt{2\left(3+\sqrt{5}\right)}\left(-15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8),x]
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]
*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2
]*x^2])/40
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(15\_R^3+8x^2+47\_R\right)\right)}{4}$	42
default	$\frac{x^2}{2}-\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	79

```
[In] int(x^9/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/4*sum(_R*ln(15*_R^3+8*x^2+47*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{20}\,\sqrt{5}\sqrt{4\,\sqrt{5}-9}\log\left(2\,x^2+\sqrt{4\,\sqrt{5}-9}\left(\sqrt{5}+3\right)\right) - \frac{1}{20}\,\sqrt{5}\sqrt{4\,\sqrt{5}-9}\log\left(2\,x^2-\sqrt{4\,\sqrt{5}-9}\left(\sqrt{5}+3\right)\right) + \frac{1}{20}\,\sqrt{5}\sqrt{-4\,\sqrt{5}-9}\log\left(2\,x^2+\left(\sqrt{5}-3\right)\sqrt{-4\,\sqrt{5}-9}\right) - \frac{1}{20}\,\sqrt{5}\sqrt{-4\,\sqrt{5}-9}\log\left(2\,x^2-\left(\sqrt{5}-3\right)\sqrt{-4\,\sqrt{5}-9}\right)$$

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/20*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(2*x^2 + sqrt(4*sqrt(5) - 9)*
(sqrt(5) + 3)) - 1/20*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(2*x^2 - sqrt(4*sqrt(5
) - 9)*(sqrt(5) + 3)) + 1/20*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(2*x^2 + (sqrt
(5) - 3)*sqrt(-4*sqrt(5) - 9)) - 1/20*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(2*x^
2 - (sqrt(5) - 3)*sqrt(-4*sqrt(5) - 9))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{x^2}{2} + 2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

```
[In] integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 +
 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \int \frac{x^9}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}+\frac{768\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192} - \frac{768\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}+\frac{x^2}{2}$$

```
[In] int(x^9/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(1
/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80)^
(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192) -
 (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5^(1/
2)/20 - 9/80)^(1/2) + x^2/2
```

3.370 $\int \frac{x^7}{1+3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1+3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] 1/40*ln(2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(2*x^4+5^(1/2)+3)*(5+3*5^(1/2
))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 646, 31}

$$\int \frac{x^7}{1+3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

```
[In] Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+3x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[In] Integrate[x^7/(1 + 3*x^4 + x^8),x]
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + S
qrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln\left(x^{8}+3x^{4}+1\right)}{8}+\frac{3\,\mathrm{arctanh}\left(\frac{\left(2x^{4}+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	33
risch	$\frac{\ln\left(2x^{4}+\sqrt{5}+3\right)}{8}+\frac{3\ln\left(2x^{4}+\sqrt{5}+3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^{4}-\sqrt{5}+3\right)}{8}-\frac{3\ln\left(2x^{4}-\sqrt{5}+3\right)\sqrt{5}}{40}$	64

```
[In] int(x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{1+3x^4+x^8}\,dx=\frac{3}{40}\,\sqrt{5}\log\left(\frac{2\,x^{8}+6\,x^{4}+\sqrt{5}(2\,x^{4}+3)+7}{x^{8}+3\,x^{4}+1}\right)+\frac{1}{8}\,\log\left(x^{8}+3\,x^{4}+1\right)$$

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) + 1/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8}\,dx=\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^{4}-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^{4}+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

```
[In] integrate(x**7/(x**8+3*x**4+1),x)
[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log
(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8} - \frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

```
[In] int(x^7/(3*x^4 + x^8 + 1),x)
[Out] log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

3.371 $\int \frac{x^5}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^5}{1+3x^4+x^8}\,dx=\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))+1/2*arctan(x^2*2^(1/2
)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1373, 1144, 209}

$$\int \frac{x^5}{1+3x^4+x^8}\,dx=\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[In] Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&\quad+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{2\sqrt{5}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\left(5-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

```
[In] Integrate[x^5/(1 + 3*x^4 + x^8),x]
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}+15\_Z^{2}+1\right)}\_R\ln\left(-10\_R^{3}+x^{2}-3\_R\right)\right)}{4}$	34
default	$\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	70

```
[In] int(x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(-10*_R^3+x^2-3*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.77

$$\int \frac{x^5}{1+3x^4+x^8}\,dx=-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(10\,x^{2}+\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-3}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(10\,x^{2}-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-3}\right)+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(10\,x^{2}+\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(10\,x^{2}-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-3}\right)$$

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] -1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(10*x^2 + sqrt(10)*sqrt(5)*sqrt(sqrt(5)
 - 3)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(10*x^2 - sqrt(10)*sqrt(5)*sqrt
(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(10*x^2 + sqrt(10)*sqr
t(5)*sqrt(-sqrt(5) - 3)) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(10*x^2 - sq
rt(10)*sqrt(5)*sqrt(-sqrt(5) - 3))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = -2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

```
[In] integrate(x**5/(x**8+3*x**4+1),x)
[Out] -2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*at
an(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \int \frac{x^5}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{20}\,x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\,x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)
*arctan(2*x^2/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = 2\operatorname{atanh}\left(\frac{60\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\operatorname{atanh}\left(\frac{60\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3} - \frac{28\sqrt{5}\,x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

```
[In] int(x^5/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x^
2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2) -
 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1/2)
*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/160)^
(1/2)
```

3.372 $\int \frac{x^3}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1+3x^4+x^8}\,dx=-\frac{\operatorname{arctanh}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 632, 212}

$$\int \frac{x^3}{1+3x^4+x^8}\,dx=-\frac{\operatorname{arctanh}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[In] Int[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTanh[(3 + 2*x^4)/Sqrt[5]]/Sqrt[5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+3x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 3+2x^4\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\log\left(-3+\sqrt{5}-2x^4\right)-\log\left(3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

```
[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]
[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{20}$	36

```
[In] int(x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

```
[In] integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2
)/20
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{8\,\sqrt{5}\,x^4+3\,\sqrt{5}}{18\,x^4+7}\right)}{10}$$

```
[In] int(x^3/(3*x^4 + x^8 + 1),x)
[Out] (5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

3.373 $\int \frac{x}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1+3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctan(x^2*2^(1/2)/(3+
5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1373, 1107, 209}

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right) - \frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[In] Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+3x^2+x^4}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} \\
&= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} - \frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[In] Integrate[x/(1 + 3*x^4 + x^8),x]
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3
 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+15\_Z^{2}+1\right)}\_R\ln\left(-15\_R^{3}+x^{2}-7\_R\right)\right)}{4}$	34
default	$-\frac{2\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	60

```
[In] int(x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(-15*_R^3+x^2-7*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.12

$$\int \frac{x}{1+3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(20\,x^2+\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{\sqrt{5}-3}\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-3}\log\left(20\,x^2-\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{\sqrt{5}-3}\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(20\,x^2+\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{5}-3}\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-3}\log\left(20\,x^2-\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{5}-3}\right)$$

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] -1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(20*x^2 + sqrt(10)*(3*sqrt(5) + 5)*sqrt
(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 3)*log(20*x^2 - sqrt(10)*(3*s
qrt(5) + 5)*sqrt(sqrt(5) - 3)) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 3)*log(20*x^
2 + sqrt(10)*(3*sqrt(5) - 5)*sqrt(-sqrt(5) - 3)) - 1/40*sqrt(10)*sqrt(-sqrt
(5) - 3)*log(20*x^2 - sqrt(10)*(3*sqrt(5) - 5)*sqrt(-sqrt(5) - 3))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

```
[In] integrate(x/(x**8+3*x**4+1),x)
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*ata
n(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x}{1+3x^4+x^8}\,dx = \int \frac{x}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan(
2*x^2/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}-\frac{72\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18} + \frac{72\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

```
[In] int(x/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/2
)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160)^
(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18) +
(72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(1/2)
/160 - 3/160)^(1/2)
```

3.374 $\int \frac{1}{x(1+3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] ln(x)-1/40*ln(2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(2*x^4-5^(1/2)+3)*(5+3*
5^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 719, 29, 646, 31}

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right) + \log(x)$$

```
[In] Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1+3x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{1+3x+x^2}\,dx, x, x^4\right)\\
&= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad -\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \log(x)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5]
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

method	result	size
default	$\ln(x)-\frac{\ln(x^8+3x^4+1)}{8}+\frac{3\,\mathrm{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	35
risch	$\ln(x)-\frac{\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(3x^4-\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4-\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)}{8}$	70

```
[In] int(1/x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(1+3x^4+x^8\right)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right) - \frac{1}{8}\log\left(x^8+3\,x^4+1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \log(x) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

```
[In] integrate(1/x/(x**8+3*x**4+1),x)
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\,(1+3x^4+x^8)}\,dx = \ln(x) - \ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right) + \ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)$$

```
[In] int(1/(x*(3*x^4 + x^8 + 1)),x)
[Out] log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
+ x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = -\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2-1/40*arctan(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)+1/2*
arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1137, 1180, 209}

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}} - \frac{1}{2x^2}$$

```
[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\,(1+3x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{-3-x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&\quad -\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{10}\sqrt{45-20\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^3 (1+3x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4 + \#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]

[Out] -1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) &]/4

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(35\_R^3+8x^2+123\_R\right)\right)}{4}$	42
default	$-\frac{1}{2x^2}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	75

[In] int(1/x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)

[Out] -1/2/x^2+1/4*sum(_R*ln(35*_R^3+8*x^2+123*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^3 (1+3x^4+x^8)}\,dx = \frac{\sqrt{5}x^2\sqrt{4\sqrt{5}-9}\log\left(2\,x^2+\sqrt{4\sqrt{5}-9}(3\sqrt{5}+7)\right)-\sqrt{5}x^2\sqrt{4\sqrt{5}-9}\log\left(2\,x^2-\sqrt{4\sqrt{5}-9}(3\sqrt{5}+7\right)}{}$$

[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")

```
[Out] 1/20*(sqrt(5)*x^2*sqrt(4*sqrt(5) - 9)*log(2*x^2 + sqrt(4*sqrt(5) - 9)*(3*sq
rt(5) + 7)) - sqrt(5)*x^2*sqrt(4*sqrt(5) - 9)*log(2*x^2 - sqrt(4*sqrt(5) -
9)*(3*sqrt(5) + 7)) + sqrt(5)*x^2*sqrt(-4*sqrt(5) - 9)*log(2*x^2 + (3*sqrt(
5) - 7)*sqrt(-4*sqrt(5) - 9)) - sqrt(5)*x^2*sqrt(-4*sqrt(5) - 9)*log(2*x^2
- (3*sqrt(5) - 7)*sqrt(-4*sqrt(5) - 9)) - 10)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 (1+3x^4+x^8)}\,dx = -2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

```
[In] integrate(1/x**3/(x**8+3*x**4+1),x)
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*at
an(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (1+3x^4+x^8)}\,dx = \int \frac{1}{(x^8+3\,x^4+1)x^3}\,dx$$

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 (1+3x^4+x^8)}\,dx = -\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)-\frac{1}{2\,x^2}$$

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1/
20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2/x
^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3\,(1+3x^4+x^8)}\,dx = 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}+\frac{12032\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}-\frac{12032\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}-\frac{1}{2\,x^2}$$

```
[In] int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (12
032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^(1
/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*
5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/
2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] -1/4/x^4-3*ln(x)+1/40*ln(2*x^4+5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(2*x^4-5^(1
/2)+3)*(15+7*5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 723, 814, 646, 31}

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = -\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

```
[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+3x+x^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{x\left(1+3x+x^2\right)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{3}{x}+\frac{8+3x}{1+3x+x^2}\right)\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}-3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8+3x}{1+3x+x^2}\,dx, x, x^4\right)
\end{aligned}$$

$$= -\frac{1}{4x^4} - 3\log(x) + \frac{1}{40}\left(15 - 7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x}\,dx, x, x^4\right) + \frac{1}{40}\left(15 + 7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x}\,dx, x, x^4\right)$$

$$= -\frac{1}{4x^4} - 3\log(x) + \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) + \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5\left(1+3x^4+x^8\right)}\,dx = \frac{1}{40}\left(-\frac{10}{x^4} - 120\log(x) + \left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

method	result
default	$-\frac{1}{4x^4} - 3\ln(x) + \frac{3\ln(x^8+3x^4+1)}{8} - \frac{7\,\text{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{1}{4x^4} - 3\ln(x) + \frac{3\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8} + \frac{7\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40} + \frac{3\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8} - \frac{7\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

```
[In] int(1/x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^5 (1+3x^4+x^8)}\,dx = \frac{7\sqrt{5}x^4 \log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)+15\,x^4\log\left(x^8+3\,x^4+1\right)-120\,x^4\log\left(x\right)-10}{40\,x^4}$$

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*
x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5 (1+3x^4+x^8)}\,dx = -3\log\left(x\right)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

```
[In] integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (1+3x^4+x^8)}\,dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)-\frac{1}{4\,x^4}+\frac{3}{8}\log\left(x^8+3\,x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 + 3
/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{3\,x^4-1}{4\,x^4} + \frac{3}{8}\log\left(x^8+3\,x^4+1\right)-\frac{3}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
- 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4\,x^4} - 3\,\ln\left(x\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)$$

```
[In] int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

3.377 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/6/x^6+3/2/x^2-1/2*arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1/
2))+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1373, 1137, 1295, 1180, 209}

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{6x^6}+\frac{3}{2x^2}$$

```
[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 (1+3x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6} + \frac{1}{6}\text{Subst}\left(\int \frac{-9-3x^2}{x^2 (1+3x^2+x^4)}\,dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{-24-9x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$

$$= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)$$

$$= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{20}\sqrt{1230+550\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6} + \frac{3}{2x^2} + \frac{1}{4}\text{RootSum}\left[1+3\#1^4 + \#1^8\&, \frac{8\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

```
[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
[Out] -1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*L
og[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\frac{3x^4}{2}-\frac{1}{6}}{x^6} + \frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+615\_Z^2+1\right)} \_R\ln\left(-90\_R^3+55x^2-2207\_R\right)\right)}{4}$	48
default	$-\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}}+\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	84

```
[In] int(1/x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (3/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-90*_R^3+55*x^2-2207*_R),_R=RootOf(25*_Z^4+
615*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(55) = 110$.

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{3\sqrt{10}x^6\sqrt{55\sqrt{5}-123}\log\left(10\,x^2+\sqrt{10}\sqrt{55\sqrt{5}-123}(9\sqrt{5}+20)\right)-3\sqrt{10}x^6\sqrt{55\sqrt{5}-123}\log\left(10\right.}{}$$

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/120*(3*sqrt(10)*x^6*sqrt(55*sqrt(5) - 123)*log(10*x^2 + sqrt(10)*sqrt(55
*sqrt(5) - 123)*(9*sqrt(5) + 20)) - 3*sqrt(10)*x^6*sqrt(55*sqrt(5) - 123)*l
og(10*x^2 - sqrt(10)*sqrt(55*sqrt(5) - 123)*(9*sqrt(5) + 20)) - 3*sqrt(10)*
x^6*sqrt(-55*sqrt(5) - 123)*log(10*x^2 + sqrt(10)*(9*sqrt(5) - 20)*sqrt(-55
*sqrt(5) - 123)) + 3*sqrt(10)*x^6*sqrt(-55*sqrt(5) - 123)*log(10*x^2 - sqrt
(10)*(9*sqrt(5) - 20)*sqrt(-55*sqrt(5) - 123)) - 180*x^4 + 20)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = 2\cdot\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)+\frac{9x^4-1}{6x^6}$$

```
[In] integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/4
0)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx=\int \frac{1}{(x^8+3\,x^4+1)x^7}\,dx$$

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx=\frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)+\frac{9\,x^4-1}{6\,x^6}$$

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) + 1
/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) + 1/
6*(9*x^4 - 1)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx=2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}-\frac{1488300\,\sqrt{5}\,x^2\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}\right)\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}-2\,\mathrm{atanh}\left(\frac{3327500\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}+\frac{1488300\,\sqrt{5}\,x^2\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}\right)\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}+\frac{\frac{3\,x^4}{2}-\frac{1}{6}}{x^6}$$

```
[In] int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425
*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((3327500*x
^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (1488
300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 255
0075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

3.378 $\int \frac{x^8}{1+3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 460

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] x+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)*2^(1/4
)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)
*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(12
3-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(
1/4)+5^(1/2)-1)*(123-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/
4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(
1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*
ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+55*5^(1/2))^(1/4)*2^
```

```
(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+5
5*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1381, 1436, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\sqrt{10}} + \frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{984-440\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}} + \frac{\sqrt[4]{984-440\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}} + \frac{\sqrt[4]{123+55\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} + x$$

```
[In] Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt
[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x
)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
+ 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x-\int \frac{1+3x^4}{1+3x^4+x^8}\,dx\\
&= x-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= x+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\
&\quad+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\
&\quad-\frac{\left(15+7\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{20\sqrt{3+\sqrt{5}}}-\frac{\left(15+7\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{20\sqrt{3+\sqrt{5}}}
\end{aligned}$$

$$= x + \frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)} - \sqrt[4]{2\left(3-\sqrt{5}\right)}x + x^2}\,dx$$

$$+ \frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)} + \sqrt[4]{2\left(3-\sqrt{5}\right)}x + x^2}\,dx$$

$$- \frac{\left(\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\sqrt[4]{3+\sqrt{5}}\right)\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}}$$

$$- \frac{\left(\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\sqrt[4]{3+\sqrt{5}}\right)\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}}$$

$$- \frac{1}{4}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)} - \sqrt[4]{2\left(3+\sqrt{5}\right)}x + x^2}\,dx$$

$$- \frac{1}{4}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)} + \sqrt[4]{2\left(3+\sqrt{5}\right)}x + x^2}\,dx$$

$$+ \frac{\sqrt[4]{123+55\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{123+55\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$=x-\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)$$
$$+\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)$$
$$+\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{123+55\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{123-55\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{123-55\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{123+55\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{123+55\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$=x-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$-\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)+\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right.}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.13

$$\int \frac{x^8}{1+3x^4+x^8} \, dx = x - \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&, \frac{\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[x^8/(1 + 3*x^4 + x^8),x]
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.10

method	result	size
default	$x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-3\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	46
risch	$x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-3\_R^4-1\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	46

```
[In] int(x^8/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.92

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20x\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{55\sqrt{5}-123}}\left(3\sqrt{5}+5\right)+20x\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20x\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\log\left(-\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-55\sqrt{5}-123}}\left(3\sqrt{5}-5\right)+20x\right) + x$$

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*log(sqrt(10)*sqrt(sqrt(2
)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt
(2)*sqrt(55*sqrt(5) - 123))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 12
3))*(3*sqrt(5) + 5) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) -
 123))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) + 5) +
 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*log(-sqrt(10)*
sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(1
0)*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(sqrt(10)*sqrt(sqrt(2)*sqrt(-55
*sqrt(5) - 123))*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(
-55*sqrt(5) - 123))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*(3*
```

```
sqrt(5) - 5) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))
*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*(3*sqrt(5) - 5) + 20*x
) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(-sqrt(10)*sqrt
(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*(3*sqrt(5) - 5) + 20*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.06

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

```
[In] integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**
5/11 + 1288*_t/55 + x)))
```

Maxima [F]

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = \int \frac{x^8}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x+\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x-\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x+\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x-\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right) + x$$

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25
*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*x^2) -
1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) + x
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.47

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$
$$- \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$

```
[In] int(x^8/(3*x^4 + x^8 + 1),x)
```

```
[Out] x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (2
^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1/4
))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4)) -
(2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1/4))
/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/4)) +
 (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123
)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2) - 123)
^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2)
- 123)^(1/4)*1i)/20
```

3.379 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 431

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$

```
[Out] -1/40*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^(1
/2)-1/40*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^
(1/2)-1/80*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(3
/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
```

```
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)-1/80*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1388, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}$$

```
[In] Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int\frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int\frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$=\frac{\left(3-\sqrt{5}\right)\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}-\frac{\left(3-\sqrt{5}\right)\int\frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int\frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}dx}{4\sqrt{10}}$$

$$-\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}dx}{4\sqrt{10}}$$

$$+\frac{\left(3+\sqrt{5}\right)^{3/4}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}dx}{8\sqrt[4]{2}\sqrt{5}}$$

$$+\frac{\left(3+\sqrt{5}\right)^{3/4}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}dx}{8\sqrt[4]{2}\sqrt{5}}$$

$$-\frac{1}{40}\left(-5+3\sqrt{5}\right)\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}dx$$

$$-\frac{1}{40}\left(-5+3\sqrt{5}\right)\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}dx$$

$$+\frac{1}{40}\left(5+3\sqrt{5}\right)\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}dx$$

$$+\frac{1}{40}\left(5+3\sqrt{5}\right)\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}dx$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}}$$

$$+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}}$$

$$+\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$

$$-\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$

$$-\frac{\left(3-\sqrt{5}\right)^{3/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}$$

$$-\frac{\left(5-3\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{20\sqrt[4]{2\left(3-\sqrt{5}\right)}}$$

$$+\frac{\left(3+\sqrt{5}\right)^{3/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}$$

$$-\frac{\left(3+\sqrt{5}\right)^{3/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}$$

$$= \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}$$
$$- \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}$$
$$- \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}}$$
$$+ \frac{\sqrt[4]{9-4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}}$$
$$+ \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$
$$- \frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{8\sqrt[4]{2}\sqrt{5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.10

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{3+2\#1^4}\&\right]$$

```
[In] Integrate[x^6/(1 + 3*x^4 + x^8),x]
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40

```
[In] int(x^6/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \frac{x^6}{1+3x^4+x^8}\,dx \\
&= \frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&\quad -\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&\quad +\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right) \\
&\quad -\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right) \\
&\quad -\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(4\sqrt{5}-9\right)^{\frac{3}{4}}\left(3\sqrt{5}+7\right)+2x\right) \\
&\quad +\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(4\sqrt{5}-9\right)^{\frac{3}{4}}\left(3\sqrt{5}+7\right)+2x\right) \\
&\quad -\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(3\sqrt{5}-7\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right) \\
&\quad +\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(3\sqrt{5}-7\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right)
\end{aligned}$$

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(sqrt(4*sqrt(5) - 9)*(3*sqrt(5)
+ 7)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5)
- 9))*log(-sqrt(4*sqrt(5) - 9)*(3*sqrt(5) + 7)*sqrt(-sqrt(4*sqrt(5) - 9)) +
 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log((3*sqrt(5) - 7)*sqrt(-
4*sqrt(5) - 9)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt
(-4*sqrt(5) - 9))*log(-(3*sqrt(5) - 7)*sqrt(-4*sqrt(5) - 9)*sqrt(-sqrt(-4*s
qrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log((4*sqrt(5) - 9
)^(3/4)*(3*sqrt(5) + 7) + 2*x) + 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log(-(4
*sqrt(5) - 9)^(3/4)*(3*sqrt(5) + 7) + 2*x) - 1/20*sqrt(5)*(-4*sqrt(5) - 9)^
(1/4)*log((3*sqrt(5) - 7)*(-4*sqrt(5) - 9)^(3/4) + 2*x) + 1/20*sqrt(5)*(-4*
sqrt(5) - 9)^(1/4)*log(-(3*sqrt(5) - 7)*(-4*sqrt(5) - 9)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-1792000t^7-4920t^3+x\right)\right)\right)$$

```
[In] integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7
 - 4920*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.55

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}+1}\right)^2+400\,x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}+1}\right)^2+400\,x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}-1}\right)^2+10000\,x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}-1}\right)^2+10000\,x^2\right)$$

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5
) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5)
 - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(10*sqrt
(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

```
[In] int(x^6/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2)
 - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) - 2
4))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)*16
i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*(4*5
^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

3.380 $\int \frac{x^4}{1+3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 451

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1
/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^
(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(1
/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
```

```
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1388, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{x^4}{1+3x^4+x^8}\,dx &= \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} \\ &- \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} \\ &- \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} \\ &+ \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\ 2^{3/4}\sqrt{5}} \\ &+ \frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &- \frac{\sqrt[4]{3-\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &- \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}} \\ &+ \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\ 2^{3/4}\sqrt{5}}\end{aligned}$$

```
[In] Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NegQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}\left(3+\sqrt{5}\right)}\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{4}\sqrt{\frac{1}{5}\left(3+\sqrt{5}\right)}\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx\right)$$

$$-\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx$$

$$+\frac{\sqrt[4]{3-\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$+\frac{\sqrt[4]{3-\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$+\frac{1}{4}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx$$

$$+\frac{1}{4}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx$$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$-\frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$=\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{3-\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{3-\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$+\frac{\sqrt[4]{3+\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$
$$-\frac{\sqrt[4]{3+\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{3-\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{3+2\#1^4}\&\right]$$

```
[In] Integrate[x^4/(1 + 3*x^4 + x^8),x]
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}}{4}$	40
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}}{4}$	40

```
[In] int(x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}+10\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right) + \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right) - \frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}+10\,x\right)$$

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(sq
rt(2)*sqrt(sqrt(5) - 3)) + 10*x) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5)
- 3))*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3)) + 10*x) - 1/40*
sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(-sqrt(2
)*sqrt(sqrt(5) - 3)) + 10*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3
))*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3)) + 10*x) + 1/40*sq
rt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(sqrt(2)*s
qrt(-sqrt(5) - 3)) + 10*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))
*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3)) + 10*x) + 1/40*sqrt
(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)*sqrt(5)*sqrt(-sqrt(2)*s
qrt(-sqrt(5) - 3)) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3)
)*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3)) + 10*x)
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-51200t^5-12t+x\right)\right)\right)$$

```
[In] integrate(x**4/(x**8+3*x**4+1),x)
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
12*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.53

$$\begin{aligned}\int \frac{x^4}{1+3x^4+x^8}\,dx = {} & \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\ & -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\ & +\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\ & +\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x+\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x-\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\ & -\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x+\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right) \\ & +\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x-\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right)\end{aligned}$$

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) + 5)*l
og(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*log(
625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*log(422
5*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*log(4225
*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{1+3x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}$$
$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20}$$

```
[In] int(x^4/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(-
 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2^(
3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/2))
/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4))/20
 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2
)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) -
(2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/2) - 3)
^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1
/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*((3
*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2))
 + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/
```

```
2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4))/20
+ (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2)*(
5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)) + (2^(3/4
)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 +
 (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4)*1i)/20
```

3.381 $\int \frac{x^2}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 427

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = -\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}-\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

```
[Out] 1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(1/
2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(
1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2^(3/4)/(3-5^(1
/2))^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2
^(3/4)/(3-5^(1/2))^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4)
)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/
4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))
^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)+1/40*ln(2*x^2+2*2^(1/4)
*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1389, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

```
[In] Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2^(
3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(
1/4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2
*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*
```

```
(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
- ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4
)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + S
qrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*
(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sq
rt[5]))^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}$$
$$- \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}}$$
$$+ \frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$
$$+ \frac{\sqrt[4]{3+\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$
$$- \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$= \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}$$
$$+ \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$
$$+ \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$- \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$
$$- \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$
$$+ \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1+2\#1^5}\&\right]$$

```
[In] Integrate[x^2/(1 + 3*x^4 + x^8),x]
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

```
[In] int(x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int \frac{x^2}{1+3x^4+x^8}\,dx \\
&= -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right) \\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right) \\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right) \\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}+5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{\sqrt{5}-3}}\sqrt{\sqrt{5}-3}+40\,x\right) \\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right) \\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right) \\
&\quad-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right) \\
&\quad+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\log\left(-\sqrt{10}\left(3\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-\sqrt{5}-3}}\sqrt{-\sqrt{5}-3}+40\,x\right)
\end{aligned}$$

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*(3*sqrt(5)*sqrt
(2) + 5*sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3) + 40*x)
+ 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*log(-sqrt(10)*(3*sqrt(5)*sq
rt(2) + 5*sqrt(2))*sqrt(sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3) + 40*x
) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(sqrt(10)*(3*sqrt(5)*
sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3) + 4
0*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*log(-sqrt(10)*(3*sqrt
(5)*sqrt(2) + 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(sqrt(5) - 3))*sqrt(sqrt(5) - 3)
 + 40*x) + 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqrt(10)*(3*s
qrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*sqrt(-sqrt(5)
- 3) + 40*x) - 1/40*sqrt(10)*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*log(-sqrt(10)
*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(2)*sqrt(-sqrt(5) - 3))*sqrt(-sqr
t(5) - 3) + 40*x) - 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*log(sqr
t(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*sqr
t(-sqrt(5) - 3) + 40*x) + 1/40*sqrt(10)*sqrt(-sqrt(2)*sqrt(-sqrt(5) - 3))*l
og(-sqrt(10)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(2)*sqrt(-sqrt(5) -
3))*sqrt(-sqrt(5) - 3) + 40*x)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{1+3x^4+x^8}\,dx \\ = \mathrm{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-6144000t^7-2240t^3+x\right)\right)\right)$$

```
[In] integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
- 2240*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \int \frac{x^2}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right)$$

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,\left(\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,\left(-\sqrt{5}-3\right)^{1/4}\,1\mathrm{i}}{20}$$

```
[In] int(x^2/(3*x^4 + x^8 + 1),x)
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7))
 - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2)
- 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i)/(
2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/
2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(
- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5^(1/2)
 - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1
/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) + (2^(3/4
)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^
(1/4)*1i)/20
```

3.382 $\int \frac{1}{1+3x^4+x^8}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 414

$$\int \frac{1}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

```
[Out] -1/20*arctan(-1+x*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/20*arctan(1+x
*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/40*ln(1+2*x^2+5^(1/2)-2*x*(5^(
1/2)+1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/40*ln(1+2*x^2+5^(1/2)+2*x*(5^(1/2)+
1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/20*arctan(-1+x*(5^(1/2)+1)^(1/2))*(20+10
*5^(1/2))^(1/2)+1/20*arctan(1+x*(5^(1/2)+1)^(1/2))*(20+10*5^(1/2))^(1/2)-1/
40*ln(-1+2*x^2+5^(1/2)-2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)+1/40*ln
(-1+2*x^2+5^(1/2)+2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1361, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}} + \frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}} + \frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} - \frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

```
[In] Int[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/Sq
rt[10] + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
```

```
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3-\sqrt{5}\right)}}$$

$$+ \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} + \frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

$$- \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3+\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{2\sqrt{10\left(3+\sqrt{5}\right)}}$$

$$- \frac{\sqrt[4]{9+4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}}$$

$$- \frac{\sqrt[4]{9+4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{4\sqrt{10}}$$

$$
\begin{aligned}
&= -\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} \\
&+ \frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} \\
&+ \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \\
&- \frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \\
&+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} \\
&- \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}}+\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} \\
&+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \\
&-\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} \\
&+\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} \\
&+\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \\
&-\frac{\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

$$
\int \frac{1}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1^3+2\#1^7}\&\right]
$$

```
[In] Integrate[(1 + 3*x^4 + x^8)^(-1),x]
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37

```
[In] int(1/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1}{1+3x^4+x^8}\,dx = &-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\left(\sqrt{5}+3\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\left(\sqrt{5}+3\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right)\\&-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\left(\sqrt{5}-3\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)\\&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\left(\sqrt{5}-3\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right)\\&-\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(\sqrt{5}+3\right)+2x\right)\\&+\frac{1}{20}\sqrt{5}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(\sqrt{5}+3\right)+2x\right)\\&-\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\left(\sqrt{5}-3\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right)\\&+\frac{1}{20}\sqrt{5}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\left(\sqrt{5}-3\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right)\end{aligned}$$

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log((sqrt(5) + 3)*sqrt(-sqrt(4*sqr
t(5) - 9)) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(-(sqrt(5) +
 3)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(-4*sqrt(5)
- 9))*log((sqrt(5) - 3)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x) + 1/20*sqrt(5)*s
qrt(-sqrt(-4*sqrt(5) - 9))*log(-(sqrt(5) - 3)*sqrt(-sqrt(-4*sqrt(5) - 9)) +
 2*x) - 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log((4*sqrt(5) - 9)^(1/4)*(sqrt(
5) + 3) + 2*x) + 1/20*sqrt(5)*(4*sqrt(5) - 9)^(1/4)*log(-(4*sqrt(5) - 9)^(1
/4)*(sqrt(5) + 3) + 2*x) - 1/20*sqrt(5)*(-4*sqrt(5) - 9)^(1/4)*log((sqrt(5)
 - 3)*(-4*sqrt(5) - 9)^(1/4) + 2*x) + 1/20*sqrt(5)*(-4*sqrt(5) - 9)^(1/4)*l
og(-(sqrt(5) - 3)*(-4*sqrt(5) - 9)^(1/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

```
[In] integrate(1/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
47*_t/2 + x)))
```

Maxima [F]

$$\int \frac{1}{1+3x^4+x^8}\,dx = \int \frac{1}{x^8+3\,x^4+1}\,dx$$

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.58

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}+1}\right)^2+10000x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}+1}\right)^2+10000x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}-1}\right)^2+400x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}-1}\right)^2+400x^2\right)$$

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10*sq
rt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10
*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*sqrt(10*
sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.97

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{64\,\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{64\,\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

```
[In] int(1/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)
^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4
))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*
5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(24*5^
(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*x*(4*
5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^
(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4
)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) +
 (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2
) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 - (5^(1/2)*
atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*
(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*
(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4)*1i
)/10
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)} dx$

Optimal result

Integrand size = 16, antiderivative size = 416

$$\int \frac{1}{x^2(1+3x^4+x^8)} dx = -\frac{1}{x} + \frac{\left(3+\sqrt{5}\right)^{5/4} \arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4} \arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$-\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right) + \frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right) - \frac{(3+\sqrt{5}}{}$$

```
[Out] -1/x+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+
1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+1/40*l
n(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^(1/4)-
1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^
(1/4)-1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(246+110*5^(1
/2))^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(
246+110*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1382, 1524, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$$
$$= \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\,2^{3/4}\sqrt{5}}$$
$$-\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)-\frac{(3+\sqrt{}}{}$$

```
[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{x} + \int \frac{x^2(-3-x^4)}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{x} + \frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx - \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\frac{1}{x} - \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} + \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} + \frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}$$

$$= -\frac{1}{x} - \frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\ 2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\ 2^{3/4}\sqrt{5}} + \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{8\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{8\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} + \frac{1}{40}\left(-5+3\sqrt{5}\right)\int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx + \frac{1}{40}\left(-5+3\sqrt{5}\right)\int \frac{}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+}$$

$$= -\frac{1}{x} - \frac{(3+\sqrt{5})^{5/4}\log\left(\sqrt{2(3-\sqrt{5})} - 2\sqrt[4]{2\left(3-\sqrt{5}\right)}x + 2x^2\right)}{8\ 2^{3/4}\sqrt{5}} + \frac{(3+\sqrt{5})^{5/4}\log\left(\sqrt{2(3-\sqrt{5})} + 2\sqrt[4]{2\left(3-\sqrt{5}\right)}x + 2x^2\right)}{8\ 2^{3/4}\sqrt{5}}$$

$$+\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)-\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\Bigg($$

$$=-\frac{1}{x}+\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} -\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)+\frac{\sqrt[4]{246-110\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{(3+}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1+2\#1^5}\&\right]$$

```
[In] Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1 + 2*#1^5) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(625\_Z^8+3075\_Z^4+1\right)}\_R\ln\left(1175\_R^7+5778\_R^3+11x\right)}{4}$	42
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(\_R^6+3\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}-\frac{1}{x}$	52

```
[In] int(1/x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+1/4*sum(_R*ln(1175*_R^7+5778*_R^3+11*x),_R=RootOf(625*_Z^8+3075*_Z^4+1
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(270) = 540.

Time = 0.26 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx = \frac{\sqrt{10}x\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\log\left(\sqrt{10}\left(47\sqrt{5}\sqrt{2}+105\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{55\sqrt{5}-123}}\sqrt{55\sqrt{5}-123}+40x\right)-\cdots}{}$$

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(sqrt(10)*x*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*log(sqrt(10)*(47*sqrt
(5)*sqrt(2) + 105*sqrt(2))*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*sqrt(55*sqr
t(5) - 123) + 40*x) - sqrt(10)*x*sqrt(sqrt(2)*sqrt(55*sqrt(5) - 123))*log(-
sqrt(10)*(47*sqrt(5)*sqrt(2) + 105*sqrt(2))*sqrt(sqrt(2)*sqrt(55*sqrt(5) -
123))*sqrt(55*sqrt(5) - 123) + 40*x) - sqrt(10)*x*sqrt(-sqrt(2)*sqrt(55*sqr
t(5) - 123))*log(sqrt(10)*(47*sqrt(5)*sqrt(2) + 105*sqrt(2))*sqrt(-sqrt(2)*
sqrt(55*sqrt(5) - 123))*sqrt(55*sqrt(5) - 123) + 40*x) + sqrt(10)*x*sqrt(-s
qrt(2)*sqrt(55*sqrt(5) - 123))*log(-sqrt(10)*(47*sqrt(5)*sqrt(2) + 105*sqrt
(2))*sqrt(-sqrt(2)*sqrt(55*sqrt(5) - 123))*sqrt(55*sqrt(5) - 123) + 40*x) -
 sqrt(10)*x*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(sqrt(10)*(47*sqrt(5)*
sqrt(2) - 105*sqrt(2))*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*sqrt(-55*sqrt(
5) - 123) + 40*x) + sqrt(10)*x*sqrt(sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(-s
qrt(10)*(47*sqrt(5)*sqrt(2) - 105*sqrt(2))*sqrt(sqrt(2)*sqrt(-55*sqrt(5) -
```

```
123))*sqrt(-55*sqrt(5) - 123) + 40*x) + sqrt(10)*x*sqrt(-sqrt(2)*sqrt(-55*s
qrt(5) - 123))*log(sqrt(10)*(47*sqrt(5)*sqrt(2) - 105*sqrt(2))*sqrt(-sqrt(2
)*sqrt(-55*sqrt(5) - 123))*sqrt(-55*sqrt(5) - 123) + 40*x) - sqrt(10)*x*sqr
t(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*log(-sqrt(10)*(47*sqrt(5)*sqrt(2) - 105
*sqrt(2))*sqrt(-sqrt(2)*sqrt(-55*sqrt(5) - 123))*sqrt(-55*sqrt(5) - 123) +
40*x) - 40)/x
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = \operatorname{RootSum}\left(40960000t^8+787200t^4+1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}+\frac{369792t^3}{11}+x\right)\right)\right) - \frac{1}{x}$$

```
[In] integrate(1/x**2/(x**8+3*x**4+1),x)
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7
/11 + 369792*_t**3/11 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = \int \frac{1}{(x^8+3\,x^4+1)x^2}\,dx$$

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x+\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x-\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x+\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x-\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right) - \frac{1}{x}$$

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sqrt(25
*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^2) - 1/
40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180625*x^2)
 - 1/x
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\,(1+3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\,\left(-55\,\sqrt{5}-123\right)^{1/4}}{2\,\left(3025\,\sqrt{5}+6765\right)}+\frac{1155\,2^{3/4}\,\sqrt{5}\,x\,\left(-55\,\sqrt{5}-123\right)^{1/4}}{2\,\left(3025\,\sqrt{5}+6765\right)}\right)\,\left(-55\,\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\,\left(55\,\sqrt{5}-123\right)^{1/4}}{2\,\left(3025\,\sqrt{5}-6765\right)}-\frac{1155\,2^{3/4}\,\sqrt{5}\,x\,\left(55\,\sqrt{5}-123\right)^{1/4}}{2\,\left(3025\,\sqrt{5}-6765\right)}\right)\,\left(55\,\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-55\,\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-55\,\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}\right)\,\left(-55\,\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20} - \frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(55\,\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(55\,\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}\right)\,\left(55\,\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$

```
[In] int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

```
[Out] - 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/(
2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/
4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) - 676
5)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(-
55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*5^(1/2)
*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/
2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1/2) - 123
)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55*5^(1/2) -
 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)^(1/4)*1i)/
20
```

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 466

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx=-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/3/x^3+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/2))^(1/4
)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/
2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/
2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+
5^(1/2))^(1/4)+5^(1/2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arct
an(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-
1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*
5^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(843+377*5^(
```

```
1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(
1/2)-1)*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1382, 1436, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = \frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}} - \frac{1}{3x^3} + \frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843+377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-9-3x^4}{1+3x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{3x^3}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}\\
&\quad+\frac{\left(-5+3\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{20\sqrt{3+\sqrt{5}}}+\frac{\left(-5+3\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{20\sqrt{3+\sqrt{5}}}
\end{aligned}
$$

$$= -\frac{1}{3x^3} - \frac{\sqrt[4]{843-377\sqrt{5}} \int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{843-377\sqrt{5}} \int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\left(3+\sqrt{5}\right)^{3/2} \int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{8\sqrt{10}}$$

$$- \frac{\left(3+\sqrt{5}\right)^{3/2} \int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{8\sqrt{10}}$$

$$+ \frac{\left(3+\sqrt{5}\right)^{7/4} \int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{16\sqrt[4]{2}\sqrt{5}}$$

$$+ \frac{\left(3+\sqrt{5}\right)^{7/4} \int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{16\sqrt[4]{2}\sqrt{5}}$$

$$+ \frac{\left(-5+3\sqrt{5}\right) \int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{20\sqrt{2\left(3+\sqrt{5}\right)}}$$

$$+ \frac{\left(-5+3\sqrt{5}\right) \int \frac{1}{\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+x^2}\,dx}{20\sqrt{2\left(3+\sqrt{5}\right)}}$$

$$= -\frac{1}{3x^3} + \frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}$$

$$- \frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}$$

$$- \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

$$+ \frac{\sqrt[4]{843-377\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\sqrt[4]{843-377\sqrt{5}}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}}$$

$$- \frac{\left(3+\sqrt{5}\right)^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}$$

$$+ \frac{\left(3+\sqrt{5}\right)^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}$$

$$= -\frac{1}{3x^3} + \frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+2x^2\right)}{4\ 2^{3/4}\sqrt{5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^8+21075\_Z^4+1\right)}\_R\ln\left(175\_R^5+5778\_R+377x\right)\right)}{4}$	40
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}-\frac{1}{3x^3}$	50

```
[In] int(1/x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3+1/4*sum(_R*ln(175*_R^5+5778*_R+377*x),_R=RootOf(625*_Z^8+21075*_Z^
4+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx$$
$$= \frac{3\sqrt{10}x^3\sqrt{\sqrt{2}\sqrt{377\sqrt{5}-843}}\log\left(\sqrt{10}\sqrt{\sqrt{2}\sqrt{377\sqrt{5}-843}}(7\sqrt{5}+15)+20x\right)-3\sqrt{10}x^3\sqrt{\sqrt{2}\sqrt{377}}$$

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(3*sqrt(10)*x^3*sqrt(sqrt(2)*sqrt(377*sqrt(5) - 843))*log(sqrt(10)*sq
rt(sqrt(2)*sqrt(377*sqrt(5) - 843))*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x
^3*sqrt(sqrt(2)*sqrt(377*sqrt(5) - 843))*log(-sqrt(10)*sqrt(sqrt(2)*sqrt(37
7*sqrt(5) - 843))*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-sqrt(2)*s
qrt(377*sqrt(5) - 843))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(377*sqrt(5) - 843))
*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(-sqrt(2)*sqrt(377*sqrt(5) -
 843))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(377*sqrt(5) - 843))*(7*sqrt(5) + 15
) + 20*x) - 3*sqrt(10)*x^3*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*log(sqrt(
10)*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*(7*sqrt(5) - 15) + 20*x) + 3*sqr
t(10)*x^3*sqrt(sqrt(2)*sqrt(-377*sqrt(5) - 843))*log(-sqrt(10)*sqrt(sqrt(2)
```

```
*sqrt(-377*sqrt(5) - 843))*(7*sqrt(5) - 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(-
sqrt(2)*sqrt(-377*sqrt(5) - 843))*log(sqrt(10)*sqrt(-sqrt(2)*sqrt(-377*sqrt
(5) - 843))*(7*sqrt(5) - 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-sqrt(2)*sqrt(-3
77*sqrt(5) - 843))*log(-sqrt(10)*sqrt(-sqrt(2)*sqrt(-377*sqrt(5) - 843))*(7
*sqrt(5) - 15) + 20*x) - 40)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^4 (1+3x^4+x^8)} dx = \text{RootSum}\left(40960000t^8+5395200t^4+1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}+\frac{23112t}{377}+x\right)\right)\right) - \frac{1}{3x^3}$$

```
[In] integrate(1/x**4/(x**8+3*x**4+1),x)
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5/
377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (1+3x^4+x^8)} dx = \int \frac{1}{(x^8+3x^4+1)x^4} dx$$

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} + \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) + \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) - \frac{1}{3\,x^3}$$

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt(65*
sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2) - 1/4
0*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 93122500*
x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) - 1))^2 +
 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt(sqrt(5)
 - 1))^2 + 53728900*x^2) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4\left(1+3x^4+x^8\right)}\,dx$$

$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{20735\,2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)(37}{20}$$

$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{20735\,2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right.}}{20}$$

$$-\frac{1}{3\,x^3}$$

$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-377\sqrt{5}-843\right)^{1/4}46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right.}}{20}$$

$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(377\sqrt{5}-843\right)^{1/4}46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)(}{20}$$

```
[In] int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*
2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843
)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*2^
(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^
(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*2^(3/
4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1
/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^(3/4)*5
^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2
) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 377*5^
(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2) + 1
508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 3
77*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2
) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2) - 84
3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1/2) - 843)^(1/4
)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)
*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4)
*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*
(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1i)/20
```

3.385 $\int \frac{x^m}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(3-5^(1/2)))/(1+m)/(
3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(
3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1389, 371}

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

```
[In] Int[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[5
])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{x^m}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^m}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}\\ &= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.60 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.91

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \frac{x^m\left(-\text{RootSum}\left[-1-\#1^2+\#1^4\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1+2\#1^3}\&\right]+\frac{\text{RootSum}\left[-1-\#\right.}{}\right.}{}$$

```
[In] Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSum[-1 - #1^2 + #1^4 &
 , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1
- m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m,
```

```
1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2
*#1^3) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeometric2F1
[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3)) & ] - RootSu
m[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeo
metric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2
)/(x/#1)^m)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

Maple [F]

$$\int \frac{x^m}{x^8 - 3x^4 + 1} dx$$

[In] `int(x^m/(x^8-3*x^4+1),x)`

[Out] `int(x^m/(x^8-3*x^4+1),x)`

Fricas [F]

$$\int \frac{x^m}{1 - 3x^4 + x^8} \, dx = \int \frac{x^m}{x^8 - 3\,x^4 + 1} \, dx$$

[In] `integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")`

[Out] `integral(x^m/(x^8 - 3*x^4 + 1), x)`

Sympy [F]

$$\int \frac{x^m}{1 - 3x^4 + x^8} \, dx = \int \frac{x^m}{(x^4 - x^2 - 1)\,(x^4 + x^2 - 1)} \, dx$$

[In] `integrate(x**m/(x**8-3*x**4+1),x)`

[Out] `Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)`

Maxima [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

```
[In] int(x^m/(x^8 - 3*x^4 + 1),x)
[Out] int(x^m/(x^8 - 3*x^4 + 1), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1-3x^4+x^8} dx = \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) + \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] 1/4*x^4+1/40*ln(-2*x^4-5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*
(15+7*5^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 717, 646, 31}

$$\int \frac{x^{11}}{1-3x^4+x^8} dx = \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right) + \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

```
[In] Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-3x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{4}\text{Subst}\left(\int \frac{-1+3x}{1-3x+x^2}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad + \frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \frac{x^4}{4} + \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) + \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

```
[In] Integrate[x^11/(1 - 3*x^4 + x^8),x]
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[
-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}+\frac{3\ln\left(x^8-3x^4+1\right)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}$	38
risch	$\frac{x^4}{4}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{3\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	69

```
[In] int(x^11/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)+\frac{3}{8}\log\left(x^8-3\,x^4+1\right)$$

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 -
 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{x^4}{4} + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

```
[In] integrate(x**11/(x**8-3*x**4+1),x)
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(5
)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right) + \frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 3
/8*log(x^8 - 3*x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right) + \frac{3}{8}\,\log\left(|x^8-3\,x^4+1|\right)$$

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3
)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{3\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{7\sqrt{5}\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{7\sqrt{5}\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

```
[In] int(x^11/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5^
(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2
))/40 + x^4/4
```

3.387 $\int \frac{x^9}{1-3x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{x^9}{1-3x^4+x^8} dx = \frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctanh(x^2*
2^(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1136, 1180, 213}

$$\int \frac{x^9}{1-3x^4+x^8} dx = -\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right) + \frac{x^2}{2}$$

```
[In] Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&\quad+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)\\
&\quad+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{20}\left(10x^2+\left(-5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

```
[In] Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*S
qrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result
default	$\frac{x^2}{2}-\frac{\ln(x^4+x^2-1)}{4}-\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln(x^4-x^2-1)}{4}-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}$
risch	$\frac{x^2}{2}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{4}+\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{10}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)}{4}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)\sqrt{5}}{10}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{4}+\frac{\ln\left(2x^2-\sqrt{5}-1\right.}{10}$

```
[In] int(x^9/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/4*ln
(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(63) = 126.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} + \left(-\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - \frac{47\sqrt{5}}{20} - 120\left(-\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3\right) + \left(-\frac{1}{4} + \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - 120\left(-\frac{1}{4} + \frac{\sqrt{5}}{10}\right)^3 + \frac{47\sqrt{5}}{20}\right) + \left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47\sqrt{5}}{20} - 120\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3 + \frac{47}{8}\right) + \left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)\log\left(x^2 - 120\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)^3 + \frac{47\sqrt{5}}{20} + \frac{47}{8}\right)$$

```
[In] integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(5)
/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 - 12
0*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5
)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2 - \sqrt{5} + 1}{2\,x^2 + \sqrt{5} + 1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2 - \sqrt{5} - 1}{2\,x^2 + \sqrt{5} - 1}\right) - \frac{1}{4}\,\log\left(x^4 + x^2 - 1\right) + \frac{1}{4}\,\log\left(x^4 - x^2 - 1\right)$$

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^4
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{1}{4}\,\log\left(\left|x^4+x^2-1\right|\right) + \frac{1}{4}\,\log\left(\left|x^4-x^2-1\right|\right)$$

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
+ 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4
*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} - \mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192} + \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\,\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right) - \mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192} - \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\,\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

```
[In] int(x^9/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)*x
^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

3.388 $\int \frac{x^7}{1-3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1-3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right) \log \left(3-\sqrt{5}-2x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right) \log \left(3+\sqrt{5}-2x^4\right)$$

```
[Out] 1/40*ln(-2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*(5+3*5^(1
/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 646, 31}

$$\int \frac{x^7}{1-3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right) \log \left(-2x^4-\sqrt{5}+3\right) + \frac{1}{40}\left(5+3\sqrt{5}\right) \log \left(-2x^4+\sqrt{5}+3\right)$$

```
[In] Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-3x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad+\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

```
[In] Integrate[x^7/(1 - 3*x^4 + x^8),x]
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + S
qrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln\left(x^{8}-3x^{4}+1\right)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{4}-3\right)\sqrt{5}}{5}\right)}{20}$	33
risch	$\frac{\ln\left(2x^{4}-\sqrt{5}-3\right)}{8}+\frac{3\ln\left(2x^{4}-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^{4}+\sqrt{5}-3\right)}{8}-\frac{3\ln\left(2x^{4}+\sqrt{5}-3\right)\sqrt{5}}{40}$	64

```
[In] int(x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\,\sqrt{5}\log\left(\frac{2\,x^{8}-6\,x^{4}-\sqrt{5}(2\,x^{4}-3)+7}{x^{8}-3\,x^{4}+1}\right)+\frac{1}{8}\,\log\left(x^{8}-3\,x^{4}+1\right)$$

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) + 1/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^{4}-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^{4}-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

```
[In] integrate(x**7/(x**8-3*x**4+1),x)
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*log
(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)+\frac{1}{8}\log\left(x^8-3\,x^4+1\right)$$

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8
 - 3*x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right)+\frac{1}{8}\log\left(|x^8-3\,x^4+1|\right)$$

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*l
og(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

```
[In] int(x^7/(x^8 - 3*x^4 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

3.389 $\int \frac{x^5}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))-1/2*arctanh(x^2*2^(1/
2)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1373, 1144, 213}

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

```
[In] Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -1/2*(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]) + (Sqrt[(3
 - Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&\quad+\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\begin{aligned}
\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\Big(&\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)\\
&-\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\Big)
\end{aligned}$$

```
[In] Integrate[x^5/(1 - 3*x^4 + x^8),x]
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] -
 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 +
Sqrt[5] + 2*x^2])/40
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\ln(x^4+x^2-1)}{8}-\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}+\frac{\ln(x^4-x^2-1)}{8}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}$
risch	$\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{8}-\frac{\ln\left(2x^2+\sqrt{5}-1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{40}$

```
[In] int(x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^4+x^2-1)-1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/8*ln(x^4-x
^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(58) = 116.

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.04

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right) + \left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right) + \left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{3}{2}\right) + \left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)$$

```
[In] integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40)
**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 + 3*
sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sqrt(
5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3
+ 3*sqrt(5)/10 + 3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right) + \frac{1}{8}\log\left(x^4-x^2-1\right)$$

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5
)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1)
 + 1/8*log(x^4 - x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}+1|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{|2\,x^2-\sqrt{5}-1|}{|2\,x^2+\sqrt{5}-1|}\right) - \frac{1}{8}\log\left(|x^4+x^2-1|\right)+\frac{1}{8}\log\left(|x^4-x^2-1|\right)$$

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqr
t(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x
^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\mathrm{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right) - \mathrm{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

```
[In] int(x^5/(x^8 - 3*x^4 + 1),x)
```

```
[Out] - atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
+ 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^(1
/2)/20 - 1/4)
```

3.390 $\int \frac{x^3}{1-3x^4+x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1-3x^4+x^8} dx = \frac{\operatorname{arctanh}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] 1/10*arctanh(1/5*(-2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 632, 212}

$$\int \frac{x^3}{1-3x^4+x^8} dx = \frac{\operatorname{arctanh}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[In] Int[x^3/(1 - 3*x^4 + x^8),x]
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-3x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, -3+2x^4\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{\log\left(3+\sqrt{5}-2x^4\right)-\log\left(-3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

```
[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{20}$	36

```
[In] int(x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)$$

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2
)/20
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right)$$

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{3\sqrt{5}-8\sqrt{5}\,x^4}{18\,x^4-7}\right)}{10}$$

```
[In] int(x^3/(x^8 - 3*x^4 + 1),x)
[Out] (5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

3.391 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1-3x^4+x^8}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctanh(x^2*2^(1/2)/(
3+5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1373, 1107, 213}

$$\int \frac{x}{1-3x^4+x^8}\,dx=\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[In] Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}\\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)\right)-\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

```
[In] Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5
] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*x^2])/40
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\ln\left(x^{4}+x^{2}-1\right)}{8}+\frac{\operatorname{arctanh}\left(\frac{\left(2x^{2}+1\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}+\frac{\ln\left(x^{4}-x^{2}-1\right)}{8}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{2}-1\right)\sqrt{5}}{5}\right)}{20}$
risch	$\frac{\ln\left(2x^{2}+\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^{2}+\sqrt{5}-1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^{2}-\sqrt{5}-1\right)}{8}-\frac{\ln\left(2x^{2}-\sqrt{5}-1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^{2}+\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^{2}+\sqrt{5}+1\right)\sqrt{5}}{40}-$

```
[In] int(x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^4+x^2-1)+1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/8*ln(x^4-x
^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2+\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2+\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(53) = 106.

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int \frac{x}{1-3x^4+x^8}\,dx = \left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right) + \left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right) + \left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

```
[In] integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**
3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt
(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)
/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3
+ 7*sqrt(5)/10 + 7/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right) + \frac{1}{8}\log\left(x^4-x^2-1\right)$$

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1
) + 1/8*log(x^4 - x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}+1|}{2x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}-1|}{|2x^2+\sqrt{5}-1|}\right) - \frac{1}{8}\log\left(|x^4+x^2-1|\right) + \frac{1}{8}\log\left(|x^4-x^2-1|\right)$$

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(
x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x}{1-3x^4+x^8}\,dx = \operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}-18} - \frac{13\sqrt{5}x^2}{8\sqrt{5}-18}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right) + \operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}+18} + \frac{13\sqrt{5}x^2}{8\sqrt{5}+18}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

```
[In] int(x/(x^8 - 3*x^4 + 1),x)
```

```
[Out] atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1/
2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1/2
) + 18))*(5^(1/2)/20 + 1/4)
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(-2*x^4-5^(1/2)+3)*(5+
3*5^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 719, 29, 646, 31}

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = -\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

```
[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1-3x+x^2)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{1-3x+x^2}\,dx, x, x^4\right)\\
&= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&\quad -\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\,(1-3x^4+x^8)}\,dx=\log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(x)-\frac{\ln(x^4-x^2-1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{\ln(x^4+x^2-1)}{8}+\frac{3\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	64
risch	$\ln(x)-\frac{\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(3x^4+\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4+\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)}{8}$	70

```
[In] int(1/x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(
x^4+x^2-1)+3/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\,(1-3x^4+x^8)}\,dx=\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)-\frac{1}{8}\log\left(x^8-3\,x^4+1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx = \log\left(x\right) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

```
[In] integrate(1/x/(x**8-3*x**4+1),x)
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right) - \frac{1}{8}\log\left(x^8-3\,x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^8
 - 3*x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right) + \frac{1}{4}\log\left(x^4\right) - \frac{1}{8}\log\left(\left|x^8-3\,x^4+1\right|\right)$$

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*l
og(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx = \ln(x)+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right) - \ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

```
[In] int(1/(x*(x^8 - 3*x^4 + 1)),x)
[Out] log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
+ x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2+1/40*arctanh(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/2
*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1137, 1180, 213}

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}-\frac{1}{2x^2}$$

```
[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{3-x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)\\
&\quad -\frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{10}\sqrt{45-20\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \frac{1}{20}\left(-\frac{10}{x^2}-\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*
Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

method	result
default	$-\frac{1}{2x^2}+\frac{\ln(x^4-x^2-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x^4+x^2-1)}{4}+\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}$
risch	$-\frac{1}{2x^2}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)}{4}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(4x^2-2-2\sqrt{5}\right)}{4}-\frac{\ln\left(4x^2-2-2\sqrt{5}\right)\sqrt{5}}{10}-\frac{\ln\left(4x^2+2+2\sqrt{5}\right)}{4}+\frac{\ln(4x}{}$

```
[In] int(1/x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*l
n(x^4+x^2-1)+1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \frac{2\sqrt{5}x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log(}{20x^2}$$

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^
2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4
- x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(70) = 140$.

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right) + \left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right) + \left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right) + \left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right) - \frac{1}{2x^2}$$

```
[In] integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1/
4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3 +
123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-1/4
 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqr
t(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right) - \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right) - \frac{1}{2x^2} - \frac{1}{4}\log\left(x^4+x^2-1\right) + \frac{1}{4}\log\left(x^4-x^2-1\right)$$

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x^4
 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right) - \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right) - \frac{1}{2x^2} - \frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right) + \frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/
4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}-7872} - \frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right) + \operatorname{atanh}\left(\frac{12736\,x^2}{3520\sqrt{5}+7872} + \frac{5696\sqrt{5}\,x^2}{3520\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right) - \frac{1}{2x^2}$$

```
[In] int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
- 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (569
6*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{x^5(1-3x^4+x^8)} \, dx = -\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) -\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] -1/4/x^4+3*ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(-2*x^4-5^
(1/2)+3)*(15+7*5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 723, 814, 646, 31}

$$\int \frac{1}{x^5(1-3x^4+x^8)} \, dx = -\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right) -\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

```
[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1-3x+x^2)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{x(1-3x+x^2)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(\frac{3}{x}+\frac{8-3x}{1-3x+x^2}\right)\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8-3x}{1-3x+x^2}\,dx, x, x^4\right)
\end{aligned}$$

$$= -\frac{1}{4x^4} + 3\log(x) + \frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) - \frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)$$

$$= -\frac{1}{4x^4} + 3\log(x) - \frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) - \frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{1}{40}\left(-\frac{10}{x^4} + 120\log(x) + \left(-15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right) - \left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result
default	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln(x^4-x^2-1)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{3\ln(x^4+x^2-1)}{8}+\frac{7\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

```
[In] int(1/x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/x^4+3*ln(x)-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/
2))-3/8*ln(x^4+x^2-1)+7/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{7\sqrt{5}x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)-15x^4\log\left(x^8-3x^4+1\right)+120x^4\log(x)-10}{40x^4}$$

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*
x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = 3\log(x)+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-\frac{1}{4x^4}$$

```
[In] integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)/
40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4} - \frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 - 3
/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1-3x^4+x^8)} \, dx = \frac{7}{40} \sqrt{5} \log \left(\frac{|2\, x^4 - \sqrt{5} - 3|}{|2\, x^4 + \sqrt{5} - 3|} \right) - \frac{3\, x^4 + 1}{4\, x^4} + \frac{3}{4} \log \left(x^4\right) - \frac{3}{8} \log \left(\left|x^8 - 3\, x^4 + 1\right|\right)$$

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(
3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5 (1-3x^4+x^8)} \, dx = 3 \ln (x) - \frac{1}{4\, x^4} + \ln \left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right) \left(\frac{7\, \sqrt{5}}{40} - \frac{3}{8}\right) - \ln \left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right) \left(\frac{7\, \sqrt{5}}{40} + \frac{3}{8}\right)$$

```
[In] int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] 3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

3.395 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/6/x^6-3/2/x^2-1/2*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1
/2))+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1373, 1137, 1295, 1180, 213}

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{6x^6}-\frac{3}{2x^2}$$

```
[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\,(1-3x^2+x^4)}\,dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{9-3x^2}{x^2\,(1-3x^2+x^4)}\,dx,x,x^2\right)\end{aligned}$$

$$= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{-24+9x^2}{1-3x^2+x^4}\,dx, x, x^2\right)$$

$$= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) - \frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)$$

$$= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{20}\sqrt{1230+550\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx = \frac{1}{120}\left(-\frac{20}{x^6} - \frac{180}{x^2} - 3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right) + 3\left(25-11\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) + 3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right) + 3\left(-25+11\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

```
[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
- 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[
5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
default	$-\frac{1}{6x^6} - \frac{3}{2x^2} + \frac{5\ln\left(x^4-x^2-1\right)}{8} + \frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20} - \frac{5\ln\left(x^4+x^2-1\right)}{8} + \frac{11\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$\frac{-\frac{3x^4}{2}-\frac{1}{6}}{x^6} + \frac{5\ln\left(11x^2-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)}{8} + \frac{11\ln\left(11x^2-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40} + \frac{5\ln\left(11x^2-\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)}{8} - \frac{11\ln\left(11x^2-\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40} - 5$

```
[In] int(1/x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6-3/2/x^2+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1
/2))-5/8*ln(x^4+x^2-1)+11/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^7 (1-3x^4+x^8)} dx = \frac{33\sqrt{5}x^6 \log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right) + 33\sqrt{5}x^6 \log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right) - 75x^6 \log(x^4+x^2-1) + 75x}{120x^6}$$

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x
^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) - 18
0*x^4 - 20)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(75) = 150.

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = \left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right) + \left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right) + \left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}+\frac{1152\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right) + \left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right) + \frac{-9x^4-1}{6x^6}$$

```
[In] integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt(
5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/8
- 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x**2
 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 -
 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)
/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7 (1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{9\,x^4+1}{6\,x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sqr
t(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x^6
 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^7 (1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{9\,x^4+1}{6\,x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
[Out] -11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*
sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4
 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (1-3x^4+x^8)}\,dx = \mathrm{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}-2550075} - \frac{1845250\,\sqrt{5}\,x^2}{1140425\,\sqrt{5}-2550075}\right)\left(\frac{11\,\sqrt{5}}{20}-\frac{5}{4}\right) + \mathrm{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}+2550075} + \frac{1845250\,\sqrt{5}\,x^2}{1140425\,\sqrt{5}+2550075}\right)\left(\frac{11\,\sqrt{5}}{20}+\frac{5}{4}\right)-\frac{\frac{3\,x^4}{2}+\frac{1}{6}}{x^6}$$

```
[In] int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(11
40425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(11
40425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550075
))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

3.396 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

```
[Out] x+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/2))^(1/4)*
2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/
2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1381, 1436, 218, 212, 209}

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984-440\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} + x$$

```
[In] Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x-\int \frac{1-3x^4}{1-3x^4+x^8}\,dx\\
&= x-\frac{1}{10}\left(-15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= x+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx\\
&\quad+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx-\\
&\quad-\frac{\left(-15-7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}--\frac{\left(-15-7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}
\end{aligned}
$$

$$
\begin{aligned}
&= x - \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} \\
&+ \frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} \\
&- \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} \\
&+ \frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int \frac{x^8}{1-3x^4+x^8}\,dx = x &+ \frac{\left(-2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} \\
&+ \frac{\left(-2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}
\end{aligned}
$$

```
[In] Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
 + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

method	result
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+55\_Z^2-1\right)}\_R\ln\left(15\_R^3+29\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-55\_Z^2-1\right)}\_R\ln\left(-15\_R^3+\right.\right.}{4}$
default	$x-\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\mathrm{arc}}{5\sqrt{2}}$

[In] int(x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)

[Out] x+1/4*sum(_R*ln(15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))+1/4*sum(_R *ln(-15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(118) = 236.

Time = 0.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.87

$$
\begin{aligned}
\int \frac{x^8}{1-3x^4+x^8}\,dx &= \frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20\,x\right) \\
&\quad -\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20\,x\right) \\
&\quad -\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20\,x\right) \\
&\quad +\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20\,x\right) \\
&\quad +\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}+11}\log\left(\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{-5\sqrt{5}+11}+20\,x\right) \\
&\quad -\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}+11}\log\left(-\sqrt{10}\left(3\sqrt{5}+5\right)\sqrt{-5\sqrt{5}+11}+20\,x\right) \\
&\quad -\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}-11}\log\left(\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-5\sqrt{5}-11}+20\,x\right) \\
&\quad +\frac{1}{40}\sqrt{10}\sqrt{-5\sqrt{5}-11}\log\left(-\sqrt{10}\left(3\sqrt{5}-5\right)\sqrt{-5\sqrt{5}-11}+20\,x\right) \\
&\quad +x
\end{aligned}
$$

[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 2

```
0*x) + 1/40*sqrt(10)*sqrt(-5*sqrt(5) + 11)*log(sqrt(10)*(3*sqrt(5) + 5)*sqr
t(-5*sqrt(5) + 11) + 20*x) - 1/40*sqrt(10)*sqrt(-5*sqrt(5) + 11)*log(-sqrt(
10)*(3*sqrt(5) + 5)*sqrt(-5*sqrt(5) + 11) + 20*x) - 1/40*sqrt(10)*sqrt(-5*s
qrt(5) - 11)*log(sqrt(10)*(3*sqrt(5) - 5)*sqrt(-5*sqrt(5) - 11) + 20*x) + 1
/40*sqrt(10)*sqrt(-5*sqrt(5) - 11)*log(-sqrt(10)*(3*sqrt(5) - 5)*sqrt(-5*sq
rt(5) - 11) + 20*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x + \operatorname{RootSum}\left(6400t^4-880t^2-1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+880t^2-1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11}+\frac{1288t}{55}+x\right)\right)\right)$$

```
[In] integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 +
 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(
-15360*_t**5/11 + 1288*_t/55 + x)))
```

Maxima [F]

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = \int \frac{x^8}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
1)/(x^4 + x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) + x$$

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs
(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sq
rt(1/2*sqrt(5) - 1/2))) + x
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-50\,\sqrt{5}-110}\,55\mathrm{i}}{2\,\left(275\,\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\,\sqrt{-50\,\sqrt{5}-110}\,33\mathrm{i}}{2\,\left(275\,\sqrt{5}+605\right)}\right)\,\sqrt{-50\,\sqrt{5}-110}\,1\mathrm{i}}{20} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{110-50\,\sqrt{5}}\,55\mathrm{i}}{2\,\left(275\,\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\,\sqrt{110-50\,\sqrt{5}}\,33\mathrm{i}}{2\,\left(275\,\sqrt{5}-605\right)}\right)\,\sqrt{110-50\,\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{50\,\sqrt{5}-110}\,55\mathrm{i}}{2\,\left(275\,\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\,\sqrt{50\,\sqrt{5}-110}\,33\mathrm{i}}{2\,\left(275\,\sqrt{5}-605\right)}\right)\,\sqrt{50\,\sqrt{5}-110}\,1\mathrm{i}}{20} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{50\,\sqrt{5}+110}\,55\mathrm{i}}{2\,\left(275\,\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\,\sqrt{50\,\sqrt{5}+110}\,33\mathrm{i}}{2\,\left(275\,\sqrt{5}+605\right)}\right)\,\sqrt{50\,\sqrt{5}+110}\,1\mathrm{i}}{20}$$

```
[In] int(x^8/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(
1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(1/
2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*5^(
1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2) - 6
05)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1/2)*5
5i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i)/(2*(
275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2) + 110)^
(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/20
```

3.397 $\int \frac{x^6}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{144-64\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{144-64\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

```
[Out] -1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1/2
)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(1/4)*
5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(
1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1388, 304, 209, 212}

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{144-64\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{144-64\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

```
[In] Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\ &= \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}-\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}\\ &\quad -\frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}\\ &= \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}\\ &\quad -\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\left(3-\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\frac{\left(-3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}-\frac{\left(-3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

```
[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((3
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqr
t[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-20\_Z^2-1\right)}\_R\ln\left(5\_R^3-7\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+20\_Z^2-1\right)}\_R\ln\left(5\_R^3+7\_R+2x\right)\right)}{4}$
default	$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\mathrm{arctanh}}{10\sqrt{2\sqrt{5}-}}$

```
[In] int(x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(5*_R^3-7*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))+1/4*sum(_R*ln(
5*_R^3+7*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(113) = 226.

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{x^6}{1-3x^4+x^8}\,dx &= -\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)\\&\quad+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)\\&\quad+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)\\&\quad-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)\\&\quad-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(\left(\sqrt{5}+1\right)\sqrt{-\sqrt{5}+2}+2x\right)\\&\quad+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{-\sqrt{5}+2}+2x\right)\\&\quad+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(\left(\sqrt{5}-1\right)\sqrt{-\sqrt{5}-2}+2x\right)\\&\quad-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\left(\sqrt{5}-1\right)\sqrt{-\sqrt{5}-2}+2x\right)\end{aligned}$$

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] -1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2*x)
+ 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2*x
```

```
) + 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqrt(sqrt(5) - 2) + 2*
x) - 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*sqrt(sqrt(5) - 2) +
2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log((sqrt(5) + 1)*sqrt(-sqrt(5) + 2)
 + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log(-(sqrt(5) + 1)*sqrt(-sqrt(5)
+ 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log((sqrt(5) - 1)*sqrt(-sqrt(
5) - 2) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(sqrt(5) - 1)*sqrt(-s
qrt(5) - 2) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.32

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \text{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right) + \text{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

```
[In] integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920
*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792
000*_t**7 + 4920*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \int \frac{x^6}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*s
qrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10)*
log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x
 - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sqrt(1
/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*sqrt(5
) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{2-\sqrt{5}}}{8\,\sqrt{5}-24}\right)\,\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{-\sqrt{5}-2}}{8\,\sqrt{5}+24}\right)\,\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}$$

```
[In] int(x^6/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(1
/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))*(
5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8*5^
(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) - 2)^
(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

3.398 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] 1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)
*5^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4)*2^(
3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4
)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1388, 218, 212, 209}

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[In] Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -1/2*(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/Sqrt[5] +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx+\frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx\\
&\quad -\frac{1}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{5}\right)}\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx-\frac{1}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{5}\right)}\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx\\
&= -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}\\
&\quad -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{-1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{-1+\sqrt{5}}\,\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{1-}}{2\sqrt{10}}$$

```
[In] Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*Ar
cTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10]
)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(10\_R^3+\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-10\_R^3+\_R+x\right)\right)}{4}$
default	$-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\mathrm{arctan}}{10\sqrt{2\sqrt{}}}$

[In] int(x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)

[Out] 1/4*sum(_R*ln(10*_R^3+_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))+1/4*sum(_R*ln(-10*_R^3+_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(119) = 238.

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\begin{aligned}\int\frac{x^4}{1-3x^4+x^8}\,dx=&-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}+1}+10x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}+1}+10x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-1}+10x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{-\sqrt{5}-1}+10x\right)\end{aligned}$$

[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] -1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) +

```
 1) + 10*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(sqr
t(5) - 1) + 10*x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*s
qrt(sqrt(5) - 1) + 10*x) + 1/40*sqrt(10)*sqrt(-sqrt(5) + 1)*log(sqrt(10)*sq
rt(5)*sqrt(-sqrt(5) + 1) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) + 1)*log(-sq
rt(10)*sqrt(5)*sqrt(-sqrt(5) + 1) + 10*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 1
)*log(sqrt(10)*sqrt(5)*sqrt(-sqrt(5) - 1) + 10*x) + 1/40*sqrt(10)*sqrt(-sqr
t(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(-sqrt(5) - 1) + 10*x)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)$$

```
[In] integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t +
 x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 +
 12*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \int \frac{x^4}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) +
10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(
abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x +
sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqrt(1/2
*sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}$$

```
[In] int(x^4/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5
^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2)
- 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(2*(
5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2) +
1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^
(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(
10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*
(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) -
1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

3.399 $\int \frac{x^2}{1-3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{x^2}{1-3x^4+x^8} \, dx = \frac{1}{20}\sqrt{-10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right) - \frac{1}{20}\sqrt{10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right) - \frac{1}{20}\sqrt{-10+10\sqrt{5}}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right) + \frac{1}{20}\sqrt{10+10\sqrt{5}}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)$$

```
[Out] 1/20*arctan(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctanh
(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctan(1/2*x*(2+2*
5^(1/2))^(1/2))*(10+10*5^(1/2))^(1/2)+1/20*arctanh(1/2*x*(2+2*5^(1/2))^(1/2
))*(10+10*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {1389, 304, 209, 212}

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[In] Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - Ar
cTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((
(3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral}=\frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}-\frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$=\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}}-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}}-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}}+\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}}$$

$$=\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

$$-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1-3x^4+x^8}\,dx=-\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2
/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*
x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}-5\_Z^{2}-1\right)}\_R\ln\left(-5\_R^{3}+3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+5\_Z^{2}-1\right)}\_R\ln\left(-5\_R^{3}-3\_R+x\right)\right)}{4}$
default	$-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}$

```
[In] int(x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(-5*_R^3+3*_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))+1/4*sum(_R*ln(-5
*_R^3-3*_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(97) = 194.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{x^2}{1-3x^4+x^8}\,dx =& -\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}+1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{-\sqrt{5}+1}+20\,x\right)\\&+\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\\&-\frac{1}{40}\sqrt{10}\sqrt{-\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}-5\right)\sqrt{-\sqrt{5}-1}+20\,x\right)\end{aligned}$$

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(sqrt(5) -
1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5) + 5)*sq
```

```
rt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(sqrt(10)*sqrt
(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-
sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(-sqrt
(5) + 1)*log(sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*x) - 1/40*sqrt(
10)*sqrt(-sqrt(5) + 1)*log(-sqrt(10)*(sqrt(5) + 5)*sqrt(-sqrt(5) + 1) + 20*
x) + 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) - 5)*sqrt(-sqrt
(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(-sqrt(5) - 1)*log(-sqrt(10)*(sqrt(5)
- 5)*sqrt(-sqrt(5) - 1) + 20*x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)$$

```
[In] integrate(x**2/(x**8-3*x**4+1),x)
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*_
t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(6144000*
_t**7 - 2240*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

```
[In] int(x^2/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) -
1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2)
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

3.400 $\int \frac{1}{1-3x^4+x^8}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}$$

```
[Out] -1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(
3/4)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/
2))^(3/4)+1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*
5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*5
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1361, 218, 212, 209}

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}$$

```
[In] Int[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
```

```
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} \\
&= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} \\
&= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}} \\
&\quad - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt{5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$
\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}
$$

```
[In] Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt
[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+20\_Z^2-1\right)}\_R\ln\left(-15\_R^3-11\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-20\_Z^2-1\right)}\_R\ln\left(15\_R^3-11\_R\right.\right.}{4}$
default	$-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\mathrm{arctan}}{10\sqrt{2\sqrt{5}}}$

```
[In] int(1/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(-15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))+1/4*sum(_R*
ln(15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(113) = 226.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$
\begin{aligned}
\int \frac{1}{1-3x^4+x^8}\,dx = &-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)\\
&+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)\\
&-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)\\
&+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)\\
&-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(\left(\sqrt{5}+3\right)\sqrt{-\sqrt{5}+2}+2x\right)\\
&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}+2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{-\sqrt{5}+2}+2x\right)\\
&-\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(\left(\sqrt{5}-3\right)\sqrt{-\sqrt{5}-2}+2x\right)\\
&+\frac{1}{20}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\left(\sqrt{5}-3\right)\sqrt{-\sqrt{5}-2}+2x\right)
\end{aligned}
$$

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2*x)
+ 1/20*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2*x
```

```
) - 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 3) + 2*
x) + 1/20*sqrt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 3) +
2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log((sqrt(5) + 3)*sqrt(-sqrt(5) + 2)
 + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) + 2)*log(-(sqrt(5) + 3)*sqrt(-sqrt(5)
+ 2) + 2*x) - 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log((sqrt(5) - 3)*sqrt(-sqrt(
5) - 2) + 2*x) + 1/20*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(sqrt(5) - 3)*sqrt(-s
qrt(5) - 2) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{1}{1-3x^4+x^8}\,dx = \text{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right) + \text{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

```
[In] integrate(1/(x**8-3*x**4+1),x)
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
+ x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 -
 47*_t/2 + x)))
```

Maxima [F]

$$\int \frac{1}{1-3x^4+x^8}\,dx = \int \frac{1}{x^8-3\,x^4+1}\,dx$$

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.87

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5*
sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 10)
*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(
x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + sqrt(
1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.45

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{2-\sqrt{5}}\,144\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{2-\sqrt{5}}\,64\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{5}-2}\,144\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{-\sqrt{5}-2}\,64\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}-2}\,144\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{\sqrt{5}-2}\,64\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}+2}\,144\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{\sqrt{5}+2}\,64\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}$$

```
[In] int(1/(x^8 - 3*x^4 + 1),x)

[Out] (5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2)
*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*1i
)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) - (5^
(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/2)*1
i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232) - (5
^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (
5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) + 2)^(1/2)
*1i)/10
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{984-440\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{5/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

```
[Out] -1/x+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4
)*5^(1/2)-1/40*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/4)*2^
(3/4)*5^(1/2)+1/40*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/
4)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1382, 1524, 304, 209, 212}

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \frac{\sqrt[4]{984-440\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{984-440\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{5/4}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{1}{x}$$

```
[In] Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{x} + \int \frac{x^2(3-x^4)}{1-3x^4+x^8}\,dx \\
&= -\frac{1}{x} + \frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx - \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx \\
&= -\frac{1}{x} - \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} + \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}} \\
&\quad + \frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}} \\
&= -\frac{1}{x} + \frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}} \\
&\quad - \frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{5/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{(3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} - \frac{(-3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}} + \frac{(3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} + \frac{(-3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}$$

```
[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 +
 Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(
1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[
10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*
Sqrt[10*(1 + Sqrt[5])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-55\_Z^2-1\right)}\_R\ln\left(-20\_R^3+47\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+55\_Z^2-1\right)}\_R\ln\left(-20\_R^3\right.\right.}{4}$
default	$\frac{\left(\sqrt{5}-3\right)\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{1}{x}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\operatorname{arcta}}{10\sqrt{2\sqrt{}}}$

```
[In] int(1/x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+1/4*sum(_R*ln(-20*_R^3+47*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))+1/4*su
m(_R*ln(-20*_R^3-47*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(118) = 236.

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^2 (1-3x^4+x^8)} \, dx = -\frac{\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}(2\sqrt{5}+5)+10x\right)-\sqrt{10}x\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-\right.}{}$$

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/40*(sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2
*sqrt(5) + 5) + 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(
5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*l
og(sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - sqrt(10)*x*sqrt(
5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x)
- sqrt(10)*x*sqrt(-5*sqrt(5) + 11)*log(sqrt(10)*(2*sqrt(5) + 5)*sqrt(-5*sqr
t(5) + 11) + 10*x) + sqrt(10)*x*sqrt(-5*sqrt(5) + 11)*log(-sqrt(10)*(2*sqrt
(5) + 5)*sqrt(-5*sqrt(5) + 11) + 10*x) - sqrt(10)*x*sqrt(-5*sqrt(5) - 11)*l
og(sqrt(10)*(2*sqrt(5) - 5)*sqrt(-5*sqrt(5) - 11) + 10*x) + sqrt(10)*x*sqrt
(-5*sqrt(5) - 11)*log(-sqrt(10)*(2*sqrt(5) - 5)*sqrt(-5*sqrt(5) - 11) + 10*
x) + 40)/x
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^2 (1-3x^4+x^8)} \, dx = \operatorname{RootSum}\left(6400t^4-880t^2-1, \left(t \mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+880t^2-1, \left(t \mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right) - \frac{1}{x}$$

```
[In] integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 3
69792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*l
og(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx=\int \frac{1}{(x^8-3\,x^4+1)x^2}\,dx$$

[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 - 2)/(x^4 - x^2 - 1), x)

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx=\frac{1}{20}\,\sqrt{50\,\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\,\sqrt{50\,\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\,\sqrt{50\,\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\,\sqrt{50\,\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\,\sqrt{50\,\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\,\sqrt{50\,\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{x}$$

[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/x

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{x}-\frac{\mathrm{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20} + \frac{\mathrm{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\mathrm{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20} - \frac{\mathrm{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

```
[In] int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/
2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*5^
(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(3025*5
^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*5^(1
/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 11
0)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(
1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 - (at
an((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (5^(1/2)*
x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^(1/2) + 1
10)^(1/2)*1i)/20 - 1/x
```

3.402 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 182

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{7/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{7/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/3/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5^(1/2))^(1/
4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5
^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*
(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4
)*2^(3/4))*(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1382, 1436, 218, 212, 209}

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{7/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{7/4}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\ 2^{3/4}\sqrt{5}} - \frac{1}{3x^3}$$

```
[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{9-3x^4}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{3x^3}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{10\sqrt{3+\sqrt{5}}}\\
&\quad+\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{4\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{4\sqrt{5}}
\end{aligned}$$

$$= -\frac{1}{3x^3} - \frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{3x^3} + \frac{\left(2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(-2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \frac{\left(2+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(-2+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
 Sqrt[5])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.40

method	result
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+145\_Z^2-1\right)}\_R\ln\left(-35\_R^3-199\_R+13x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-145\_Z^2-1\right)}\_R\ln\left(35\right.\right.}{4}$
default	$-\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}-2\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}}{5\sqrt{2\sqrt{5}-}}$

```
[In] int(1/x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3+1/4*sum(_R*ln(-35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4+145*_Z^2-1))
+1/4*sum(_R*ln(35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4-145*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = -\frac{3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}-29}(7\sqrt{5}+15)+20x\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(-\sqrt{10}\sqrt{\right.}}{}$$

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/120*(3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*sqrt(5) -
 29)*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(-s
qrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15) + 20*x) - 3*sqrt(10)*x^3*sqr
t(13*sqrt(5) + 29)*log(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) - 15) + 20
*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sqrt(5) +
29)*(7*sqrt(5) - 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(-13*sqrt(5) + 29)*log(sq
rt(10)*(7*sqrt(5) + 15)*sqrt(-13*sqrt(5) + 29) + 20*x) - 3*sqrt(10)*x^3*sqr
t(-13*sqrt(5) + 29)*log(-sqrt(10)*(7*sqrt(5) + 15)*sqrt(-13*sqrt(5) + 29) +
 20*x) - 3*sqrt(10)*x^3*sqrt(-13*sqrt(5) - 29)*log(sqrt(10)*(7*sqrt(5) - 15
)*sqrt(-13*sqrt(5) - 29) + 20*x) + 3*sqrt(10)*x^3*sqrt(-13*sqrt(5) - 29)*lo
g(-sqrt(10)*(7*sqrt(5) - 15)*sqrt(-13*sqrt(5) - 29) + 20*x) + 40)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$
\begin{aligned}
&\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx \\
&= \operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right) \\
&\quad+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^3}
\end{aligned}
$$

```
[In] integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 2
3112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log
(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

Maxima [F]

$$
\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx=\int \frac{1}{(x^8-3\,x^4+1)x^4}\,dx
$$

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((2
*x^2 - 3)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 (1-3x^4+x^8)} \, dx = -\frac{1}{20} \sqrt{130 \sqrt{5} - 290} \arctan \left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}} \right) + \frac{1}{20} \sqrt{130 \sqrt{5} + 290} \arctan \left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}} \right) - \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log \left(\left| x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}} \right| \right) + \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log \left(\left| x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}} \right| \right) + \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log \left(\left| x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}} \right| \right) - \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log \left(\left| x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}} \right| \right) - \frac{1}{3 x^3}$$

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqrt(
5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) - 2
90)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290)*lo
g(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log(abs(
x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^4 (1-3x^4+x^8)}\,dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{-130\sqrt{5}-290}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\sqrt{-130\sqrt{5}-290}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{-130\sqrt{5}-290}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{290-130\sqrt{5}}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\sqrt{290-130\sqrt{5}}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{290-130\sqrt{5}}\,1\mathrm{i}}{20} - \frac{1}{3x^3} - \frac{\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{13\sqrt{5}-29}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{13\sqrt{5}+29}\,1\mathrm{i}}{20}$$

```
[In] int(1/(x^4*(x^8 - 3*x^4 + 1)),x)
[Out] (atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417)) +
 (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 196417
)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1/2)
*20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^(1/2
)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)/20 -
 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735i)/(2*
(87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*463
71i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20 - (10^(1
/2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(1/2) + 19
6417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*(87841*5^(
1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20
```

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{2889+1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] -1/5/x^5-3/x+1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5^(1/2
))^(1/4)*5^(1/2)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5
^(1/2))^(1/4)*5^(1/2)-1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(2889+12
92*5^(1/2))^(1/4)*5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(28
89+1292*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1382, 1518, 1524, 304, 209, 212}

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = \frac{\sqrt[4]{2889-1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889+1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889-1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{2889+1292\sqrt{5}}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{1}{5x^5} - \frac{3}{x}$$

```
[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5x^5}+\frac{1}{5}\int\frac{15-5x^4}{x^2\left(1-3x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{5}\int\frac{x^2(-40+15x^4)}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int\frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int\frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\\
&= -\frac{1}{5x^5}-\frac{3}{x}-\frac{\left(7-3\sqrt{5}\right)\int\frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}+\frac{\left(7-3\sqrt{5}\right)\int\frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}\\
&\quad+\frac{\left(7+3\sqrt{5}\right)\int\frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{2\sqrt{10}}-\frac{\left(7+3\sqrt{5}\right)\int\frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{2\sqrt{10}}
\end{aligned}$$

$$= -\frac{1}{5x^5} - \frac{3}{x} + \frac{\sqrt[4]{46224 - 20672\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{46224 + 20672\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{46224 - 20672\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{4\sqrt{5}} + \frac{\sqrt[4]{46224 + 20672\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^6 (1 - 3x^4 + x^8)}\, dx = -\frac{1}{5x^5} - \frac{3}{x} + \frac{(-7 - 3\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{(7 - 3\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}} - \frac{(-7 - 3\sqrt{5})\,\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{(7 - 3\sqrt{5})\,\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*S
qrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5
])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 +
Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
risch	$\frac{-3x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+380\_Z^2-1\right)}\_R\ln\left(-55\_R^3-843\_R+34x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-380\_Z^2-1\right)}\_R\ln\left(\right.\right.}{4}$
default	$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+$

```
[In] int(1/x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-3*x^4-1/5)/x^5+1/4*sum(_R*ln(-55*_R^3-843*_R+34*x),_R=RootOf(25*_Z^4+380*
_Z^2-1))+1/4*sum(_R*ln(-55*_R^3+843*_R+34*x),_R=RootOf(25*_Z^4-380*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(121) = 242.

Time = 0.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx =$$
$$-\frac{\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}\left(5\sqrt{5}+11\right)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}\left(5\right.\right.}{}$$

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5*sqrt(
5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt(5) -
38)*(5*sqrt(5) + 11) + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(sqrt(17
*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*
log(-sqrt(17*sqrt(5) + 38)*(5*sqrt(5) - 11) + 2*x) - sqrt(5)*x^5*sqrt(-17*s
qrt(5) + 38)*log((5*sqrt(5) + 11)*sqrt(-17*sqrt(5) + 38) + 2*x) + sqrt(5)*x
^5*sqrt(-17*sqrt(5) + 38)*log(-(5*sqrt(5) + 11)*sqrt(-17*sqrt(5) + 38) + 2*
x) + sqrt(5)*x^5*sqrt(-17*sqrt(5) - 38)*log((5*sqrt(5) - 11)*sqrt(-17*sqrt(
5) - 38) + 2*x) - sqrt(5)*x^5*sqrt(-17*sqrt(5) - 38)*log(-(5*sqrt(5) - 11)*
sqrt(-17*sqrt(5) - 38) + 2*x) + 60*x^4 + 4)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx$$
$$= \operatorname{RootSum}\left(6400t^4 - 6080t^2 - 1, \left(t \mapsto t \log \left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right)$$
$$+ \operatorname{RootSum}\left(6400t^4 + 6080t^2 - 1, \left(t \mapsto t \log \left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right)$$
$$+ \frac{-15x^4 - 1}{5x^5}$$

```
[In] integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
- 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(
_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4 - 1
)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = \int \frac{1}{(x^8 - 3\,x^4 + 1)x^6} \, dx$$

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2
*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = \frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{15x^4+1}{5x^5}$$

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(8
5*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5) -
 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx$$
$$= -\frac{3\,x^4+\frac{1}{5}}{x^5} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-85\,\sqrt{5}-190}\,372096\mathrm{i}}{2550408\,\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\,\sqrt{-85\,\sqrt{5}-190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{-85\,\sqrt{5}-190}\,1\mathrm{i}}{10}$$
$$+ \frac{\mathrm{atan}\left(\frac{x\,\sqrt{190-85\,\sqrt{5}}\,372096\mathrm{i}}{2550408\,\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\,\sqrt{190-85\,\sqrt{5}}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{190-85\,\sqrt{5}}\,1\mathrm{i}}{10}$$
$$+ \frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}-190}\,372096\mathrm{i}}{2550408\,\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}-190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{85\,\sqrt{5}-190}\,1\mathrm{i}}{10}$$
$$- \frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}+190}\,372096\mathrm{i}}{2550408\,\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}+190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{85\,\sqrt{5}+190}\,1\mathrm{i}}{10}$$

```
[In] int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5
^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)))
*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3720
96i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2)*83
2048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)/10 +
 (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))
)*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/2)*37209
6i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1/2)*83204
8i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)/10 - (3*x
^4 + 1/5)/x^5
```

3.404 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 189

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

```
[Out] -1/7/x^7-1/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(39603-17711*5^
(1/2))^(1/4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*
(39603-17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(
1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*
x*(3+5^(1/2))^(1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1382, 1518, 1436, 218, 212, 209}

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{1}{7x^7} - \frac{1}{x^3}$$

```
[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{21-7x^4}{x^4\left(1-3x^4+x^8\right)}\,dx\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{21}\int \frac{-168+63x^4}{1-3x^4+x^8}\,dx\\ &= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx\end{aligned}$$

$$= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}\,dx}{10\sqrt{3+\sqrt{5}}} - \frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}}\,dx}{10\sqrt{3+\sqrt{5}}}$$
$$+ \frac{1}{2}\sqrt{\frac{1}{5}\left(123+55\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx$$
$$+ \frac{1}{2}\sqrt{\frac{1}{5}\left(123+55\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx$$

$$= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}$$
$$+ \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$
$$- \frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}$$
$$+ \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{7x^7} - \frac{1}{x^3} + \frac{\left(11+5\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}$$
$$+ \frac{\left(11-5\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$
$$- \frac{\left(-11-5\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}$$
$$- \frac{\left(-11+5\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

```
[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(
2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]
*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

method	result
risch	$\frac{-x^4-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-995\_Z^2-1\right)}\_R\ln\left(90\_R^3-3571\_R+89x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+995\_Z^2-1\right)}\_R\ln\left(-\right.\right.}{4}$
default	$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(11+5\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right.}{10\sqrt{2\sqrt{5}+2}}$

```
[In] int(1/x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-x^4-1/7)/x^7+1/4*sum(_R*ln(90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4-995*_Z
^2-1))+1/4*sum(_R*ln(-90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4+995*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(133) = 266.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx = -\frac{7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}-199}\left(9\sqrt{5}+20\right)+10x\right)-7\sqrt{10}x^7\sqrt{89\sqrt{5}-199}\log\left(-\sqrt{\right.}}{}$$

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/280*(7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5)
- 199)*(9*sqrt(5) + 20) + 10*x) - 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log
(-sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) + 10*x) - 7*sqrt(10)*x^7
*sqrt(89*sqrt(5) + 199)*log(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 20
) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(89*sqr
t(5) + 199)*(9*sqrt(5) - 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(-89*sqrt(5) + 19
9)*log(sqrt(10)*(9*sqrt(5) + 20)*sqrt(-89*sqrt(5) + 199) + 10*x) - 7*sqrt(1
0)*x^7*sqrt(-89*sqrt(5) + 199)*log(-sqrt(10)*(9*sqrt(5) + 20)*sqrt(-89*sqrt
```

```
(5) + 199) + 10*x) - 7*sqrt(10)*x^7*sqrt(-89*sqrt(5) - 199)*log(sqrt(10)*(9
*sqrt(5) - 20)*sqrt(-89*sqrt(5) - 199) + 10*x) + 7*sqrt(10)*x^7*sqrt(-89*sq
rt(5) - 199)*log(-sqrt(10)*(9*sqrt(5) - 20)*sqrt(-89*sqrt(5) - 199) + 10*x)
 + 280*x^4 + 40)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = \operatorname{RootSum}\left(6400t^4-15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \frac{-7x^4-1}{7x^7}$$

```
[In] integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
- 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t
, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7*x*
*7)
```

Maxima [F]

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^8}\,dx$$

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2*
integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{7x^4+1}{7x^7}$$

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*sqr
t(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5)
- 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*lo
g(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx$$
$$= -\frac{x^4+\frac{1}{7}}{x^7} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\,\sqrt{5}-199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}+165580139\right)}\right)\sqrt{-89\,\sqrt{5}-199}\,1\mathrm{i}}{20}$$
$$+ \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{199-89\,\sqrt{5}}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{199-89\,\sqrt{5}}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}-165580139\right)}\right)\sqrt{199-89\,\sqrt{5}}\,1\mathrm{i}}{20}$$
$$- \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}-199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}-199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}-165580139\right)}\right)\sqrt{89\,\sqrt{5}-199}\,1\mathrm{i}}{20}$$
$$- \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}+199}\,6677047\mathrm{i}}{2\,\left(74049691\,\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\,\sqrt{5}+199}\,14930373\mathrm{i}}{10\,\left(74049691\,\sqrt{5}+165580139\right)}\right)\sqrt{89\,\sqrt{5}+199}\,1\mathrm{i}}{20}$$

```
[In] int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(7404969
1*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*14
930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)*1i
)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^(1/2
)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(199 -
 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(199 - 8
9*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 199)^(1/
2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(89*5
^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(89*5^(
1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) + 199)^(1
/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(89*
5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(89*5^
(1/2) + 199)^(1/2)*1i)/20
```

3.405 $\int \frac{x^3}{2+3x^4+x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

```
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 630, 31}

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

```
[In] Int[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\log\left(1+x^4\right) - \frac{1}{4}\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = \frac{1}{4}\log\left(1+x^4\right) - \frac{1}{4}\log\left(2+x^4\right)$$

```
[In] Integrate[x^3/(2 + 3*x^4 + x^8),x]
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(x^4+1)}{4} - \frac{\ln(x^4+2)}{4}$	18
norman	$\frac{\ln(x^4+1)}{4} - \frac{\ln(x^4+2)}{4}$	18
risch	$\frac{\ln(x^4+1)}{4} - \frac{\ln(x^4+2)}{4}$	18
parallelrisch	$\frac{\ln(x^4+1)}{4} - \frac{\ln(x^4+2)}{4}$	18

```
[In] int(x^3/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = \frac{\log\left(x^4+1\right)}{4}-\frac{\log\left(x^4+2\right)}{4}$$

```
[In] integrate(x**3/(x**8+3*x**4+2),x)
[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\,\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{256}{9\,(144\,x^4+160)}-\frac{7}{9}\right)}{2}$$

```
[In] int(x^3/(3*x^4 + x^8 + 2),x)
[Out] -atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{x^4}{4} + \frac{1}{4}\log\left(1+x^4\right) - \log\left(2+x^4\right)$$

```
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 717, 646, 31}

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{x^4}{4} + \frac{1}{4}\log\left(x^4+1\right) - \log\left(x^4+2\right)$$

```
[In] Int[x^11/(2 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{2+3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-2-3x}{2+3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)$$

```
[In] Integrate[x^11/(2 + 3*x^4 + x^8),x]
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
norman	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
risch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
parallelrisch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23

```
[In] int(x^11/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{x^4}{4}+\frac{\log\left(x^4+1\right)}{4}-\log\left(x^4+2\right)$$

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{\ln\left(x^4+1\right)}{4} - \ln\left(x^4+2\right) + \frac{x^4}{4}$$

```
[In] int(x^11/(3*x^4 + x^8 + 2),x)
[Out] log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

```
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1371, 648, 632, 210, 642}

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{1}{10}\log\left(x^{10}+x^5+2\right) - \frac{\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[In] Int[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{x}{2+x+x^2}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{10}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\right) + \frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{2+x+x^2}\,dx, x, x^5\right)\\
&= \frac{1}{10}\log\left(2+x^5+x^{10}\right) + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

```
[In] Integrate[x^9/(2 + x^5 + x^10),x]
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(x^{10}+x^5+2)}{10}-\frac{\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	31
risch	$\frac{\ln(4x^{10}+4x^5+8)}{10}-\frac{\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	35

```
[In] int(x^9/(x^10+x^5+2),x,method=_RETURNVERBOSE)
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

```
[In] integrate(x**9/(x**10+x**5+2),x)
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Giac [A] (verification not implemented)

none

Time = 1.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

```
[In] int(x^9/(x^5 + x^10 + 2),x)
[Out] log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2 \arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1366, 632, 210}

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2 \arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[In] Int[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)\\ &= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[In] Integrate[x^4/(2 + x^5 + x^10),x]
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19
risch	$\frac{2\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19

```
[In] int(x^4/(x^10+x^5+2),x,method=_RETURNVERBOSE)
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)$$

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

```
[In] integrate(x**4/(x**10+x**5+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)$$

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Giac [A] (verification not implemented)

none

Time = 1.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)$$

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx=\frac{2\,\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

```
[In] int(x^4/(x^5 + x^10 + 2),x)
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^5+x^{10})} \, dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$\int \frac{1}{x(1+x^5+x^{10})} \, dx = -\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log(x)$$

```
[In] Int[1/(x*(1 + x^5 + x^10)),x]
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x(1+x+x^2)}\,dx,x,x^5\right)\\
&= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)
\end{aligned}$$

$$= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx$$
$$= \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x+x^2\right) - \frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&, \frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#1}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

```
[In] Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln(x) - \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$\ln(x) - \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1-i\sqrt{3}\right)x^2+2x-1+i\sqrt{3}\right)}{5} - \frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x\right.}{5}$

```
[In] int(1/x/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log(x)$$

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \log(x) - \frac{\log(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

```
[In] integrate(1/x/(x**10+x**5+1),x)
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \frac{1}{5}\log\left(x^5\right)$$

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
1/5*log(x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log\left(|x|\right)$$

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \ln(x) - \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

```
[In] int(1/(x*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

3.410 $\int \frac{1}{x^6(1+x^5+x^{10})} dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^6 (1 + x^5 + x^{10})} dx = -\frac{1}{5x^5} - \frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} - \log(x) + \frac{1}{10}\log\left(1 + x^5 + x^{10}\right)$$

```
[Out] -1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\int \frac{1}{x^6 (1 + x^5 + x^{10})} dx = -\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{1}{5x^5} + \frac{1}{10}\log\left(x^{10} + x^5 + 1\right) - \log(x)$$

```
[In] Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/5*1/x^5 - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x^2\,(1+x+x^2)}\,dx, x, x^5\right)\\ &= -\frac{1}{5x^5}+\frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{x\,(1+x+x^2)}\,dx, x, x^5\right)\end{aligned}$$

$$= -\frac{1}{5x^5} + \frac{1}{5}\text{Subst}\left(\int \left(-\frac{1}{x} + \frac{x}{1+x+x^2}\right) dx, x, x^5\right)$$

$$= -\frac{1}{5x^5} - \log(x) + \frac{1}{5}\text{Subst}\left(\int \frac{x}{1+x+x^2} dx, x, x^5\right)$$

$$= -\frac{1}{5x^5} - \log(x) - \frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2} dx, x, x^5\right) + \frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2} dx, x, x^5\right)$$

$$= -\frac{1}{5x^5} - \log(x) + \frac{1}{10}\log\left(1+x^5+x^{10}\right) + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x^5\right)$$

$$= -\frac{1}{5x^5} - \frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} - \log(x) + \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^6(1+x^5+x^{10})} dx = \frac{1}{30}\left(-\frac{6}{x^5} + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 30\log(x) + 3\log\left(1+x+x^2\right) + 6\text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7 +\#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1+\log(x-\#1)\#1^2-3\log(x-\#1)\#1^3+2\log(x-\#1)\#1^4 +}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8}\right.\right.$$

```
[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x
^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]
 + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#
1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#
1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{5x^5} - \ln(x) + \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$-\frac{1}{5x^5} - \ln(x) + \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x-1-i\sqrt{3}\right)}{5} + \frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1\right.\right.}{5}$

```
[In] int(1/x^6/(x^10+x^5+1),x,method=_RETURNVERBOSE)
[Out] -1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = -\frac{2\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)-3x^5\log\left(x^{10}+x^5+1\right)+30x^5\log(x)+6}{30x^5}$$

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5
 + 1) + 30*x^5*log(x) + 6)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = -\log(x)+\frac{\log(x^{10}+x^5+1)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

```
[In] integrate(1/x**6/(x**10+x**5+1),x)
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15 - 1/(5*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x
^5 + 1) - 1/5*log(x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6 (1 + x^5 + x^{10})} \, dx = -\frac{1}{15} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^5 + 1) \right) + \frac{x^5 - 1}{5 x^5} + \frac{1}{10} \log \left(x^{10} + x^5 + 1 \right) - \log \left(|x| \right)$$

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (1 + x^5 + x^{10})} \, dx = \frac{\ln \left(x^{10} + x^5 + 1 \right)}{10} - \ln \left(x \right) - \frac{\sqrt{3} \operatorname{atan} \left(\frac{2 \sqrt{3} x^5}{3} + \frac{\sqrt{3}}{3} \right)}{15} - \frac{1}{5 x^5}$$

```
[In] int(1/(x^6*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15 - 1/(5*x^5)
```

3.411 $\int \frac{1}{x+x^6+x^{11}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{x+x^6+x^{11}} \, dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1608, 1371, 719, 29, 648, 632, 210, 642}

$$\int \frac{1}{x+x^6+x^{11}} \, dx = -\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log(x)$$

```
[In] Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx\\ &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\end{aligned}$$

$$= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right) + \frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^5\right)$$

$$= \log(x) - \frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right) - \frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5\right)$$

$$= \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right) + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx$$
$$= \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x+x^2\right) - \frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&, \frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#1}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

```
[In] Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln(x) - \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$\ln(x) - \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1-i\sqrt{3}\right)x^2+2x-1+i\sqrt{3}\right)}{5} - \frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x\right.}{5}$

```
[In] int(1/(x^11+x^6+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^5+1)\right) - \frac{1}{10}\,\log\left(x^{10}+x^5+1\right) + \log\left(x\right)$$

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \log\left(x\right) - \frac{\log\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

```
[In] integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Maxima [F]

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \int \frac{1}{x^{11}+x^6+x}\,dx$$

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6 -
 x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x) -
 1/10*log(x^2 + x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^5+1)\right) - \frac{1}{10}\,\log\left(x^{10}+x^5+1\right) + \log\left(|x|\right)$$

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="giac")
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \ln\left(x\right) - \frac{\ln\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

```
[In] int(1/(x + x^6 + x^11),x)
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx = -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c} + \frac{b(b^4-5ab^2c+5a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5}$$

```
[Out] -b*(-2*a*c+b^2)*x/c^4+1/2*(-a*c+b^2)*x^2/c^3-1/3*b*x^3/c^2+1/4*x^4/c+1/2*(a
^2*c^2-3*a*b^2*c+b^4)*ln(c*x^2+b*x+a)/c^5+b*(5*a^2*c^2-5*a*b^2*c+b^4)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1368, 715, 648, 632, 212, 642}

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx = \frac{b(5a^2c^2-5ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{(a^2c^2-3ab^2c+b^4)\log\left(a+bx+cx^2\right)}{2c^5} - \frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

```
[In] Int[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^5}{a+bx+cx^2}\,dx$$

$$= \int \left(-\frac{b(b^2-2ac)}{c^4}+\frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}+\frac{ab(b^2-2ac)+(b^4-3ab^2c+a^2c^2)\,x}{c^4\,(a+bx+cx^2)}\right)\,dx$$

$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\int \frac{ab(b^2-2ac)+(b^4-3ab^2c+a^2c^2)x}{a+bx+cx^2}\,dx}{c^4}$$

$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c} + \frac{(b^4-3ab^2c+a^2c^2)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^5}-\frac{(b(b^4-5ab^2c+5a^2c^2))\int \frac{1}{a+bx+cx^2}\,dx}{2c^5}$$

$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5} + \frac{(b(b^4-5ab^2c+5a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^5}$$

$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c} + \frac{b(b^4-5ab^2c+5a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \frac{cx(-12b^3+6b^2cx-4bc(-6a+cx^2)+3c^2x(-2a+cx^2))-\frac{12b(b^4-5ab^2c+5a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+6(b^4-3a}{12c^5}$$

```
[In] Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
- (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)])
/(12*c^5)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{1}{4}c^3x^4-\frac{1}{3}bc^2x^3-\frac{1}{2}ac^2x^2+\frac{1}{2}b^2cx^2+2abcx-b^3x}{c^4}+\frac{\frac{(a^2c^2-3ab^2c+b^4)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-2cba^2+ab^3-\frac{(a^2c^2-3ab^2c+b^4)b}{2c}\right)\arctan\left(\frac{\cdot}{\sqrt{\cdot}}\right.}{\sqrt{4ac-b^2}}}{c^4}$
risch	Expression too large to display

[In] int(x^3/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)

[Out] 1/c^4*(1/4*c^3*x^4-1/3*b*c^2*x^3-1/2*a*c^2*x^2+1/2*b^2*c*x^2+2*a*b*c*x-b^3*x)+1/c^4*(1/2*(a^2*c^2-3*a*b^2*c+b^4)/c*ln(c*x^2+b*x+a)+2*(-2*c*b*a^2+a*b^3-1/2*(a^2*c^2-3*a*b^2*c+b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.17

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left[\frac{3\,(b^2c^4-4\,ac^5)x^4-4\,(b^3c^3-4\,abc^4)x^3+6\,(b^4c^2-5\,ab^2c^3+4\,a^2c^4)x^2+6\,(b^5-5\,ab^3c+5\,a^2bc^2)\sqrt{b^2-4}}{}\right.$$

[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")

[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(144) = 288$.

Time = 0.73 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.12

$$\int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}}\,dx$$
$$= -\frac{bx^3}{3c^2} + x^2\left(-\frac{a}{2c^2} + \frac{b^2}{2c^3}\right) + x\left(\frac{2ab}{c^3} - \frac{b^3}{c^4}\right) + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2 - 5ab^2c + b^4)}{2c^5\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2 - 5ab^3c + b^5}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2 - 5ab^2c + b^4)}{2c^5\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{a^2c^2 - 3ab^2c + b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2 - 4a^2b^2c + ab^4 - 4ac^5\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right) +}{5a^2bc^2 - 5ab^3c + b^5}\right.$$
$$+\frac{x^4}{4c}$$

```
[In] integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b**
3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5
*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**
3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a**2*
c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c
 + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b
**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*
c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**2)*(5*a*
*2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**
2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c
**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*sqrt(-
4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (
a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**
5)) + x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{3\,c^3x^4 - 4\,bc^2x^3 + 6\,b^2cx^2 - 6\,ac^2x^2 - 12\,b^3x + 24\,abcx}{12\,c^4} + \frac{(b^4 - 3\,ab^2c + a^2c^2)\log\left(cx^2 + bx + a\right)}{2\,c^5} - \frac{(b^5 - 5\,ab^3c + 5\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^5}$$

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a
*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = x\left(\frac{b\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\,\left(4\,a\,c^6-b^2\,c^5\right)} - \frac{b\,x^3}{3\,c^2} - \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c^5\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^3/(c + a/x^2 + b/x),x)
```

```
[Out] x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2/
(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*b^
4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4*a*
c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2))
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{(b^4-4ab^2c+2a^2c^2)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

```
[Out] (-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)
/c^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1368, 715, 648, 632, 212, 642}

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=-\frac{(2a^2c^2-4ab^2c+b^4)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

```
[In] Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{a+bx+cx^2}\,dx \\
&= \int \left(\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{a(b^2-ac)+b(b^2-2ac)\,x}{c^3\,(a+bx+cx^2)}\right)dx \\
&= \frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x}{a+bx+cx^2}\,dx}{c^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(b^2-ac)\,x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b(b^2-2ac)) \int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4} + \frac{(b^4-4ab^2c+2a^2c^2) \int \frac{1}{a+bx+cx^2}\,dx}{2c^4} \\
&= \frac{(b^2-ac)\,x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4} \\
&\quad - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^4} \\
&= \frac{(b^2-ac)\,x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} \\
&\quad - \frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$
\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx(6b^2-6ac-3bcx+2c^2x^2) + \frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 3(b^3-2abc)\log(a+x(b+cx))}{6c^4}
$$

```
[In] Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^
2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*
a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{-\frac{1}{3}c^2x^3+\frac{1}{2}bc\,x^2+acx-b^2x}{c^3}+\frac{\frac{\left(2abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(c\,a^2-b^2a-\frac{\left(2abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^3}$	128
risch	Expression too large to display	1138

```
[In] int(x^2/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*c^2*x^3+1/2*b*c*x^2+a*c*x-b^2*x)+1/c^3*(1/2*(2*a*b*c-b^3)/c*ln
(c*x^2+b*x+a)+2*(c*a^2-b^2*a-1/2*(2*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.25

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2+3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4}}{cx^2+bx+a}\right.}{6\,(b^2c^4-4\,ac^5)}\right.$$

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*
a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2
*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a
))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 -
 4*a*c^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.

Time = 0.60 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.22

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx^2}{2c^2}+x\left(-\frac{a}{c^2}+\frac{b^2}{c^3}\right)+\left(\frac{b(2ac-b^2)}{2c^4}\right.$$
$$\left.-\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x+\frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4}-\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2c}\right.$$
$$+\left(\frac{b(2ac-b^2)}{2c^4}\right.$$
$$\left.+\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x+\frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4}+\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2c}\right.$$
$$+\frac{x^3}{3c}$$

```
[In] integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) - s
qrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)
))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))
) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c +
 b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4
*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 +
4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*
c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*
c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{2\,c^2x^3-3\,bcx^2+6\,b^2x-6\,acx}{6\,c^3}-\frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,c^4}+\frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*l
og(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x^3}{3\,c}-x\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)-\frac{b\,x^2}{2\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,a^2\,b\,c^2-6\,a\,b^3\,c+b^5\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^2/(c + a/x^2 + b/x),x)
```

```
[Out] x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*(
b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c -
b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c
^4*(4*a*c - b^2)^(1/2))
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=-\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1368, 715, 648, 632, 212, 642}

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[In] Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3}{a+bx+cx^2}\,dx\\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3}
\end{aligned}$$

$$= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx(-2b+cx) - \frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (b^2-ac)\log(a+x(b+cx))}{2c^3}$$

```
[In] Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{(-ac+b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(ab-\frac{(-ac+b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(12a^2b\,c^2-7a\,b^3c+b\cdots\right.}{\cdots}$

```
[In] int(x/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*
(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+(b^4\cdots}{2\,(b^2c^3-4\,ac^4)}\right.$$

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.47 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

```
[In] integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx^2-2\,bx}{2\,c^2}+\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x^2}{2\,c}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{b\,x}{c^2}+\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x/(c + a/x^2 + b/x),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x}{c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1354, 717, 648, 632, 212, 642}

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[In] Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{a+bx+cx^2}\,dx \\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c} \\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2} \\
&= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2} \\
&= \frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c}+\frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}$$

```
[In] Integrate[(c + a/x^2 + b/x)^(-1),x]
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^2c^2+6a\,b^2c-b^4-2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,cx-\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)ab}{c(4ac-b^2)}+\frac{\ln\left(-8a^2c^2+6a\,b^2c-b^4-2\sqrt{-(4ac-\right.}{}$

```
[In] int(1/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
[Out] x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log(cx^2}{2\,(b^2c^2-4\,ac^3)}\right.$$

$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log(cx^2+bx+a}{2\,(b^2c^2-4\,ac^3)}$$

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.34 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x}{c}$$

```
[In] integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c} - \frac{b\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c} + \frac{b^3\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

```
[In] int(1/(c + a/x^2 + b/x),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x}{a+bx+cx^2}\,dx\\
&= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}\\
&= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log(a+x(b+cx))}{2c}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x),x]
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln(c\,x^2+bx+a)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)b^2}{2c(4ac-b^2)}+\frac{\ln\left(-2\sqrt{\right.}}{}$

```
[In] int(1/(c+a/x^2+b/x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)},\frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{arctan}}{}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
```

```
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right)}{b}\right)$$

```
[In] integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

```
[In] int(1/(x*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\operatorname{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] 2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1366, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x^2),x]
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)\\ &= 2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^2),x]
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

```
[In] int(1/(c+a/x^2+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.33

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\ -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(32) = 64.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

```
[In] integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

```
[In] int(1/(x^2*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\end{aligned}$$

$$= \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx\right)}{a}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^3}\,dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x) + \log(a + x(b + cx))}{2a}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{a} + \frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2} - \frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}$
risch	$-\frac{2\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b^2(4ac-b^2)}\,ac-2\sqrt{-b^2(4ac-b^2)}\,b^2\right)x+12cb\,a^2-3a\,b^3-\sqrt{-b^2(4ac-b^2)}\,ab\right)c}{4ac-b^2} + \frac{\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b\right.\right.}{}$

```
[In] int(1/(c+a/x^2+b/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a+1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(x\right)}{2\,(ab^2-4\,a^2c)},\right.$$

[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(54) = 108.

Time = 4.46 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)} - \frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+}{9abc^2-2b^3}\right. + \left(\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)} - \frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4}{9abc^2-2b^3c}\right. + \frac{\log\left(x\right)}{a}$$

[In] integrate(1/(c+a/x**2+b/x)/x**3,x)

[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**

```
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,aca}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=\frac{\ln(x)}{a}-\ln\left(b\,c-\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)+3\,c^2\,x\right)\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-\ln\left(\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-b\,c-3\,c^2\,x\right)\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)$$

```
[In] int(1/(x^3*(c + a/x^2 + b/x)),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx=-\frac{1}{ax}-\frac{(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx=-\frac{(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+bx+cx^2)}\,dx$$

$$= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx}{a}$$

$$= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)dx}{a}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} + \frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^2}$$

$$= -\frac{1}{ax} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \frac{-\frac{2a}{x} + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x) + b\log(a+x(b+cx))}{2a^2}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{ax} - \frac{b\ln(x)}{a^2} + \frac{\frac{b\ln(c\,x^2+bx+a)}{2} + \frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{1}{ax} - \frac{b\ln(x)}{a^2} + \left(\sum_{\_R=\text{RootOf}\left((4a^3c-a^2b^2)\_Z^2+(-4abc+b^3)\_Z+c^2\right)} \_R\ln\left(\left((6a^3c-2a^2b^2)\_R^2-2\_Rabc\right.\right.\right.$

```
[In] int(1/(c+a/x^2+b/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+a\right)}{2\,(a^2b^2-4\,a^3c)x}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \frac{b\log\left(cx^2+bx+a\right)}{2\,a^2} - \frac{b\log\left(|x|\right)}{a^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2} - \frac{1}{ax}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$
$$= \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,\right.}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{1}{a\,x}$$
$$-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\right.}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{b\,\ln\left(x\right)}{a^2}$$

```
[In] int(1/(x^4*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3} + \frac{\log(x)\,(b^2-ac)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c
)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{2ax^2}+\frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx}{a} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{(b^2-ac)\log(x)}{a^3}+\frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx}{a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b(b^2-3ac))\int \frac{1}{a+bx+cx^2}\,dx}{2a^3}-\frac{(b^2-ac)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3} \\
&\quad +\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^3} \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} \\
&\quad +\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}
$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{\left(-ac+b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{b}{a^{2}x}+\frac{\frac{\left(a\,c^{2}-b^{2}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(a\,c^{2}-b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}$	128
risch	Expression too large to display	2265

```
[In] int(1/(c+a/x^2+b/x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2+(-a*c+b^2)*ln(x)/a^3+b/a^2/x+1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2
+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c^2)x}{2\,(a^3b^2-4\,a^4c)x^2}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)/x**5,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = -\frac{(b^2-ac)\log(cx^2+bx+a)}{2a^3} + \frac{(b^2-ac)\log(|x|)}{a^3} - \frac{(b^3-3abc)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^3} + \frac{2abx-a^2}{2a^3x^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-4}\right.}{4\,c}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b^2-}\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln(x)\,\left(a\,c-b^2\right)}{a^3}$$

```
[In] int(1/(x^5*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2
*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x} +\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

```
[In] Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c +
2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c])
```

```
- (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*
a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^4(a+bx+cx^2)}\,dx \\
&= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx}{a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} \\
&\quad + \frac{(b(b^2-2ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4} + \frac{(b^4-4ab^2c+2a^2c^2)\int \frac{1}{a+bx+cx^2}\,dx}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4} \\
&\quad - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} \\
&\quad - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \frac{-\frac{2a^3}{x^3} + \frac{3a^2b}{x^2} + \frac{6a(-b^2+ac)}{x} + \frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 6(b^3-2abc)\log(x) + 3(b^3-2abc)\log(a+x}{6a^4}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{1}{3a\,x^3}-\frac{-ac+b^2}{x\,a^3}+\frac{b(2ac-b^2)\ln(x)}{a^4}+\frac{b}{2a^2x^2}+\frac{\frac{(-2ab\,c^2+b^3c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(a^2c^2-3a\,b^2c+b^4-\frac{(-2ab\,c^2+b^3c)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{a^4}$
risch	Expression too large to display

[In] int(1/(c+a/x^2+b/x)/x^6,x,method=_RETURNVERBOSE)

[Out] -1/3/a/x^3-(-a*c+b^2)/x/a^3+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2+1/a^4*(1/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/2*(-2*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$$
$$=\left[\frac{3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\,(b^5-6\,ab^3}{6}\right.$$
$$-\frac{6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\,(b^5-6\,ab^3c+8\,a^2}{6\,(a^4}$$

[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")

[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx &= \frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{(b^3-2\,abc)\log\left(|x|\right)}{a^4} \\ &\quad + \frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4} \\ &\quad + \frac{3\,a^2bx-2\,a^3-6\,(ab^2-a^2c)x^2}{6\,a^4x^3}\end{aligned}$$

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \ln\Big(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\Big(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,\left(a\,c-b^2\right)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

```
[In] int(1/(x^6*(c + a/x^2 + b/x)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 196

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} \, dx = -\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)} + \frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)} - \frac{bx^3}{c\,(b^2-4ac)} + \frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

```
[Out] -b*(-11*a*c+3*b^2)*x/c^3/(-4*a*c+b^2)+1/2*(-8*a*c+3*b^2)*x^2/c^2/(-4*a*c+b^
2)-b*x^3/c/(-4*a*c+b^2)+x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^2*
c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2
)^(3/2)+1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/c^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

$= \{1368, 752, 814, 648, 632, 212, 642\}$

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{b(30a^2c^2-20ab^2c+3b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{3/2}} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4} - \frac{bx(3b^2-11ac)}{c^3\left(b^2-4ac\right)} + \frac{x^2(3b^2-8ac)}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{c\left(b^2-4ac\right)} + \frac{x^4(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[In] Int[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
```

```
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5}{(a+bx+cx^2)^2}\,dx\\
&= \frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= \frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\\
&\quad + \frac{\int \left(\frac{b(3b^2-11ac)}{c^3} - \frac{(3b^2-8ac)x}{c^2} + \frac{3bx^2}{c} - \frac{ab(3b^2-11ac)+(b^2-4ac)(3b^2-2ac)x}{c^3(a+bx+cx^2)}\right)dx}{-b^2+4ac}\\
&= -\frac{b(3b^2-11ac)\,x}{c^3(b^2-4ac)} + \frac{(3b^2-8ac)\,x^2}{2c^2(b^2-4ac)} - \frac{bx^3}{c(b^2-4ac)}\\
&\quad + \frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{ab(3b^2-11ac)+(b^2-4ac)(3b^2-2ac)x}{a+bx+cx^2}\,dx}{c^3(b^2-4ac)}\\
&= -\frac{b(3b^2-11ac)\,x}{c^3(b^2-4ac)} + \frac{(3b^2-8ac)\,x^2}{2c^2(b^2-4ac)} - \frac{bx^3}{c(b^2-4ac)} + \frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\\
&\quad + \frac{(3b^2-2ac)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4} - \frac{(b(3b^4-20ab^2c+30a^2c^2))\int \frac{1}{a+bx+cx^2}\,dx}{2c^4(b^2-4ac)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)} + \frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)} - \frac{bx^3}{c\,(b^2-4ac)} \\
&\quad + \frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4} \\
&\quad + \frac{(b(3b^4-20ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^4\,(b^2-4ac)} \\
&= -\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)} + \frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)} - \frac{bx^3}{c\,(b^2-4ac)} + \frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} \\
&\quad + \frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$
\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx
$$

$$
= \frac{-4bcx + c^2x^2 + \frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + (3b^2-2ac)\log(a}{2c^4}
$$

```
[In] Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(
-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2
) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+2bx}{c^3}+\frac{\frac{-\frac{b\left(5a^2c^2-5a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}-\frac{a\left(2a^2c^2-4a\,b^2c+b^4\right)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\left(-8a^2c^2+14a\,b^2c-3b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(11cb\,a^2-3a\,b^3-\frac{\left(-8a^2c^2+1\right.}{\sqrt{}}\right.}{4ac-b^2}}{c^3}$
risch	Expression too large to display

```
[In] int(x/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/2*c*x^2+2*b*x)+1/c^3*((-b*(5*a^2*c^2-5*a*b^2*c+b^4)/c/(4*a*c-b^2
)*x-a/c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*
```

```
(1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)/c*ln(c*x^2+b*x+a)+2*(11*c*b*a^2-3*a*b^3-
1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(188) = 376$.

Time = 0.29 (sec) , antiderivative size = 1029, normalized size of antiderivative = 5.25

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$
$$= \left[\frac{2\,ab^6 - 16\,a^2b^4c + 36\,a^3b^2c^2 - 16\,a^4c^3 + (b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^4 - 3\,(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x^3 -}{}\right.$$

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 -
 (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20
*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2
*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2
*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b
^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*
x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2
*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*
a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b
^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(180) = 360.

Time = 1.35 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.16

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{2bx}{c^3}+\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8}{30}\right.$$
$$+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(30a^2c^2-20ab^2c+3b^4\right)}{2c^4\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8ab}{30a^2b}\right.$$
$$+\frac{-2a^3c^2+4a^2b^2c-ab^4+x\left(-5a^2bc^2+5ab^3c-b^5\right)}{4a^2c^5-ab^2c^4+x^2\cdot\left(4ac^6-b^2c^5\right)+x\left(4abc^5-b^3c^4\right)}+\frac{x^2}{2c^2}$$

```
[In] integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2
*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c
**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c
**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b
**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-b*sqrt(-
(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
)/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(b*s
qrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c
**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
```

```
3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (-2
*a**3*c**2 + 4*a**2*b**2*c - a*b**4 + x*(-5*a**2*b*c**2 + 5*a*b**3*c - b**5
))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**2*c**5) + x*(4*a*b*c**5
- b**3*c**4)) + x**2/(2*c**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} + \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,c^4} + \frac{c^2x^2-4\,bcx}{2\,c^4} + \frac{ab^4-4\,a^2b^2c+2\,a^3c^2+(b^5-5\,ab^3c+5\,a^2bc^2)x}{(cx^2+bx+a)(b^2-4\,ac)c^4}$$

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c
^4)
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.95

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x^2}{2\,c^2}-\frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c+3\,b^8\right)}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)}-\frac{2\,b\,x}{c^3}+\frac{b\,\mathrm{atan}\left(\frac{c^4\,\left(\frac{2\,b\,x\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^3}-\frac{b\,\left(b^3\,c^3-4\,a\,b\,c^4\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^7\,{\left(4\,a\,c-b^2\right)}^4}\right)\,{\left(4\,a\,c-b^2\right)}^{5/2}}{30\,a^2\,b\,c^2-20\,a\,b^3\,c+3\,b^5}\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(x/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*(
b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3*x
) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2
*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*
c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*c^2 -
20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30*a^2*b*
c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c - b^2)^(3
/2))
```

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx = \frac{2(b^2-3ac)x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}$$

```
[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*
c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1354, 752, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx = -\frac{2(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3}$$

```
[In] Int[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
```

```
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right)dx}{-b^2+4ac}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2}\,dx}{c^2\,(b^2-4ac)}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{(b^4-6ab^2c+6a^2c^2)\int \frac{1}{a+bx+cx^2}\,dx}{c^3\,(b^2-4ac)}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3} - \frac{(2(b^4-6ab^2c+6a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^3\,(b^2-4ac)}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{cx + \frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))} - \frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+x(b+cx))}{c^3}$$

```
[In] Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{(2a^2c^2-4a\,b^2c+b^4)x}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\frac{(4abc-b^3)\ln(c\,x^2+bx+a)}{c}+\frac{4\left(3c\,a^2-b^2a-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

```
[In] int(1/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*x+
a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(146) = 292.

Time = 0.30 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$$

$$= \left[-\frac{ab^5 - 7\,a^2b^3c + 12\,a^3bc^2 - (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^3 - (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^2 + (ab^4 - 6\,a^2b^2c -}{} \right.$$

$$-\frac{ab^5 - 7\,a^2b^3c + 12\,a^3bc^2 - (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^3 - (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^2 + 2\,(ab^4 - 6\,a^2b^2c}{}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
```

```
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.

Time = 1.07 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.61

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \left(-\frac{b}{c^3} - \frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\cdots}{\cdots}\right) + \left(-\frac{b}{c^3} + \frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\cdots}{\cdots}\right) + \frac{-3a^2bc+ab^3+x(2a^2c^2-4ab^2c+b^4)}{4a^2c^4-ab^2c^3+x^2\cdot(4ac^5-b^2c^4)+x\left(4abc^4-b^3c^3\right)} + \frac{x}{c^2}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
```

```
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
```

```
((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,\left(b^3-3\,a\,b\,c\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^3\,x^2+b\,c^2\,x+a\,c^2} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)} - \frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(1/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx$

Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx = -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx+cx^2)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 752, 787, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} dx = \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{bx}{c(b^2-4ac)} + \frac{\log(a+bx+cx^2)}{2c^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx}{c(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{(b(b^2-6ac))\int \frac{1}{a+bx+cx^2}\,dx}{2c^2(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^2} + \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^2(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx+cx^2)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b\left(3ac-b^{2}\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{a\left(2ac-b^{2}\right)}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{\frac{\left(4ac-b^{2}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ab-\frac{\left(4ac-b^{2}\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c(4ac-b^{2})}$
risch	$\frac{\frac{b\left(3ac-b^{2}\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{a\left(2ac-b^{2}\right)}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{8\ln\left(-24a^{2}b\,c^{2}+10a\,b^{3}c-b^{5}-2\sqrt{-b^{2}(4ac-b^{2})(6ac-b^{2})^{2}}\,cx-\sqrt{-b^{2}(4ac-b^{2})(6ac-b^{2})^{2}}\,b\right)a^{2}}{(4ac-b^{2})^{2}}-\frac{4\ln}{}$

```
[In] int(1/(c+a/x^2+b/x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+
a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-b^
2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{2\,(ab^4c^2-8\,a^2b^2c^3}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.76 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6ac-b^2\right)}{2c^2\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} + \frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.}{6abc-b^3}\right) + \left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6ac-b^2\right)}{2c^2\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)} + \frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.}{6abc-b^3}\right) + \frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot(4ac^4-b^2c^3)+x\,(4abc^3-b^3c^2)}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x,x)
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(1/(x*(c + a/x^2 + b/x)^2),x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx=\frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{4a\operatorname{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)-4*a*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1366, 628, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx=\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\operatorname{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}+\frac{(2a)\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}\\
&= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{(4a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)}{b^2-4ac}\\
&= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx=\frac{b^2x+a(b-2cx)}{c\left(-b^2+4ac\right)(a+x(b+cx))}+\frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}
$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

```
[In] int(1/(c+a/x^2+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.45

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = \left[-\frac{ab^3-4\,a^2bc+2\,(ac^2x^2+abcx+a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}, -\frac{ab^3-4\,a^2bc-4\,(ac^2x^2+abcx+a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right]$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(60) = 120.

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + 2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + \frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,a}\right)}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} dx = \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1368, 652, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} dx = \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx \\
&= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac} \\
&= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2b)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac} \\
&= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2x^3}\,dx=\frac{2a+bx}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left((-8a\,c^2+2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left((8a\,c^2-2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	148

```
[In] int(1/(c+a/x^2+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$=\left[\frac{2\,ab^2-8\,a^2c-(bcx^2+b^2x+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^3-4\,abc)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\frac{2\,ab^2}{\ldots}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx=\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx=-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{4c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1368, 628, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = \frac{4c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx\\
&= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\
&= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2x^4}\,dx=-\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	144

[In] int(1/(c+a/x^2+b/x)^2/x^4,x,method=_RETURNVERBOSE)

[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx$$
$$=\left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")

[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(61) = 122.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = -2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) +2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx = \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 754, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx = \frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} - \frac{\left(b(b^2-6ac)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx+cx^2\right)}{2a^2} + \frac{\left(b(b^2-6ac)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{\ln(x)}{a^2}-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(4a\,c^2-b^2c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(5abc-b^3-\frac{(4a\,c^2-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$	177
risch	Expression too large to display	2292

```
[In] int(1/(c+a/x^2+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^2-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x
+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2
*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.

Time = 0.34 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$$
$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
```

```
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} \, dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}$$
$$+ \frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2\right.}{}$$
$$+ \frac{\ln\left(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2\right.}{}$$

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3}+\frac{b\log(a+bx+cx^2)}{a^3}$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 754, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} dx = \frac{b\log(a+bx+cx^2)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2(b^2-3ac)}{a^2x(b^2-4ac)} - \frac{2(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} + \frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

[In] Int[1/((c + a/x^2 + b/x)^2*x^6),x]

[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
```

```
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 (a+bx+cx^2)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{\int \frac{-2(b^2-3ac)-2bcx}{x^2(a+bx+cx^2)}\,dx}{a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{\int \left(\frac{2(-b^2+3ac)}{ax^2} - \frac{2b(-b^2+4ac)}{a^2x} + \frac{2(-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x)}{a^2(a+bx+cx^2)}\right)dx}{a(b^2-4ac)}$$

$$= -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x}{a+bx+cx^2}\,dx}{a^3(b^2-4ac)}$$

$$= -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \frac{(b^4-6ab^2c+6a^2c^2)\int \frac{1}{a+bx+cx^2}\,dx}{a^3(b^2-4ac)}$$

$$= -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3} - \frac{(2(b^4-6ab^2c+6a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^3(b^2-4ac)}$$

$$= -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{1}{a^2x}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2c}{\sqrt{4a}}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}$
risch	$\frac{-\frac{2c(3ac-b^2)x^2}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x}{a^2(4ac-b^2)}-\frac{1}{a}}{x(cx^2+bx+a)}-\frac{2b\ln(x)}{a^3}+2\left(\sum_{\_R=\mathrm{RootOf}((64a^6c^3-48a^5b^2c^2+12a^4b^4c-a^3b^6)\_Z^2+(-64bc^3a^3+48b^3c^2a\right.$

```
[In] int(1/(c+a/x^2+b/x)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2
)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x
^2+b*x+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(144) = 288.

Time = 0.40 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$$
$$= \left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^2 + ((b^4c - 6\,ab^2c^2 + 6\,a^2c^3)x^3 + (b^5 - 6\,ab^3c +}{}\right.$$
$$-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^2 + 2\,((b^4c - 6\,ab^2c^2 + 6\,a^2c^3)x^3 + (b^5 - 6\,ab^3c -}{}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} - \frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)} + \frac{b\log\left(cx^2+bx+a\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$$

$$= \ln\left(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}-\ln\left(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
```

```
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} dx = -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2}+\frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{1368, 754, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = -\frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4} + \frac{b(3b^2-11ac)}{a^3x\left(b^2-4ac\right)}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)} + \frac{b(30a^2c^2-20ab^2c+3b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}} + \frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] -1/2*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b
^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^
2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2
- 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3 (a+bx+cx^2)^2}\,dx \\
&= \frac{b^2-2ac+bcx}{a(b^2-4ac)\,x^2(a+bx+cx^2)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx}{a(b^2-4ac)} \\
&= \frac{b^2-2ac+bcx}{a(b^2-4ac)\,x^2(a+bx+cx^2)} \\
&\quad - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac)}{a^3x} + \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx}{a(b^2-4ac)} \\
&= -\frac{3b^2-8ac}{2a^2(b^2-4ac)\,x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)\,x^2(a+bx+cx^2)} \\
&\quad + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{\int \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a+bx+cx^2}\,dx}{a^4(b^2-4ac)}
\end{aligned}
$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4} - \frac{(b(3b^4-20ab^2c+30a^2c^2))\int\frac{1}{a+bx+cx^2}\,dx}{2a^4(b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4} + \frac{(b(3b^4-20ab^2c+30a^2c^2))\operatorname{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^4(b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$

$$= \frac{-\frac{a^2}{x^2} + \frac{4ab}{x} + \frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2(3b^2-2ac)\log(x) + (-}{2a^4}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$-\frac{1}{2a^{2}x^{2}}+\frac{\left(-2ac+3b^{2}\right)\ln\left(x\right)}{a^{4}}+\frac{2b}{a^{3}x}+\frac{\frac{\frac{acb\left(3ac-b^{2}\right)x}{4ac-b^{2}}-\frac{a\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)}{4ac-b^{2}}}{c\,x^{2}+bx+a}+\frac{\frac{\left(8a^{2}c^{3}-14b^{2}a\,c^{2}+3b^{4}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(19a^{2}b\,c^{2}-\cdots\right.}{\cdots}}{\cdots}}{a^{4}}$
risch	Expression too large to display

```
[In] int(1/(c+a/x^2+b/x)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x+1/a^4*((a*c*b*(3*a*c-b^2)/(
4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*
c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^2+b*x+a)+2*(19*a^2*b*
c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(194) = 388.

Time = 0.46 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
```

```
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**7,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx &= -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} \\ &\quad-\frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{(3\,b^2-2\,ac)\log\left(|x|\right)}{a^4} \\ &\quad-\frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}\end{aligned}$$

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$
$$= \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+307\,a^3\,b^4\,c^2-\right.}{}$$
$$-\frac{\ln\left(x\right)\,\left(2\,a\,c-3\,b^2\right)}{a^4}-\frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,\left(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4\right)}{2\,a^3\,\left(4\,a\,c-b^2\right)}-\frac{b\,c\,x^3\,\left(11\,a\,c-3\,b^2\right)}{a^3\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^3+a\,x^2}$$
$$+\frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+307\,a^3\,b^4\,c^2\right.}{}$$

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 238

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx &= \frac{3(b^4-7ab^2c+10a^2c^2)\,x}{c^3\,(b^2-4ac)^2} - \frac{3b(b^2-6ac)\,x^2}{2c^2\,(b^2-4ac)^2} \\ &\quad + \frac{x^5(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)\,x)}{c\,(b^2-4ac)^2\,(a+bx+cx^2)} \\ &\quad - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{5/2}} \\ &\quad - \frac{3b\log\left(a+bx+cx^2\right)}{2c^4}\end{aligned}$$

```
[Out] 3*(10*a^2*c^2-7*a*b^2*c+b^4)*x/c^3/(-4*a*c+b^2)^2-3/2*b*(-6*a*c+b^2)*x^2/c^
2/(-4*a*c+b^2)^2+1/2*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+x^3*(a*(-10
*a*c+b^2)+b*(-7*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^3+3
0*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4
*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{1354, 752, 832, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{3x(10a^2c^2-7ab^2c+b^4)}{c^3\left(b^2-4ac\right)^2} - \frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\left(b^2-4ac\right)^{5/2}} - \frac{3bx^2(b^2-6ac)}{2c^2\left(b^2-4ac\right)^2} + \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

```
[In] Int[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{x^6}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int \frac{x^4(10a+2bx)}{(a+bx+cx^2)^2}\,dx}{2(b^2-4ac)}$$

$$= \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{\int \frac{x^2(6a(b^2-10ac)+6b(b^2-6ac)x)}{a+bx+cx^2}dx}{2c(b^2-4ac)^2}$$

$$= \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{\int \left(-\frac{6(b^4-7ab^2c+10a^2c^2)}{c^2} + \frac{6b(b^2-6ac)x}{c} + \frac{6\left(a(b^4-7ab^2c+10a^2c^2)+b(b^2-4ac)^2x\right)}{c^2(a+bx+cx^2)}\right)dx}{2c(b^2-4ac)^2}$$

$$= \frac{3(b^4-7ab^2c+10a^2c^2)x}{c^3(b^2-4ac)^2} - \frac{3b(b^2-6ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{3\int \frac{a(b^4-7ab^2c+10a^2c^2)+b(b^2-4ac)^2x}{a+bx+cx^2}dx}{c^3(b^2-4ac)^2}$$

$$= \frac{3(b^4-7ab^2c+10a^2c^2)x}{c^3(b^2-4ac)^2} - \frac{3b(b^2-6ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{(3b)\int \frac{b+2cx}{a+bx+cx^2}dx}{2c^4} + \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\int \frac{1}{a+bx+cx^2}dx}{2c^4(b^2-4ac)^2}$$

$$= \frac{3(b^4-7ab^2c+10a^2c^2)x}{c^3(b^2-4ac)^2} - \frac{3b(b^2-6ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{3b\log(a+bx+cx^2)}{2c^4} - \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b+2cx\right)}{c^4(b^2-4ac)^2}$$

$$= \frac{3(b^4-7ab^2c+10a^2c^2)x}{c^3(b^2-4ac)^2} - \frac{3b(b^2-6ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)x)}{c(b^2-4ac)^2(a+bx+cx^2)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4(b^2-4ac)^{5/2}} - \frac{3b\log(a+bx+cx^2)}{2c^4}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$$

$$= \frac{2c^2x+\frac{b^7-14ab^5c+61a^2b^3c^2-78a^3bc^3-6b^6cx+48ab^4c^2x-102a^2b^2c^3x+36a^3c^4x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{-b^6x+a^2b^2c(5b-9cx)-ab^4(b-6cx)+a^3c^2(-5b+2cx)}{(b^2-4ac)(a+x(b+cx))^2}+\cdots}{2c^5}$$

```
[In] Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.68

method	result
default	$\frac{x}{c^3}-\frac{\frac{-\frac{3\left(6c^3a^3-17a^2b^2c^2+8a\,b^4c-b^6\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(42c^3a^3+41a^2b^2c^2-34a\,b^4c+5b^6\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{a\left(14c^3a^3-71a^2b^2c^2+38a\,b^4c-5b^6\right)x}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\,a^2\left(58a^2c^2-36a\cdots\right)}{2c\left(16a^2c^2-8a\,b\cdots\right)}}{\left(c\,x^2+bx+a\right)^2}\cdots}{c^3}$
risch	Expression too large to display

```
[In] int(1/(c+a/x^2+b/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3-1/c^3*((-3*(6*a^3*c^3-17*a^2*b^2*c^2+8*a*b^4*c-b^6)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^3+1/2*b*(42*a^3*c^3+41*a^2*b^2*c^2-34*a*b^4*c+5*b^6)/(16*a^2*c^
2-8*a*b^2*c+b^4)/c*x^2-a/c*(14*a^3*c^3-71*a^2*b^2*c^2+38*a*b^4*c-5*b^6)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*a^2/c*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*
a^2*b*c^2-8*a*b^3*c+b^5)/c*ln(c*x^2+b*x+a)+2*(10*a^3*c^2-7*a^2*b^2*c+b^4*a-
1/2*(16*a^2*b*c^2-8*a*b^3*c+b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(228) = 456$.

Time = 0.29 (sec) , antiderivative size = 1926, normalized size of antiderivative = 8.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4
+ 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^
3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5
*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*
c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c +
10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c
 + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^
4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a
^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*
x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*
```

```
x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x
^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
+ (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 -
 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^
5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*
a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(236) = 472$.

Time = 3.04 (sec) , antiderivative size = 1714, normalized size of antiderivative = 7.20

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
+ 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
```

```
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
- 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}}+\frac{x}{c^3}-\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,c^4} - \frac{5\,a^2b^5-36\,a^3b^3c+58\,a^4bc^2+6\,(b^6c-8\,ab^4c^2+17\,a^2b^2c^3-6\,a^3c^4)x^3+(5\,b^7-34\,ab^5c+41\,a^2b^3c^2+42}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^4}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx=\frac{x}{c^3}$$

$$-\frac{\frac{3x^3\left(-6a^3c^3+17a^2b^2c^2-8ab^4c+b^6\right)}{16a^2c^2-8ab^2c+b^4}+\frac{x^2\left(42a^3bc^3+41a^2b^3c^2-34ab^5c+5b^7\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{a^2\left(58a^2bc^2-36ab^3c+5b^5\right)}{2c\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{ax\left(-14a^3c^3+7\cdots\right)}{c\left(16a^2\cdots\right)}}{a^2c^3+c^5x^4+x^2\left(b^2c^3+2ac^4\right)+2bc^4x^3+2abc^3x}$$

$$+\frac{\ln\left(cx^2+bx+a\right)\left(-3072a^5bc^5+3840a^4b^3c^4-1920a^3b^5c^3+480a^2b^7c^2-60ab^9c+3b^{11}\right)}{2\left(1024a^5c^9-1280a^4b^2c^8+640a^3b^4c^7-160a^2b^6c^6+20ab^8c^5-b^{10}c^4\right)}$$

$$+\frac{3\,\mathrm{atan}\left(\frac{\left(\frac{3x\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)}{c^3\left(4ac-b^2\right)^5}+\frac{3\left(16a^2bc^5-8ab^3c^4+b^5c^3\right)\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)}{2c^7\left(4ac-b^2\right)^5\left(16a^2c^2-8ab^2c+b^4\right)}\right)\left(32a^2c^6\left(4ac-b^2\right)^{5/2}\cdots\right)}{-60a^3c^3+90a^2b^2c^2-30ab^4c+3b^6}\cdots\right)}{c^4\left(4ac-b^2\right)^{5/2}}$$

```
[In] int(1/(c + a/x^2 + b/x)^3,x)
```

```
[Out] x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*b^
5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2 - 3
6*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3*c^3
 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c
^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) + (log(
a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 + 20*a*b^
8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (3*atan(((
(3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*c - b^2)^5
) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5/2) - 16*a*b^
2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*c^2 - 30*a*b^4
*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*(4*a*c - b^2)^(
5/2))
```

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x} dx$

Optimal result

Integrand size = 18, antiderivative size = 190

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x} dx = -\frac{b(b^2-7ac)x}{c^2(b^2-4ac)^2}+\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}+\frac{x^2(a(b^2-16ac)+b(b^2-10ac)x)}{2c(b^2-4ac)^2(a+bx+cx^2)}+\frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{5/2}}+\frac{\log(a+bx+cx^2)}{2c^3}$$

```
[Out] -b*(-7*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+
b*x+a)^2+1/2*x^2*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^
2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1368, 752, 832, 787, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x} dx = \frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{5/2}}-\frac{bx(b^2-7ac)}{c^2(b^2-4ac)^2}+\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{2c(b^2-4ac)^2(a+bx+cx^2)}+\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}+\frac{\log(a+bx+cx^2)}{2c^3}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
```

```
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int \frac{x^3(8a+bx)}{(a+bx+cx^2)^2}\,dx}{2(b^2-4ac)}$$

$$= \frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c(b^2-4ac)^2(a+bx+cx^2)} - \frac{\int \frac{x(2a(b^2-16ac)+2b(b^2-7ac)x)}{a+bx+cx^2}\,dx}{2c(b^2-4ac)^2}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c(b^2-4ac)^2(a+bx+cx^2)} - \frac{\int \frac{-2ab(b^2-7ac)+(2ac(b^2-16ac)-2b^2(b^2-7ac))x}{a+bx+cx^2}\,dx}{2c^2(b^2-4ac)^2}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} - \frac{(b(b^4-10ab^2c+30a^2c^2))\int \frac{1}{a+bx+cx^2}\,dx}{2c^3\,(b^2-4ac)^2}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^3} + \frac{(b(b^4-10ab^2c+30a^2c^2))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^3\,(b^2-4ac)^2}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{5/2}} + \frac{\log(a+bx+cx^2)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$

$$= \frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx-30ab^3c^2x+50a^2bc^3x}{(b^2-4ac)^2(a+x(b+cx))} + \frac{2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx)}{(b^2-4ac)(a+x(b+cx))^2} - \frac{2bc(b^4-10ab^2c+30a^2c^2)\,\mathrm{arctan}}{(-b^2+4ac)^{5/2}}}{2c^4}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^
2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^
5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.88

method	result
default	$\frac{\frac{b\left(25a^2c^2-15a\,b^2c+2b^4\right)x^3}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(32c^3a^3+11a^2b^2c^2-19a\,b^4c+3b^6\right)x^2}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{ab\left(31a^2c^2-22a\,b^2c+3b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{3a^2\left(8a^2c^2-7a\,b^2c+b^4\right)}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{\left(16a^2c^2-8a\,b^2\right.}{}$
risch	Expression too large to display

```
[In] int(1/(c+a/x^2+b/x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(
32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*
a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)
^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^4)/c*ln(c*
x^2+b*x+a)+2*(-7*c*b*a^2+a*b^3-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 1603, normalized size of antiderivative = 8.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^
3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c
 - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 12
4*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b
^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^
3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 -
128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*
b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2
*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 3
1*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5
 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^
4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a
^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^
```

```
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5
- 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 +
 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3
- 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*
b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(180) = 360$.

Time = 1.87 (sec) , antiderivative size = 1510, normalized size of antiderivative = 7.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-
(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)
**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(3
```

```
0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30
*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a*
*2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3
*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*
a**2*b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c*
*3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c
**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 +
 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3)x^3+(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3)x^2}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^3}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 +
b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15
*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*
a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x +
 a)^2*(b^2 - 4*a*c)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.26

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$
$$= \frac{\frac{3\,a^2\,(8\,a^2\,c^2-7\,a\,b^2\,c+b^4)}{2\,c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6)}{2\,c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{b\,x^3\,(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4)}{c^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a\,b\,x\,(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4)}{c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$
$$-\frac{\ln(c\,x^2+b\,x+a)\,(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10})}{2\,(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\,(30\,a^2\,c^2-10\,a\,b^2\,c+b^4)}{c^2\,(4\,a\,c-b^2)^5}+\frac{b^2\,(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2)\,(30\,a^2\,c^2-10\,a\,b^2\,c+b^4)}{2\,c^5\,(4\,a\,c-b^2)^5\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)\left(32\,a^2\,c^5\,(4\,a\,c-b^2)^{5/2}+2\,b^4\,c^3\,(4\,a\,c-b^2)^{5/2}-16\right.}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right.}{c^3\,(4\,a\,c-b^2)^{5/2}}$$

```
[In] int(1/(x*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c^2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2*c))
/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 +
 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 + 160*a^
2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(1024*a^5*
c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 - 1280*a^
4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2*(4*a*c -
 b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*
c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a
*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx = \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\text{arctanh}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)^2-3*a*(b+2*a/x)/(-4*a*c+b^2)^2/(c+
a/x^2+b/x)+12*a^2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1366, 628, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx = \frac{12a^2\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} - \frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)} + \frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{\left(12a^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{1}{2}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx+16ab^2c^2x-20a^2c^3x}{c^3\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)} + \frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2} + \frac{24a^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*
x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a
^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(105) = 210.

Time = 0.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.34

method	result
default	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6a^2\ln\left(\left(32a^2c^3-16b^2a\,c^2+2b^4\right.\right.}{(}$

```
[In] int(1/(c+a/x^2+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a^
2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a*b
^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 953, normalized size of antiderivative = 8.59

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx$$
$$= \left[-\frac{a^2b^5 - 14\,a^3b^3c + 40\,a^4bc^2 + 2\,(b^6c - 12\,ab^4c^2 + 42\,a^2b^2c^3 - 40\,a^3c^4)x^3 + (b^7 - 12\,ab^5c + 30\,a^2b^3c^2 + 8}{2\,(a^2b^6c^2 - 12\,a^3b^4c^3 + 48\,a^4b^2c^4 - 64\,a^5c^5 + (b^6c^4 - 12\,ab^4c^5 + 48\,a^2b^2c^6 - 64\,a^3c^7)x^4}\right.$$
$$-\frac{a^2b^5 - 14\,a^3b^3c + 40\,a^4bc^2 + 2\,(b^6c - 12\,ab^4c^2 + 42\,a^2b^2c^3 - 40\,a^3c^4)x^3 + (b^7 - 12\,ab^5c + 30\,a^2b^3c^2 + 8}{2\,(a^2b^6c^2 - 12\,a^3b^4c^3 + 48\,a^4b^2c^4 - 64\,a^5c^5 + (b^6c^4 - 12\,ab^4c^5 + 48\,a^2b^2c^6 - 64\,a^3c^7)x^4 + 2\,(}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 +
 (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^
6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^
4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3
*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 1
28*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b
*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4
*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^
2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b
^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(
b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10
*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*
c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(94) = 188.

Time = 0.82 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.93

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = -6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a\cdots}{12a^2c}\right. +6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6a^2b\cdots}{12a^2c}\right. +\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2\cdot(2a^2bc^2+8ab^3c-b^5)\cdots}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\cdot(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3\cdot(64a^2bc^5-32ab^3c^4+4b^5c^3)+x^2\cdot(\cdots}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4*c
*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6*a*
*2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**5*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**
5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**3*(-2
0*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*a*b**3*c
 - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4*c**4 - 1
6*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b**2*c**5 +
 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2
*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**
2*b**3*c**3 + 4*a*b**5*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3*x
^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*c*x
 + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

```
[Out] (12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2
)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 + 10*
a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a
*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^5 + 8
*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2*c^2 -
 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^
2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} dx = -\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3bx(2a+bx)}{2(b^2-4ac)^2(a+bx+cx^2)} + \frac{6ab\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3/2*b*x*(b*x+2*a)/(-4*a*c+b
^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 742, 736, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} dx = \frac{6ab\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{3bx(2a+bx)}{2(b^2-4ac)^2(a+bx+cx^2)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a +
b*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Dist[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3}{(a+bx+cx^2)^3}\,dx\\ &= -\frac{x^3(b+2cx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}+\frac{(3b)\int \frac{x^2}{(a+bx+cx^2)^2}\,dx}{2\,(b^2-4ac)}\\ &= -\frac{x^3(b+2cx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}+\frac{3bx(2a+bx)}{2\,(b^2-4ac)^2\,(a+bx+cx^2)}-\frac{(3ab)\int \frac{1}{a+bx+cx^2}\,dx}{(b^2-4ac)^2}\end{aligned}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6ab)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{\left(b^2-4ac\right)^2}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3x^3}\,dx = -\frac{8a^3c+b^4x^2+abx(2b^2+bcx+6c^2x^2)+a^2(b^2+10bcx+16c^2x^2)}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2} - \frac{6ab\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10
*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(99) = 198.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-$ 3ab ln((32a²c³−16b²ac²+2b⁴c)x−(− … / (−4ac+b …

```
[In] int(1/(c+a/x^2+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-
1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 872, normalized size of antiderivative = 8.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$$
$$= \left[-\frac{a^2b^4+4\,a^3b^2c-32\,a^4c^2+6\,(ab^3c^2-4\,a^2bc^3)x^3+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^2-6\,(abc^3x^4+\cdots}{2\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^4+2\,(b^7c^2-12\,ab\cdots}\right.$$
$$-\frac{a^2b^4+4\,a^3b^2c-32\,a^4c^2+6\,(ab^3c^2-4\,a^2bc^3)x^3+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^2-12\,(abc\cdots}{2\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^4+2\,(b^7c^2-12\,ab\cdots}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2
*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a
^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*
a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^
3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b
^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)
*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3
*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c -
 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^
4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2
*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64
*a^4*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(102) = 204.

Time = 0.66 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.79

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = 3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab\cdots}{6abc}\right. - 3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3ab\cdots}{6abc}\right. + \frac{-8a^3c-a^2b^2-6abc^2x^3+x^2(-16a^2c^2-ab^2c-b^4)+x(-1\cdots}{32a^4c^3-16a^3b^2c^2+2a^2b^4c+x^4\cdot(32a^2c^5-16ab^2c^4+2b^4c^3)+x^3\cdot(64a^2bc^4-32ab^3c^3+4b^5c^2)+x^2\cdot(6\cdots}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**
2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c**3*s
qrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) +
 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2
)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x**3 + x
**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3))/(32*a*
*4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 - 16*a*b**
2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4*b**5*c**2
) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 3
2*a**2*b**3*c**2 + 4*a*b**5*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^2+bx+a)^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^
2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.53

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{\frac{x^2\,(16\,a^2\,c^2+a\,b^2\,c+b^4)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a^2\,(b^2+8\,a\,c)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,a\,b\,c\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{a\,b\,x\,(b^2+5\,a\,c)}{c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,a\,b\,\mathrm{atan}\left(\frac{\left(\frac{3\,a\,b^2}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,a\,b\,c\,x}{(4\,a\,c-b^2)^{5/2}}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{3\,a\,b}\right)}{(4\,a\,c-b^2)^{5/2}}$$

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3)/
(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) -
 (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^2)^(5
/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx = \frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3ab+(b^2+2ac)x}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{2(b^2+2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+(3*a*b+(2*a*c+b^2)*x)/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 752, 652, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx = -\frac{2(2ac+b^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{x(2ac+b^2)+3ab}{(b^2-4ac)^2(a+bx+cx^2)}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{\int \frac{2a-2bx}{(a+bx+cx^2)^2}\,dx}{2(b^2-4ac)}$$

$$= \frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3ab+(b^2+2ac)x}{(b^2-4ac)^2(a+bx+cx^2)} + \frac{(b^2+2ac)\int \frac{1}{a+bx+cx^2}\,dx}{(b^2-4ac)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2(b^2+2ac)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2(b^2+2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3x^4}\,dx = \frac{1}{2}\left(\frac{b^2x+a(b-2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2}+\frac{\left(b^2+2ac\right)\left(b+2cx\right)}{c\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)} + \frac{4(b^2+2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2
*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.83

method	result
default	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{2\left(2ac+b^2\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{\left(c\,x^2+bx+a\right)^2}-\frac{2\ln\left(\left(32a^2c^3-16b^2a\,c^2+2b^4c\right)x+\left(-4ac+b^2\right)^{\frac{5}{2}}+\right.}{\left(-4ac+b^2\right)^{\frac{5}{2}}}$

```
[In] int(1/(c+a/x^2+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(109) = 218$.

Time = 0.28 (sec) , antiderivative size = 887, normalized size of antiderivative = 7.71

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx$$
$$= \left[\frac{6\,a^2b^3 - 24\,a^3bc + 2\,(b^4c - 2\,ab^2c^2 - 8\,a^2c^3)x^3 + 3\,(b^5 - 2\,ab^3c - 8\,a^2bc^2)x^2 + 2\,((b^2c^2 + 2\,ac^3)x^4 + a^2b^2}{2\,(a^2b^6 - 12\,a^3b^4c + 48\,a^4b^2c^2 - 64\,a^5c^3 + (b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5)x^4 + 2}\right.$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2
+ 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 -
22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x
^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)
*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*
b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(107) = 214$.

Time = 0.75 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = -\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\cdot(2ac$$

$$+b^2)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right.$$

$$+\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\cdot(2ac$$

$$+b^2)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(}{4ac^2+2b^2c}\right.$$

$$+\frac{6a^2b+x^3\cdot(4ac^2+2b^2c)+x^2\cdot(6abc+3b^3)+x(-4a^2c+1}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/(
4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c*
*2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**
3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b**2*c) +
x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2)
 + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 -
 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^
2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{\frac{3\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{a\,x\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2\,a\,c\right)\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{b^2+2\,a\,c}\right)\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c
 + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^2)*(
b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6bc\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 652, 628, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{6bc\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}} + \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{(3b)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(3bc)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6bc)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{\left(b^2-4ac\right)^2}$$

$$= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \frac{\frac{\left(b^2-4ac\right)(2a+bx)}{(a+x(b+cx))^2} - \frac{3b(b+2cx)}{a+x(b+cx)} - \frac{12bc\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x
*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

method	result
default	$\frac{-bx-2a}{2(4ac-b^2)(cx^2+bx+a)^2} - \frac{3b\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)} + \frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{2(4ac-b^2)}$
risch	$\frac{-\frac{3bc^2x^3}{16a^2c^2-8ab^2c+b^4} - \frac{9b^2cx^2}{2\left(16a^2c^2-8ab^2c+b^4\right)} - \frac{\left(5ac+b^2\right)bx}{16a^2c^2-8ab^2c+b^4} - \frac{a\left(8ac+b^2\right)}{2\left(16a^2c^2-8ab^2c+b^4\right)}}{\left(cx^2+bx+a\right)^2} - \frac{3bc\ln\left(\left(32a^2c^3-16b^2ac^2+2b^4c\right)x-(-4ac\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

```
[In] int(1/(c+a/x^2+b/x)^3/x^5,x,method=_RETURNVERBOSE)
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/2*b/(4*a*c-b^2)*((2*c*x+b)/(4*
a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 788, normalized size of antiderivative = 7.65

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$

$$= \left[-\frac{ab^4 + 4\,a^2b^2c - 32\,a^3c^2 + 6\,(b^3c^2 - 4\,abc^3)x^3 + 9\,(b^4c - 4\,ab^2c^2)x^2 - 6\,(bc^3x^4 + 2\,b^2c^2x^3 +}{2\,(a^2b^6 - 12\,a^3b^4c + 48\,a^4b^2c^2 - 64\,a^5c^3 + (b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5)x^4 + 2\,(b^7c - 12\,ab^5c^2}\right.$$

$$-\frac{ab^4 + 4\,a^2b^2c - 32\,a^3c^2 + 6\,(b^3c^2 - 4\,abc^3)x^3 + 9\,(b^4c - 4\,ab^2c^2)x^2 - 12\,(bc^3x^4 + 2\,b^2c^2x}{2\,(a^2b^6 - 12\,a^3b^4c + 48\,a^4b^2c^2 - 64\,a^5c^3 + (b^6c^2 - 12\,ab^4c^3 + 48\,a^2b^2c^4 - 64\,a^3c^5)x^4 + 2\,(b^7c - 12\,ab^5c^2 +}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(
b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2
*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5
+ a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a
^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*
c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(95) = 190$.

Time = 0.60 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$
$$= 3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b}{6bc^2}\right.$$
$$-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b}{6bc^2}\right.$$
$$+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x(-10abc-2b^3)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*
c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c**4*
sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2 - 6*b*
c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2
*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*
b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*
b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*
c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{\frac{8\,c\,a^2+a\,b^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b^2\,c\,x^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,b\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,b\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2
) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a*c -
 b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx = \frac{-b-2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12c^2 \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/2*(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(c
*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1368, 628, 632, 212}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} \, dx = -\frac{12c^2 \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((
b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx+cx^2)^3}\,dx \\
&= -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{(3c)\int \frac{1}{(a+bx+cx^2)^2}\,dx}{b^2-4ac} \\
&= -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} + \frac{(6c^2)\int \frac{1}{a+bx+cx^2}\,dx}{(b^2-4ac)^2} \\
&= -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} \\
&\quad - \frac{(12c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{(b^2-4ac)^2} \\
&= -\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx=\frac{-\frac{(b+2cx)(b^2-6bcx-2c(5a+3cx^2))}{(a+x(b+cx))^2}+\frac{24c^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{9b\,c^2x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16a\,b^2c+2b^4}}{(c\,x^2+bx+a)^2}-\frac{6c^2\ln\left(\left(32a^2c^3-16b^2a\,c^2+2b^4c\right)x+\left(-4ac+b^2\right)^{\frac{5}{2}}\right.}{\left(-4ac+b^2\right)^{\frac{5}{2}}}$

```
[In] int(1/(c+a/x^2+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*c
-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$$

$$=\left[-\frac{b^5-14\,ab^3c+40\,a^2bc^2-12\,(b^2c^3-4\,ac^4)x^3-18\,(b^3c^2-4\,abc^3)x^2-12\,(c^4x^4+2\,bc^3x^3+}{2\,(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5)x^4+2\,(b^7c-12\,ab^5c^2}\right.$$

$$-\frac{b^5-14\,ab^3c+40\,a^2bc^2-12\,(b^2c^3-4\,ac^4)x^3-18\,(b^3c^2-4\,abc^3)x^2+24\,(c^4x^4+2\,bc^3}{2\,(a^2b^6-12\,a^3b^4c+48\,a^4b^2c^2-64\,a^5c^3+(b^6c^2-12\,ab^4c^3+48\,a^2b^2c^4-64\,a^3c^5)x^4+2\,(b^7c-12\,ab^5c^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.

Time = 0.70 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = -6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+6b}{12c^3}\right. \\ +6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6b^6c}{12c^3}\right. \\ +\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
```

```
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.77

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,c^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*
c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2
 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c - b^2)
^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2)))/(4*a
*c - b^2)^(5/2)
```

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 185

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} dx = \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log\left(a+bx+cx^2\right)}{2a^3}$$

```
[Out] 1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c+
16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a^2
*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)
^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used

= {1368, 754, 836, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = -\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3} + \frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b(30a^2c^2-10ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}} + \frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{\int \frac{2(b^2-4ac)^2+2bc(b^2-7ac)x}{x(a+bx+cx^2)}\,dx}{2a^2(b^2-4ac)^2}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{\int \left(\frac{2(-b^2+4ac)^2}{ax} + \frac{2\left(-b(b^4-9ab^2c+23a^2c^2)-c(b^2-4ac)^2x\right)}{a(a+bx+cx^2)}\right)dx}{2a^2(b^2-4ac)^2}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} + \frac{\int \frac{-b(b^4-9ab^2c+23a^2c^2)-c(b^2-4ac)^2x}{a+bx+cx^2}\,dx}{a^3(b^2-4ac)^2}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3} - \frac{(b(b^4-10ab^2c+30a^2c^2))\int \frac{1}{a+bx+cx^2}\,dx}{2a^3(b^2-4ac)^2}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^4-10ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^3(b^2-4ac)^2}$$

$$= \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log(a+bx+cx^2)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$

$$= \frac{\frac{a^2(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))^2} + \frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx-14abc^2x)}{(b^2-4ac)^2(a+x(b+cx))} - \frac{2b(b^4-10ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}} + 2\log(x) - \log(a}{2a^3}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^
4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(
a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)
])/(2*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(175) = 350$.

Time = 0.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.90

method	result
default	$\frac{\ln(x)}{a^3} - \frac{\frac{\frac{ab\,c^2(7ac-b^2)x^3}{16a^2c^2-8a\,b^2c+b^4} - \frac{ac(16a^2c^2-29a\,b^2c+4b^4)x^2}{2(16a^2c^2-8a\,b^2c+b^4)} + \frac{ab(a^2c^2+6a\,b^2c-b^4)x}{16a^2c^2-8a\,b^2c+b^4} - \frac{3a^2(8a^2c^2-7a\,b^2c+b^4)}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^2+bx+a)^2} + \frac{(16a^2c^3-8b^2a\,c^2+b^4c)\ln(c}{2c}}{a^3}$
risch	Expression too large to display

```
[In] int(1/(c+a/x^2+b/x)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^3-1/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*a*
c*(16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(a^2*c^2
+6*a*b^2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4
)*(1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(23*a^2*b*c^2-9*a
*b^3*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(175) = 350$.

Time = 0.52 (sec) , antiderivative size = 1985, normalized size of antiderivative = 10.73

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b
^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*
(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
```

```
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^
6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -
128*a^7*c^4)*x^2 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^
3)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*
b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2
+ 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*
c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2
 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*
x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^7 - 10*a^2*b^5*c
+ 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 +
 (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^2 + 2*(a^4*b^
7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$
$$= -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^4-8\,a^4b^2c+16\,a^5c^2)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^3} + \frac{\log\left(|x|\right)}{a^3}$$
$$+ \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(ab^3c^2-7\,a^2bc^3)x^3+(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3)x^2+2\,(ab^5-6\,a^2b^3c}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^3}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 +
b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2
- 4*a*c)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \frac{\ln(x)}{a^3} + \frac{\frac{3\,(8\,a^2\,c^2-7\,a\,b^2\,c+b^4)}{2\,a\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(16\,a^2\,c^3-29\,a\,b^2\,c^2+4\,b^4\,c)}{2\,a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{b\,x\,(a^2\,c^2+6\,a\,b^2\,c-b^4)}{a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{b\,c^2\,x^3\,(7\,a\,c-b^2)}{a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{\ln\left(1536\,a^6\,c^5-2\,b^{11}\,x-2\,a\,b^{10}+2\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^5}+39\,a^2\,b^8\,c+2\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^5}-303\,a^3\,b^6 \cdots\right.}{\cdots} + \frac{\ln\left(2\,a\,b^{10}+2\,b^{11}\,x-1536\,a^6\,c^5+2\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^8\,c+2\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^5}+303\,a^3\,b^6 \cdots\right.}{\cdots}$$

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(1
536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) + 39*a
^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160*a^4*b^
4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c^3*x - 25
60*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^9*c*x + 2
016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*
(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c - b^2)^5)^(1/
2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20*a*b^8*c + 30
*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))
/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a^6*c^5 + 2*a*b^
5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2
) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-
(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^
7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c -
 b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c -
b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^10 - 1024*a^5
*c^5 + b^5*(-(4*a*c - b^2)^5)^(1/2) + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1
280*a^4*b^2*c^4 - 20*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a
*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5)
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx = & -\frac{3(b^2-5ac)(b^2-2ac)}{a^3(b^2-4ac)^2 x}+\frac{b^2-2ac+bcx}{2a(b^2-4ac)x(a+bx+cx^2)^2} \\ & +\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)x}{2a^2(b^2-4ac)^2 x(a+bx+cx^2)} \\ & -\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{5/2}} \\ & -\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx+cx^2)}{2a^4}\end{aligned}$$

```
[Out] -3*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/2*(b*c*x-2*a*c+b^2)/a/(
-4*a*c+b^2)/x/(c*x^2+b*x+a)^2+1/2*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*
c+b^2)*x)/a^2/(-4*a*c+b^2)^2/x/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c^2-
10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(5/2
)-3*b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used

$= \{1368, 754, 836, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \frac{3b\log\left(a+bx+cx^2\right)}{2a^4} - \frac{3b\log(x)}{a^4} - \frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3x\left(b^2-4ac\right)^2} + \frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{2a^2x\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}} + \frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}+\frac{\int\frac{6\left(b^2-5ac\right)\left(b^2-2ac\right)+6bc\left(b^2-6ac\right)x}{x^2\left(a+bx+cx^2\right)}\,dx}{2a^2\left(b^2-4ac\right)^2}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}+\frac{\int\left(\frac{6\left(b^2-5ac\right)\left(b^2-2ac\right)}{ax^2}-\frac{6b\left(-b^2+4ac\right)^2}{a^2x}+\frac{6\left(b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc\left(b^2-4ac\right)^2x\right)}{a^2\left(a+bx+cx^2\right)}\right)dx}{2a^2\left(b^2-4ac\right)^2}$$

$$=-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x}+\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{3\int\frac{b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc\left(b^2-4ac\right)^2x}{a+bx+cx^2}\,dx}{a^4\left(b^2-4ac\right)^2}$$

$$=-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x}+\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{(3b)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}+\frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\int\frac{1}{a+bx+cx^2}\,dx}{2a^4\left(b^2-4ac\right)^2}$$

$$=-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x}+\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^4\left(b^2-4ac\right)^2}$$

$$=-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x}+\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)}-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{5/2}}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$$
$$= \frac{-\frac{2a}{x}+\frac{a^2(b^3-3abc+b^2cx-2ac^2x)}{(-b^2+4ac)(a+x(b+cx))^2}-\frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx-26ab^2c^2x+28a^2c^3x)}{(b^2-4ac)^2(a+x(b+cx))}+\frac{6(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\arctan\left(\frac{\ldots}{\sqrt{\ldots}}\right)}{(-b^2+4ac)^{5/2}}\ldots}{2a^4}$$

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.69

method	result
default	$-\frac{1}{a^3x}-\frac{3b\ln(x)}{a^4}-\frac{\frac{a c^2(14a^2c^2-13a b^2c+2b^4)x^3}{16a^2c^2-8a b^2c+b^4}+\frac{abc(74a^2c^2-55a b^2c+8b^4)x^2}{32a^2c^2-16a b^2c+2b^4}+\frac{a(18c^3a^3+7a^2b^2c^2-12a b^4c+2b^6)x}{16a^2c^2-8a b^2c+b^4}+\frac{a^2b(58a^2c^2-36\ldots}{32a^2c^2-16a\ldots}}{(c x^2+bx+a)^2}\ldots$
risch	$\frac{-\frac{3c^2(10a^2c^2-7a b^2c+b^4)x^4}{a^3(16a^2c^2-8a b^2c+b^4)}-\frac{3bc(46a^2c^2-29a b^2c+4b^4)x^3}{2(16a^2c^2-8a b^2c+b^4)a^3}-\frac{(50c^3a^3+7a^2b^2c^2-18a b^4c+3b^6)x^2}{a^3(16a^2c^2-8a b^2c+b^4)}-\frac{b(122a^2c^2-68a b^2c+9b^4)x}{2a^2(16a^2c^2-8a b^2c+b^4)}-\frac{1}{a}}{x(c x^2+bx+a)^2}-\frac{3b\ln\ldots}{a^4}\ldots$

```
[In] int(1/(c+a/x^2+b/x)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3/x-3*b*ln(x)/a^4-1/a^4*((a*c^2*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^3+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^2+a*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x+1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*
b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2*b*c^
3+8*a*b^3*c^2-b^5*c)/c*ln(c*x^2+b*x+a)+2*(10*c^3*a^3-23*a^2*b^2*c^2+9*a*b^4
*c-b^6-1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(229) = 458$.

Time = 0.67 (sec) , antiderivative size = 2280, normalized size of antiderivative = 9.54

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Too large to display}$$

[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")

[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(

```
b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b
^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12
*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c
 + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2
+ 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^4-8\,a^5b^2c+16\,a^6c^2)\sqrt{-b^2+4\,ac}} + \frac{3\,b\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{2\,a^3b^4-16\,a^4b^2c+32\,a^5c^2+6\,(ab^4c^2-7\,a^2b^2c^3+10\,a^3c^4)x^4+3\,(4\,ab^5c-29\,a^2b^3c^2+46\,a^3bc^3)x^3+2\,(3}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^4x}$$

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 1255, normalized size of antiderivative = 5.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = -\frac{\frac{1}{a}+\frac{x^2\,(50\,a^3\,c^3+7\,a^2\,b^2\,c^2-18\,a\,b^4\,c+3\,b^6)}{a^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,(122\,a^2\,b\,c^2-68\,a\,b^3\,c+9\,b^5)}{2\,a^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,x^3\,(46\,a^2\,b\,c^3-29\,a\,b^3\,c^2+4\,b^5\,c)}{2\,a^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,c^2\,x^4\,(10\,a^2\,c^2-7\,a}{a^3\,(16\,a^2\,c^2-8\,a\,b^2}}{x^3\,(b^2+2\,a\,c)+a^2\,x+c^2\,x^5+2\,a\,b\,x^2+2\,b\,c\,x^4} - \frac{3\,b\,\ln(x)}{a^4} - \frac{3\,\ln\left(2\,a\,b^{11}+2\,b^{12}\,x+2\,a\,b^6\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^9\,c-1696\,a^6\,b\,c^5+320\,a^6\,c^6\,x+2\,b^7\,x\,\sqrt{-(4\,a\,c-}\right.}{} - \frac{3\,\ln\left(2\,a\,b^{11}+2\,b^{12}\,x-2\,a\,b^6\,\sqrt{-(4\,a\,c-b^2)^5}-39\,a^2\,b^9\,c-1696\,a^6\,b\,c^5+320\,a^6\,c^6\,x-2\,b^7\,x\,\sqrt{-(4\,a\,c-}\right.}{}$$

```
[In] int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*a^
```

```
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2
*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*a^2*
c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c + b^2)
+ a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*log(2*a*
b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^
6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^
2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b
^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x + 39*a^3
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) -
 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)
^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*
b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-(4*a*c - b^2)^5
)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*log(2*a*b^11 + 2*b
^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^6*b*c^5 +
320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^2 - 1170*a
^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/2) + 17*a^
2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b^6*c^3*x +
 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x - 39*a^3*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) + 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) + 58*a^3*b*
c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)^5)^(1/2))
*(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*b^7*c^2 -
640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 + 20*a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) -
20*a*b^9*c - 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a*b^4*c*(-(4*a*c
- b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5)
```

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1368, 715, 646, 31}

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[In] Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{2+13x+15x^2}\,dx \\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\,(2+13x+15x^2)}\right)dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375} \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

```
[In] Integrate[x^2/(15 + 2/x^2 + 13/x),x]
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^{3}}{45}-\frac{13x^{2}}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^{2}}{450}+\frac{x^{3}}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^{2}}{450}+\frac{x^{3}}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^{2}}{450}+\frac{x^{3}}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31

```
[In] int(x^2/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x^{3}}{45}-\frac{13x^{2}}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

```
[In] integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(|5\,x+1|\right) - \frac{16}{567}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

```
[In] int(x^2/(13/x + 2/x^2 + 15),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1368, 715, 646, 31}

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = \frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[In] Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3}{2+13x+15x^2}\,dx\\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\,(2+13x+15x^2)}\right)\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[In] Integrate[x/(15 + 2/x^2 + 13/x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^{2}}{30}-\frac{13x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8\ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^{2}}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26

```
[In] int(x/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{30}\,x^{2}-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x^{2}}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

```
[In] integrate(x/(15+2/x**2+13/x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x + 1\right) + \frac{8}{189}\,\log\left(3\,x + 2\right)$$

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x + 1|\right) + \frac{8}{189}\,\log\left(|3\,x + 2|\right)$$

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}}\,dx = \frac{8\,\ln\left(x + \frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x + \frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

```
[In] int(x/(13/x + 2/x^2 + 15),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1354, 717, 646, 31}

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[In] Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{2+13x+15x^2}\,dx \\
&= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx \\
&= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx \\
&= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

```
[In] Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21

```
[In] int(1/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
[Out] 1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

```
[In] integrate(1/(15+2/x**2+13/x),x)
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x+1\right) - \frac{4}{63}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x+1|\right) - \frac{4}{63}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{x}{15} - \frac{4\,\ln\left(x+\frac{2}{3}\right)}{63} + \frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

```
[In] int(1/(13/x + 2/x^2 + 15),x)
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 646, 31}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = \frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

```
[In] int(1/(15+2/x^2+13/x)/x,x,method=_RETURNVERBOSE)
[Out] -1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x,x)
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{1}{35}\,\log\left(|5\,x+1|\right)+\frac{2}{21}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

```
[In] int(1/(x*(13/x + 2/x^2 + 15)),x)
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)$$

```
[Out] 1/7*ln(5+1/x)-1/7*ln(3+2/x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1366, 630, 31}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{15+13x+2x^2}\,dx, x, \frac{1}{x}\right)\\ &= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3+2x}\,dx, x, \frac{1}{x}\right)\right)+\frac{2}{7}\text{Subst}\left(\int \frac{1}{10+2x}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18

```
[In] int(1/(15+2/x^2+13/x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x**2,x)
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

```
[In] int(1/(x^2*(13/x + 2/x^2 + 15)),x)
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 719, 29, 646, 31}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x(2+13x+15x^2)}\,dx \\
&= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx \\
&= \frac{\log(x)}{2} + \frac{45}{14}\int \frac{1}{10+15x}\,dx - \frac{75}{7}\int \frac{1}{3+15x}\,dx \\
&= \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

```
[In] int(1/(15+2/x^2+13/x)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x**3,x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = -\frac{5}{7}\,\log\left(5\,x+1\right)+\frac{3}{14}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = -\frac{5}{7}\,\log\left(|5\,x+1|\right)+\frac{3}{14}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = \frac{3\,\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

```
[In] int(1/(x^3*(13/x + 2/x^2 + 15)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 723, 814}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^2\,(2+13x+15x^2)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\,(2+13x+15x^2)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\ &= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln(x+\frac{1}{5})x+9\ln(x+\frac{2}{3})x+14}{28x}$	27

```
[In] int(1/(15+2/x^2+13/x)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = -\frac{1}{2\,x} + \frac{25}{7}\,\log\left(|5\,x+1|\right) - \frac{9}{28}\,\log\left(|3\,x+2|\right) - \frac{13}{4}\,\log\left(|x|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = \frac{25\,\ln\left(x+\frac{1}{5}\right)}{7} - \frac{9\,\ln\left(x+\frac{2}{3}\right)}{28} - \frac{13\,\ln\left(x\right)}{4} - \frac{1}{2\,x}$$

```
[In] int(1/(x^4*(13/x + 2/x^2 + 15)),x)
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 723, 814}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3(2+13x+15x^2)}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2(2+13x+15x^2)}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx \\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^{2}}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^{2}}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
parallelrisch	$\frac{973\ln(x)x^{2}-1000\ln(x+\frac{1}{5})x^{2}+27\ln(x+\frac{2}{3})x^{2}-14+182x}{56x^{2}}$	36
norman	$\frac{-\frac{1}{4}x^{2}+\frac{13}{4}x^{3}}{x^{4}}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	37

```
[In] int(1/(15+2/x^2+13/x)/x^5,x,method=_RETURNVERBOSE)
[Out] (13/4*x-1/4)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = -\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x**5,x)
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log\left(5\,x+1\right) + \frac{27}{56}\log\left(3\,x+2\right) + \frac{139}{8}\log\left(x\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log\left(|5\,x+1|\right) + \frac{27}{56}\log\left(|3\,x+2|\right) + \frac{139}{8}\log\left(|x|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{27\,\ln\left(x+\frac{2}{3}\right)}{56} - \frac{125\,\ln\left(x+\frac{1}{5}\right)}{7} + \frac{139\,\ln\left(x\right)}{8} + \frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

```
[In] int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 723, 814}

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^4 (2+13x+15x^2)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3 (2+13x+15x^2)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx \\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln\left(x+\frac{1}{5}\right)x^3+243\ln\left(x+\frac{2}{3}\right)x^3+56+5838x^2-546x}{336x^3}$	41
norman	$\frac{-\frac{1}{6}x^2+\frac{13}{8}x^3-\frac{139}{8}x^4}{x^5}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	42

```
[In] int(1/(15+2/x^2+13/x)/x^6,x,method=_RETURNVERBOSE)
[Out] (-139/8*x^2+13/8*x-1/6)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

```
[In] integrate(1/(15+2/x**2+13/x)/x**6,x)
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right)-\frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

```
[In] int(1/(x^6*(13/x + 2/x^2 + 15)),x)
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5}{2}a^{3/2}b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) + \frac{5(b^4-24ab^2c-48a^2c^2)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{128c^{3/2}}$$

```
[Out] -5/24*(a+c/x^2+b/x)^(3/2)*(7*b+6*c/x)+(a+c/x^2+b/x)^(5/2)*x+5/2*a^(3/2)*b*a
rctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))+5/128*(-48*a^2*c^2-24*a*b
^2*c+b^4)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(3/2)-5/64*(
b*(44*a*c+b^2)+2*c*(12*a*c+b^2)/x)*(a+c/x^2+b/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

$= \{1356, 746, 828, 857, 635, 212, 738\}$

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \frac{5}{2}a^{3/2}b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right) + \frac{5(-48a^2c^2-24ab^2c+b^4)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}} - \frac{5\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{64c} + x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} - \frac{5}{24}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}$$

```
[In] Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
```

```
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x-\frac{5}{2}\text{Subst}\left(\int \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x\\
&\quad+\frac{5\text{Subst}\left(\int \frac{\left(-8abc-c\left(b^2+12ac\right)x\right)\sqrt{a+bx+cx^2}}{x}\,dx, x, \frac{1}{x}\right)}{16c}
\end{aligned}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x-\frac{5\text{Subst}\left(\int \frac{32a^2bc^2-\frac{1}{2}c(b^4-24ab^2c-48a^2c^2)x}{x\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{64c^2}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x-\frac{1}{2}(5a^2b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right) + \frac{(5(b^4-24ab^2c-48a^2c^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{128c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x+(5a^2b)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) + \frac{(5(b^4-24ab^2c-48a^2c^2))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{64c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x + \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)+\frac{5(b^4-24ab^2c-48a^2c^2)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{128c^{3/2}}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\frac{\sqrt{a+\frac{c+bx}{x^2}}\left(15(b^4-24ab^2c-48a^2c^2)\,x^4\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{c+x(b+ax)}(48c^3+15b^3x^3\right.\right.}{192c^{3/2}x^3\sqrt{}}$$

```
[In] Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

[Out] -1/192*(Sqrt[a + (c + b*x)/x^2]*(15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*Arc
Tanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[c + x*(b
+ a*x)]*(48*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 +
278*a*b*x - 96*a^2*x^2)) + 480*a^(3/2)*b*c*x^4*Log[b + 2*a*x - 2*Sqrt[a]*Sq
rt[c + x*(b + a*x)]])))/(c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(556abcx^3+15b^3x^3+216ac^2x^2+118b^2cx^2+136bc^2x+48c^3)\sqrt{\frac{ax^2+bx+c}{x^2}}}{192x^3c}+\frac{\left(384cba^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx+c}\right)+128a^3c\left(\frac{\sqrt{a x}}{}\right)\right)}{}$
default	$\frac{\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{5}{2}}x\left(6a^{\frac{3}{2}}(ax^2+bx+c)^{\frac{7}{2}}b^3x^3-6a^{\frac{3}{2}}(ax^2+bx+c)^{\frac{5}{2}}b^4x^4+600a^{\frac{7}{2}}\sqrt{ax^2+bx+c}\,bc^3x^5-30a^{\frac{5}{2}}\sqrt{ax^2+bx+c}\,b^3c^2x^5+960\ln\left(\right)\right)}{}$

[In] int((a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/192*(556*a*b*c*x^3+15*b^3*x^3+216*a*c^2*x^2+118*b^2*c*x^2+136*b*c^2*x+48
c^3)/x^3/c((a*x^2+b*x+c)/x^2)^(1/2)+1/128/c*(384*c*b*a^(3/2)*ln((1/2*b+a*
x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))+128*a^3*c*(1/a*(a*x^2+b*x+c)^(1/2)-1/2*b/a^
(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2)))-(240*a^2*c^2+120*a*b^2*c
-5*b^4)/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x))*((a*x^2+b*x+
c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.70

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \left[\frac{960\, a^{\frac{3}{2}} b c^2 x^3 \log\left(-8\, a^2 x^2 - 8\, abx - b^2 - 4\, ac - 4\,(2\, ax^2 + bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right) - 15\,(b^4 - 24 \right.$$

$$-\frac{1920\sqrt{-a}abc^2x^3\arctan\left(\frac{(2\,ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^2x^2+abx+ac)}\right) + 15\,(b^4 - 24\,ab^2c - 48\,a^2c^2)\sqrt{c}x^3\log\left(-\frac{8\,bcx+(b^2+4\,ac)x^2}{}\right.}{}$$

$$-\frac{960\sqrt{-a}abc^2x^3\arctan\left(\frac{(2\,ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^2x^2+abx+ac)}\right) + 15\,(b^4 - 24\,ab^2c - 48\,a^2c^2)\sqrt{-c}x^3\arctan\left(\frac{(bx^2+2\,cx)\sqrt{-c}}{2\,(acx^2+b}\right.}{384}$$

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2
*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c -
 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x
^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4
- 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108
*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)
/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt(
(a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 -
(15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2
+ b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8
*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2
)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192
*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*
b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(9
60*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2
)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (1
5*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c^2*x^3)]
```

Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+c/x**2+b/x)**(5/2),x)
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \text{Timed out}$$

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} dx$$

```
[In] int((a + b/x + c/x^2)^(5/2),x)
[Out] int((a + b/x + c/x^2)^(5/2), x)
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{8\sqrt{c}}$$

```
[Out] (a+c/x^2+b/x)^(3/2)*x+3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/
2))*a^(1/2)-3/8*(4*a*c+b^2)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/
2))/c^(1/2)-3/4*(3*b+2*c/x)*(a+c/x^2+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1356, 746, 828, 857, 635, 212, 738}

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}} + \frac{3}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

```
[In] Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{3}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x+\frac{3\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{8c}$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right) - \frac{1}{8}\left(3\left(b^2+4ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + (3ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - \frac{1}{4}\left(3\left(b^2+4ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}x$$
$$+\frac{3}{2}\sqrt{a}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)-\frac{3(b^2+4ac)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{8\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \frac{\sqrt{a+\frac{c+bx}{x^2}}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{c}\left((2c+x(5b-4ax))\sqrt{c+x(b+ax)}\right.\right.}{4\sqrt{c}x\sqrt{c+x(b+ax)}}$$

```
[In] Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[a]*x - Sqrt[c +
 x*(b + a*x)])/Sqrt[c]] - Sqrt[c]*((2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b +
a*x)] + 6*Sqrt[a]*b*x^2*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])))
/(4*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{(5bx+2c)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{4x}+\frac{\left(a\sqrt{a\,x^2+bx+c}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)}{2}-\frac{3\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)a}{2}-\frac{3\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{}}{x}\right.}{8\sqrt{c}}\right.}{\sqrt{a\,x^2+bx+c}}$
default	$-\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)x^2-2a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}b\,x^3-4a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}c\,x^2-6a^{\frac{5}{2}}\sqrt{a\,x^2+bx+c}\,bc\,x\right.}{}$

```
[In] int((a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(5*b*x+2*c)/x*((a*x^2+b*x+c)/x^2)^(1/2)+(a*(a*x^2+b*x+c)^(1/2)+3/2*a^(
1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))-3/2*c^(1/2)*ln((2*c+b*x+
2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*a-3/8/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^
2+b*x+c)^(1/2))/x)*b^2)*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.89

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx = \left[\frac{12\sqrt{a}bcx\log\left(-8a^2x^2-8abx-b^2-4ac-4(2ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+3(b^2+4ac)\sqrt{\ldots}}{16c\ldots}\right.$$

$$-\frac{24\sqrt{-a}bcx\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)-3(b^2+4ac)\sqrt{c}x\log\left(-\frac{8bcx+(b^2+4ac)x^2+8c^2-4(bx^2+2cx)\sqrt{c}\sqrt{\frac{ax\ldots}{}}}{x^2}\ldots\right.}{16cx}$$

$$-\frac{12\sqrt{-a}bcx\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)-3(b^2+4ac)\sqrt{-c}x\arctan\left(\frac{(bx^2+2cx)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(acx^2+bcx+c^2)}\right)-2(4a\ldots}{8cx}$$

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2
 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*lo
g(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a
*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b*x)*s
qrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)
*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2
)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt(
(a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x -
2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b*c*x*arctan(1
/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x +
a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt
((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x -
 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx=\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(3/2),x)
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx=\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx=\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\,dx$$

```
[In] int((a + b/x + c/x^2)^(3/2),x)
[Out] int((a + b/x + c/x^2)^(3/2), x)
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x+\frac{b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}}-\sqrt{c}\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

```
[Out] 1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(1/2)-arctanh(1/
2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)+x*(a+c/x^2+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1356, 746, 857, 635, 212, 738}

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\frac{b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&\quad - c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)\\
&\quad - (2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)\\
&= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}} - \sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$
\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx
= \frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(2\sqrt{a}\sqrt{c+x(b+ax)}+4\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-b\log\left(b+2ax-2\sqrt{a}\sqrt{c+x(b\right.\right.}{2\sqrt{a}\sqrt{c+x(b+ax)}}
$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(2*Sqrt[a]*Sqrt[c + x*(b + a*x)] + 4*Sqrt[a]*Sqr
t[c]*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - b*Log[b + 2*a*x
 - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+b x +c}{x^{2}}}\, x \left(-2 \sqrt{c}\, \ln \left(\frac{2 c +b x +2 \sqrt{c}\, \sqrt{a\,x^{2}+b x +c}}{x}\right) \sqrt{a}+b \ln \left(\frac{2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right)+2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}\right)}{2 \sqrt{a\,x^{2}+b x +c}\, \sqrt{a}}$	121

```
[In] int((a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*
x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 590, normalized size of antiderivative = 5.62

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx = \left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac-4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+2\,a\sqrt{c}\log\left(-\right.}{4\,a}\right.$$

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) +
2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*s
qrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x +
 c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x
 + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2
))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
 sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2)
- sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2
)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-
c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(1/2),x)
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x + c/x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=x\sqrt{\frac{1}{x^2}}\,\sqrt{a\,x^2+b\,x+c}-\sqrt{c}\,x\,\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^2+b\,x+c}+b\,x}{x}\right)\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^2+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

```
[In] int((a + b/x + c/x^2)^(1/2),x)
[Out] x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(c
 + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(c
+ b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}-\frac{b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(3/2)+x*(a+c/x^
2+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1356, 744, 738, 212}

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

```
[In] Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a} - \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a}\\
&= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a} - \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \frac{2\sqrt{a}(c+x(b+ax))-b\sqrt{c+x(b+ax)}\operatorname{arctanh}\left(\frac{b+2ax}{2\sqrt{a}\sqrt{c+x(b+ax)}}\right)}{2a^{3/2}x\sqrt{a+\frac{c+bx}{x^2}}}$$

```
[In] Integrate[1/Sqrt[a + c/x^2 + b/x],x]
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/
(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\sqrt{a\,x^2+bx+c}\left(2\sqrt{a\,x^2+bx+c}\,a^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right)}{2\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x\,a^{\frac{5}{2}}}$	88
risch	$\frac{a\,x^2+bx+c}{a\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)\sqrt{a\,x^2+bx+c}}{2a^{\frac{3}{2}}\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}$	97

```
[In] int(1/(a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*(a*x^2+b
*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}}{}\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

[In] integrate(1/(a+c/x**2+b/x)**(1/2),x)

[Out] Integral(1/sqrt(a + b/x + c/x**2), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a + b/x + c/x^2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\text{Exception raised: TypeError}$$

[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\frac{x\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2\,x}}{\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\,a^{3/2}}$$

```
[In] int(1/(a + b/x + c/x^2)^(1/2),x)
[Out] (x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
c/x^2)^(1/2))))/(2*a^(3/2))
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\frac{(3b^2-8ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a^2(b^2-4ac)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a(b^2-4ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

```
[Out] -3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(5/2)-2*(b^2-2*
a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(1/2)+(-8*a*c+3*b^2)*x*(a+c/x^2+b
/x)^(1/2)/a^2/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1356, 754, 820, 738, 212}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=-\frac{3b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2(b^2-4ac)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

```
[In] Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}+\frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{2a^2}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a^2}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a^2\left(b^2-4ac\right)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{a}(-3b^3x+10abcx+4ac(2c+ax^2)-b^2(3c+ax^2))-3b(b^2-4ac)\sqrt{c+x(b+ax)}\log\left(a^2\left(b+2ax-\right.\right.}{2a^{5/2}\left(b^2-4ac\right)x\sqrt{a+\frac{c+bx}{x^2}}}$$

```
[In] Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] -1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a
*x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*Log[a^2*(b + 2*a*x - 2*Sqr
t[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x
^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

method	result
default	$\frac{(a\,x^2+bx+c)\left(8a^{\frac{7}{2}}c\,x^2-2a^{\frac{5}{2}}b^2x^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x+16a^{\frac{5}{2}}c^2-6a^{\frac{3}{2}}b^2c-12\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{a\,x^2+bx+c}\,a^2bc+3\ln\left(\frac{2\sqrt{}}{}\right.}{2a^{\frac{7}{2}}\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x^3(4ac-b^2)}$
risch	$\frac{a\,x^2+bx+c}{a^2\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}+\frac{\left(\frac{3bx}{2a^2\sqrt{a\,x^2+bx+c}}-\frac{b^2}{4a^3\sqrt{a\,x^2+bx+c}}-\frac{b^3x}{2a^2\left(4ac-b^2\right)\sqrt{a\,x^2+bx+c}}-\frac{b^4}{4a^3\left(4ac-b^2\right)\sqrt{a\,x^2+bx+c}}-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+b}\right.}{2a^{\frac{5}{2}}}\right.}{\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}$

```
[In] int(1/(a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*c*x^2-2*a^(5/2)*b^2*x^2+20*a^(5/2)*b*c
*x-6*a^(3/2)*b^3*x+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*(a*x^2+b*x+c
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(2*(
a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)/((a
*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.50

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\left[\frac{3\,(b^3c-4\,abc^2+(ab^3-4\,a^2bc)x^2+(b^4-4\,ab^2c)x)\sqrt{a}\log\left(-8\,a^2x^2-8\,abx-b^2-\right.}{4\,(a^3b^2c-4\,a}\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)
*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)
*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*
a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/
2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sq
rt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2
*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*
x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c -
4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx= -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(a^{\frac{5}{2}}b^2-4\,a^{\frac{7}{2}}c\right)} +\frac{\left(\frac{(ab^2-4\,a^2c)x}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}\right)x+\frac{3\,b^2c-8\,ac^2}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}}{\sqrt{ax^2+bx+c}} +\frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
[Out] -1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a
)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(a^(5/2)*b
^2 - 4*a^(7/2)*c) + (((a*b^2 - 4*a^2*c)*x/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x))
 + (3*b^3 - 10*a*b*c)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))*x + (3*b^2*c - 8*a
*c^2)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))/sqrt(a*x^2 + b*x + c) + 3/2*b*log(
abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) + b))/(a^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/x + c/x^2)^(3/2),x)
[Out] int(1/(a + b/x + c/x^2)^(3/2), x)
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 220

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx = \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3(b^2-4ac)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a(b^2-4ac)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2(b^2-4ac)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{5b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{7/2}}$$

```
[Out] -2/3*(b^2-2*a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(3/2)-5/2*b*arctanh(1
/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(7/2)-2/3*(5*b^4-32*a*b^2*c+32*
a^2*c^2+b*c*(-28*a*c+5*b^2)/x)*x/a^2/(-4*a*c+b^2)^2/(a+c/x^2+b/x)^(1/2)+1/3
*(128*a^2*c^2-100*a*b^2*c+15*b^4)*x*(a+c/x^2+b/x)^(1/2)/a^3/(-4*a*c+b^2)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {1356, 754, 836, 820, 738, 212}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{5b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}} - \frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}} + \frac{x(128a^2c^2-100ab^2c+15b^4)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2} - \frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

```
[In] Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2 (a+bx+cx^2)^{5/2}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a(b^2-4ac)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(-5b^2+16ac)-3bcx}{x^2(a+bx+cx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a(b^2-4ac)} \\
&= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a(b^2-4ac)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2(b^2-4ac)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{\frac{1}{4}(15b^4-100ab^2c+128a^2c^2)+\frac{1}{2}bc(5b^2-28ac)x}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{3a^2(b^2-4ac)^2}
\end{aligned}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{(5b)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{2a^3}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{(5b)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a^3}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5b^2-28ac)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{7/2}}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12

$$\int\frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{a}(c+x(b+ax))\left(15b^6x^2+8a^2bc^2x(39c+32ax^2)-2ab^3cx(105c+74ax^2)+10b^5(3cx+2ax^3)+3b^4(5c^2-30acx^2+a^2x^{\cdots}\right.}{(b^2-4ac)^2}}{\cdots}$$

```
[In] Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((2*Sqrt[a]*(c + x*(b + a*x))*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2)
 - 2*a*b^3*c*x*(105*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2
 - 30*a*c*x^2 + a^2*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*
a*b^2*c*(25*c^2 - 12*a*c*x^2 + 6*a^2*x^4)))/(b^2 - 4*a*c)^2 + 15*b*(c + x*(
b + a*x))^(5/2)*Log[a^3*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])/(6*
a^(7/2)*x^5*(a + (c + b*x)/x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.71

method	result
default	$-\frac{(a x^2+bx+c)\left(-96a^{\frac{13}{2}}c^2x^4+48a^{\frac{11}{2}}b^2cx^4-512a^{\frac{11}{2}}bc^2x^3-6a^{\frac{9}{2}}b^4x^4-384a^{\frac{11}{2}}c^3x^2+296a^{\frac{9}{2}}b^3cx^3-96a^{\frac{9}{2}}b^2c^2x^2-40a^{\frac{7}{2}}b^5x^3-624a\right.}{}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(a*x^2+b*x+c)*(-96*a^(13/2)*c^2*x^4+48*a^(11/2)*b^2*c*x^4-512*a^(11/2)
*b*c^2*x^3-6*a^(9/2)*b^4*x^4-384*a^(11/2)*c^3*x^2+296*a^(9/2)*b^3*c*x^3-96*
a^(9/2)*b^2*c^2*x^2-40*a^(7/2)*b^5*x^3-624*a^(9/2)*b*c^3*x+180*a^(7/2)*b^4*
c*x^2-256*a^(9/2)*c^4+420*a^(7/2)*b^3*c^2*x-30*a^(5/2)*b^6*x^2+200*a^(7/2)*
b^2*c^3-60*a^(5/2)*b^5*c*x+240*(a*x^2+b*x+c)^(3/2)*ln(1/2*(2*(a*x^2+b*x+c)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b*c^2-120*(a*x^2+b*x+c)^(3/2)*ln(1/2*(2
*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^3*c+15*(a*x^2+b*x+c)^(
3/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^5-30*a^(
5/2)*b^4*c^2)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(198) = 396.

Time = 0.41 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.91

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{15\,(b^5c^2-8\,ab^3c^3+16\,a^2bc^4+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x^4+2\,(ab^6-8\,a^2b^4c+}{}\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6
*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*
sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^
5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b
^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 5
2*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((
a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4
 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*
```

```
c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*
x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*
a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35
*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^
3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+c/x**2+b/x)**(5/2),x)
[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(198) = 396$.

Time = 0.37 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.27

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx =$$
$$-\frac{\left(15\,b^5\sqrt{c}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c^{\frac{3}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^{\frac{5}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4c\right.}{6\left(a^{\frac{7}{2}}b^4\sqrt{c}-8\,a^{\frac{9}{2}}b^2c^{\frac{3}{2}}+16\,a^{\frac{11}{2}}c^{\frac{5}{2}}\right)}$$
$$+\frac{\left(\left(\left(\frac{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2)x}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}+\frac{4\,(5\,ab^5-37\,a^2b^3c+64\,a^3bc^2)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{3\,(5\,b^6-30\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x}\right.\right.}{3\,(ax^2+bx+c)^{\frac{3}{2}}}$$
$$+\frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b^5*sqrt(c)*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c^(3/2)*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) + 240*a^2*b*c^(5/2)*log(abs(b - 2*sqrt(a)*sqr
t(c))) + 30*sqrt(a)*b^4*c - 200*a^(3/2)*b^2*c^2 + 256*a^(5/2)*c^3)*sgn(x)/(
a^(7/2)*b^4*sqrt(c) - 8*a^(9/2)*b^2*c^(3/2) + 16*a^(11/2)*c^(5/2)) + 1/3*((
((3*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*x/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sg
n(x) + 16*a^5*c^2*sgn(x)) + 4*(5*a*b^5 - 37*a^2*b^3*c + 64*a^3*b*c^2)/(a^3*
b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + 3*(5*b^6 - 30*a*b
^4*c + 16*a^2*b^2*c^2 + 64*a^3*c^3)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) +
16*a^5*c^2*sgn(x)))*x + 6*(5*b^5*c - 35*a*b^3*c^2 + 52*a^2*b*c^3)/(a^3*b^4*
sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + (15*b^4*c^2 - 100*a*b
^2*c^3 + 128*a^2*c^4)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn
(x)))/(a*x^2 + b*x + c)^(3/2) + 5/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b
*x + c))*sqrt(a) + b))/(a^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(5/2), x)
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}x}{a+\frac{b}{x}}-\frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\log\left(\frac{1}{x}\right)}{a+\frac{b}{x}}$$

```
[Out] a*x*(a^2+b^2/x^2+2*a*b/x)^(1/2)/(a+b/x)-b*ln(1/x)*(a^2+b^2/x^2+2*a*b/x)^(1/
2)/(a+b/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1356, 660, 45}

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

```
[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^2}\,dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}} \\
&= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right)\,dx, x, \frac{1}{x}\right)}{ab+\frac{b^2}{x}} \\
&= \frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}} + \frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\log(x)}{a+\frac{b}{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = \frac{x\sqrt{\frac{(b+ax)^2}{x^2}}(ax+b\log(x))}{b+ax}$$

```
[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{\frac{a^{2} x^{2}+2 a b x+b^{2}}{x^{2}}}\, x \left(a x +b \ln \left(x \right)\right)}{a x +b}$	40
risch	$\frac{\sqrt{\frac{\left(a x +b \right)^{2}}{x^{2}}}\, x^{2} a}{a x +b}+\frac{\sqrt{\frac{\left(a x +b \right)^{2}}{x^{2}}}\, x b \ln \left(x \right)}{a x +b}$	52

```
[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=ax+b\log\left(x\right)$$

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
[Out] a*x + b*log(x)
```

Sympy [F]

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\int \sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\, dx = ax + b\log{(x)}$$

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
[Out] a*x + b*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\, dx = ax\mathrm{sgn}\left(ax^2 + bx\right) + b\log{(|x|)}\,\mathrm{sgn}\left(ax^2 + bx\right)$$

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\, dx = x\,\sqrt{\frac{1}{x^2}}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} - x\,\ln\left(\frac{2\,\sqrt{b^2}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} + 2\,b^2 + 2\,a\,b\,x}{x}\right)\,\sqrt{b^2}\,\sqrt{\frac{1}{x^2}} + \frac{a\,b\,x\,\ln\left(\frac{a\,b + \sqrt{a^2}\,\sqrt{a^2\,x^2 + 2\,a\,b\,x + b^2} + a^2\,x}{\sqrt{a^2}}\right)\,\sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}$$

```
[In] int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
[Out] x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^2
 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/2)
 + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x)/(
a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1354, 1136, 1180, 211}

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[In] Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\_R^2b-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{x}{c}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

```
[In] int(1/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-2\,(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\,a\right.\right.}{}$$

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3a\right.\right.\right.$$
$$+\frac{x}{c}$$

```
[In] integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\int \frac{1}{c+\frac{b}{x^2}+\frac{a}{x^4}}\,dx$$

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(143)=286$.

Time = 0.79 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx = \text{Too large to display}$$

```
[In] int(1/(c + a/x^4 + b/x^2),x)
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
```

```
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 631

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
```

```
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1354, 1381, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{x}{c}$$

```
[In] Int[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\text{integral} = \int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$
\begin{aligned}
&= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}
$$

$$= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

```
[In] Integrate[(c + a/x^6 + b/x^3)^(-1),x]
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59

```
[In] int(1/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. 2(495) = 990.

Time = 0.39 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
[Out] -1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*s
qrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^
8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1
/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*
c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2
 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5*c^4 -
 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*
c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b
^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*(sqrt(-3
)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*
b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a*b^4 - 4*a^2*b
^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3
*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3) - (b^5*c^4 -
```

8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) + (1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3) + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3) + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 2*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - 6*x)/c

Sympy [A] (verification not implemented)

Time = 54.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$
$$=\text{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5c\right.\right.$$
$$+\frac{x}{c}$$

```
[In] integrate(1/(c+a/x**6+b/x**3),x)
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4))) +
 x/c
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \text{Too large to display}$$

```
[In] int(1/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
```

```
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 376

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1354, 1381, 1436, 218, 214, 211}

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{x}{c}$$

```
[In] Int[(c + a/x^8 + b/x^4)^(-1),x]
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^8}{a+bx^4+cx^8}\,dx \\
&= \frac{x}{c}-\frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}
\end{aligned}$$

$$= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= \frac{x}{c} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

```
[In] Integrate[(c + a/x^8 + b/x^4)^(-1),x]
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59

```
[In] int(1/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. 2(296) = 592.

Time = 0.46 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*
x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqr
t(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 +
 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^
4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^
3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
```

^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^

```
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))) + c*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((a*b^4 - 3*a^2*
b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b
^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c
^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(-sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a
^2*c^7)))*log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13
*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-sqrt(1/2)*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
) + 4*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(c+a/x**8+b/x**4),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
[Out] integrate(1/(c + b/x^4 + a/x^8), x)
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

```
[In] int(1/(c + a/x^8 + b/x^4),x)
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
```

^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)
(-(b^9 + b^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16
*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c
^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4
*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*
b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4
*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/

(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i - 2*atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120 *a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2

*c^8)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - 2*atan(((((16*(a^3
*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
c^2(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)
/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*
a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^
2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
+ 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (((16*(a^3*b^6 - 4*a^6
*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^
(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/
c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*
a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4
*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*
c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c
^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 -
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*
c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c +
13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1

```
3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6
 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a
*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))
^(1/4)*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*
a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3
*b^2*c^8)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + x/c
```

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x} dx = 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\,\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

```
[Out] -2*arctanh(1/2*(2*a+b*x^(1/2))/a^(1/2)/(a+c*x+b*x^(1/2))^(1/2))*a^(1/2)+b*a
rctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+c*x+b*x^(1/2))^(1/2))/c^(1/2)+2*(a+c*
x+b*x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1371, 748, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x} dx = -2\sqrt{a}\,\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}} + 2\sqrt{a+b\sqrt{x}+cx}$$

```
[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{a+b\sqrt{x}+cx} - \text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{a+b\sqrt{x}+cx} + (2a)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right) + b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{a+b\sqrt{x}+cx} - (4a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right) + (2b)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right)$$
$$= 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\sqrt{a+b\sqrt{x}+cx} + 4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}-\sqrt{a+b\sqrt{x}+cx}}{\sqrt{a}}\right) - \frac{b\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}$$

```
[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[x] - Sqrt[a +
 b*Sqrt[x] + c*x])/Sqrt[a]] - (b*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a + b
*Sqrt[x] + c*x]])/Sqrt[c]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$2\sqrt{a+cx+b\sqrt{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$	84
default	$2\sqrt{a+cx+b\sqrt{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}\right)$	84

```
[In] int((a+c*x+b*x^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^
(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2
))/x^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

```
[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

```
[In] int((a + c*x + b*x^(1/2))^(1/2)/x,x)
[Out] int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx=-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

```
[Out] -1/160*b*(b+2*c*x^(1/2))^5/c^4+1/192*(b+2*c*x^(1/2))^6/c^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {28, 196, 45}

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx=\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

```
[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -1/160*(b*(b + 2*c*Sqrt[x])^5)/c^4 + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4\,dx}{c^2}\\
&= \frac{2\text{Subst}\left(\int x\left(\frac{b}{2}+cx\right)^4\,dx,x,\sqrt{x}\right)}{c^2}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{b\left(\frac{b}{2}+cx\right)^4}{2c}+\frac{\left(\frac{b}{2}+cx\right)^5}{c}\right)\,dx,x,\sqrt{x}\right)}{c^2}\\
&= -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2\,dx=\frac{15b^4x+80b^3cx^{3/2}+180b^2c^2x^2+192bc^3x^{5/2}+80c^4x^3}{240c^2}$$

```
[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
[Out] (15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^4
*x^3)/(240*c^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\frac{8c^4x^3}{3}+\frac{32b\,c^3x^{\frac{5}{2}}}{5}+6b^2c^2x^2+\frac{8b^3c\,x^{\frac{3}{2}}}{3}+\frac{b^4x}{2}}{8c^2}$	50
default	$\frac{b^2x^2}{2}+\frac{b\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)}{2c}+\frac{\left(\frac{b^2}{c}+4cx\right)^3}{192c}$	52
trager	$\frac{\frac{\left(16c^4x^2+36b^2x\,c^2+16c^4x+3b^4+36b^2c^2+16c^4\right)(x-1)}{3}+\frac{16bc\,x^{\frac{3}{2}}\left(12c^2x+5b^2\right)}{15}}{16c^2}$	73

[In] int((1/4/c*b^2+c*x+b*x^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] 1/8/c^2*(8/3*c^4*x^3+32/5*b*c^3*x^(5/2)+6*b^2*c^2*x^2+8/3*b^3*c*x^(3/2)+1/2*b^4*x)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\,(12\,bc^3x^2+5\,b^3cx)\sqrt{x}}{240\,c^2}$$

[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="fricas")

[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c*x)*sqrt(x))/c^2

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

[In] integrate((1/4*b**2/c+c*x+b*x**(1/2))**2,x)

[Out] b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 + c**2*x**3/3

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{1}{3}c^2x^3+\frac{4}{5}bcx^{\frac{5}{2}}+\frac{1}{2}b^2x^2+\frac{b^4x}{16c^2}+\frac{\left(3cx^2+4bx^{\frac{3}{2}}\right)b^2}{12c}$$

```
[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^
2 + 4*b*x^(3/2))*b^2/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{80c^4x^3+192bc^3x^{\frac{5}{2}}+180b^2c^2x^2+80b^3cx^{\frac{3}{2}}+15b^4x}{240c^2}$$

```
[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
+ 15*b^4*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{3b^2x^2}{4}+\frac{c^2x^3}{3}+\frac{b^4x}{16c^2}+\frac{b^3x^{3/2}}{3c}+\frac{4bcx^{5/2}}{5}$$

```
[In] int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
```

```
[Out] (3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*b
*c*x^(5/2))/5
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a(a+b\sqrt{x})\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[Out] -2*a*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b^2/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)+2*(a
^2+b^2*x+2*a*b*x^(1/2))^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1355, 654, 622, 31}

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a(a+b\sqrt{x})\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{\left(2a\left(a+b\sqrt{x}\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\\
&= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$
\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}
$$

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$-\frac{2(a+b\sqrt{x})(a\ln(a+b\sqrt{x})-b\sqrt{x})}{\sqrt{(a+b\sqrt{x})^2}\, b^2}$	41
default	$\frac{(a+b\sqrt{x})(2b\sqrt{x}+a\ln(b\sqrt{x}-a)-a\ln(a+b\sqrt{x})-a\ln(b^2x-a^2))}{\sqrt{a^2+b^2x+2ab\sqrt{x}}\, b^2}$	75

```
[In] int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(a+b*x^(1/2))*(a*ln(a+b*x^(1/2))-b*x^(1/2))/((a+b*x^(1/2))^2)^(1/2)/b^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\left(\begin{cases} -\frac{a\left(\frac{a}{b}+\sqrt{x}\right)\log\left(\frac{a}{b}+\sqrt{x}\right)}{b\sqrt{b^2\left(\frac{a}{b}+\sqrt{x}\right)^2}}+\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} & \text{for } b^2\neq 0 \\ \frac{-a^2\sqrt{a^2+2ab\sqrt{x}}+\frac{\left(a^2+2ab\sqrt{x}\right)^{\frac{3}{2}}}{3}}{2a^2b^2} & \text{for } ab\neq 0 \\ \frac{x}{2\sqrt{a^2}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
[Out] 2*Piecewise((-a*(a/b + sqrt(x))*log(a/b + sqrt(x))/(b*sqrt(b**2*(a/b + sqrt
(x))**2)) + sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x)/b**2, Ne(b**2, 0)), ((-a**2
*sqrt(a**2 + 2*a*b*sqrt(x)) + (a**2 + 2*a*b*sqrt(x))**(3/2)/3)/(2*a**2*b**2
), Ne(a*b, 0)), (x/(2*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\mathrm{sgn}(a)\,\mathrm{sgn}(b)+|a|\right|\right)}{b^2}+\frac{2\,\sqrt{b^2x}}{b^2\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(b)}$$

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
[Out] -2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)/
(b^2*sgn(a)*sgn(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)
[Out] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)
```

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{10b^3}$$

```
[Out] 3/8*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-2/3*a*(a+
b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/10*(a+b*x^(1/3))^9
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{7/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^7\,dx,x,\sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^7}{b^2}-\frac{2a\left(ab+b^2x\right)^8}{b^3}+\frac{\left(ab+b^2x\right)^9}{b^4}\right)dx,x,\sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3}\\
&\quad+\frac{3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{10b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}\left(120a^7x+630a^6bx^{4/3}+1512a^5b^2x^{5/3}+2100a^4b^3x^2+1800a^3b^4x^{7/3}+945\right.}{120\left(a+b\sqrt[3]{x}\right)^7}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (((a + b*x^(1/3))^2)^(7/2)*(120*a^7*x + 630*a^6*b*x^(4/3) + 1512*a^5*b^2*x^
(5/3) + 2100*a^4*b^3*x^2 + 1800*a^3*b^4*x^(7/3) + 945*a^2*b^5*x^(8/3) + 280
*a*b^6*x^3 + 36*b^7*x^(10/3)))/(120*(a + b*x^(1/3))^7)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{7}{2}}x\left(36b^7x^{\frac{7}{3}}+280a\,b^6x^2+945a^2b^5x^{\frac{5}{3}}+1800b^4a^3x^{\frac{4}{3}}+2100a^4b^3x+1512b^2a^5x^{\frac{2}{3}}+630a^6b\,x^{\frac{1}{3}}+120a^7\right)}{120\left(a+b\,x^{\frac{1}{3}}\right)^7}$
default	$\frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}\left(36b^7x^{\frac{10}{3}}+945a^2b^5x^{\frac{8}{3}}+1800b^4a^3x^{\frac{7}{3}}+1512b^2a^5x^{\frac{5}{3}}+630a^6b\,x^{\frac{4}{3}}+280a\,b^6x^3+2100a^4b^3x^2+120a^7\right.}{120\left(a+b\,x^{\frac{1}{3}}\right)^7}$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*((a+b*x^(1/3))^2)^(7/2)*x*(36*b^7*x^(7/3)+280*a*b^6*x^2+945*a^2*b^5*x
^(5/3)+1800*b^4*a^3*x^(4/3)+2100*a^4*b^3*x+1512*b^2*a^5*x^(2/3)+630*a^6*b*x
^(1/3)+120*a^7)/(a+b*x^(1/3))^7
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{7}{3}\,ab^6x^3+\frac{35}{2}\,a^4b^3x^2+a^7x+\frac{63}{40}\left(5\,a^2b^5x^2+8\,a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\left(2\,b^7x^3+100\,a^3b^4x^2+35\,a^6bx\right)x^{\frac{1}{3}}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2
*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.70

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^9}{360b^3}-\frac{a^8\sqrt[3]{x}}{360b^2}+\frac{a^7x^{\frac{2}{3}}}{360b}+\frac{119a^6x}{360}+\frac{511a^5bx^{\frac{4}{3}}}{360}+\frac{1001a^4b^2x^{\frac{5}{3}}}{360}+\frac{1099a^3b\cdots}{360}\right.\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{13}{2}}}{13}}{4a^3b^3}\\ \frac{x\left(a^2\right)^{\frac{7}{2}}}{3}\end{cases}\right.$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] 3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**9/(360*b**3) -
 a**8*x**(1/3)/(360*b**2) + a**7*x**(2/3)/(360*b) + 119*a**6*x/360 + 511*a*
*5*b*x**(4/3)/360 + 1001*a**4*b**2*x**(5/3)/360 + 1099*a**3*b**3*x**2/360 +
 701*a**2*b**4*x**(7/3)/360 + 61*a*b**5*x**(8/3)/90 + b**6*x**3/10), Ne(b**
2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(9/2)/9 - 2*a**2*(a**2 + 2*a*b*x**(
1/3))**(11/2)/11 + (a**2 + 2*a*b*x**(1/3))**(13/2)/13)/(4*a**3*b**3), Ne(a*
b, 0)), (x*(a**2)**(7/2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2}-\frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}a}{30\,b^3}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(9
/2)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \frac{3}{10}\,b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{7}{3}\,ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{8}\,a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+15\,a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{35}{2}\,a^4b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{5}\,a^5b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{21}{4}\,a^6bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^7x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) + 6
3/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3) +
 a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x^(1
/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \int {\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{7/2} \,d x$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}$$

```
[Out] 1/2*a^2*(a+b*x^(1/3))^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/7*a*(a+
b*x^(1/3))^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/8*(a+b*x^(1/3))^7*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5\,dx,x,\sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)dx,x,\sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3}\\
&\quad+\frac{3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}\left(56a^5x+210a^4bx^{4/3}+336a^3b^2x^{5/3}+280a^2b^3x^2+120ab^4x^{7/3}+21b^5x^{8/3}\right)}{56\left(a+b\sqrt[3]{x}\right)^5}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (((a + b*x^(1/3))^2)^(5/2)*(56*a^5*x + 210*a^4*b*x^(4/3) + 336*a^3*b^2*x^(5
/3) + 280*a^2*b^3*x^2 + 120*a*b^4*x^(7/3) + 21*b^5*x^(8/3)))/(56*(a + b*x^(
1/3))^5)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{5}{2}} x\left(21b^{5}x^{\frac{5}{3}}+120b^{4}a\,x^{\frac{4}{3}}+280a^{2}b^{3}x+336a^{3}b^{2}x^{\frac{2}{3}}+210b\,a^{4}x^{\frac{1}{3}}+56a^{5}\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	76
default	$\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{\frac{5}{2}}\left(21b^{5}x^{\frac{8}{3}}+120b^{4}a\,x^{\frac{7}{3}}+336a^{3}b^{2}x^{\frac{5}{3}}+210b\,a^{4}x^{\frac{4}{3}}+280a^{2}b^{3}x^{2}+56a^{5}x\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	87

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*((a+b*x^(1/3))^2)^(5/2)*x*(21*b^5*x^(5/3)+120*b^4*a*x^(4/3)+280*a^2*b^
3*x+336*a^3*b^2*x^(2/3)+210*b*a^4*x^(1/3)+56*a^5)/(a+b*x^(1/3))^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=5\,a^2b^3x^2+a^5x+\frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=3\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^7}{168b^3}-\frac{a^6\sqrt[3]{x}}{168b^2}+\frac{a^5x^{\frac{2}{3}}}{168b}+\frac{55a^4x}{168}+\frac{155a^3bx^{\frac{4}{3}}}{168}+\frac{181a^2b^2x^{\frac{5}{3}}}{168}+\frac{33ab^3x^2}{56}+\right.\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}}{4a^3b^3}\\ \frac{x\left(a^2\right)^{\frac{5}{2}}}{3}\end{cases}$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] 3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**7/(168*b**3) -
 a**6*x**(1/3)/(168*b**2) + a**5*x**(2/3)/(168*b) + 55*a**4*x/168 + 155*a**
3*b*x**(4/3)/168 + 181*a**2*b**2*x**(5/3)/168 + 33*a*b**3*x**2/56 + b**4*x*
*(7/3)/8), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 - 2*a**2*(
a**2 + 2*a*b*x**(1/3))**(9/2)/9 + (a**2 + 2*a*b*x**(1/3))**(11/2)/11)/(4*a*
*3*b**3), Ne(a*b, 0)), (x*(a**2)**(5/2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2}-\frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/
2)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{3}{8}\,b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}\,ab^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^3b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}\,a^4bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^5x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
+ 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) +
 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{5/2}\,dx$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}$$

```
[Out] 3/4*a^2*(a+b*x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/5*a*(a+
b*x^(1/3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+1/2*(a+b*x^(1/3))^5*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1355, 659}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rule 659

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^3}{b^2}-\frac{2a(ab+b^2x)^4}{b^3}+\frac{(ab+b^2x)^5}{b^4}\right)dx, x, \sqrt[3]{x}\right)}{b^3\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3} + \frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx = \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{3/2}\left(20a^3x+45a^2bx^{4/3}+36ab^2x^{5/3}+10b^3x^2\right)}{20\left(a+b\sqrt[3]{x}\right)^3}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (((a + b*x^(1/3))^2)^(3/2)*(20*a^3*x + 45*a^2*b*x^(4/3) + 36*a*b^2*x^(5/3)
+ 10*b^3*x^2))/(20*(a + b*x^(1/3))^3)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}} x\left(10b^{3}x+36b^{2}a\,x^{\frac{2}{3}}+45a^{2}b\,x^{\frac{1}{3}}+20a^{3}\right)}{20\left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	54
default	$\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(36b^{2}a\,x^{\frac{5}{3}}+45a^{2}b\,x^{\frac{4}{3}}+10b^{3}x^{2}+20a^{3}x\right)}{20\left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	65

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/20*((a+b*x^(1/3))^2)^(3/2)*x*(10*b^3*x+36*b^2*a*x^(2/3)+45*a^2*b*x^(1/3)+
20*a^3)/(a+b*x^(1/3))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^5}{60b^3}-\frac{a^4\sqrt[3]{x}}{60b^2}+\frac{a^3x^{\frac{2}{3}}}{60b}+\frac{19a^2x}{60}+\frac{13abx^{\frac{4}{3}}}{30}+\frac{b^2x^{\frac{5}{3}}}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x\left(a^2\right)^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] 3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**5/(60*b**3) -
a**4*x**(1/3)/(60*b**2) + a**3*x**(2/3)/(60*b) + 19*a**2*x/60 + 13*a*b*x**(
4/3)/30 + b**2*x**(5/3)/6), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(
5/2)/5 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 + (a**2 + 2*a*b*x**(1/3))*
*(9/2)/9)/(4*a**3*b**3), Ne(a*b, 0)), (x*(a**2)**(3/2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^3}{4\,b^3}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2}-\frac{7\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a}{10\,b^3}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2
)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{5}\,ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{4}\,a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4
*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}dx$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

```
[Out] a*x*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))+3/4*b*x^(4/3)*(a^2+
2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2(ab+b^2x)\,dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int (abx^2+b^2x^3)\,dx,x,\sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\
&= \frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{\operatorname{csgn}\left(a+b\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right)^{2}\left(3b^{2}x^{\frac{2}{3}}-2ab\,x^{\frac{1}{3}}+a^{2}\right)}{4b^{3}}$	42
default	$\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}}\,\left(3b\,x^{\frac{4}{3}}+4ax\right)}{4a+4b\,x^{\frac{1}{3}}}$	43

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*csgn(a+b*x^(1/3))*(a+b*x^(1/3))^2*(3*b^2*x^(2/3)-2*a*b*x^(1/3)+a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/4*b*x^(4/3) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^3}{12b^3}-\frac{a^2\sqrt[3]{x}}{12b^2}+\frac{ax^{\frac{2}{3}}}{12b}+\frac{x}{4}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x\sqrt{a^2}}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
[Out] 3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**3/(12*b**3) -
a**2*x**(1/3)/(12*b**2) + a*x**(2/3)/(12*b) + x/4), Ne(b**2, 0)), ((a**4*(a
**2 + 2*a*b*x**(1/3))**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(5/2)/5 +
(a**2 + 2*a*b*x**(1/3))**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x*sqrt(a**2)
/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}a^2x^{\frac{1}{3}}}{2\,b^2} + \frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}a^3}{2\,b^3} + \frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4\,b^2} - \frac{5\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a}{4\,b^3}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a/b
^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.30

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + ax\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,(a^2+b^2\,x^{2/3}}{4\,b^3}$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^2
*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3/2*(a
+b*x^(1/3))*x^(2/3)/b/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*a^2*(a+b*x^(1
/3))*ln(a+b*x^(1/3))/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = -\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
```

```
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)\left(b\left(-2a+b\sqrt[3]{x}\right)\sqrt[3]{x}+2a^2\log\left(a+b\sqrt[3]{x}\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]
))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{3\left(a+b\,x^{\frac{1}{3}}\right)\left(b^{2} x^{\frac{2}{3}}+2 a^{2} \ln\left(a+b\,x^{\frac{1}{3}}\right)-2 a b\,x^{\frac{1}{3}}\right)}{2 \sqrt{\left(a+b\,x^{\frac{1}{3}}\right)^{2}}\, b^{3}}$	52
default	$\frac{\left(a+b\,x^{\frac{1}{3}}\right)\left(3 b^{2} x^{\frac{2}{3}}-6 a b\,x^{\frac{1}{3}}+2 a^{2} \ln\left(b^{3} x+a^{3}\right)-2 a^{2} \ln\left(b^{2} x^{\frac{2}{3}}-a b\,x^{\frac{1}{3}}+a^{2}\right)+4 a^{2} \ln\left(a+b\,x^{\frac{1}{3}}\right)\right)}{2 \sqrt{a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}}\, b^{3}}$	101

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*(a+b*x^(1/3))*(b^2*x^(2/3)+2*a^2*ln(a+b*x^(1/3))-2*a*b*x^(1/3))/((a+b*x
^(1/3))^2)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx=\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\left(\begin{cases} \frac{a^2\left(\frac{a}{b}+\sqrt[3]{x}\right)\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^2\sqrt{b^2\left(\frac{a}{b}+\sqrt[3]{x}\right)^2}} + \left(-\frac{3a}{2b^3}+\frac{\sqrt[3]{x}}{2b^2}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}} & \text{for } b^2 \neq \\ \frac{a^4\sqrt{a^2+2ab\sqrt[3]{x}} - \frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3} + \frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}}{4a^3b^3} & \text{for } ab \neq \\ \frac{x}{3\sqrt{a^2}} & \text{otherw} \end{cases}\right.$$

[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)

[Out] 3*Piecewise((a**2*(a/b + x**(1/3))*log(a/b + x**(1/3))/(b**2*sqrt(b**2*(a/b + x**(1/3))**2)) + (-3*a/(2*b**3) + x**(1/3)/(2*b**2))*sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), Ne(b**2, 0)), ((a**4*sqrt(a**2 + 2*a*b*x**(1/3)) - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(3/2)/3 + (a**2 + 2*a*b*x**(1/3))**(5/2)/5)/(4*a**3*b**3), Ne(a*b, 0)), (x/(3*sqrt(a**2)), True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\,a^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3} + \frac{3\,x^{\frac{2}{3}}}{2\,b} - \frac{3\,ax^{\frac{1}{3}}}{b^2}$$

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")

[Out] 3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(bx^{\frac{2}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right) - 2\,ax^{\frac{1}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2} + \frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3
*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx=\int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] 6*a/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2*a^2/b^3/(a+b*x^(1/3))/(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*(a+b*x^(1/3))*ln(a+b*x^(1/3))/b^3/(a^2
+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&\quad+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(
a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{3\left(2\ln\left(a+b\,x^{\frac{1}{3}}\right)b^{2}x^{\frac{2}{3}}+4\ln\left(a+b\,x^{\frac{1}{3}}\right)ab\,x^{\frac{1}{3}}+2a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)+4ab\,x^{\frac{1}{3}}+3a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{2b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}}}$
default	$\left(8x^{3}\ln\left(b^{3}x+a^{3}\right)a^{3}b^{9}-8x^{3}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)a^{3}b^{9}+16x^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)a^{3}b^{9}-27x^{\frac{4}{3}}a^{8}b^{4}+3x^{\frac{2}{3}}a^{10}b^{2}-6x^{\frac{1}{3}}a^{11}b+2\ln(b^{3}$

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(2*ln(a+b*x^(1/3))*b^2*x^(2/3)+4*ln(a+b*x^(1/3))*a*b*x^(1/3)+2*a^2*ln(a
+b*x^(1/3))+4*a*b*x^(1/3)+3*a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3\left(6\,a^3b^3x+3\,a^6+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+\left(4\,ab^5x+a^4b^2\right.\right.}{2\left(b^9x^2+2\,a^3b^6x+a^6b^3\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3} + \frac{6\,ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2} + \frac{9\,a^2}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
[Out] 3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(b
^5*(x^(1/3) + a/b)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)} + \frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a
^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\int \frac{1}{{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{3/2}} \,dx$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3/4*a^2/b^3/(a+b*x^(1/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2*a/b^3/
(a+b*x^(1/3))^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2/b^3/(a+b*x^(1/3))
/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^2}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}\right)}{4b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
[Out] ((a + b*x^(1/3))*(-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3)))/(4*b^3*((a + b*x^(
1/3))^2)^(5/2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result
derivativedivides	$-\frac{\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{4b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{5}{2}}}$
default	$-\frac{\left(6x^{\frac{22}{3}}b^{22}+45x^{\frac{20}{3}}a^2b^{20}-36x^{\frac{19}{3}}a^3b^{19}+144x^{\frac{17}{3}}a^5b^{17}-189x^{\frac{16}{3}}a^6b^{16}+126x^{\frac{14}{3}}a^8b^{14}-276x^{\frac{13}{3}}a^9b^{13}-36x^{\frac{11}{3}}a^{11}b^{11}-1\cdots\right.}{4b^3\left(b^2x^{\frac{2}{3}}\cdots\right.}$

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)^
(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=\frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\left(5\,a^2b^8x^2-4\,a^5b^5x\right)x^{\frac{2}{3}}-3\left(2\,b^{10}x^3-20\,a\cdots\right.}{4\left(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)
*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4
 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
[Out] -3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*(
x^(1/3) + a/b)^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4 b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^(
1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,\left(a+b\,x^{1/3}\right)^5}$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x^
(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -1/2*a^2/b^3/(a+b*x^(1/3))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/5*a/b^
3/(a+b*x^(1/3))^4/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/4/b^3/(a+b*x^(1/3
))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -1/2*a^2/(b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) +
(6*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{7/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^7}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^9(a+bx)^7}-\frac{2a}{b^9(a+bx)^6}+\frac{1}{b^9(a+bx)^5}\right)\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&\quad -\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}\right)}{20b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}}$$

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]

[Out] ((a + b*x^(1/3))*(-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3)))/(20*b^3*((a + b*x
^(1/3))^2)^(7/2))

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(15b^2x^{\frac{2}{3}}+6abx^{\frac{1}{3}}+a^2\right)\left(a+bx^{\frac{1}{3}}\right)}{20b^3\left(\left(a+bx^{\frac{1}{3}}\right)^2\right)^{\frac{7}{2}}}$
default	$-\frac{\left(280x^9a^5b^{27}-540x^{\frac{29}{3}}a^3b^{29}-84x^{\frac{31}{3}}ab^{31}-2106x^{\frac{26}{3}}a^6b^{26}+567x^{\frac{28}{3}}a^4b^{28}-792x^{\frac{23}{3}}a^9b^{23}+3996x^{\frac{25}{3}}a^7b^{25}+7344x^{\frac{20}{3}}a^{12}\right.}{}$

[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/20*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2
)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=$$
$$-\frac{280\,a^2b^{12}x^4-1400\,a^5b^9x^3+735\,a^8b^6x^2-14\,a^{11}b^3x+a^{14}+3\left(5\,b^{14}x^4-210\,a^3b^{11}x^3+483\,a^6b^8x^2-112\,a^9b\right.}{20\left(b^{21}x^6+6\,a^3b^{18}x^5+15\,a^6b^{15}x^4+20\,a^9b^{12}x^3+15\,a^{12}b^9x^2+\right.}$$

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")

[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*
x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5
*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^

```
10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^
12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = -\frac{3}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}+\frac{6\,a}{5\,b^8\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^5}-\frac{a^2}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
[Out] -3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^9
*(x^(1/3) + a/b)^6)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx = -\frac{15\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}+a^2}{20\left(bx^{\frac{1}{3}}+a\right)^6 b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*x
^(1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+15\,b^2\,x^{2/3}+6\,a\,b\,x^{1/3}\right)}{20\,b^3\,\left(a+b\,x^{1/3}\right)^7}$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*x
^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3/8*a^2/b^3/(a+b*x^(1/3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/7*a/b^
3/(a+b*x^(1/3))^6/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-1/2/b^3/(a+b*x^(1/3
))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2}{(a^2+2abx+b^2x^2)^{9/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{11}(a+bx)^9}-\frac{2a}{b^{11}(a+bx)^8}+\frac{1}{b^{11}(a+bx)^7}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&\quad -\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}\right)}{56b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{9/2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
[Out] ((a + b*x^(1/3))*(-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3)))/(56*b^3*((a + b*x
^(1/3))^2)^(9/2))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(28b^2x^{\frac{2}{3}}+8ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{56b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{9}{2}}}$
default	$\frac{\left(-a^{42}+202496a^{27}b^{15}x^5+31696a^{30}b^{12}x^4-11704a^{33}b^9x^3-3844a^{36}b^6x^2+40a^{39}b^3x-46480a^9b^{33}x^{11}-190568a^{12}b^{30}x^{10}\right.}{}$

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/56*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2
)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(107) = 214.

Time = 0.41 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=$$

$$-\frac{28\,b^{18}x^6-2856\,a^3b^{15}x^5+18186\,a^6b^{12}x^4-20608\,a^9b^9x^3+4200\,a^{12}b^6x^2-48\,a^{15}b^3x+a^{18}-27\,(8\,ab^{17}x^5-}{56\,(b^{27}x^8+8\,a^3b^{24}x^7+28\,a^6b^{21}x^6+56\,a}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9
*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^
4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) +
```

```
27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x
^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 +
56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 +
8*a^21*b^6*x + a^24*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{1}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,a}{7\,b^{10}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{3\,a^2}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
[Out] -1/2/(b^9*(x^(1/3) + a/b)^6) + 6/7*a/(b^10*(x^(1/3) + a/b)^7) - 3/8*a^2/(b^
11*(x^(1/3) + a/b)^8)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(bx^{\frac{1}{3}}+a\right)^8b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*x
^(1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+28\,b^2\,x^{2/3}+8\,a\,b\,x^{1/3}\right)}{56\,b^3\,{\left(a+b\,x^{1/3}\right)}^9}$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 28*b^2*x^(2/3) + 8*a*b*x
^(1/3)))/(56*b^3*(a + b*x^(1/3))^9)
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3/10*a^2/b^3/(a+b*x^(1/3))^9/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2/3*a/b
^3/(a+b*x^(1/3))^8/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/8/b^3/(a+b*x^(1/
3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2}{(a^2+2abx+b^2x^2)^{11/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^{11}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}}-\frac{2a}{b^{13}(a+bx)^{10}}+\frac{1}{b^{13}(a+bx)^9}\right)\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&\quad+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}\right)}{120b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{11/2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
[Out] ((a + b*x^(1/3))*(-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3)))/(120*b^3*((a + b
*x^(1/3))^2)^(11/2))
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

method	result
derivativedivides	$-\frac{\left(45b^2x^{\frac{2}{3}}+10ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{120b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{11}{2}}}$
default	$-\frac{\left(-5834520x^{\frac{34}{3}}a^{18}b^{34}+2207250x^{\frac{38}{3}}a^{14}b^{38}-13023000x^{\frac{31}{3}}a^{21}b^{31}+6925500x^{\frac{35}{3}}a^{17}b^{35}-12561075x^{\frac{28}{3}}a^{24}b^{28}+8650665\right.}{}$

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x,method=_RETURNVERBOSE)
[Out] -1/120*(45*b^2*x^(2/3)+10*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))
^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(107) = 214.

Time = 0.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.50

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=\frac{440\,ab^{21}x^7-25630\,a^4b^{18}x^6+186252\,a^7b^{15}x^5-326150\,a^{10}b^{12}x^4+154000}{}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a
^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x -
a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*
a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7
 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*
```

```
b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3
*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^
15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*
a^27*b^6*x + a^30*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=-\frac{3}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}+\frac{2\,a}{3\,b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3\,a^2}{10\,b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
[Out] -3/8/(b^11*(x^(1/3) + a/b)^8) + 2/3*a/(b^12*(x^(1/3) + a/b)^9) - 3/10*a^2/(
b^13*(x^(1/3) + a/b)^10)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx=-\frac{45\,b^2x^{\frac{2}{3}}+10\,abx^{\frac{1}{3}}+a^2}{120\left(bx^{\frac{1}{3}}+a\right)^{10}b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn(
b*x^(1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = -\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+45\,b^2\,x^{2/3}+10\,a\,b\,x^{1/3}\right)}{120\,b^3\,\left(a+b\,x^{1/3}\right)^{11}}$$

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 45*b^2*x^(2/3) + 10*a*b*
x^(1/3)))/(120*b^3*(a + b*x^(1/3))^11)
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal result

Integrand size = 30, antiderivative size = 77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m \,\text{Hypergeometric2F1}\left(3(1+m),-2p,\right.}{1+m}$$

```
[Out] (a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x*(d*x)^m*hypergeom([-2*p, 3+3*m],[4+3*m]
,-b*x^(1/3)/a)/(1+m)/((1+b*x^(1/3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1370, 350, 348, 66}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \,\text{Hypergeometric2F1}\left(3(m+1),-2p,\right.}{m+1}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m
), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 350

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}(dx)^m\,dx\\
&= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^m\,dx\\
&= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}\right.\right.\\
&\qquad\left.\left.+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\operatorname{Subst}\left(\int x^{-1+3(1+m)}\left(1+\frac{bx}{a}\right)^{2p}dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m\,{}_2F_1\left(3(1+m),-2p;4+3m;-\frac{b\sqrt[3]{x}}{a}\right)}{1+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}x(dx)^m\,\text{Hypergeometric2F1}\left(3(1+m),-2p,1+3(1\right.}{1+m}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
[Out] (((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*(
1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p (dx)^m\,dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int (dx)^m\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
[Out] Integral((d*x)**m*((a + b*x**(1/3))**2)**p, x)
```

Maxima [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int (d\,x)^m\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
[Out] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)
```

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 468

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2+p)} + \frac{210a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(5+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(7+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(4+p)} + \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(9+2p)}$$

```
[Out] 3*a^9*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+2*p)-12*a^9*
(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(p+1)+84*a^9*(1+b*x
^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+2*p)-84*a^9*(1+b*x^(1/
3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(2+p)+210*a^9*(1+b*x^(1/3)/a)
^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(5+2*p)-84*a^9*(1+b*x^(1/3)/a)^6*(
a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+p)+84*a^9*(1+b*x^(1/3)/a)^7*(a^2+2*
a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(7+2*p)-12*a^9*(1+b*x^(1/3)/a)^8*(a^2+2*a*b*
x^(1/3)+b^2*x^(2/3))^p/b^9/(4+p)+3*a^9*(1+b*x^(1/3)/a)^9*(a^2+2*a*b*x^(1/3)
+b^2*x^(2/3))^p/b^9/(9+2*p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1370, 272, 45}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)} - \frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)} + \frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)} - \frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+3)} + \frac{210a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+5)} - \frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+2)} + \frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+3)} - \frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+1)} + \frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
```

```
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^2\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^8\left(1+\frac{bx}{a}\right)^{2p}dx, x, \sqrt[3]{x}\right)$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^8\left(1+\frac{bx}{a}\right)^{2p}}{b^8}-\frac{8a^8\left(1+\frac{bx}{a}\right)^{1+2p}}{b^8}+\frac{28a^8\left(1+\frac{bx}{a}\right)^{2+2p}}{b^8}-\frac{56a^8\left(1+\frac{bx}{a}\right)^{3+2p}}{b^8}+\right.\right.$$

$$= \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2+p)} + \frac{210a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(5+2p)} - \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+p)} + \frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(7+2p)} - \frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(4+p)} + \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(9+2p)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.44

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(\frac{a^8}{1+2p} - \frac{4a^7\left(a+b\sqrt[3]{x}\right)}{1+p} + \frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{3+2p} - \frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{2+p} + \frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{5+2p} - \frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right.}{b^9}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3
))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/
3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1
/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9
+ 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p x^2dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.24

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(2520\,a^9+(16\,b^9p^8+288\,b^9p^7+2184\,b^9p^6+9072\,b^9p^5+22449\,b^9p^4+33642\,b^9p^3+2953\right.}{}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

```
[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 224
49*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 +
28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 13
7*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a
^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^
8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 +
 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 +
35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260*a^8*b*p
 + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 +
812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4
+ 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^
9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx=\int x^2\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
[Out] Integral(x**2*((a + b*x**(1/3))**2)**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx=\frac{3\left(\left(16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+33642\,p^3+29531\,p^2+13698\,p+2520\right.\right.}{}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*
p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6
769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 +
350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 +
 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20
*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^
2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2
 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2
*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 3
61840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(414) = 828.

Time = 0.32 (sec) , antiderivative size = 1564, normalized size of antiderivative = 3.34

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx=\text{Too large to display}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

```
[Out] 3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1/3
) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3) +
2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^5*x^3 +
 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 2800*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^
5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^4*x^(8/3) -
5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3) + 4760*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^4*x^(4/3)
- 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3*x^(7/3) + 6300*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p*a^4*b^5*p^3*x^(5/3) + 3267*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*
p^2*x^(8/3) + 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^3*x^(4/3
) - 3528*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^2*x^(7/3) - 3360*(
b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*b^9*p*x^3 - 4200*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^2*
x^(5/3) + 630*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p*x^(8/3) + 4620*
(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^2*x^(4/3) - 720*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p*x^(7/3) - 5040*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
3*b^6*p*x^2 + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*x^3 + 5040*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p^2*x^(2/3) - 1008*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p*x^(5/3) + 1260*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*a^5*b^4*p*x^(4/3) - 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6*b^3*p*x + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p*x^(2/3)
 - 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^8*b*p*x^(1/3) + 2520*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p
^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.66

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p \left(\frac{3\,x^3\,(16\,p^8 + 288\,p^7 + 2184\,p^6 + 9072\,p^5 + 22449\,p^4 + 3}{32\,p^9 + 720\,p^8 + 6960\,p^7 + 37800\,p^6 + 126546\,p^5 + 269325\,p^4}\right.$$

```
[In] int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642*
p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322*p
 + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 + 361
840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 361840*p^3
 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2268
0)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 1288*p^5 +
224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^
4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) + (84*a^
3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*(128322*p +
 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))/(b^6*(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p
^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 293175*p^2 + 361840*
p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2
2680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p + 293175*p^2 + 36184
0*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 +
 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(b^5*(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680)) - (504*a^4*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 +
 4*p^4 + 6))/(b^4*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546
*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)))
```

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal result

Integrand size = 26, antiderivative size = 315

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=-\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)}-\frac{30a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(3+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2+p)}-\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(5+2p)}+\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(3+p)}$$

```
[Out] -3*a^6*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+2*p)+15/2*a
^6*(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(p+1)-30*a^6*(1+
b*x^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+2*p)+15*a^6*(1+b*x^
(1/3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(2+p)-15*a^6*(1+b*x^(1/3)/
a)^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(5+2*p)+3/2*a^6*(1+b*x^(1/3)/a)^
6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1370, 272, 45}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(p+2)}-\frac{30a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+3)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+1)}-\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^5\left(1+\frac{bx}{a}\right)^{2p}dx,x,\sqrt[3]{x}\right)$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^5\left(1+\frac{bx}{a}\right)^{2p}}{b^5}+\frac{5a^5\left(1+\frac{bx}{a}\right)^{1+2p}}{b^5}-\frac{10a^5\left(1+\frac{bx}{a}\right)^{2+2p}}{b^5}+\frac{10a^5\left(1+\frac{bx}{a}\right)^{3+2p}}{b^5}\right.\right.$$

$$= -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)}$$
$$-\frac{30a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(3+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2+p)}$$
$$-\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(5+2p)}+\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(3+p)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left(-\frac{2a^5}{1+2p}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{1+p}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{3+2p}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{2+p}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{5+2p}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right)\ldots}{2b^6}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x
^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^
(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b*x^
(1/3))^2)^p)/(2*b^6)
```

Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p x dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=$$

$$-\frac{3\left(30\,a^6-(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6)x^2-20\,(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p)x\ldots\right.}{2\ldots}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^
6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(3
0*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 +
 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12
*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 8
12*b^6*p^2 + 441*b^6*p + 90*b^6)
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\int x\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
[Out] Integral(x*((a + b*x**(1/3))**2)**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.63

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left((8p^5+60p^4+170p^3+225p^2+137p+30)b^6x^2+2(4p^5+20p^4+35p^3+25p^2+6p)a\right.}{2(8p^6+8}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 +
 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2
 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)
*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p
^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(275) = 550.

Time = 0.32 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.37

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left(8\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^pb^6p^5x^2+8\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^pab^5p^5x^{\frac{5}{3}}+60\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}\right.\right.}{}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
[Out] 3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^4*
```

```
x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3) + 17
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^
3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 50*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p*x^2 -
 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) + 12*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x^2 - 30*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 4
41*b^6*p + 90*b^6)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.24

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x^2\,\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)}{2\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{}{b^6\,(}\right.$$

```
[In] int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*
p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350*p^4
 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(
2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (30*a^
3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*p^2 + 20*
p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6
+ 90)))
```

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(3+2p)}$$

```
[Out] 3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b*
x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(p+1)+3*(a+b*x^(1/3))^3*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1355, 660, 45}

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^p\,dx, x, \sqrt[3]{x}\right)\\
&= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{2p}\,dx, x, \sqrt[3]{x}\right)\\
&= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}\right.\right.\\
&\quad \left.\left.+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^{2p}}{b^2}-\frac{2a(ab+b^2x)^{1+2p}}{b^3}+\frac{(ab+b^2x)^{2+2p}}{b^4}\right)dx, x, \sqrt[3]{x}\right)\\
&= \frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}\\
&\quad +\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(3+2p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(1+2p)\sqrt[3]{x}+b^2\left(1+3p+2p^2\right)x^{2/3}\right)}{b^3(1+p)(1+2p)(3+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2
*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [F]

$$\int \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=-\frac{3\left(2\,a^{2}bpx^{\frac{1}{3}}-a^{3}-(2\,b^{3}p^{2}+3\,b^{3}p+b^{3})x-(2\,ab^{2}p^{2}+ab^{2}p)x^{\frac{2}{3}}\right)\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}}{4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
+ a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12*b
^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left((2\,p^2+3\,p+1)b^3x+(2\,p^2+p)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
+ a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a\right.\right.}{}$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

```
[Out] 3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2*(b
^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4*b^3*
p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{3\,x\,(2\,p^2+3\,p+1)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,(4\,p^3+12\,p^2+11\,p+3)}-\frac{6}{b^2\,(4\,p^3+}\right.$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*p
^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^(1
/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(11*p
 + 12*p^2 + 4*p^3 + 3)))
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}$$

```
[Out] -3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([1, 1+2*p],[
2+2*p],1+b*x^(1/3)/a)/(1+2*p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1370, 272, 67}

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometri
c2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x}\,dx\\
&= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{b\sqrt[3]{x}}{a}\right)}{a(1+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
[Out] (-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 +
 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

Maple [F]

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x}dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
[Out] Integral(((a + b*x**(1/3))**2)**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x}\,dx$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x,x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 75

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(4,1+2p,2(\right.}{a^3(1+2p)}$$

```
[Out] 3*b^3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([4, 1+2*p
],[2+2*p],1+b*x^(1/3)/a)/a^3/(1+2*p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1370, 272, 67}

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(4,2p+1,2(\right.}{a^3(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeome
tric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
```

```
|| GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x^2}\,dx\\ &= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^4}\,dx,x,\sqrt[3]{x}\right)\\ &= \frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\text{Hypergeometric2F1}\left(4,1+2p,2+2p,1+\frac{b\sqrt[3]{x}}{a}\right)}{a^4(1+2p)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

Maple [F]

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x^2}dx$$

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2}\,dx = \int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2}\,dx = \int \frac{\left((a+b\sqrt[3]{x})^2\right)^p}{x^2}\,dx$$

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
[Out] Integral(((a + b*x**(1/3))**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2}\,dx = \int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x^2}\,dx$$

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2,x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)$

Optimal result

Integrand size = 77, antiderivative size = 146

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)\,dx = -\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax}+\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}}-\frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^3\sqrt[3]{x}}$$

```
[Out] -(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a/x+b*(1-p)*(a+b*x^(1/3))*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^2/x^(2/3)-b^2*(1-2*p)*(1-p)*(a+b*x^(1/3
))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x^(1/3)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used

$= \{1370, 272, 67\}$

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x} \right) dx = \frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right.}{a^3(2p} + \frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(4,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x
^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
)^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 +
2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = -\frac{(2b^3(1-2p)(1-p)p)\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx}{3a^3} + \int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$$

$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x^2}dx - \frac{\left(2b^3(1-2p)(1-p)p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x}dx}{3a^3}$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^4}dx,x,\sqrt[3]{x}\right) - \frac{\left(2b^3(1-2p)(1-p)p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}dx,x,\sqrt[3]{x}\right)}{a^3}$$

$$= \frac{2b^3(1-2p)(1-p)p\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\ {}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)} + \frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\ {}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int\left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = \frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p(1-3p+2p^2)\text{Hype}\right.}{}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 -
 p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeom
etric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4,
 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

Maple [F]

$$\int \left(\frac{\left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}}{x^{2}}-\frac{2 b^{3} \left(1-2 p \right) \left(-p +1\right) p \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}}{3 a^{3} x}\right)d x$$

[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(-p+1)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)

[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(-p+1)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)\,dx = -\frac{\left(a^{2} b p x^{\frac{1}{3}} + a^{3} + (2 \, b^{3} p^{2} - 3 \, b^{3} p + b^{3}) x + 2 \, (a b^{2} p^{2} - a b^{2} p) x^{\frac{2}{3}}\right) \left(b^{2} x^{\frac{2}{3}} + 2 \, a b x^{\frac{1}{3}} + a^{2}\right)^{p}}{a^{3} x}$$

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")

[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)

Sympy [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)\,dx = -\frac{\int \left(- \frac{3 a^{3} \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x^{2}}\right)\, dx + \int \frac{2 b^{3} p \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x}\, dx + \int \left(- \frac{6 b^{3} p^{2} \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x}\right)\, dx + \int \frac{4 b^{3} p}{}}{3 a^{3}}$$

[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)

[Out] -(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + Integral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(4*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)

Maxima [F]

$$\int \left(\frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{3a^3x}\right)dx = \int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} + -$$

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")

[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)

Giac [F]

$$\int \left(\frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{3a^3x}\right)dx = \int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} + -$$

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")

[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \left(\frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{3a^3x}\right)dx = -\frac{(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3})^p\left(\frac{b^3\,x\,(2\,p^2-3\,p+1)}{a^3}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^2\,p\,x^{2/3}\,(p-1)}{a^2}+1\right)}{x}$$

[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)

[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (b*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\begin{aligned}\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=&-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\&+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\&+\frac{4\left(a+b\sqrt[4]{x}\right)\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\end{aligned}$$

```
[Out] -12*a^2/b^4/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+2*a^3/b^4/(a+b*x^(1/4))/(
a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+4*(a+b*x^(1/4))*x^(1/4)/b^3/(a^2+2*a*b
*x^(1/4)+b^2*x^(1/2))^(1/2)-12*a*(a+b*x^(1/4))*ln(a+b*x^(1/4))/b^4/(a^2+2*a
*b*x^(1/4)+b^2*x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\begin{aligned}\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=&-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\&-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\&+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\end{aligned}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, \sqrt[4]{x}\right)\\
&= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^3}\,dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\
&= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\
&= -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\\
&\quad+\frac{4\left(a+b\sqrt[4]{x}\right)\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}+2b^3x^{3/4}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b
*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^
2])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

method	result	siz
derivativedivides	$-\frac{2\left(6\ln\left(a+b\,x^{\frac{1}{4}}\right)a\,b^{2}\sqrt{x}-2b^{3}x^{\frac{3}{4}}+12\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{2}b\,x^{\frac{1}{4}}-4a\,b^{2}\sqrt{x}+6\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{3}+4a^{2}b\,x^{\frac{1}{4}}+5a^{3}\right)\left(a+b\,x^{\frac{1}{4}}\right)}{b^{4}\left(\left(a+b\,x^{\frac{1}{4}}\right)^{2}\right)^{\frac{3}{2}}}$	10
default	Expression too large to display	14

```
[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(6*ln(a+b*x^(1/4))*a*b^2*x^(1/2)-2*b^3*x^(3/4)+12*ln(a+b*x^(1/4))*a^2*b*
x^(1/4)-4*a*b^2*x^(1/2)+6*ln(a+b*x^(1/4))*a^3+4*a^2*b*x^(1/4)+5*a^3)*(a+b*x
^(1/4))/b^4/((a+b*x^(1/4))^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.99 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{2\left(9\,a^5b^4x-5\,a^9-6\left(ab^8x^2-2\,a^5b^4x+a^9\right)\log\left(bx^{\frac{1}{4}}+a\right)-2\left(3\,a^2b^7x-a\right.\right.}{b^{12}x^2-2\,a^4b^8x+}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x)
+ 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^
8*b^4)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^2}-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{b^4}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^4}-\frac{24\,a^2x^{\frac{1}{4}}}{b^5\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}-\frac{22\,a^3}{b^6\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
[Out] 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
+ a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2*x
^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)^{3/2}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
[Out] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{6\left(a+b\sqrt[6]{x}\right)\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

```
[Out] -60*a^2/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+3/2*a^5/b^6/(a+b*x^(1/6))
^3/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)-10*a^4/b^6/(a+b*x^(1/6))^2/(a^2+2*
a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+30*a^3/b^6/(a+b*x^(1/6))/(a^2+2*a*b*x^(1/6)+
b^2*x^(1/3))^(1/2)+6*(a+b*x^(1/6))*x^(1/6)/b^5/(a^2+2*a*b*x^(1/6)+b^2*x^(1/
3))^(1/2)-30*a*(a+b*x^(1/6))*ln(a+b*x^(1/6))/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(
1/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\text{integral} = 6\text{Subst}\left(\int \frac{x^5}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, \sqrt[6]{x}\right)$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \frac{x^5}{(ab+b^2x)^5}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}} - \frac{a^5}{b^{10}(a+bx)^5} + \frac{5a^4}{b^{10}(a+bx)^4} - \frac{10a^3}{b^{10}(a+bx)^3} + \frac{10a^2}{b^{10}(a+bx)^2} - \frac{5a}{b^{10}(a+bx)}\right)\,dx, x,$$
$$\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}$$
$$= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{6\left(a+b\sqrt[6]{x}\right)\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}+12b^5x^{5/6}-}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 4
8*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)
])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

method	result
derivativedivides	$-\frac{\left(60\ln\left(a+b\,x^{\frac{1}{6}}\right)a\,b^4x^{\frac{2}{3}}-12b^5x^{\frac{5}{6}}+240\ln\left(a+b\,x^{\frac{1}{6}}\right)a^2b^3\sqrt{x}-48a\,b^4x^{\frac{2}{3}}+360\ln\left(a+b\,x^{\frac{1}{6}}\right)a^3b^2x^{\frac{1}{3}}+48a^2b^3\sqrt{x}+240\ln\right.}{2b^6\left(\left(a+b\,x^{\frac{1}{6}}\right)^2\right)^{\frac{5}{2}}}$
default	Expression too large to display

```
[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(60*ln(a+b*x^(1/6))*a*b^4*x^(2/3)-12*b^5*x^(5/6)+240*ln(a+b*x^(1/6))*a
^2*b^3*x^(1/2)-48*a*b^4*x^(2/3)+360*ln(a+b*x^(1/6))*a^3*b^2*x^(1/3)+48*a^2*
b^3*x^(1/2)+240*ln(a+b*x^(1/6))*a^4*b*x^(1/6)+252*a^3*b^2*x^(1/3)+60*ln(a+b
*x^(1/6))*a^5+248*a^4*b*x^(1/6)+77*a^5)*(a+b*x^(1/6))/b^6/((a+b*x^(1/6))^2)
^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{12\,b^5x^{\frac{5}{6}}+48\,ab^4x^{\frac{2}{3}}-48\,a^2b^3\sqrt{x}-252\,a^3b^2x^{\frac{1}{3}}-248\,a^4bx^{\frac{1}{6}}-77\,a^5}{2\left(b^{10}x^{\frac{2}{3}}+4\,ab^9\sqrt{x}+6\,a^2b^8x^{\frac{1}{3}}+4\,a^3b^7x^{\frac{1}{6}}+a^4b^6\right)} - \frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^6}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*x
^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^
2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{30\,a\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} + \frac{6\,x^{\frac{1}{6}}}{b^5\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} - \frac{120\,a^2b^3\sqrt{x}+300\,a^3b^2x^{\frac{1}{3}}+260\,a^4bx^{\frac{1}{6}}+77\,a^5}{2\left(bx^{\frac{1}{6}}+a\right)^4b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -30*a*log(abs(b*x^(1/6) + a))/(b^6*sgn(b*x^(1/6) + a)) + 6*x^(1/6)/(b^5*sgn
(b*x^(1/6) + a)) - 1/2*(120*a^2*b^3*sqrt(x) + 300*a^3*b^2*x^(1/3) + 260*a^4
*b*x^(1/6) + 77*a^5)/((b*x^(1/6) + a)^4*b^6*sgn(b*x^(1/6) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)^{5/2}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = -\frac{2b^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(a+\frac{b}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{a+\frac{b}{\sqrt{x}}}+\frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}x}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\log\left(\sqrt{x}\right)}{a+\frac{b}{\sqrt{x}}}$$

```
[Out] a^3*x*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))+3*a*b^2*ln(x)*(a^2+b^2/
x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))-2*b^3*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/
(a+b/x^(1/2))/x^(1/2)+6*a^2*b*x^(1/2)*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/
x^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

```
[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x\,dx, x, \sqrt{x}\right)$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^3}{x^2}\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int\left(3a^2b^4+\frac{b^6}{x^2}+\frac{3ab^5}{x}+a^3b^3x\right)\,dx,x,\sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= -\frac{2b^4\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(ab+\frac{b^2}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt{x}}} + \frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}x}{a+\frac{b}{\sqrt{x}}}+\frac{3ab^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\log(x)}{ab+\frac{b^2}{\sqrt{x}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int\left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}\,dx = \frac{\sqrt{\frac{\left(b+a\sqrt{x}\right)^2}{x}}\left(-2b^3+6a^2bx+a^3x^{3/2}+3ab^2\sqrt{x}\log(x)\right)}{b+a\sqrt{x}}$$

```
[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt
[x]*Log[x]))/(b + a*Sqrt[x])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.36

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x+b^2+2ab\sqrt{x}}{x}\right)^{\frac{3}{2}}x\left(a^3x^{\frac{3}{2}}+3b^2a\ln(x)\sqrt{x}+6a^2bx-2b^3\right)}{\left(a\sqrt{x}+b\right)^3}$	65
default	$\frac{\left(\frac{a^2x^{\frac{3}{2}}+b^2\sqrt{x}+2abx}{x^{\frac{3}{2}}}\right)^{\frac{3}{2}}\left(x^{\frac{5}{2}}a^3+3x^{\frac{3}{2}}\ln(x)a\,b^2+6a^2b\,x^2-2b^3x\right)}{\left(a\sqrt{x}+b\right)^3}$	71

```
[In] int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] ((a^2*x+b^2+2*a*b*x^(1/2))/x)^(3/2)*x*(a^3*x^(3/2)+3*b^2*a*ln(x)*x^(1/2)+6*
a^2*b*x-2*b^3)/(a*x^(1/2)+b)^3
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
[Out] Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

Maxima [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2\,ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
[Out] a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = a^3x\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}\left(x\right) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}\left(x\right) + 6\,a^2b\sqrt{x}\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}\left(x\right) - \frac{2\,b^3\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}\left(x\right)}{\sqrt{x}}$$

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x)
)*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*
sqrt(x))*sgn(x)/sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}\,dx=\int \left(a^2+\frac{b^2}{x}+\frac{2\,a\,b}{\sqrt{x}}\right)^{3/2}\,dx$$

```
[In] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
[Out] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 391

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = -\frac{3b^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x} - \frac{63a^2b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{105a^3b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{63a^5b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}} + \frac{21a^6b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} + \frac{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{105a^4b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] -3/4*b^7*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(4/3)-7*a*b^
6*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x-63/2*a^2*b^5*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-105*a^3*b^4*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+63*a^5*b^2*x^(1/3)*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+21/2*a^6*b*x^(2/3)*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^7*x*(a^2+b^2/x^(2/3)+2*a*b/x^(
1/3))^(1/2)/(a+b/x^(1/3))+35*a^4*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)/(a+b/x^(1/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx=-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{105a^4b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}$$

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^2\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^7}{x^5}\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(21a^5b^9+\frac{b^{14}}{x^5}+\frac{7ab^{13}}{x^4}+\frac{21a^2b^{12}}{x^3}+\frac{35a^3b^{11}}{x^2}+\frac{35a^4b^{10}}{x}+7a^6b^8x+a^7b\right.}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x}$$
$$- \frac{63a^2b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{105a^3b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}}$$
$$+ \frac{63a^5b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{21a^6b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$+ \frac{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{35a^4b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\left(-3b^7-28ab^6\sqrt[3]{x}-126a^2b^5x^{2/3}-420a^3b^4x+252a^5b^2x^{5/3}+42a^6bx^2+4a^7x^{7/3}\right.}{4\left(b+a\sqrt[3]{x}\right)x}$$

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x
^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3)
 + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} x \left(4 a^{7} x^{\frac{7}{3}}+42 x^{2} a^{6} b+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{4}{3}}+252 b^{2} a^{5} x^{\frac{5}{3}}-420 b^{4} a^{3} x-126 a^{2} b^{5} x^{\frac{2}{3}}-28 a\, b^{6} x^{\frac{1}{3}}-3 b^{7}\right)}{4 \left(b+a\, x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} \left(42 a^{6} b\, x^{3}+252 b^{2} a^{5} x^{\frac{8}{3}}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{7}{3}}+4 a^{7} x^{\frac{10}{3}}-28 a\, b^{6} x^{\frac{4}{3}}-420 b^{4} a^{3} x^{2}-126 a^{2} b^{5} x^{\frac{5}{3}}-3 b^{7} x\right)}{4 \left(b+a\, x^{\frac{1}{3}}\right)^{7}}$

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/4*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*x*(4*a^7*x^(7/3)+42*x^2
*a^6*b+140*a^4*b^3*ln(x)*x^(4/3)+252*b^2*a^5*x^(5/3)-420*b^4*a^3*x-126*a^2*
b^5*x^(2/3)-28*a*b^6*x^(1/3)-3*b^7)/(b+a*x^(1/3))^7
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} dx$$

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}\,dx = 35\,a^4b^3\log\left(x\right) + \frac{4\,a^7x^{\frac{7}{3}}+42\,a^6bx^2+252\,a^5b^2x^{\frac{5}{3}}-420\,a^3b^4x-126\,a^2b^5x^{\frac{2}{3}}-28\,ab^6x^{\frac{1}{3}}-3\,b^7}{4\,x^{\frac{4}{3}}}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3)
 - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}\,dx = a^7x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+35\,a^4b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x) + \frac{21}{2}\,a^6bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+63\,a^5b^2x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x) - \frac{420\,a^3b^4x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+126\,a^2b^5x^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+28\,ab^6x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+3}{4\,x^{\frac{4}{3}}}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3)
)*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/
3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}\,dx=\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{7/2}\,dx$$

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 291

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=-\frac{3b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{15ab^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}+\frac{30a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{30a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] -3/2*b^5*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-15*a*b
^4*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+30*a^3*b^2*x
^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+15/2*a^4*b*x^(2/
3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^5*x*(a^2+b^2/x^(2/
3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+10*a^2*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = -\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(ab+\frac{b^2}{x}\right)^5x^2\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\frac{(b^2+abx)^5}{x^3}\,dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(10a^3b^7+\frac{b^{10}}{x^3}+\frac{5ab^9}{x^2}+\frac{10a^2b^8}{x}+5a^4b^6x+a^5b^5x^2\right)dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{15ab^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{30a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{15a^4b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{10a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(-3b^5-30ab^4\sqrt[3]{x}+60a^3b^2x+15a^4bx^{4/3}+2a^5x^{5/3}+20a^2b^3x^{2/3}\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}x}$$

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/
3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^2/
x^(2/3)]*x)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}}a^2+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x\left(2a^5x^{\frac{5}{3}}+15ba^4x^{\frac{4}{3}}+20a^2b^3\ln(x)x^{\frac{2}{3}}+60a^3b^2x-30b^4ax^{\frac{1}{3}}-3b^5\right)}{2\left(b+ax^{\frac{1}{3}}\right)^5}$	91
default	$\frac{\left(\frac{x^{\frac{2}{3}}a^2+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x\left(2a^5x^{\frac{5}{3}}+15ba^4x^{\frac{4}{3}}+20a^2b^3\ln(x)x^{\frac{2}{3}}+60a^3b^2x-30b^4ax^{\frac{1}{3}}-3b^5\right)}{2\left(b+ax^{\frac{1}{3}}\right)^5}$	91

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(2*a^5*x^(5/3)+15*b*a
^4*x^(4/3)+20*a^2*b^3*ln(x)*x^(2/3)+60*a^3*b^2*x-30*b^4*a*x^(1/3)-3*b^5)/(b
+a*x^(1/3))^5
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}\,dx=10\,a^2b^3\log\left(x\right)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.44

$$\begin{aligned}\int \Bigg(a^2&+\frac{b^2}{x^{2/3}}\\&+\frac{2ab}{\sqrt[3]{x}}\Bigg)^{5/2}\,dx=a^5x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)\\&+\frac{15}{2}\,a^4bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+30\,a^3b^2x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)\\&-\frac{3\left(10\,ab^4x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+b^5\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)\right)}{2\,x^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2
/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx=\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{5/2} dx$$

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 189

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx=\frac{9ab^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log\left(\sqrt[3]{x}\right)}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 9*a*b^2*x^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+9/2*a^2
*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^3*x*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+b^3*ln(x)*(a^2+b^2/x^(2/3)+2*
a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

= {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} + \frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}} + \frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}} + \frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x^2\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^3}{x}\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(3ab^5+\frac{b^6}{x}+3a^2b^4x+a^3b^3x^2\right)\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{9ab^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(18ab^2\sqrt[3]{x}+9a^2bx^{2/3}+2a^3x+2b^3\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
[Out] ((b + a*x^(1/3))*(18*a*b^2*x^(1/3) + 9*a^2*b*x^(2/3) + 2*a^3*x + 2*b^3*Log[
x]))/(2*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.37

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 a^{2} b \,x^{\frac{2}{3}}+2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 a^{2} b \,x^{\frac{2}{3}}+2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(2*a^3*x+9*a^2*b*x^(2
/3)+2*b^3*ln(x)+18*b^2*a*x^(1/3))/(b+a*x^(1/3))^3
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}\,dx=\int \left(a^{2}+\frac{2ab}{\sqrt[3]{x}}+\frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.16

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3x+b^3\log(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9ab^2x^{\frac{1}{3}}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+b^3\log(|x|)\,\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+9ab^2x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sg
n(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(
a*x + b*x^(2/3))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{3/2} dx$$

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{3b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 3/2*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a*x*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 1369, 14}

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(ab+\frac{b^2}{x}\right)x^2\,dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int\left(b^2x+abx^2\right)\,dx, x, \sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

$$= \frac{3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{\left(3b+2a\sqrt[3]{x}\right)\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}x}{2\left(b+a\sqrt[3]{x}\right)}$$

```
[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] ((3*b + 2*a*x^(1/3))*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x)/(2*(b + a*x^(1/3)))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x \left(2 a \,x^{\frac{1}{3}}+3 b \right)}{2 b +2 a \,x^{\frac{1}{3}}}$	47
default	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x^{\frac{1}{3}} \left(3 b \,x^{\frac{2}{3}}+2 a x \right)}{2 b +2 a \,x^{\frac{1}{3}}}$	50

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x*(2*a*x^(1/3)+3*b)/(b+
a*x^(1/3))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\int \sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}\,dx$$

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=ax+\frac{3}{2}\,bx^{\frac{2}{3}}$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
[Out] a*x + 3/2*b*x^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=ax\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)+\frac{3}{2}\,bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)$$

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{x\,\left(a+\frac{3\,b}{2\,x^{1/3}}\right)\,\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}{a+\frac{b}{x^{1/3}}}$$

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
[Out] (x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
+ b/x^(1/3))
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{3b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

```
[Out] 3*b^2*(a+b/x^(1/3))*x^(1/3)/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3/2*b
*(a+b/x^(1/3))*x^(2/3)/a^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a+b/x^(1/
3))*x/a/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3*b^3*(a+b/x^(1/3))*ln(b+a*x^
(1/3))/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

= {1355, 1369, 269, 45}

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx = -\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} - \frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{x}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^3}{b^2+abx}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{b}{a^3}-\frac{x}{a^2}+\frac{x^2}{ab}-\frac{b^2}{a^3(b+ax)}\right)\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(6ab^2\sqrt[3]{x}-3a^2bx^{2/3}+2a^3x-6b^3\log\left(b+a\sqrt[3]{x}\right)\right)}{2a^4\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

```
[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b
 + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result	size
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3a^{2}b\,x^{\frac{2}{3}}+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3a^{2}b\,x^{\frac{2}{3}}+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(b+a*x^(1/3))*(-2*a^3*x+3*a^2*b*x^(2/3)+6*b^3*ln(b+a*x^(1/3))-6*b^2*a*
x^(1/3))/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)/a^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\int \frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=-\frac{3\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3
))/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a
^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 300

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}+\frac{18b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{9b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

```
[Out] 3/2*b^5*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-15*b^4*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))
^(1/2)+18*b^2*(a+b/x^(1/3))*x^(1/3)/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/
2)-9/2*b*(a+b/x^(1/3))*x^(2/3)/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a
+b/x^(1/3))*x/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-30*b^3*(a+b/x^(1/3)
)*ln(b+a*x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2} - \frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)} - \frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} + \frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} - \frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} + \frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^5}{(b^2+abx)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{6}{a^5b}-\frac{3x}{a^4b^2}+\frac{x^2}{a^3b^3}-\frac{b^2}{a^5(b+ax)^3}+\frac{5b}{a^5(b+ax)^2}-\frac{10}{a^5(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2}-\frac{15\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}+\frac{18\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{9\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{30\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(-27b^5+6ab^4\sqrt[3]{x}+63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}\right.}{2a^6\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right)^{3/2}x}$$

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]

[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(-2a^5x^{\frac{5}{3}}+5b\,a^4x^{\frac{4}{3}}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^3x^{\frac{2}{3}}-20a^3b^2x+120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^4x^{\frac{1}{3}}-63b^3x^{\frac{2}{3}}a^2+60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^5-6b^4\right.}{2a^6x\left(\frac{x^{\frac{2}{3}}a^2+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$
default	$\frac{\left(2a^5x^{\frac{5}{3}}-5b\,a^4x^{\frac{4}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^3x^{\frac{2}{3}}+63b^3x^{\frac{2}{3}}a^2-120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^4x^{\frac{1}{3}}+6b^4a\,x^{\frac{1}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^5+20a^3b^2x\right.}{2a^6x\left(\frac{x^{\frac{2}{3}}a^2+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$

[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-2*a^5*x^(5/3)+5*b*a^4*x^(4/3)+60*ln(b+a*x^(1/3))*a^2*b^3*x^(2/3)-20*a^3*b^2*x+120*ln(b+a*x^(1/3))*a*b^4*x^(1/3)-63*b^3*x^(2/3)*a^2+60*ln(b+a*x^(1/3))*b^5-6*b^4*a*x^(1/3)+27*b^5)*(b+a*x^(1/3))/a^6/x/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}-\frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*b^
3*log(a*x^(1/3) + b)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = -\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x + b*x^(2/3))*sgn(x)) - 3/2*(10
*a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x + b*x^(2/3))*sgn(x))
 + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x + b*x^
(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2}}\,dx$$

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 410

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{45b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{15b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

```
[Out] 3/4*b^7*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-7*b^6*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3)
)^(1/2)+63/2*b^5*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x
^(1/3))^(1/2)-105*b^4*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*
b/x^(1/3))^(1/2)+45*b^2*(a+b/x^(1/3))*x^(1/3)/a^7/(a^2+b^2/x^(2/3)+2*a*b/x^
(1/3))^(1/2)-15/2*b*(a+b/x^(1/3))*x^(2/3)/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3
```

```
))^(1/2)+(a+b/x^(1/3))*x/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-105*b^3*
(a+b/x^(1/3))*ln(b+a*x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4} - \frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3} + \frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2} - \frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)} - \frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} + \frac{45b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} - \frac{15bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}} + \frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^7}{(b^2+abx)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{15}{a^7b^3}-\frac{5x}{a^6b^4}+\frac{x^2}{a^5b^5}-\frac{b^2}{a^7(b+ax)^5}+\frac{7b}{a^7(b+ax)^4}-\frac{21}{a^7(b+ax)^3}+\frac{35}{a^7b(b+ax)^2}-\frac{}{a^7}\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^7+\frac{b^8}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7\left(ab^6+\frac{b^7}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3}$$
$$+ \frac{63\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{105\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}$$
$$+ \frac{45\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{15\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$
$$+ \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{105\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(-319b^7-856ab^6\sqrt[3]{x}-444a^2b^5x^{2/3}+544a^3b^4x+556a^4b^3x^{4/3}+\right.}{4a^8\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right.}$$

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*
a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7
*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^
(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.49

method	result
derivativedivides	$-\frac{\left(-4a^{7}x^{\frac{7}{3}}+14x^{2}a^{6}b+420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{4}b^{3}x^{\frac{4}{3}}-84b^{2}a^{5}x^{\frac{5}{3}}+1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{3}b^{4}x-556a^{4}b^{3}x^{\frac{4}{3}}+2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b\right.}{4a^{8}x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x}{x^{\frac{2}{3}}}\right.}$
default	$\frac{\left(4a^{7}x^{\frac{7}{3}}+84b^{2}a^{5}x^{\frac{5}{3}}-420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{4}b^{3}x^{\frac{4}{3}}+556a^{4}b^{3}x^{\frac{4}{3}}-2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b^{5}x^{\frac{2}{3}}-444a^{2}b^{5}x^{\frac{2}{3}}-1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b\right.}{4a^{8}x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+}{x^{\frac{2}{3}}}\right.}$

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-4*a^7*x^(7/3)+14*x^2*a^6*b+420*ln(b+a*x^(1/3))*a^4*b^3*x^(4/3)-84*b^
2*a^5*x^(5/3)+1680*ln(b+a*x^(1/3))*a^3*b^4*x-556*a^4*b^3*x^(4/3)+2520*ln(b+
a*x^(1/3))*a^2*b^5*x^(2/3)-544*b^4*a^3*x+1680*ln(b+a*x^(1/3))*a*b^6*x^(1/3)
+444*a^2*b^5*x^(2/3)+420*ln(b+a*x^(1/3))*b^7+856*a*b^6*x^(1/3)+319*b^7)*(b+
a*x^(1/3))/a^8/x^(5/3)/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{4\,a^7x^{\frac{7}{3}} - 14\,a^6bx^2 + 84\,a^5b^2x^{\frac{5}{3}} + 556\,a^4b^3x^{\frac{4}{3}} + 544\,a^3b^4x - 444\,a^2b^5x^{\frac{2}{3}} - 856}{4\left(a^{12}x^{\frac{4}{3}} + 4\,a^{11}bx + 6\,a^{10}b^2x^{\frac{2}{3}} + 4\,a^9b^3x^{\frac{1}{3}} + a^8b^4\right)} - \frac{105\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/3
) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a^1
2*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^4)
- 105*b^3*log(a*x^(1/3) + b)/a^8
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.36

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = -\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{420\,a^3b^4x + 1134\,a^2b^5x^{\frac{2}{3}} + 1036\,ab^6x^{\frac{1}{3}} + 319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^{10}x - 15\,a^9bx^{\frac{2}{3}} + 90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x + b*x^(2/3))*sgn(x)) - 1/4*(4
20*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x^(
1/3) + b)^4*a^8*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a^10*x - 15*a^9*b*x^(
2/3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{5/2}}\,dx$$

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\begin{aligned}\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = &-\frac{4b^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(a+\frac{b}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{a+\frac{b}{\sqrt[4]{x}}} \\ &+\frac{20a^3b^2\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4b\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x^{3/4}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)} \\ &+\frac{a^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20ab^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\log\left(\sqrt[4]{x}\right)}{a+\frac{b}{\sqrt[4]{x}}}\end{aligned}$$

```
[Out] -4*b^5*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))/x^(1/4)+40*a^2*b
^3*x^(1/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20/3*a^4*b*x
^(3/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+a^5*x*(a^2+2*a*b
/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+5*a*b^4*ln(x)*(a^2+2*a*b/x^(1/4)+
b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20*a^3*b^2*x^(1/2)*(a^2+2*a*b/x^(1/4)+b^2/
x^(1/2))^(1/2)/(a+b/x^(1/4))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = -\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)} + \frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}} + \frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}} + \frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}} + \frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)} + \frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

```
[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 4\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^3\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^3\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x^2}\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(10a^2b^8+\frac{b^{10}}{x^2}+\frac{5ab^9}{x}+10a^3b^7x+5a^4b^6x^2+a^5b^5x^3\right)\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= -\frac{4b^6\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{ab+\frac{b^2}{\sqrt[4]{x}}}+\frac{20a^3b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt[4]{x}}}$$
$$+\frac{20a^4b^2\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x^{3/4}}{3\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x}{a+\frac{b}{\sqrt[4]{x}}}+\frac{5ab^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[4]{x}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=\frac{\sqrt{\frac{\left(b+a\sqrt[4]{x}\right)^2}{\sqrt{x}}}\left(-12b^5+120a^2b^3\sqrt{x}+60a^3b^2x^{3/4}+20a^4bx+3a^5x^{5/4}+15ab^4\sqrt[4]{x}\log(x)\right)}{3\left(b+a\sqrt[4]{x}\right)}$$

```
[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^
2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b +
a*x^(1/4)))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^2\sqrt{x}+b^2+2ab\,x^{\frac{1}{4}}}{\sqrt{x}}\right)^{\frac{5}{2}} x\left(3a^5x^{\frac{5}{4}}+20x\,a^4b+60a^3b^2x^{\frac{3}{4}}+15b^4a\ln(x)x^{\frac{1}{4}}+120a^2b^3\sqrt{x}-12b^5\right)}{3\left(a\,x^{\frac{1}{4}}+b\right)^5}$	91
default	$\frac{\left(\frac{a^2x^{\frac{3}{4}}+b^2x^{\frac{1}{4}}+2ab\sqrt{x}}{x^{\frac{3}{4}}}\right)^{\frac{5}{2}} x\left(3a^5x^{\frac{5}{4}}+20x\,a^4b+60a^3b^2x^{\frac{3}{4}}+15b^4a\ln(x)x^{\frac{1}{4}}+120a^2b^3\sqrt{x}-12b^5\right)}{3\left(a\,x^{\frac{1}{4}}+b\right)^5}$	95

```
[In] int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a^2*x^(1/2)+b^2+2*a*b*x^(1/4))/x^(1/2))^(5/2)*x*(3*a^5*x^(5/4)+20*x*a
^4*b+60*a^3*b^2*x^(3/4)+15*b^4*a*ln(x)*x^(1/4)+120*a^2*b^3*x^(1/2)-12*b^5)/
(a*x^(1/4)+b)^5
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \text{Timed out}$$

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \text{Timed out}$$

```
[In] integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = 5\,ab^4\log\left(x\right) + \frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 120
*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = a^5 x \operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 5\,ab^4\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + \frac{20}{3}\,a^4 b x^{\frac{3}{4}}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 20\,a^3b^2\sqrt{x}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) + 40\,a^2b^3x^{\frac{1}{4}}\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x) - \frac{4\,b^5\operatorname{sgn}\left(ax+bx^{\frac{3}{4}}\right)\operatorname{sgn}(x)}{x^{\frac{1}{4}}}$$

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4)
)*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt
(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*s
gn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)^{5/2} dx$$

```
[In] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 291

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx=\frac{25ab^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{50a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^4b\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{4/5}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x}{a+\frac{b}{\sqrt[5]{x}}}+\frac{5b^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\log\left(\sqrt[5]{x}\right)}{a+\frac{b}{\sqrt[5]{x}}}$$

```
[Out] 25*a*b^4*x^(1/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25*a^2
*b^3*x^(2/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+50/3*a^3*b
^2*x^(3/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25/4*a^4*b*x
^(4/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+a^5*x*(a^2+b^2/x
^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+b^5*ln(x)*(a^2+b^2/x^(2/5)+2*a*b/
x^(1/5))^(1/2)/(a+b/x^(1/5))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)} + \frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}$$

```
[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 5\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^4\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^4\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x}\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(5ab^9+\frac{b^{10}}{x}+10a^2b^8x+10a^3b^7x^2+5a^4b^6x^3+a^5b^5x^4\right)\,dx, x, \sqrt[5]{}\right.}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{25ab^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{ab+\frac{b^2}{\sqrt[5]{x}}}+\frac{25a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{ab+\frac{b^2}{\sqrt[5]{x}}}+\frac{50a^3b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$
$$+\frac{25a^4b^2\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{4/5}}{4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x}{a+\frac{b}{\sqrt[5]{x}}}+\frac{b^6\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[5]{x}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[5]{x}\right)\left(300ab^4\sqrt[5]{x}+300a^2b^3x^{2/5}+200a^3b^2x^{3/5}+75a^4bx^{4/5}+12a^5x+12b^5\log(x)\right)}{12\sqrt{\frac{\left(b+a\sqrt[5]{x}\right)^2}{x^{2/5}}}\sqrt[5]{x}}$$

```
[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] ((b + a*x^(1/5))*(300*a*b^4*x^(1/5) + 300*a^2*b^3*x^(2/5) + 200*a^3*b^2*x^(
3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x + 12*b^5*Log[x]))/(12*Sqrt[(b + a*x^(1/5
))^2/x^(2/5)]*x^(1/5))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x^{\frac{2}{5}}+2abx^{\frac{1}{5}}+b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}x\left(12a^5x+75ba^4x^{\frac{4}{5}}+200a^3b^2x^{\frac{3}{5}}+300a^2b^3x^{\frac{2}{5}}+12b^5\ln(x)+300b^4ax^{\frac{1}{5}}\right)}{12\left(ax^{\frac{1}{5}}+b\right)^5}$	91
default	$\frac{\left(\frac{a^2x^{\frac{2}{5}}+2abx^{\frac{1}{5}}+b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}x\left(12a^5x+75ba^4x^{\frac{4}{5}}+200a^3b^2x^{\frac{3}{5}}+300a^2b^3x^{\frac{2}{5}}+12b^5\ln(x)+300b^4ax^{\frac{1}{5}}\right)}{12\left(ax^{\frac{1}{5}}+b\right)^5}$	91

```
[In] int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(12*a^5*x+75*b*a^4*x
^(4/5)+200*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*b^4*a*x^(1/
5))/(a*x^(1/5)+b)^5
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
[Out] Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = a^5x+b^5\log(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}+\frac{50}{3}a^3b^2x^{\frac{3}{5}}+25a^2b^3x^{\frac{2}{5}}+25ab^4x^{\frac{1}{5}}$$

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^3
*x^(2/5) + 25*a*b^4*x^(1/5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.43

$$\begin{aligned}\int \Bigg(a^2&+\frac{b^2}{x^{2/5}}\\&+\frac{2ab}{\sqrt[5]{x}}\Bigg)^{5/2} dx = a^5x\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+b^5\log(|x|)\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)\\&+\frac{25}{4}a^4bx^{\frac{4}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+\frac{50}{3}a^3b^2x^{\frac{3}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)\\&+25a^2b^3x^{\frac{2}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)+25ab^4x^{\frac{1}{5}}\operatorname{sgn}\left(ax+bx^{\frac{4}{5}}\right)\operatorname{sgn}(x)\end{aligned}$$

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sg
n(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5
)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn
(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx=\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)^{5/2} dx$$

```
[In] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 222

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

```
[Out] 20*a/b^5/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-5/4*a^4/b^5/(a+b*x^(1/5))^3/
(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)+20/3*a^3/b^5/(a+b*x^(1/5))^2/(a^2+2*a
*b*x^(1/5)+b^2*x^(2/5))^(1/2)-15*a^2/b^5/(a+b*x^(1/5))/(a^2+2*a*b*x^(1/5)+b
^2*x^(2/5))^(1/2)+5*(a+b*x^(1/5))*ln(a+b*x^(1/5))/b^5/(a^2+2*a*b*x^(1/5)+b^
2*x^(2/5))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

```
[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 5\text{Subst}\left(\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, \sqrt[5]{x}\right)\\
&= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \frac{x^4}{(ab+b^2x)^5}\,dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}\\
&= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right)dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} \\
&\quad + \frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} \\
&\quad - \frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

$$
\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\right.}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}
$$

```
[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*
(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b
*x^(1/5))^2])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{5\left(12\ln\left(a+b\,x^{\frac{1}{5}}\right)b^4x^{\frac{4}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a\,b^3x^{\frac{3}{5}}+72\ln\left(a+b\,x^{\frac{1}{5}}\right)a^2b^2x^{\frac{2}{5}}+48a\,b^3x^{\frac{3}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a^3b\,x^{\frac{1}{5}}+108a^2b^2x^{\frac{2}{5}}\right.}{12b^5\left(\left(a+b\,x^{\frac{1}{5}}\right)^2\right)^{\frac{5}{2}}}$
default	Expression too large to display

```
[In] int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*(12*ln(a+b*x^(1/5))*b^4*x^(4/5)+48*ln(a+b*x^(1/5))*a*b^3*x^(3/5)+72*ln
(a+b*x^(1/5))*a^2*b^2*x^(2/5)+48*a*b^3*x^(3/5)+48*ln(a+b*x^(1/5))*a^3*b*x^(
1/5)+108*a^2*b^2*x^(2/5)+12*ln(a+b*x^(1/5))*a^4+88*a^3*b*x^(1/5)+25*a^4)*(a
+b*x^(1/5))/b^5/((a+b*x^(1/5))^2)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\left(300\,a^5b^{15}x^3+100\,a^{15}b^5x+25\,a^{20}+12\left(b^{20}x^4+4\,a^5b^{15}x^3+6\,a^{10}b^{10}x^2+\right.\right.}{}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^
15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*
b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^
2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*
a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (20
7*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^
25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\left(48\,ab^3x^{\frac{3}{5}}+108\,a^2b^2x^{\frac{2}{5}}+88\,a^3bx^{\frac{1}{5}}+25\,a^4\right)}{12\left(b^9x^{\frac{4}{5}}+4\,ab^8x^{\frac{3}{5}}+6\,a^2b^7x^{\frac{2}{5}}+4\,a^3b^6x^{\frac{1}{5}}+a^4b^5\right)} + \frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^5}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/(
b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4
*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\,\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^5\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)} + \frac{5\left(48\,ab^2x^{\frac{3}{5}}+108\,a^2bx^{\frac{2}{5}}+88\,a^3x^{\frac{1}{5}}+\frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^4 b^4\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5)
 + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sg
n(b*x^(1/5) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)^{5/2}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
[Out] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 391

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = -\frac{6b^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(a+\frac{b}{\sqrt[6]{x}}\right)\sqrt[6]{x}} + \frac{126a^2b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{105a^3b^4\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{70a^4b^3\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{63a^5b^2\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)} + \frac{42a^6b\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{5/6}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)} + \frac{a^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x}{a+\frac{b}{\sqrt[6]{x}}} + \frac{42ab^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\log\left(\sqrt[6]{x}\right)}{a+\frac{b}{\sqrt[6]{x}}}$$

```
[Out] -6*b^7*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))/x^(1/6)+126*a^2*
b^5*x^(1/6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+105*a^3*b^4
*x^(1/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+63/2*a^5*b^2*x
```

```
^(2/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+42/5*a^6*b*x^(5/
6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+a^7*x*(a^2+b^2/x^(1/
3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+7*a*b^6*ln(x)*(a^2+b^2/x^(1/3)+2*a*b/
x^(1/6))^(1/2)/(a+b/x^(1/6))+70*a^4*b^3*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/
2)*x^(1/2)/(a+b/x^(1/6))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = -\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)} + \frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)} + \frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)} + \frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}} + \frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

```
[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = 6\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^5\,dx, x, \sqrt[6]{x}\right)$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^5\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^7}{x^2}\,dx, x, \sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(21a^2b^{12}+\frac{b^{14}}{x^2}+\frac{7ab^{13}}{x}+35a^3b^{11}x+35a^4b^{10}x^2+21a^5b^9x^3+7a^6b^{8}\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= -\frac{6b^8\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)\sqrt[6]{x}} + \frac{126a^2b^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}} + \frac{105a^3b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}} + \frac{70a^4b^4\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt{x}}{ab+\frac{b^2}{\sqrt[6]{x}}} + \frac{63a^5b^3\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)} + \frac{42a^6b^2\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{5/6}}{5\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)} + \frac{a^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x}{a+\frac{b}{\sqrt[6]{x}}} + \frac{7ab^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\log(x)}{ab+\frac{b^2}{\sqrt[6]{x}}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[6]{x}\right)^2}{\sqrt[3]{x}}}\left(-60b^7+1260a^2b^5\sqrt[3]{x}+1050a^3b^4\sqrt{x}+700a^4b^3x^{2/3}+315a^5b^2x^{5/6}+84a^6bx+\right.}{10\left(b+a\sqrt[6]{x}\right)}$$

```
[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3
*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*
a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.30

$$\frac{\left(\frac{a^2\sqrt{x}+2ab\,x^{\frac{1}{3}}+x^{\frac{1}{6}}b^2}{\sqrt{x}}\right)^{\frac{7}{2}} x\left(10a^7x^{\frac{7}{6}}+84a^6bx+315b^2a^5x^{\frac{5}{6}}+700a^4b^3x^{\frac{2}{3}}+1050b^4a^3\sqrt{x}+70a\,b^6\ln(x)\,x^{\frac{1}{6}}+126\right.}{10\left(a\,x^{\frac{1}{6}}+b\right)^7}$$

```
[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

```
[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+x^(1/6)*b^2)/x^(1/2))^(7/2)*x*(10*a^7*x^(7
/6)+84*a^6*b*x+315*b^2*a^5*x^(5/6)+700*a^4*b^3*x^(2/3)+1050*b^4*a^3*x^(1/2)
+70*a*b^6*ln(x)*x^(1/6)+1260*a^2*b^5*x^(1/3)-60*b^7)/(a*x^(1/6)+b)^7
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = 7\,ab^6\log\left(x\right) + \frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)
/x^(1/6)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx = a^7x\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 7\,ab^6\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + \frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + \frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 70\,a^4b^3\sqrt{x}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 105\,a^3b^4x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 126\,a^2b^5x^{\frac{1}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) - \frac{6\,b^7\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{6}}}$$

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6)
)*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^
(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))
*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(
1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6
)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx = \int \left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)^{7/2}\,dx$$

```
[In] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log\left(b+cx^n\right)}{c^3n}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+b^2*ln(b+c*x^n)/c^3/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 45}

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1+3n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log\left(b+cx^n\right)}{c^3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx = \frac{cx^n(-2b+cx^n)+2b^2\log\left(b+cx^n\right)}{2c^3n}$$

```
[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2cn}-\frac{b\,x^n}{c^2n}+\frac{b^2\ln\left(x^n+\frac{b}{c}\right)}{c^3n}$	47
norman	$\left(\frac{e^{3n\ln(x)}}{2cn}-\frac{b\,e^{2n\ln(x)}}{c^2n}\right)e^{-n\ln(x)}+\frac{b^2\ln\left(c\,e^{n\ln(x)}+b\right)}{c^3n}$	62

```
[In] int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/2/c/n*(x^n)^2-b*x^n/c^2/n+b^2/c^3/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \frac{c^2x^{2\,n} - 2\,bcx^n + 2\,b^2\log\left(cx^n + b\right)}{2\,c^3n}$$

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

Sympy [A] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{4n-1}}{3bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{b^2\log\left(\frac{b}{c}+x^n\right)}{c^3n} - \frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(3*b*n*x**n), Eq
(c, 0)), (log(x)/(b + c), Eq(n, 0)), (b**2*log(b/c + x**n)/(c**3*n) - b*x**
n/(c**2*n) + x**(2*n)/(2*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n} + \frac{cx^{2\,n} - 2\,bx^n}{2\,c^2n}$$

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

```
[In] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}$$

```
[Out] x^n/c/n-b*ln(b+c*x^n)/c^2/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 45}

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}$$

```
[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1+2n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x}{b+cx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{x^n}{cn}-\frac{b\log\left(b+cx^n\right)}{c^2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{cx^n-b\log\left(cn(b+cx^n)\right)}{c^2n}$$

```
[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] (c*x^n - b*Log[c*n*(b + c*x^n)])/(c^2*n)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^n}{cn}-\frac{b\ln\left(x^n+\frac{b}{c}\right)}{c^2n}$	31
norman	$\frac{{\rm e}^{n\ln(x)}}{cn}-\frac{b\ln\left(c\,{\rm e}^{n\ln(x)}+b\right)}{c^2n}$	33

```
[In] int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] x^n/c/n-b/c^2/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \frac{cx^n - b\log\left(cx^n + b\right)}{c^2 n}$$

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 8.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for}\: c = 0 \wedge n = 0 \\ \frac{x x^{-n} x^{3n-1}}{2bn} & \text{for}\: c = 0 \\ \frac{\log(x)}{b+c} & \text{for}\: n = 0 \\ -\frac{b\log\left(\frac{b}{c}+x^n\right)}{c^2 n} + \frac{x^n}{cn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(2*b*n*x**n), Eq
(c, 0)), (log(x)/(b + c), Eq(n, 0)), (-b*log(b/c + x**n)/(c**2*n) + x**n/(c
*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2 n}$$

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}} \, dx = \int \frac{x^{3\,n-1}}{cx^{2\,n} + bx^n} \, dx$$

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}} \, dx = \int \frac{x^{3\,n-1}}{b\,x^n + c\,x^{2\,n}} \, dx$$

```
[In] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 15

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

```
[Out] ln(b+c*x^n)/c/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1598, 266}

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

```
[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b + c*x^n]/(c*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1+n}}{b+cx^n}\,dx\\ &= \frac{\log(b+cx^n)}{cn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx = \frac{\log(b+cx^n)}{cn}$$

```
[In] Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
[Out] Log[b + c*x^n]/(c*n)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln(c\,e^{n\ln(x)}+b)}{cn}$	18
risch	$\frac{\ln\left(x^n+\frac{b}{c}\right)}{cn}$	18

```
[In] int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx = \frac{\log(cx^n+b)}{cn}$$

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^n + b)/(c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(10) = 20$.

Time = 1.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{2n-1}}{bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(b*n*x**n), Eq(c
, 0)), (log(x)/(b + c), Eq(n, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/(c
*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^n + b)/c)/(c*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n} + bx^n}\,dx$$

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

```
[In] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

```
[Out] ln(x)/b-ln(b+c*x^n)/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1598, 272, 36, 29, 31}

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

```
[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x(b+cx^n)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{bn} - \frac{c\text{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^n\right)}{bn}\\ &= \frac{\log(x)}{b} - \frac{\log(b+cx^n)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx = \frac{\log(x^n) - \log(bn(b+cx^n))}{bn}$$

```
[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
[Out] (Log[x^n] - Log[b*n*(b + c*x^n)])/(b*n)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
norman	$\frac{\ln(x)}{b}-\frac{\ln\left(c\,{\mathrm e}^{n\ln(x)}+b\right)}{b n}$	26
risch	$\frac{\ln(x)}{b}-\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{b n}$	26

[In] int(x^(-1+n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)

[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\frac{n\log(x)-\log(cx^n+b)}{bn}$$

[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] (n*log(x) - log(c*x^n + b))/(b*n)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.

Time = 1.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\begin{cases}\tilde{\infty}\log(x) & \text{for } b=0\wedge c=0\wedge n=0\\ -\frac{xx^{-2n}x^{n-1}}{cn} & \text{for } b=0\\ \frac{\log(x)}{b} & \text{for } c=0\\ \frac{\log(x)}{b+c} & \text{for } n=0\\ \frac{2\log(x)}{b}-\frac{\log\left(\frac{bx^n}{c}+x^{2n}\right)}{bn} & \text{otherwise}\end{cases}$$

[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)

[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(n - 1)/(c*n
*x**(2*n)), Eq(b, 0)), (log(x)/b, Eq(c, 0)), (log(x)/(b + c), Eq(n, 0)), (2
*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)}{b\,n}$$

```
[In] int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] -(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}$$

```
[Out] -1/2/b/n/(x^(2*n))+c/b^2/n/(x^n)+c^2*ln(x)/b^3-c^2*ln(b+c*x^n)/b^3/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 46}

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

```
[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*1/(b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^
n])/(b^3*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1-2n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx = -\frac{bx^{-2n}(b-2cx^n)-2c^2\log(x^n)+2c^2\log(b+cx^n)}{2b^3n}$$

```
[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
[Out] -1/2*((b*(b - 2*c*x^n))/x^(2*n) - 2*c^2*Log[x^n] + 2*c^2*Log[b + c*x^n])/(b
^3*n)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{c\,x^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln\left(x^n+\frac{b}{c}\right)}{b^3n}$	58
norman	$\left(\frac{c\,\mathrm{e}^{n\ln(x)}}{b^2n}-\frac{1}{2bn}+\frac{c^2\ln(x)\mathrm{e}^{2n\ln(x)}}{b^3}\right)\mathrm{e}^{-2n\ln(x)}-\frac{c^2\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^3n}$	69

```
[In] int(x^(-1-n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

[Out] c/b^2/n/(x^n)-1/2/b/n/(x^n)^2+c^2*ln(x)/b^3-c^2/b^3/n*ln(x^n+b/c)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \frac{2\,c^2nx^{2\,n}\log\left(x\right) - 2\,c^2x^{2\,n}\log\left(cx^n + b\right) + 2\,bcx^n - b^2}{2\,b^3nx^{2\,n}}$$

[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b^2)/(b^3*n*x^(2*n))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 29.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \tilde{\infty}\log\left(x\right) & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{xx^{-2n}x^{-n-1}}{3cn} & \text{for}\: b = 0 \\ -\frac{xx^{-n}x^{-n-1}}{2bn} & \text{for}\: c = 0 \\ \frac{\log\left(x\right)}{b+c} & \text{for}\: n = 0 \\ -\frac{x^{-2n}}{2bn} + \frac{cx^{-n}}{b^2n} + \frac{c^2\log\left(x^n\right)}{b^3n} - \frac{c^2\log\left(\frac{b}{c} + x^n\right)}{b^3n} & \text{otherwise} \end{cases}$$

[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)

[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(3*c*n*x**(2*n)), Eq(b, 0)), (-x*x**(-n - 1)/(2*b*n*x**n), Eq(c, 0)), (log(x)/(b + c), Eq(n, 0)), (-1/(2*b*n*x**(2*n)) + c/(b**2*n*x**n) + c**2*log(x**n)/(b**3*n) - c**2*log(b/c + x**n)/(b**3*n), True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \frac{c^2\log(x)}{b^3} - \frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n} + \frac{2\,cx^n - b}{2\,b^2nx^{2\,n}}$$

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*
x^(2*n))
```

Giac [F]

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{n+1}\,(b\,x^n + c\,x^{2\,n})}\,dx$$

```
[In] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}$$

```
[Out] -1/3/b/n/(x^(3*n))+1/2*c/b^2/n/(x^(2*n))-c^2/b^3/n/(x^n)-c^3*ln(x)/b^4+c^3*
ln(b+c*x^n)/b^4/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 46}

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = \frac{c^3\log(b+cx^n)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

```
[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/3*1/(b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])
/b^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1-3n}}{b+cx^n}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{bx^{-3n}(2b^2-3bcx^n+6c^2x^{2n})+6c^3\log(x^n)-6c^3\log(b+cx^n)}{6b^4n}$$

```
[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
[Out] -1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*Log[x^n] - 6*
c^3*Log[b + c*x^n])/(b^4*n)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{c^2 x^{-n}}{b^3 n}+\frac{c\,x^{-2n}}{2b^2 n}-\frac{x^{-3n}}{3bn}-\frac{c^3 \ln(x)}{b^4}+\frac{c^3 \ln\left(x^n+\frac{b}{c}\right)}{b^4 n}$	75
norman	$\left(-\frac{1}{3bn}+\frac{c\,\mathrm{e}^{n\ln(x)}}{2b^2 n}-\frac{c^2 \mathrm{e}^{2n\ln(x)}}{b^3 n}-\frac{c^3 \ln(x)\mathrm{e}^{3n\ln(x)}}{b^4}\right)\mathrm{e}^{-3n\ln(x)}+\frac{c^3 \ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^4 n}$	88

```
[In] int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] -c^2/b^3/n/(x^n)+1/2*c/b^2/n/(x^n)^2-1/3/b/n/(x^n)^3-c^3*ln(x)/b^4+c^3/b^4/
n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{6\,c^3nx^{3\,n}\log\left(x\right)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2*
n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{c^3\log\left(x\right)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*b
*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

Giac [F]

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}} \, dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n} + bx^n} \, dx$$

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}} \, dx = \int \frac{1}{x^{2\,n+1}\,(b\,x^n + c\,x^{2\,n})} \, dx$$

```
[In] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5n}$$

```
[Out] -1/4/b/n/(x^(4*n))+1/3*c/b^2/n/(x^(3*n))-1/2*c^2/b^3/n/(x^(2*n))+c^3/b^4/n/
(x^n)+c^4*ln(x)/b^5-c^4*ln(b+c*x^n)/b^5/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 46}

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx=-\frac{c^4\log(b+cx^n)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

```
[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/4*1/(b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b
^4*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{-1-4n}}{b+cx^n}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5} - \frac{c}{b^2x^4} + \frac{c^2}{b^3x^3} - \frac{c^3}{b^4x^2} + \frac{c^4}{b^5x} - \frac{c^5}{b^5(b+cx)}\right)dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-4n}}{4bn} + \frac{cx^{-3n}}{3b^2n} - \frac{c^2x^{-2n}}{2b^3n} + \frac{c^3x^{-n}}{b^4n} + \frac{c^4\log(x)}{b^5} - \frac{c^4\log(b+cx^n)}{b^5n}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx = -\frac{bx^{-4n}(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n})-12c^4\log(x^n)+12c^4\log(b+cx^n)}{12b^5n}$$

```
[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] -1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n)
 - 12*c^4*Log[x^n] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{c^{3} x^{-n}}{b^{4} n}-\frac{c^{2} x^{-2 n}}{2 b^{3} n}+\frac{c \,x^{-3 n}}{3 b^{2} n}-\frac{x^{-4 n}}{4 b n}+\frac{c^{4} \ln \left(x \right)}{b^{5}}-\frac{c^{4} \ln \left(x^{n}+\frac{b}{c}\right)}{b^{5} n}$	90
norman	$\left(\frac{c^{3} {\mathrm e}^{3 n \ln \left(x \right)}}{b^{4} n}-\frac{1}{4 b n}+\frac{c \,{\mathrm e}^{n \ln \left(x \right)}}{3 b^{2} n}-\frac{c^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{2 b^{3} n}+\frac{c^{4} \ln \left(x \right) {\mathrm e}^{4 n \ln \left(x \right)}}{b^{5}}\right) {\mathrm e}^{-4 n \ln \left(x \right)}-\frac{c^{4} \ln \left(c \,{\mathrm e}^{n \ln \left(x \right)}+b \right)}{b^{5} n}$	105

```
[In] int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/b^4/n/(x^n)-1/2*c^2/b^3/n/(x^n)^2+1/3*c/b^2/n/(x^n)^3-1/4/b/n/(x^n)^4+c
^4*ln(x)/b^5-c^4/b^5/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx = \frac{12\,c^{4}nx^{4\,n}\log\left(x\right)-12\,c^{4}x^{4\,n}\log\left(cx^{n}+b\right)+12\,bc^{3}x^{3\,n}-6\,b^{2}c^{2}x^{2\,n}+4\,b^{3}cx^{n}-3\,b^{4}}{12\,b^{5}nx^{4\,n}}$$

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x^
(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{c^4\log(x)}{b^5} - \frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n} + \frac{12\,c^3x^{3\,n} - 6\,bc^2x^{2\,n} + 4\,b^2cx^n - 3\,b^3}{12\,b^4nx^{4\,n}}$$

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6*
b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

Giac [F]

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-3\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{3\,n+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-3n/4}}{3bn}+\frac{\sqrt{2}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}$$

```
[Out] -4/3/b/n/(x^(3/4*n))+1/2*c^(3/4)*ln(-b^(1/4)*c^(1/4)*x^(1/4*n)*2^(1/2)+b^(1
/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-1/2*c^(3/4)*ln(b^(1/4)*c^(1/4)*x^(
1/4*n)*2^(1/2)+b^(1/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-c^(3/4)*arctan(
-1+c^(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n-c^(3/4)*arctan(1+c^
(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1598, 369, 352, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \frac{\sqrt{2}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{b^{7/4}n} + \frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n} - \frac{4x^{-3n/4}}{3bn}$$

```
[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
```

```
b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{-1-\frac{3n}{4}}}{b+cx^n}\,dx \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{c\int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n}\,dx}{b} \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{bn}
\end{aligned}
$$

$$=-\frac{4x^{-3n/4}}{3bn}-\frac{(2c)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n}-\frac{(2c)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n}$$

$$=-\frac{4x^{-3n/4}}{3bn}-\frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n}-\frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n}+\frac{c^{3/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{\sqrt{2}b^{7/4}n}+\frac{c^{3/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,x^{1+\frac{1}{4}(-4+n)}\right)}{\sqrt{2}b^{7/4}n}$$

$$=-\frac{4x^{-3n/4}}{3bn}+\frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{\left(\sqrt{2}c^{3/4}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x^{1+\frac{1}{4}(-4+n)}}{\sqrt[4]{b}}\right)}{b^{7/4}n}+\frac{\left(\sqrt{2}c^{3/4}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x^{1+\frac{1}{4}(-4+n)}}{\sqrt[4]{b}}\right)}{b^{7/4}n}$$

$$=-\frac{4x^{-3n/4}}{3bn}+\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}+\frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{c}x^{n/2}\right)}{\sqrt{2}b^{7/4}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-3n/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{cx^n}{b}\right)}{3bn}$$

```
[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23

method	result	size
risch	$-\frac{4x^{-\frac{3n}{4}}}{3bn}+\left(\sum_{\_R=\text{RootOf}\left(b^7n^4\_Z^4+c^3\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{b^2n\_R}{c}\right)\right)$	54

```
[In] int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n
^4+c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{3\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}\,n-1}}{x}\right)-3\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}-cxx^{\frac{1}{4}\,n-1}}{x}\right)}{3}$$

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*
n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^
4))^(1/4)*log(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 3*I*
```

```
b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((I*b^2*n*(-c^3/(b^7*n^4))^
(1/4) + c*x*x^(1/4*n - 1))/x) - 3*I*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^
(1/4)*log((-I*b^2*n*(-c^3/(b^7*n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) + 4)/(b*
n*x^3*x^(3/4*n - 3))
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-n}x^{\frac{n}{4}-1}}{b+cx^n}\,dx$$

```
[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/4 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}\,n}+\sqrt{2}(x^n)^{\frac{1}{4}}\left(\frac{b}{c}\right.\right.}{b^2}}{6\,n}$$

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] -1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*(
x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^3)^
(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2 - 3*
sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqr
t(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

```
[In] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}c^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}$$

```
[Out] -3/2/b/n/(x^(2/3*n))-c^(2/3)*ln(b^(1/3)+c^(1/3)*x^(1/3*n))/b^(5/3)/n+1/2*c^
(2/3)*ln(b^(2/3)-b^(1/3)*c^(1/3)*x^(1/3*n)+c^(2/3)*x^(2/3*n))/b^(5/3)/n+c^(
2/3)*arctan(1/3*(b^(1/3)-2*c^(1/3)*x^(1/3*n))/b^(1/3)*3^(1/2))*3^(1/2)/b^(5
/3)/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1598, 369, 352, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \frac{\sqrt{3}c^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

```
[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{-1-\frac{2n}{3}}}{b+cx^n}\,dx \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n}\,dx}{b} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{(3c)\text{Subst}\left(\int \frac{1}{b+cx^3}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{bn} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{c}x}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n} \\
&\quad -\frac{c\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n} \\
&\quad +\frac{c^{2/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2c^{2/3}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{2b^{5/3}n} \\
&\quad -\frac{(3c)\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,x^{1+\frac{1}{3}(-3+n)}\right)}{2b^{4/3}n} \\
&= -\frac{3x^{-2n/3}}{2bn}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n} \\
&\quad -\frac{\left(3c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{c}x^{1+\frac{1}{3}(-3+n)}}{\sqrt[3]{b}}\right)}{b^{5/3}n}
\end{aligned}$$

$$= -\frac{3x^{-2n/3}}{2bn} + \frac{\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = -\frac{3x^{-2n/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{cx^n}{b}\right)}{2bn}$$

```
[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{3x^{-\frac{2n}{3}}}{2bn}+\left(\sum_{\_R=\text{RootOf}\left(b^5n^3\_Z^3+c^2\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{b^2n\_R}{c}\right)\right)$	54

```
[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(_Z^3*b^5*n
^3+c^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2}{2bnx^2x^{\frac{2}{3}n-2}}$$

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x
*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c^2
/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3*n
- 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^
2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{3}-1}}{b+cx^n}\,dx$$

```
[In] integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(n/3 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n}\,dx$$

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx$$

$$= \frac{\frac{2\,c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}\,n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2} - \frac{2\,\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}\,n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2} - \frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}\,n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2} - \frac{3}{b(x^n)^{\frac{2}{3}}}}{2\,n}$$

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(-
b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3))/
b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(2/3
))/b^2 - 3/(b*(x^n)^(2/3)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

```
[In] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}}{bn}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

```
[Out] -2/b/n/(x^(1/2*n))+2*arctan(b^(1/2)/(x^(1/2*n))/c^(1/2))*c^(1/2)/b^(3/2)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 352, 199, 327, 211}

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

```
[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2x^{-n/2}}{bn} + \frac{(2c)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-n/2}}{bn} + \frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{cx^n}{b}\right)}{bn}$$

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^2n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^2n}$	79

```
[In] int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/2))-1/b^
2*(-b*c)^(1/2)/n*ln(x^(1/2*n)+1/c*(-b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.02

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}n-1}}, \frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}n-1}}\right]$$

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqr
t(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2
*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/
2*n - 1))]
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{-n}x^{\frac{n}{2}-1}}{b + cx^n}\, dx$$

```
[In] integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/2 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\, dx$$

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = -\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}b} + \frac{1}{b\sqrt{x^n}}\right)}{n}$$

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] -2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{\frac{n}{2}-1}}{b\,x^n + c\,x^{2\,n}}\, dx$$

```
[In] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}$$

```
[Out] -2/3/b/n/(x^(3/2*n))+2*c/b^2/n/(x^(1/2*n))-2*c^(3/2)*arctan(b^(1/2)/(x^(1/2
*n))/c^(1/2))/b^(5/2)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1598, 369, 352, 199, 327, 211}

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2c^{3/2}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

```
[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{-1-\frac{3n}{2}}}{b+cx^n}\,dx \\
&= -\frac{2x^{-3n/2}}{3bn}-\frac{c\int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx}{b} \\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx,x,x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx,x,x^{-n/2}\right)}{bn} \\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx,x,x^{-n/2}\right)}{b^2n} \\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = -\frac{2x^{-3n/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{cx^n}{b}\right)}{3bn}$$

```
[In] Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{2c\,x^{-\frac{n}{2}}}{b^2n}-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^3n}-\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^3n}$	97

```
[In] int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/
2*n)+1/c*(-b*c)^(1/2))-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \left[-\frac{2\,bx^3x^{-\frac{3}{2}\,n-3}-6\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n},\right.$$
$$\left.-\frac{2\left(bx^3x^{-\frac{3}{2}\,n-3}-3\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}\,n-1}}\right)\right)}{3\,b^2n}\right]$$

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((
b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2) +
 c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt
(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-n}x^{-\frac{n}{2}-1}}{b+cx^n}\,dx$$

```
[In] integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n/2 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*
n*x^(3/2*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-4n/3}}{4bn}+\frac{3cx^{-n/3}}{b^2n}+\frac{\sqrt{3}c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n}$$

```
[Out] -3/4/b/n/(x^(4/3*n))+3*c/b^2/n/(x^(1/3*n))-c^(4/3)*ln(c^(1/3)+b^(1/3)/(x^(1
/3*n)))/b^(7/3)/n+1/2*c^(4/3)*ln(c^(2/3)+b^(2/3)/(x^(2/3*n))-b^(1/3)*c^(1/3
)/(x^(1/3*n)))/b^(7/3)/n+c^(4/3)*arctan(1/3*(1-2*b^(1/3)/c^(1/3)/(x^(1/3*n)
))*3^(1/2))*3^(1/2)/b^(7/3)/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1598, 369, 352, 199, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \frac{\sqrt{3}c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n}-\frac{c^{4/3}\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

```
[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{-1-\frac{4n}{3}}}{b+cx^n}\,dx\\
&= -\frac{3x^{-4n/3}}{4bn}-\frac{c\int \frac{x^{-1-\frac{n}{3}}}{b+cx^n}\,dx}{b}\\
&= -\frac{3x^{-4n/3}}{4bn}+\frac{(3c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}}\,dx,x,x^{-n/3}\right)}{bn}\\
&= -\frac{3x^{-4n/3}}{4bn}+\frac{(3c)\text{Subst}\left(\int \frac{x^3}{c+bx^3}\,dx,x,x^{-n/3}\right)}{bn}\\
&= -\frac{3x^{-4n/3}}{4bn}+\frac{3cx^{-n/3}}{b^2n}-\frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{c+bx^3}\,dx,x,x^{-n/3}\right)}{b^2n}
\end{aligned}$$

$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{b}x}\,dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3}\text{Subst}\left(\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{b^2n}$$

$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2b^{2/3}x}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{2b^{7/3}n} - \frac{(3c^{5/3})\,\text{Subst}\left(\int \frac{1}{c^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x+b^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{2b^2n}$$

$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n} - \frac{(3c^{4/3})\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}\right)}{b^{7/3}n}$$

$$= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-4n/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},-\frac{cx^n}{b}\right)}{4bn}$$

```
[In] Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3c\,x^{-\frac{n}{3}}}{b^2 n}-\frac{3x^{-\frac{4n}{3}}}{4bn}+\left(\sum_{\_R=\mathrm{RootOf}\left(b^7 n^3 \_Z^3+c^4\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{b^5 n^2 \_R^2}{c^3}\right)\right)$	73

```
[In] int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3
*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3\,bx^4x^{-\frac{4}{3}\,n-4}-12\,cxx^{-\frac{1}{3}\,n-1}-4\,\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bxx^{-\frac{1}{3}\,n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right)-4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}\,n-1}-(\cdots}{x}\right.}{4\,b^2n}$$

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(
1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c)
- 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1
/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))
/x^2))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{3}-1}}{b+cx^n}\,dx$$

```
[In] integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n/3 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*
n*x^(4/3*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = &-\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}\\&-\frac{\sqrt{2}c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}\\&+\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n}\\&-\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n}\end{aligned}$$

```
[Out] -4/5/b/n/(x^(5/4*n))+4*c/b^2/n/(x^(1/4*n))+1/2*c^(5/4)*ln(-b^(1/4)*c^(1/4)*
2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(9/4)/n*2^(1/2)-1/2*c^(5
/4)*ln(b^(1/4)*c^(1/4)*2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(
9/4)/n*2^(1/2)+c^(5/4)*arctan(1-b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2
)/b^(9/4)/n-c^(5/4)*arctan(1+b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2)/b
^(9/4)/n
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1598, 369, 352, 199, 327, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\frac{\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{b^{9/4}n}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}b^{9/4}n}+\frac{4cx^{-n/4}}{b^2n}-\frac{4x^{-5n/4}}{5bn}$$

```
[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{-1-\frac{5n}{4}}}{b+cx^n}\,dx$$

$$= -\frac{4x^{-5n/4}}{5bn} - \frac{c\int \frac{x^{-1-\frac{n}{4}}}{b+cx^n}\,dx}{b}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}}\,dx, x, x^{-n/4}\right)}{bn}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\text{Subst}\left(\int \frac{x^4}{c+bx^4}\,dx, x, x^{-n/4}\right)}{bn}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(4c^2)\,\text{Subst}\left(\int \frac{1}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(2c^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}x^2}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n} - \frac{(2c^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{b}x^2}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} + \frac{c^{5/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{c^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}+x^2}\,dx, x, x^{-n/4}\right)}{b^{5/2}n} - \frac{c^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}+x^2}\,dx, x, x^{-n/4}\right)}{b^{5/2}n}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{(\sqrt{2}c^{5/4})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} + \frac{(\sqrt{2}c^{5/4})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} + \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}x^{-n/2}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-5n/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\frac{cx^n}{b}\right)}{5bn}$$

```
[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{4c\,x^{-\frac{n}{4}}}{b^2n}-\frac{4x^{-\frac{5n}{4}}}{5bn}+\left(\sum_{\_R=\text{RootOf}\left(b^9n^4\_Z^4+c^5\right)}\_R\ln\left(x^{\frac{n}{4}}+\frac{b^7n^3\_R^3}{c^4}\right)\right)$	73

```
[In] int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^4
*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx =$$
$$-\frac{4\,bx^5x^{-\frac{5}{4}\,n-5}+5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}\,n-1}}{x}\right)-5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}-cxx^{-\frac{1}{4}}}{x}\right.}{}$$

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-
c^5/(b^9*n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1
/4)*log(-(b^2*n*(-c^5/(b^9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) + 5*I*b^2*n
*(-c^5/(b^9*n^4))^(1/4)*log((I*b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1/4*n
 - 1))/x) - 5*I*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((-I*b^2*n*(-c^5/(b^9*n^4))
^(1/4) + c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{4}-1}}{b+cx^n}\,dx$$

```
[In] integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(-n/4 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2*
n*x^(5/4*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{4}+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.506 $\int x^{-1-n(-1+p)}(bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 37

$$\int x^{-1-n(-1+p)}(bx^n+cx^{2n})^p\,dx=\frac{x^{-n(1+p)}(bx^n+cx^{2n})^{1+p}}{cn(1+p)}$$

```
[Out] (b*x^n+c*x^(2*n))^(p+1)/c/n/(p+1)/(x^(n*(p+1)))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2039}

$$\int x^{-1-n(-1+p)}(bx^n+cx^{2n})^p\,dx=\frac{x^{-n(p+1)}(bx^n+cx^{2n})^{p+1}}{cn(p+1)}$$

```
[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=\frac{x^{-n(1+p)}(bx^n+cx^{2n})^{1+p}}{cn(1+p)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^{-np}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{cn(1+p)}$$

```
[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

Maple [F]

$$\int x^{-1-n(-1+p)}\left(b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cxx^{-np+n-1}x^n+bxx^{-np+n-1})(cx^{2\,n}+bx^n)^p}{(cnp+cn)x^n}$$

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c
*n*p + c*n)*x^n)
```

Sympy [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(p-1)-1}(x^n(b+cx^n))^p\,dx$$

```
[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
[Out] Integral(x**(-n*(p - 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^n+b)e^{(-np\log(x)+p\log(cx^n+b)+p\log(x^n))}}{cn(p+1)}$$

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

Giac [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n\right)^p x^{-n(p-1)-1}\,dx$$

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,(p-1)+1}}\,dx$$

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

3.507 $\int x^{-1-n(1+2p)}(bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

```
[Out] -(b*x^n+c*x^(2*n))^(p+1)/b/n/(p+1)/(x^(2*n*(p+1)))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2039}

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

```
[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=-\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-n(1+2p)}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{bn(1+p)}$$

```
[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

Maple [F]

$$\int x^{-1-n(1+2p)}\left(b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
[Out] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{(cxx^{-2\,np-n-1}x^n+bxx^{-2\,np-n-1})(cx^{2\,n}+bx^n)^p}{bnp+bn}$$

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^
p/(b*n*p + b*n)
```

Sympy [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(2p+1)-1}(x^n(b+cx^n))^p\,dx$$

```
[In] integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
[Out] Integral(x**(-n*(2*p + 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n\right)^p x^{-n(2\,p+1)-1}\,dx$$

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n\right)^p x^{-n(2\,p+1)-1}\,dx$$

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,(2\,p+1)+1}}\,dx$$

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

3.508 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 112

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = -\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}$$

```
[Out] -1/6*a*(a+b*x^n)^6*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/7*
(a+b*x^n)^7*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1369, 272, 45}

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = \frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}-\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}$$

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/6*(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b^2 + b^3*
x^n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b
^3*x^n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}(ab+b^2x^n)^5\,dx}{b^4(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x(ab+b^2x)^5\,dx,x,x^n\right)}{b^4n(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^5}{b}+\frac{(ab+b^2x)^6}{b^2}\right)dx,x,x^n\right)}{b^4n(ab+b^2x^n)}\\
&= -\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n(ab^2+b^3x^n)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n(ab^2+b^3x^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}dx=\frac{x^{2n}\left((a+bx^n)^2\right)^{5/2}(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n})}{42n(a+bx^n)^5}$$

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
[Out] (x^(2*n)*((a + b*x^n)^2)^(5/2)*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n)
 + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x^n
)^5)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.86

method	result
risch	$\frac{\sqrt{(a+b x^n)^2}\, b^5 x^{7n}}{7(a+b x^n)n}+\frac{5\sqrt{(a+b x^n)^2}\, b^4 a x^{6n}}{6(a+b x^n)n}+\frac{2\sqrt{(a+b x^n)^2}\, a^2 b^3 x^{5n}}{(a+b x^n)n}+\frac{5\sqrt{(a+b x^n)^2}\, a^3 b^2 x^{4n}}{2(a+b x^n)n}+\frac{5\sqrt{(a+b x^n)^2}\, b a^4 x^{3n}}{3(a+b x^n)n}+$

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*b^4*a/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+5/
2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*b*a^4/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}x^{2\,n-1}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{5/2}\,dx$$

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.509 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 112

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}$$

```
[Out] -1/4*a*(a+b*x^n)^4*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/5*
(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1369, 272, 45}

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)} - \frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}$$

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -1/4*(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b^2 + b^3*
x^n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b
^3*x^n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}(ab+b^2x^n)^3\,dx}{b^2(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\text{Subst}\left(\int x(ab+b^2x)^3\,dx,x,x^n\right)}{b^2n(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^3}{b}+\frac{(ab+b^2x)^4}{b^2}\right)dx,x,x^n\right)}{b^2n(ab+b^2x^n)}\\
&= -\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n(ab^2+b^3x^n)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n(ab^2+b^3x^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{x^{2n}\left((a+bx^n)^2\right)^{3/2}(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n})}{20n(a+bx^n)^3}$$

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] (x^(2*n)*((a + b*x^n)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) +
4*b^3*x^(3*n)))/(20*n*(a + b*x^n)^3)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3} x^{5 n}}{5\left(a+b\,x^{n}\right) n}+\frac{3 \sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2} a\,x^{4 n}}{4\left(a+b\,x^{n}\right) n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2} b\,x^{3 n}}{\left(a+b\,x^{n}\right) n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3} x^{2 n}}{2\left(a+b\,x^{n}\right) n}$	135

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*b^2*a/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*(x^n)^3+1/2*((
a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^{2\,n-1}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 99

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}$$

```
[Out] 1/2*a*x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a+b*x^n)+1/3*b^2*x^(3*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1369, 14}

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}$$

```
[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}(ab+b^2x^n)\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(abx^{-1+2n}+b^2x^{-1+3n}\right)\,dx}{ab+b^2x^n}\\ &= \frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{2n}\sqrt{(a+bx^n)^2}(3a+2bx^n)}{6n\,(a+bx^n)}$$

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^{3n}}{3(a+b\,x^n)n}+\frac{\sqrt{(a+b\,x^n)^2}\,a\,x^{2n}}{2(a+b\,x^n)n}$	64

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*
x^n)*a/n*(x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Giac [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^{2\,n-1}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x^n*(a+b*x^n)/b/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-a*(a+b*x^n)*ln(a+b*x^n)
/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1369, 272, 45}

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^n)\int \frac{x^{-1+2n}}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \frac{x}{ab+b^2x}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a}{b^2(a+bx)}\right)\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(a+bx^n)\,(bx^n-a\log(bn(a+bx^n)))}{b^2n\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] ((a + b*x^n)*(b*x^n - a*Log[b*n*(a + b*x^n)]))/(b^2*n*Sqrt[(a + b*x^n)^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,x^{n}}{\left(a+b\,x^{n}\right)b n}-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a\ln\left(x^{n}+\frac{a}{b}\right)}{\left(a+b\,x^{n}\right)b^{2} n}$	71

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/b^2/n
*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.27

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{bx^n-a\log\left(bx^n+a\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^{2n-1}}{\sqrt{\left(a+bx^n\right)^2}}\,dx$$

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.512 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 48

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\frac{x^{2n}}{2an\,(a+bx^n)\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^(2*n)/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1369, 270}

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\frac{x^{2n}}{2an\,(a+bx^n)\,\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{x^{-1+2n}}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= \frac{x^{2n}}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{(-a-2bx^n)\,(a+bx^n)}{2b^2n\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] ((-a - 2*b*x^n)*(a + b*x^n))/(2*b^2*n*((a + b*x^n)^2)^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,(2b\,x^n+a)}{2(a+b\,x^n)^3b^2n}$	37

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^{2\,n-1}}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/4*a/b^2/n/(a+b*x^n)^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/3/b^2/n/(a+b*x^
n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^4(ab+b^2x^n)) \int \frac{x^{-1+2n}}{(ab+b^2x^n)^5} \, dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{(b^4(ab+b^2x^n)) \operatorname{Subst}\left(\int \frac{x}{(ab+b^2x)^5} \, dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{(b^4(ab+b^2x^n)) \operatorname{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right) \, dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{a}{4b^2n\,(a+bx^n)^3\,\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n\,(a+bx^n)^2\,\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}} \, dx = \frac{(-a-4bx^n)\,(a+bx^n)}{12b^2n\,\left((a+bx^n)^2\right)^{5/2}}$$

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
[Out] ((-a - 4*b*x^n)*(a + b*x^n))/(12*b^2*n*((a + b*x^n)^2)^(5/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,(4b\,x^{n}+a)}{12(a+b\,x^{n})^{5}b^{2}n}$	37

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \int \frac{x^{2\,n-1}}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{5/2}}\,dx$$

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.514 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/6*a/b^2/n/(a+b*x^n)^5/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/5/b^2/n/(a+b*x^n)^4/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1369, 272, 45}

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^6(ab+b^2x^n)\right)\int \frac{x^{-1+2n}}{(ab+b^2x^n)^7}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{\left(b^6(ab+b^2x^n)\right)\text{Subst}\left(\int \frac{x}{(ab+b^2x)^7}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{\left(b^6(ab+b^2x^n)\right)\text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7}+\frac{1}{b^8(a+bx)^6}\right)\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{(-a-6bx^n)\,(a+bx^n)}{30b^2n\left((a+bx^n)^2\right)^{7/2}}$$

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
[Out] ((-a - 6*b*x^n)*(a + b*x^n))/(30*b^2*n*((a + b*x^n)^2)^(7/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(6 b\,x^{n}+a\right)}{30 \left(a+b\,x^{n}\right)^{7} b^{2} n}$	37

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = -\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas
")
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \int \frac{x^{2\,n-1}}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{7/2}}\,dx$$

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

3.515 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 108

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\,(ab+b^2x^n)}$$

```
[Out] a*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+b^2*x^(1+
n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1369, 14, 20, 30}

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\,(ab+b^2x^n)} + \frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\,(a+bx^n)}$$

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int(dx)^m\left(ab+b^2x^n\right)\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(ab(dx)^m+b^2x^n(dx)^m\right)\,dx}{ab+b^2x^n}\\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)}+\frac{\left(b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n}\\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)}+\frac{\left(b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+n}\,dx}{ab+b^2x^n}\\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)}+\frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x(dx)^m\sqrt{(a+bx^n)^2}(a(1+m+n)+b(1+m)x^n)}{(1+m)(1+m+n)\,(a+bx^n)}$$

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1
 + m + n)*(a + b*x^n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x(mb\,x^n+am+an+b\,x^n+a)d^m x^m \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(idx)m(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\ \mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}}}{(a+b\,x^n)(1+m)(1+m+n)}$	99

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(1+m+n)*d^m
*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(
I*d)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$
$$=\frac{(bm+b)xx^ne^{(m\log(d)+m\log(x))}+(am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2\,m+1}$$

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [F]

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int (dx)^m\sqrt{(a+bx^n)^2}\,dx$$

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ad^m(m+n+1)xx^m+bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2+m(n+2)+n+1}$$

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
m*(n + 2) + n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$
$$=\frac{bmxx^ne^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)+bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+}{}$$

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*n*
x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int (d\,x)^m\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.516 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}$$

```
[Out] 1/3*a*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(3+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 14}

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)(ab+b^2x^n)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}$$

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2(ab+b^2x^n)\,dx}{ab+b^2x^n}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (abx^2+b^2x^{2+n})\,dx}{ab+b^2x^n}\\
&= \frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{x^3\sqrt{(a+bx^n)^2}(a(3+n)+3bx^n)}{3(3+n)(a+bx^n)}$$

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\,x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^3x^n}{(a+b\,x^n)(3+n)}$	61

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(3+
n)*x^3*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\sqrt{(a+bx^n)^2}\,dx$$

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n\mathrm{sgn}(bx^n+a)+anx^3\mathrm{sgn}(bx^n+a)+3\,ax^3\mathrm{sgn}(bx^n+a)}{3\,(n+3)}$$

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^
n + a))/(n + 3)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}$$

```
[Out] 1/2*a*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(2+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)(ab+b^2x^n)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x(ab+b^2x^n)\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (abx+b^2x^{1+n})\,dx}{ab+b^2x^n}\\ &= \frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{x^2\sqrt{(a+bx^n)^2}(a(2+n)+2bx^n)}{2(2+n)(a+bx^n)}$$

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\,x^2}{2a+2b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^2x^n}{(a+b\,x^n)(2+n)}$	61

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(2+
n)*x^2*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [F]

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x\sqrt{(a+bx^n)^2}\,dx$$

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n+a(n+2)x^2}{2\,(n+2)}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n\mathrm{sgn}(bx^n+a)+anx^2\mathrm{sgn}(bx^n+a)+2\,ax^2\mathrm{sgn}(bx^n+a)}{2\,(n+2)}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^
n + a))/(n + 2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\,(ab+b^2x^n)}$$

```
[Out] a*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(1+n)*(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1357}

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\,(ab+b^2x^n)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int(2ab+2b^2x^n)\,dx}{2ab+2b^2x^n}\\
&= \frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)(ab+b^2x^n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x\sqrt{(a+bx^n)^2}(a+an+bx^n)}{(1+n)(a+bx^n)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,ax}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,bx\,x^n}{(a+b\,x^n)(1+n)}$	56

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(1+n)*x*x
^n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.23

$$\int\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bxx^n+(an+a)x}{n+1}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F]

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.22

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \frac{a(n+1)x + bxx^n}{n+1}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.28

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \left(ax + \frac{bx^{n+1}}{n+1}\right)\operatorname{sgn}(bx^n + a)$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \int \sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

```
[Out] b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a*ln(x)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\frac{ab+b^2x^n}{x}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(\frac{ab}{x}+b^2x^{-1+n}\right)\,dx}{ab+b^2x^n}\\ &= \frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.45

$$\int\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{\sqrt{(a+bx^n)^2}(bx^n+a\log(x^n))}{n\,(a+bx^n)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*Log[x^n]))/(n*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\ln(x)}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)n}$	54

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} \, dx = \frac{an \log(x) + bx^n}{n}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
[Out] (a*n*log(x) + b*x^n)/n
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} \, dx = \int \frac{\sqrt{(a + bx^n)^2}}{x} \, dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} \, dx = a \log(x) + \frac{bx^n}{n}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
[Out] a*log(x) + b*x^n/n
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} \, dx = \int \frac{\sqrt{b^2x^{2n} + 2abx^n + a^2}}{x} \, dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\,dx = \int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x}\,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)
```

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}$$

```
[Out] -a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-b^2*x^(-1+n)*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^2}\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^2}+b^2x^{-2+n}\right)\,dx}{ab+b^2x^n}\\ &= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=\frac{\sqrt{(a+bx^n)^2}(a-an+bx^n)}{(-1+n)x\,(a+bx^n)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,a}{(a+b\,x^n)x}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)(-1+n)x}$	61

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b/x
*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = -\frac{an - bx^n - a}{(n-1)x}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \int \frac{\sqrt{(a + bx^n)^2}}{x^2}\,dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = -\frac{a(n-1) - bx^n}{(n-1)x}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^2}\,dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2} \, dx = \int \frac{\sqrt{a^2 + b^2 \, x^{2\,n} + 2\,a\,b\,x^n}}{x^2} \, dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}$$

```
[Out] -1/2*a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-b^2*x^(-2+n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 14}

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x^2*(a + b*x^n)) - (b^2*x^(-2
 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^3}\,dx}{ab+b^2x^n} \\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^3}+b^2x^{-3+n}\right)\,dx}{ab+b^2x^n} \\
&= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\,(ab+b^2x^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \frac{\sqrt{(a+bx^n)^2}(-a(-2+n)+2bx^n)}{2(-2+n)x^2\,(a+bx^n)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n)
)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,a}{2(a+b\,x^n)x^2}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)(-2+n)x^2}$	61

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+
n)*b/x^2*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} \, dx = -\frac{an - 2\,bx^n - 2\,a}{2\,(n-2)x^2}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} \, dx = \int \frac{\sqrt{(a + bx^n)^2}}{x^3} \, dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} \, dx = -\frac{a(n-2) - 2\,bx^n}{2\,(n-2)x^2}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} \, dx = \int \frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{x^3} \, dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^3}\,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 30, antiderivative size = 238

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{1+2n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+2n)\left(ab+b^2x^n\right)} + \frac{b^4x^{1+3n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+3n)\left(ab+b^2x^n\right)}$$

```
[Out] a^3*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+3*a^2*b
^2*x^(1+n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)+
3*a*b^3*x^(1+2*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+2*n)/(a*b+
b^2*x^n)+b^4*x^(1+3*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+3*n)/
(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1369, 276, 20, 30}

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\left(ab+b^2x^n\right)} + \frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\left(ab+b^2x^n\right)} + \frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\left(a+bx^n\right)}$$

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int(dx)^m\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3(dx)^m+3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x^{3n}(dx)^m\right)dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)}+\frac{\left(3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx)^m\,dx}{ab+b^2x^n}\\
&\quad+\frac{\left(3ab^3\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{2n}(dx)^m\,dx}{ab+b^2x^n}+\frac{\left(b^4\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{3n}(dx)^m\,dx}{ab+b^2x^n}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3 (dx)^{1+m} \sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} \\
&\quad + \frac{\left(3a^2b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+n}\,dx}{ab+b^2x^n} \\
&\quad + \frac{\left(3ab^3x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+2n}\,dx}{ab+b^2x^n} \\
&\quad + \frac{\left(b^4x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^{m+3n}\,dx}{ab+b^2x^n} \\
&= \frac{a^3 (dx)^{1+m} \sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)} \\
&\quad + \frac{3ab^3x^{1+2n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+2n)\left(ab+b^2x^n\right)} + \frac{b^4x^{1+3n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+3n)\left(ab+b^2x^n\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.38

$$
\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{x(dx)^m \left(\left(a+bx^n\right)^2\right)^{3/2} \left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3ab^2x^{2n}}{1+m+2n}+\frac{b^3x^{3n}}{1+m+3n}\right)}{\left(a+bx^n\right)^3}
$$

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) +
 (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n
)^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.10

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x\left(a^3+3m\,a^3+3a\,b^2m^3x^{2n}+6b^3mn\,x^{3n}+9a\,b^2m^2x^{2n}+9a\,b^2n^2x^{2n}+9m\,b^2a\,x^{2n}+3b^3m^2n\,x^{3n}+2b^3m\,n^2x^{3n}+12b^2a\,x^{2n}\right.}{}$

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(a^3+3*m*a^3+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*
(x^n)^3+3*a^2*b*m^3*x^n+9*m*a^2*b*x^n+15*a^2*b*n*x^n+9*a*b^2*m^2*(x^n)^2+9*
a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*b^2*a*(x^n)^2+3*b^3*
```

m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3+12*b^2*a*(x^n)^2*n+a^3*m^3+3*a^3*m^2+(x^n)^3*b^3+9*a*b^2*m*n^2*(x^n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*n*x^n+11*a^3*m*n^2+12*a^3*m*n+6*a^3*m^2*n+3*b^3*m^2*(x^n)^3+b^3*m^3*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^n)^3*n+11*a^3*n^2+6*a^3*n+3*(x^n)^2*a*b^2+3*x^n*a^2*b+6*a^3*n^3+12*a*b^2*m^2*n*(x^n)^2)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*d^m*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.64

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\,(b^3m+b^3)n^2+3\,(b^3m^2+2\,b^3m+b^3)n)xx^{3\,n}e^{(m\log(d)+m\log(}}{}$$

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 + 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int (dx)^m \left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}} dx$$

[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)

[Out] Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.16

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \frac{(m^3 + 3m^2(2n+1) + 6n^3 + (11n^2 + 12n + 3)m + 11n^2 + 6n + 1)a^3d^mxx^m + (m^3 + 3n$$

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)
*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n +
 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 +
 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m
^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^
(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6
*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. $2(230) = 460$.

Time = 0.41 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.42

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x
^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3
*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2
*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2
*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*
```

m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a^3*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*e^

```
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n*x*e^(m*log(d) + m*log(x))*sg
n(b*x^n + a) + 15*a^2*b*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b
^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n
)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + b^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*x*e^(m*log(d) + m
*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 3*a*b^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a))/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*
m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*m + 6*n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \int (d\,x)^m \left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2} dx$$

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.523 $\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)(ab+b^2x^n)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}+\frac{3ab^3x^{3+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+2n)(ab+b^2x^n)}$$

```
[Out] 1/3*a^3*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+1/3*b^4*x^(3+3*n)*(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(3+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)+3*a*b^3*x^(3+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(3+2*n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 276}

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)(ab+b^2x^n)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)(ab+b^2x^n)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)(ab+b^2x^n)}+\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}$$

```
[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^3b^3x^2+3ab^5x^{2(1+n)}+3a^2b^4x^{2+n}+b^6x^{2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\
&= \frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}\\
&\quad+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{3+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+2n)\left(ab+b^2x^n\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

$$
\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^3\sqrt{\left(a+bx^n\right)^2}\left(a^3(9+18n+11n^2+2n^3)+9a^2b(3+5n+2n^2)\,x^n+9ab^2(3+4n+n^2)\,x^2\right.}{3(1+n)(3+n)(3+2n)\left(a+bx^n\right)}
$$

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*
n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^
(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3} x^{3}}{3a+3b\,x^{n}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3} x^{3} x^{3n}}{3\left(a+b\,x^{n}\right)\left(1+n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2} a\,x^{3} x^{2n}}{\left(a+b\,x^{n}\right)\left(3+2n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2} b\,x^{3} x^{n}}{\left(a+b\,x^{n}\right)\left(3+n\right)}$	146

[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^3+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(3+2*n)*x^3*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(3+n)*x^3*x^n

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,b^3n^2+9\,b^3n+9\,b^3)x^3x^{3\,n}+9\,(ab^2n^2+4\,ab^2n+3\,ab^2)x^3x^{2\,n}+9\,(2\,a^2bn^2+5\,a^2bn+3\,a^2b)}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n + 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)

Sympy [F]

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^2\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}\,dx$$

[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)

[Out] Integral(x**2*((a + b*x**n)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,n^2+9\,n+9)b^3x^3x^{3\,n}+9\,(n^2+4\,n+3)ab^2x^3x^{2\,n}+9\,(2\,n^2+5\,n+3)a^2bx^3x^n+(2\,n^3+1}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*n
) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x^3
)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,b^3n^2x^3x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^3x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^3x^n\mathrm{sgn}(bx^n+a)+2\,a^3n^}{}$$

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n + a
) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^n +
 a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a) + 9
*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a) + 27*
a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^3*sgn(
b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx=\int x^2\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.524 $\int x(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}+\frac{b^4x^{2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+3n)\left(ab+b^2x^n\right)}$$

```
[Out] 1/2*a^3*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+3/2*a*b^3*x^(2+2*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(2+n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)+b^4*x^(2+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+3*n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 276}

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)\left(ab+b^2x^n\right)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)\left(ab+b^2x^n\right)}+\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}$$

```
[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x(ab+b^2x^n)^3\,dx}{b^2(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^3b^3x+3a^2b^4x^{1+n}+3ab^5x^{1+2n}+b^6x^{1+3n}\right)dx}{b^2(ab+b^2x^n)}\\
&= \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)(ab+b^2x^n)}\\
&\quad+\frac{3a^2b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}+\frac{b^4x^{2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+3n)(ab+b^2x^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.59

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{x^2\sqrt{(a+bx^n)^2}\left(a^3(4+12n+11n^2+3n^3)+6a^2b(2+5n+3n^2)x^n+3ab^2(4+8n+3n^2)x^{2n}\right.}{2(1+n)(2+n)(2+3n)(a+bx^n)}$$

```
[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*
n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*
x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}x^{2}}{2a+2b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{2}x^{3n}}{(a+b\,x^{n})(2+3n)}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a\,b^{2}x^{2}x^{2n}}{2(a+b\,x^{n})(1+n)}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b\,x^{2}x^{n}}{(a+b\,x^{n})(2+n)}$	145

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3
/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2*x^2/(1+n)*(x^n
)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(2+n)*x^2*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,(b^3n^2+3\,b^3n+2\,b^3)x^2x^{3\,n}+3\,(3\,ab^2n^2+8\,ab^2n+4\,ab^2)x^2x^{2\,n}+6\,(3\,a^2bn^2+5\,a^2bn+2\,a}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Sympy [F]

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,(n^2+3\,n+2)b^3x^2x^{3\,n}+3\,(3\,n^2+8\,n+4)ab^2x^2x^{2\,n}+6\,(3\,n^2+5\,n+2)a^2bx^2x^n+(3\,n^3+}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(2
*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x
^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^2x^n\mathrm{sgn}(bx^n+a)+3\,a^3n^3x}{}$$

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n + a
) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^n +
 a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a) + 4
*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a) + 12*
a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^2*sgn(
b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.525 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{a^3x(a^2+2abx^n+b^2x^{2n})^{3/2}}{(a+bx^n)^3} + \frac{3a^2b^4x^{1+n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+n)\,(ab+b^2x^n)^3} + \frac{3ab^5x^{1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+2n)\,(ab+b^2x^n)^3} + \frac{b^6x^{1+3n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+3n)\,(ab+b^2x^n)^3}$$

```
[Out] a^3*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(a+b*x^n)^3+3*a^2*b^4*x^(1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+n)/(a*b+b^2*x^n)^3+3*a*b^5*x^(1+2*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+2*n)/(a*b+b^2*x^n)^3+b^6*x^(1+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(3/2)/(1+3*n)/(a*b+b^2*x^n)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1357, 250}

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{b^6x^{3n+1}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(3n+1)\,(ab+b^2x^n)^3} + \frac{3ab^5x^{2n+1}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(2n+1)\,(ab+b^2x^n)^3} + \frac{3a^2b^4x^{n+1}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(n+1)\,(ab+b^2x^n)^3} + \frac{a^3x(a^2+2abx^n+b^2x^{2n})^{3/2}}{(a+bx^n)^3}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(2ab+2b^2x^n\right)^3\,dx}{\left(2ab+2b^2x^n\right)^3}\\
&= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^n+24ab^5x^{2n}+8b^6x^{3n}\right)dx}{\left(2ab+2b^2x^n\right)^3}\\
&= \frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{\left(a+bx^n\right)^3}+\frac{3a^2b^4x^{1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+n)\left(ab+b^2x^n\right)^3}\\
&\quad+\frac{3ab^5x^{1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+2n)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{1+3n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(1+3n)\left(ab+b^2x^n\right)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.59

$$
\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{x\sqrt{\left(a+bx^n\right)^2}\left(a^3\left(1+6n+11n^2+6n^3\right)+3a^2b\left(1+5n+6n^2\right)x^n+3ab^2\left(1+4n+3n^2\right)x^{2n}\right.}{(1+n)(1+2n)(1+3n)\left(a+bx^n\right)}
$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n +
 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(
3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}x}{a+b\,x^{n}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x\,x^{3n}}{\left(a+b\,x^{n}\right)\left(1+3n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}ax\,x^{2n}}{\left(a+b\,x^{n}\right)\left(1+2n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{2}bx\,x^{n}}{\left(a+b\,x^{n}\right)\left(1+n\right)}$	138

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/(1+3*
n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(1+2*n)*x*(x^n)^2+3*((a+
b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,b^3n^2+3\,b^3n+b^3)xx^{3\,n}+3\,(3\,ab^2n^2+4\,ab^2n+ab^2)xx^{2\,n}+3\,(6\,a^2bn^2+5\,a^2bn+a^2b)xx^{n}}{6\,n^3+11\,n^2+6\,n+1}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.49

$$\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} \, dx = \frac{(2\,n^2 + 3\,n + 1)b^3xx^{3\,n} + 3\,(3\,n^2 + 4\,n + 1)ab^2xx^{2\,n} + 3\,(6\,n^2 + 5\,n + 1)a^2bxx^n + (6\,n^3 + 1}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3*
(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 +
11*n^2 + 6*n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28

$$\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} \, dx = \frac{6\,a^3n^3x\mathrm{sgn}(bx^n + a) + 2\,b^3n^2xx^{3\,n}\mathrm{sgn}(bx^n + a) + 9\,ab^2n^2xx^{2\,n}\mathrm{sgn}(bx^n + a) + 18\,a^2bn^2xx^n\mathrm{s}}{}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] (6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*
n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3*n
^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*
n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x^n +
 a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a) + 3*a
^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1
)
```

Mupad [F(-1)]

Timed out.

$$\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} \, dx = \int \left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2} \,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.526 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx=\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

```
[Out] 3*a^2*b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3/2*a*b^3*x
^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^4*x^(3*n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a^3*ln(x)*(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1369, 272, 45}

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx=\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
+ (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\frac{(ab+b^2x^n)^3}{x}\,dx}{b^2(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\frac{(ab+b^2x)^3}{x}\,dx,x,x^n\right)}{b^2n(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^n\right)}{b^2n(ab+b^2x^n)}\\
&= \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(ab+b^2x^n)}\\
&\quad+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)}+\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.34

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} \, dx = \frac{\sqrt{(a + bx^n)^2}(bx^n(18a^2 + 9abx^n + 2b^2x^{2n}) + 6a^3 \log (x^n))}{6n\,(a + bx^n)}$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)) + 6*a^3*Lo
g[x^n]))/(6*n*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a^3\ln(x)}{a+b\,x^n} + \frac{\sqrt{(a+b\,x^n)^2}\,b^3x^{3n}}{3(a+b\,x^n)n} + \frac{3\sqrt{(a+b\,x^n)^2}\,b^2a\,x^{2n}}{2(a+b\,x^n)n} + \frac{3\sqrt{(a+b\,x^n)^2}\,a^2b\,x^n}{(a+b\,x^n)n}$	127

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b
^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/n*(x^n)^2+3*((a+b*x^n)
^2)^(1/2)/(a+b*x^n)*a^2*b/n*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} \, dx = \frac{6\,a^3n\log{(x)} + 2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [F]

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx = \int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx = a^3\log(x) + \frac{2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Giac [F]

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx = \int \frac{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx = \int \frac{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}{x}\,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

3.527 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx = -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x(a+bx^n)} - \frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)(ab+b^2x^n)} - \frac{3ab^3x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)(ab+b^2x^n)} - \frac{b^4x^{-1+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)(ab+b^2x^n)}$$

```
[Out] -a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-3*a^2*b^2*x^(-1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)-3*a*b^3*x^(-1+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(1-2*n)/(a*b+b^2*x^n)-b^4*x^(-1+3*n)*(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)/(1-3*n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx = -\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)(ab+b^2x^n)} - \frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)(ab+b^2x^n)} - \frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)(ab+b^2x^n)} - \frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x(a+bx^n)}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\frac{(ab+b^2x^n)^3}{x^2}\,dx}{b^2\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(\frac{a^3b^3}{x^2}+3a^2b^4x^{-2+n}+3ab^5x^{2(-1+n)}+b^6x^{-2+3n}\right)dx}{b^2\left(ab+b^2x^n\right)}\\ &= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\left(a+bx^n\right)}-\frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\left(ab+b^2x^n\right)}\\ &\quad-\frac{3ab^3x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\left(ab+b^2x^n\right)}-\frac{b^4x^{-1+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx=\frac{\sqrt{(a+bx^n)^2}\left(a^3(1-6n+11n^2-6n^3)+3a^2b(1-5n+6n^2)\,x^n+3ab^2(1-\right.}{(-1+n)(-1+2n)(-1+3n)x\,(a+b}$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6
*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*
n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}}{(a+b\,x^{n})x}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{3n}}{(a+b\,x^{n})(-1+3n)x}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b^{2}a\,x^{2n}}{(a+b\,x^{n})(-1+2n)x}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b\,x^{n}}{(a+b\,x^{n})(-1+n)x}$	147

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+3*n)*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)*b^2*a/x*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a^2*b/x*x^n

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.62

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx = -\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-(2\,b^3n^2-3\,b^3n+b^3)x^{3\,n}-3\,(3\,ab^2n^2-4\,ab^2n+ab^2)x^{2\,n}-3\,(6\,a^2bn^2-5\,a}{(6\,n^3-11\,n^2+6\,n-1)x}$$

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")

[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)

Sympy [F]

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx = \int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)

[Out] Integral(((a + b*x**n)**2)**(3/2)/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = \frac{(2n^2 - 3n + 1)b^3x^{3n} + 3(3n^2 - 4n + 1)ab^2x^{2n} + 3(6n^2 - 5n + 1)a^2bx^n}{(6n^3 - 11n^2 + 6n - 1)x}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*n
^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^2
+ 6*n - 1)*x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = \int \frac{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}}{x^2} dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = \int \frac{(a^2 + b^2x^{2n} + 2abx^n)^{3/2}}{x^2} dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

3.528 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 218

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx = -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2(a+bx^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)(ab+b^2x^n)} - \frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)(ab+b^2x^n)} - \frac{b^4x^{-2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)(ab+b^2x^n)}$$

```
[Out] -1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-3/2*a*b^3*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(x^(2-2*n))/(a*b+b^2*x^n)-3*a^2*b^2*x^(-2+n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)-b^4*x^(-2+3*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-3*n)/(a*b+b^2*x^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 276}

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx = -\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)(ab+b^2x^n)} - \frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)(ab+b^2x^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)(ab+b^2x^n)} - \frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2(a+bx^n)}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x^2*(a + b*x^n)) - (3*a*b^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x
^n)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*
(a*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(
(2 - 3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\frac{(ab+b^2x^n)^3}{x^3}\,dx}{b^2\,(ab+b^2x^n)}\\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(\frac{a^3b^3}{x^3}+3a^2b^4x^{-3+n}+b^6x^{3(-1+n)}+3ab^5x^{-3+2n}\right)dx}{b^2\,(ab+b^2x^n)}\\
&= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\,(ab+b^2x^n)}\\
&\quad-\frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\,(ab+b^2x^n)}-\frac{b^4x^{-2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\,(ab+b^2x^n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$
\int\frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx=\frac{\sqrt{(a+bx^n)^2}\left(a^3(4-12n+11n^2-3n^3)+6a^2b(2-5n+3n^2)\,x^n+3ab^2(4\right.}{2(-2+n)(-1+n)(-2+3n)x^2\,(a+}
$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3
*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}}{2(a+b\,x^{n})x^{2}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{3n}}{(a+b\,x^{n})(-2+3n)x^{2}}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b^{2}a\,x^{2n}}{2(a+b\,x^{n})(-1+n)x^{2}}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b\,x^{n}}{(a+b\,x^{n})(-2+n)x^{2}}$	145

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-
2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b^2*a/x^2*(
x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)*a^2*b/x^2*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx =$$
$$-\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\,(b^3n^2-3\,b^3n+2\,b^3)x^{3\,n}-3\,(3\,ab^2n^2-8\,ab^2n+4\,ab^2)x^{2\,n}-6\,(3\,a^2bn}{2\,(3\,n^3-11\,n^2+12\,n-4)x^2}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{2\,(n^2 - 3\,n + 2)b^3x^{3\,n} + 3\,(3\,n^2 - 8\,n + 4)ab^2x^{2\,n} + 6\,(3\,n^2 - 5\,n + 2)a^2bx^n}{2\,(3\,n^3 - 11\,n^2 + 12\,n - 4)x^2}$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6*
(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11
*n^2 + 12*n - 4)*x^2)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}{x^3}\,dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(dx)^{1+m}(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/d/(1+m
)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1369, 371}

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(dx)^{m+1}(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(ab+b^2x^n)\int \frac{(dx)^m}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(dx)^{1+m}(a+bx^n)\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x(dx)^m(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*
x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{(d\,x)^m}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}} dx$

Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}} dx = \frac{x^3(a+bx^n) \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}} dx = \frac{x^3(a+bx^n) \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ab+b^2x^n)\int \frac{x^2}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3(a+bx^n)\;{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x**2/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 371}

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(ab+b^2x^n)\int \frac{x}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{x}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x}{\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x*(a+b*x^n)*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b^2*x^(2*
n))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1357, 251}

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2ab+2b^2x^n)\int \frac{1}{2ab+2b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{1}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (a+b*x^n)*ln(x)/a/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a+b*x^n)*ln(a+b*x^n)/a
/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1369, 272, 36, 29, 31}

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(ab+b^2x^n)\int \frac{1}{x(ab+b^2x^n)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{(ab+b^2x^n)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{abn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(b(ab+b^2x^n))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= \frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(a+bx^n)\left(\log(x^n)-\log(an(a+bx^n))\right)}{an\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] ((a + b*x^n)*(Log[x^n] - Log[a*n*(a + b*x^n)]))/(a*n*Sqrt[(a + b*x^n)^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,\ln(x)}{(a+b\,x^n)a} - \frac{\sqrt{(a+b\,x^n)^2}\,\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)an}$	66

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n*ln(
x^n+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{n\log(x)-\log(bx^n+a)}{an}$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}x}\,dx$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{x\,\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a^2+2*a*b*x^n+b^2*
x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(ab + b^2x^n)\int \frac{1}{x^2(ab+b^2x^n)}\,dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

$$= -\frac{(a + bx^n)\,{}_2F_1\left(1, -\frac{1}{n}; -\frac{1-n}{n}; -\frac{bx^n}{a}\right)}{ax\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = -\frac{(a + bx^n)\,\text{Hypergeometric2F1}\left(1, -\frac{1}{n}, 1 - \frac{1}{n}, -\frac{bx^n}{a}\right)}{ax\sqrt{(a + bx^n)^2}}$$

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a*
x*Sqrt[(a + b*x^n)^2]))
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a^2 + 2ab\,x^n + b^2x^{2n}}}dx$$

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x
^n + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(
a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ab+b^2x^n)\int \frac{1}{x^3(ab+b^2x^n)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a*x^2
*Sqrt[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}dx$$

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^3}\,dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x
^n + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x^3\sqrt{(a+bx^n)^2}}\,dx$$

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^3}\,dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x^3}\,dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x^3\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.536 $\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx = \frac{(dx)^{1+m}(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/d/(1
+m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1369, 371}

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx = \frac{(dx)^{m+1}(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^n))\int \frac{(dx)^m}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(dx)^{1+m}\,(a+bx^n)\;{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x(dx)^m\,(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3(1+m)\sqrt{(a+bx^n)^2}}$$

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*
x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F]

$$\int \frac{(dx)^m}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
+ a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e^(
m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{(d\,x)^m}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.537 $\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([3, 3/n],[(3+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(b^2(ab+b^2x^n))\int \frac{x^2}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3(a+bx^n)\ {}_2F_1\left(3,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^3(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a^3\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] (x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F]

$$\int \frac{x^2}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*x
^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] (2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*
(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n
 + a^4*n^2)
```

Giac [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\, dx = \int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.538 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 371}

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(b^2(ab+b^2x^n))\int \frac{x}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2(a+bx^n)\ {}_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^2(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
[Out] (x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F]

$$\int \frac{x}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^(
2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(x/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n - 1
)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Giac [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.539 $\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\,(a^2+2abx^n+b^2x^{2n})^{3/2}}$$

```
[Out] x*(a+b*x^n)^3*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n+b^2*x
^(2*n))^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1357, 251}

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\,(a^2+2abx^n+b^2x^{2n})^{3/2}}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2ab+2b^2x^n)^3 \int \frac{1}{(2ab+2b^2x^n)^3}\,dx}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\\ &= \frac{x(a+bx^n)^3\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*((a + b*x^n)^2)^(3/2))
```

Maple [F]

$$\int \frac{1}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2*
n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*n
 - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Giac [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.540 $\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\left(a+bx^n\right)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/a^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)+(a+b*x^n)*ln(x)/a^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a
+b*x^n)*ln(a+b*x^n)/a^3/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1369, 272, 46}

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b^2(ab+b^2x^n))\int \frac{1}{x(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{(b^2(ab+b^2x^n))\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^3}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{(b^2(ab+b^2x^n))\,\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&= \frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}} \\
&\quad+\frac{(a+bx^n)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{a(3a+2bx^n)+2(a+bx^n)^2\log\left(x^n\right)-2(a+bx^n)^2\log\left(a+bx^n\right)}{2a^3n\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
[Out] (a*(3*a + 2*b*x^n) + 2*(a + b*x^n)^2*Log[x^n] - 2*(a + b*x^n)^2*Log[a + b*x
^n])/(2*a^3*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\ln\left(x\right)}{\left(a+b\,x^{n}\right)a^{3}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(2b\,x^{n}+3a\right)}{2\left(a+b\,x^{n}\right)^{3}a^{2}n}-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\ln\left(x^{n}+\frac{a}{b}\right)}{\left(a+b\,x^{n}\right)a^{3}n}$	104

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3
*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{2\,b^2nx^{2\,n}\log\left(x\right)+2\,a^2n\log\left(x\right)+3\,a^2+2\left(2\,abn\log\left(x\right)+ab\right)x^n-2\left(b^2x^2\right.}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a^
3 - log((b*x^n + a)/b)/(a^3*n)
```

Giac [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.541 $\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([3, -1/n],[(-1+n)/n],-b*x^n/a)/a^3/x/(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{1}{x^2(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= -\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = -\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
[Out] -(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(
a^3*x*((a + b*x^n)^2)^(3/2)))
```

Maple [F]

$$\int \frac{1}{x^2\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*x
^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2)*
x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{x^2 \left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/2
*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x^n
 + a^4*n^2*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.542 $\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([3, -2/n],[(-2+n)/n],-b*x^n/a)/a^3/x^2/(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1369, 371}

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(
a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(b^2(ab+b^2x^n)) \int \frac{1}{x^3(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$= -\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\,(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
[Out] -1/2*((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a^3
*x^2*((a + b*x^n)^2)^(3/2))
```

Maple [F]

$$\int \frac{1}{x^3\,(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}dx$$

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}\, dx = \int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^3}\, dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*x
^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3)*
x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}\, dx = \int \frac{1}{x^3 \left(\left(a + bx^n\right)^2\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}\, dx = \int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^3}\, dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2*
b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x^n
 + a^4*n^2*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}\, dx = \int \frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{3}{2}} x^3}\, dx$$

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.543 $\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx$

Optimal result

Integrand size $=36$, antiderivative size $=52$

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

```
[Out] x*(a+b*x^(1/(-1-2*p)))*(a^2+2*a*b*x^(1/(-1-2*p))+b^2/(x^(2/(1+2*p))))^p/a
```

Rubi [A] (verified)

Time $=0.01$ (sec) , antiderivative size $=52$, normalized size of antiderivative $=1.00$, number of steps used $=2$, number of rules used $=2$, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used $=\{1357, 197\}$

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

```
[In] Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
```

```
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \left(\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{-2p}\right)\int\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{2p}dx$$
$$=\frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{x^{\frac{2p}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)\left(x^{-\frac{2}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)^2\right)^p}{a}$$

```
[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x
^(2/(1 + 2*p)))^p)/a
```

Maple [F]

$$\int\left(a^2+b^2x^{-\frac{2}{1+2p}}+2ab\,x^{-\frac{1}{1+2p}}\right)^p dx$$

```
[In] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
[Out] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.52

$$\int\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p dx=\frac{\left(axx^{\left(\frac{1}{2p+1}\right)}+bx\right)\left(\frac{a^2x^{\frac{2}{2p+1}}+2abx^{\left(\frac{1}{2p+1}\right)}+b^2}{x^{\frac{2}{2p+1}}}\right)^p}{ax^{\left(\frac{1}{2p+1}\right)}}$$

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="f
ricas")
[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) +
 b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2\,p+1}\right)}}\right)^p\,dx$$

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="m
axima")
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

Giac [F]

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,ab}{x^{\left(\frac{1}{2\,p+1}\right)}}\right)^p\,dx$$

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="g
iac")
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\,dx=\int \left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,a\,b}{x^{\frac{1}{2\,p+1}}}\right)^p\,dx$$

```
[In] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p,x)
[Out] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p, x)
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}} dx$

Optimal result

Integrand size = 33, antiderivative size = 43

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}} dx=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}$$

```
[Out] x*(a+b*x^n)/a/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1357, 197}

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}} dx=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(2ab+2b^2x^n\right)^{-\frac{-1-n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\right)\int\left(2ab+2b^2x^n\right)^{\frac{-1-n}{n}}\,dx\\ &= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\frac{x(a+bx^n)\left((a+bx^n)^2\right)^{-\frac{1+n}{2n}}}{a}$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
norman	$\left(x+\frac{bx\,\mathrm{e}^{n\ln(x)}}{a}\right)\mathrm{e}^{\frac{(1+n)\ln\left(\frac{1}{\sqrt{a^2+2ab\,\mathrm{e}^{n\ln(x)}+b^2\mathrm{e}^{2n\ln(x)}}}\right)}{n}}$	51

```
[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x,method=_RETURNVERBOSE)
[Out] (x+b/a*x*exp(n*ln(x)))/exp(1/2*(1+n)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(n*
ln(x))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\frac{bxx^n+ax}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{n+1}{2n}}a}$$

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="frica
s")
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(37) = 74$.

Time = 4.98 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.72

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx = \begin{cases} x(a^2+2abx^n+b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}+\frac{bxx^n(a^2+2abx^n+b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}}{a} & \text{for } a\neq 0\\ -x(b^2x^{2n})^{-1-\frac{1}{n}}(b^2x^{2n})^{\frac{1}{2}+\frac{1}{2n}}+x(b^2x^{2n})^{-\frac{1}{2}-\frac{1}{2n}}-\frac{x(b^2x^{2n})^{-1-\frac{1}{n}}(b^2x^{2n})^{\frac{1}{2}+\frac{1}{2n}}}{n} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

```
[Out] Piecewise((x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1/2 - 1/(2*n)) + b*x*x*
*n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1/2 - 1/(2*n))/a, Ne(a, 0)), (-x*
(b**2*x**(2*n))**(-1 - 1/n)*(b**2*x**(2*n))**(1/2 + 1/(2*n)) + x*(b**2*x**(
2*n))**(-1/2 - 1/(2*n)) - x*(b**2*x**(2*n))**(-1 - 1/n)*(b**2*x**(2*n))**(1
/2 + 1/(2*n))/n, True))
```

Maxima [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx = \int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{n+1}{2\,n}}}\,dx$$

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

Giac [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx = \int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{n+1}{2\,n}}}\,dx$$

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{\frac{n}{2}+\frac{1}{2}}{n}}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)
```

3.545 $\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2 (1+p)}}\right)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2 (1+p)}}\right)^p \, dx=\frac{2 (1+p) x \left(a+b x^{-\frac{1}{2 (1+p)}}\right) \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2 (1+p)}}\right)^p}{a (1+2 p)}-\frac{x \left(a+b x^{-\frac{1}{2 (1+p)}}\right)^2 \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2 (1+p)}}\right)^p}{a^2 (1+2 p)}$$

```
[Out] 2*(p+1)*x*(a+b/(x^(1/2/(p+1))))*(a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1/2/(p+1))
))^p/a/(1+2*p)-x*(a+b/(x^(1/2/(p+1))))^2*(a^2+b^2/(x^(1/(p+1)))+2*a*b/(x^(1
/2/(p+1))))^p/a^2/(1+2*p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1357, 198, 197}

$$\int \left(a^2+b^2 x^{-\frac{1}{1+p}}+2 a b x^{-\frac{1}{2 (1+p)}}\right)^p \, dx=\frac{2 (p+1) x \left(a+b x^{-\frac{1}{2 (p+1)}}\right) \left(a^2+2 a b x^{-\frac{1}{2 (p+1)}}+b^2 x^{-\frac{1}{p+1}}\right)^p}{a (2 p+1)}-\frac{x \left(a+b x^{-\frac{1}{2 (p+1)}}\right)^2 \left(a^2+2 a b x^{-\frac{1}{2 (p+1)}}+b^2 x^{-\frac{1}{p+1}}\right)^p}{a^2 (2 p+1)}$$

```
[In] Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{2p}dx\\
&= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}\\
&\quad-\frac{\left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int\left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{1+2p}dx}{2ab(1+2p)}\\
&= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}\\
&\quad-\frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a^2(1+2p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \frac{x^{\frac{p}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)\left(x^{-\frac{1}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)^2\right)^p\left(-b+a(1+2p)x^{\frac{1}{2+2p}}\right)}{a^2(1+2p)}$$

```
[In] Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
[Out] (x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

Maple [F]

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2ab\,x^{-\frac{1}{2(1+p)}}\right)^p dx$$

```
[In] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
[Out] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+(2\,a^2p+a^2)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{(2\,a^2p+a^2)x^{\left(\frac{1}{p+1}\right)}}$$

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fri
cas")
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a
*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p +
a^2)*x^(1/(p + 1)))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\int \left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="max
ima")
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

Giac [F]

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\int \left(a^2+\frac{2\,ab}{x^{\frac{1}{2\,(p+1)}}}+\frac{b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p\,dx$$

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gia
c")
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx=\int \left(\frac{b^2}{x^{\frac{1}{p+1}}}+a^2+\frac{2\,a\,b}{x^{\frac{1}{2\,(p+1)}}}\right)^p\,dx$$

```
[In] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)
[Out] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a^2(1+n)}$$

```
[Out] x*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a/(1+n)+n*x*(a+b*x^n)^2*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a^2/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1357, 198, 197}

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \left(\left(2ab+2b^2x^n\right)^{-\frac{-1-2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\right)\int\left(2ab+2b^2x^n\right)^{\frac{-1-2n}{n}}\,dx$$

$$=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{\left(n\left(2ab+2b^2x^n\right)^{-\frac{-1-2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\right)\int\left(2ab+2b^2x^n\right)^{1+\frac{-1-2n}{n}}\,dx}{2ab(1+n)}$$

$$=\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a^2(1+n)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$$
$$=\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

Maple [F]

$$\int \left(a^2+2ab\,x^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx$$

[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)

[Out] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx = \frac{b^2nxx^{2\,n}+(2\,abn+ab)xx^n+(a^2n+a^2)x}{(a^2n+a^2)(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{2\,n+1}{2\,n}}}$$

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fricas")

[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))

Sympy [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx = \begin{cases} \frac{x}{2\sqrt{\frac{b^2}{x^2}}} \\ -x\left(b^2x^{2n}\right)^{-2-\frac{1}{n}}\left(b^2x^{2n}\right)^{1+\frac{1}{2n}}+x\left(b^2x^{2n}\right)^{-1-\frac{1}{2n}}-\frac{x\left(b^2x^{2n}\right)^{-2-\frac{1}{n}}\left(b^2x^{2n}\right)^{1+\frac{1}{2n}}}{2n} \\ \int \frac{1}{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}\,dx \\ \frac{a^2nx\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{a^2x\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{2abnxx^n\left(a^2+2abx^n+b^2x^{2n}\right)^{-1-\frac{1}{2n}}}{a^2n+a^2}+\frac{abxx^n\left(a^2+2abx^n+b^2x^{2n}\right)^{-1}}{a^2n+a^2} \end{cases}$$

[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)

[Out] Piecewise((x/(2*sqrt(b**2/x**2)), Eq(a, 0) & Eq(n, -1)), (-x*(b**2*x**(2*n))**(-2 - 1/n)*(b**2*x**(2*n))**(1 + 1/(2*n)) + x*(b**2*x**(2*n))**(-1 - 1/(2*n)) - x*(b**2*x**(2*n))**(-2 - 1/n)*(b**2*x**(2*n))**(1 + 1/(2*n))/(2*n), Eq(a, 0)), (Integral(1/sqrt(a**2 + 2*a*b/x + b**2/x**2), x), Eq(n, -1)), (a**2*n*x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*n))/(a**2*n + a**2) + a**2*x*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*n))/(a**2*n + a*

```
*2) + 2*a*b*n*x*x**n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*n))/(a
**2*n + a**2) + a*b*x*x**n*(a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-1 - 1/(2*
n))/(a**2*n + a**2) + b**2*n*x*x**(2*n)*(a**2 + 2*a*b*x**n + b**2*x**(2*n))
**(-1 - 1/(2*n))/(a**2*n + a**2), True))
```

Maxima [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

Giac [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{2\,n+1}{2\,n}}}\,dx$$

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{\frac{n+\frac{1}{2}}{n}}}\,dx$$

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 117

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{(dx)^{-2n(1+p)}(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(1+2p)} + \frac{(dx)^{-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^{1+p}}{2a^2dn(1+p)(1+2p)}$$

```
[Out] -(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/a/d/n/(1+2*p)/((d*x)^(2*n*(p+1)))+
1/2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(p+1)/a^2/d/n/(p+1)/(1+2*p)/((d*x)^(2*n*(p+
1)))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1370, 279, 270}

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \frac{\left(\frac{bx^n}{a}+1\right)^2 (dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)} - \frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

```
[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^
(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*
d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int (dx)^{-1-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^{2p} dx \\
&= -\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)} \\
&\quad+\frac{\left((-2n(1+p)+n(1+2p))\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int (dx)^{-1-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^{1+2p}}{n(1+2p)} \\
&= -\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)} \\
&\quad+\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(1+3p+2p^2\right)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p dx = -\frac{x(dx)^{-1-2n(1+p)}\left((a+bx^n)^2\right)^p\left(1+\frac{bx^n}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-2p,-2(1+p),1-2(1+p),-\frac{bx^n}{a}\right)}{2n(1+p)}$$

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
-2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p))
```

Maple [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2ab\,x^n + b^2x^{2n}\right)^p dx$$

```
[In] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

```
[Out] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = -\frac{\left(2\,abpxx^n e^{(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x))} - b^2xx^{2\,n} e^{(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x))} + (2\,a^2p + a^2)}{2\,(2\,a^2np^2 + 3\,a^2np + a^2n)}$$

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x))
+ (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = \int (dx)^{-2n(p+1)-1} \left((a + bx^n)^2\right)^p \, dx$$

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Integral((d*x)**(-2*n*(p + 1) - 1)*((a + b*x**n)**2)**p, x)
```

Maxima [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = \int \left(b^2x^{2n} + 2\,abx^n + a^2\right)^p (dx)^{-2\,n(p+1)-1} \, dx$$

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
maxima")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Giac [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = \int \left(b^2x^{2n} + 2\,abx^n + a^2\right)^p (dx)^{-2\,n(p+1)-1} \, dx$$

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1-2n(1+p)} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = \int \frac{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^p}{(d\,x)^{2\,n\,(p+1)+1}} \, dx$$

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

3.548 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(1+p)}$$

```
[Out] -a^2*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+2*p)+1/2*a^2*(1+b*x
^n/a)^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1370, 272, 45}

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{a^2\left(\frac{bx^n}{a}+1\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int x^{-1+2n}\left(1+\frac{bx^n}{a}\right)^{2p}\,dx\\ &= \frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^n\right)}{n}\\ &= \frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2b^2n(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.52

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^p(-a+b(1+2p)x^n)}{2b^2n(1+p)(1+2p)}$$

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
+ 2*p))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{\left(-2 b^{2} p\, x^{2 n}-2 a p\, x^{n} b-b^{2} x^{2 n}+a^{2}\right) \left(a+b\, x^{n}\right)^{2 p} {\mathrm e}^{-\frac{i \operatorname{csgn}\left(i \left(a+b\, x^{n}\right)^{2}\right) \pi p \left(-\operatorname{csgn}\left(i \left(a+b\, x^{n}\right)^{2}\right)+\operatorname{csgn}\left(i \left(a+b\, x^{n}\right)\right)\right)^{2}}{2}}}{2 \left(1+2 p\right) \left(1+p\right) n\, b^{2}}$	113

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(1+2*p)/(1+p)/n/b^2*(a+
b*x^n)^(2*p)*exp(-1/2*I*csgn(I*(a+b*x^n)^2)*Pi*p*(-csgn(I*(a+b*x^n)^2)+csgn
(I*(a+b*x^n)))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int x^{-1+2n} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \frac{(2\,abpx^n - a^2 + (2\,b^2p + b^2)x^{2\,n})(b^2x^{2\,n} + 2\,abx^n + a^2)^p}{2\,(2\,b^2np^2 + 3\,b^2np + b^2n)}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
+ a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n} \left(a^2 + 2abx^n + b^2x^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{\left(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2\right)(bx^n+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2n}$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2 +
 3*p + 1)*b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^px^{2\,n-1}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p\,dx$$

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b(b^2-3ac)\,\mathrm{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+1/2*(-a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/c^3/n+b*(
-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c^3/n/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 715, 648, 632, 212, 642}

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\frac{b(b^2-3ac)\,\mathrm{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)x}{c^2(a+bx+cx^2)}\right)\,dx, x, x^n\right)}{n}\end{aligned}$$

$$= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^n\right)}{c^2n}$$

$$= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} - \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^3n} + \frac{(b^2-ac)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^3n}$$

$$= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n} + \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{c^3n}$$

$$= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n} + \frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{cx^n(-2b+cx^n) - \frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2c^3n}$$

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c^3
*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. 2(103) = 206.

Time = 0.35 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.77

method	result
risch	$-\frac{\ln(x)a}{c^2} + \frac{\ln(x)b^2}{c^3} + \frac{x^{2n}}{2cn} - \frac{b\,x^n}{c^2n} + \frac{4n^2\ln(x)a^2c^2}{4a\,c^4n^2-b^2c^3n^2} - \frac{5n^2\ln(x)a\,b^2c}{4a\,c^4n^2-b^2c^3n^2} + \frac{n^2\ln(x)b^4}{4a\,c^4n^2-b^2c^3n^2} - \frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-36a}}{2}\right.}{c(4a}$

```
[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-b
^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2*c
+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*
a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a
*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*
c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^
2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2
*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3
+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/c^3/(4*a*c-
b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*
c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b
^8)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-(b^2c^2-4\,ac^3)x^{2\,n}+2\,(b^3c-4}{2\,(b^2c^3-4\,ac^4)n}\right.$$

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 -
5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n),
 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*
x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) -
2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b
*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a*
b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x),
 x)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{4\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}} dx = \frac{x^n}{cn} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}n} - \frac{b\log(a+bx^n+cx^{2n})}{2c^2n}$$

```
[Out] x^n/c/n-1/2*b*ln(a+b*x^n+c*x^(2*n))/c^2/n-(-2*a*c+b^2)*arctanh((b+2*c*x^n)/
(-4*a*c+b^2)^(1/2))/c^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 717, 648, 632, 212, 642}

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}} dx = -\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}} - \frac{b\log(a+bx^n+cx^{2n})}{2c^2n} + \frac{x^n}{cn}$$

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\
&= \frac{x^n}{cn} + \frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^n\right)}{cn}\\
&= \frac{x^n}{cn} - \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^2n} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2c^2n}\\
&= \frac{x^n}{cn} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{c^2n}\\
&= \frac{x^n}{cn} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx=\frac{2cx^n+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+x^n(b+cx^n)\right)}{2c^2n}$$

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + x^n*(b + c*x^n)])/(2*c^2*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(81) = 162.

Time = 0.29 (sec) , antiderivative size = 664, normalized size of antiderivative = 7.63

method	result
risch	$-\frac{b\ln(x)}{c^2}+\frac{x^n}{cn}+\frac{4n^2\ln(x)abc}{4a\,c^3n^2-b^2c^2n^2}-\frac{n^2\ln(x)b^3}{4a\,c^3n^2-b^2c^2n^2}-\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16c^3a^3+20a^2b^2c^2-8a\,b^4c+b^6}}{2c\left(2ac-b^2\right)}\right)ab}{(4ac-b^2)cn}+\ln\left(x^n-\right.$

```
[In] int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^3
*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-2*a*b*c+b^3+
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2/(4
*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^
4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b
*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-1
6*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/(4*a*c-b^2)/c/n*ln(x^n+1/2*
(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*a*b+1/2/(4*a*c-b^2)/c^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^
2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/(4*a*c-b^2)/c^2/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*
c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.28

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^2-4\,ac^3)n}, -\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^2-4\,ac^3)n}\right]$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2
*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n
+ a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a
))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b
^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2
 - 4*a*c^3)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*
n) + b*c^2*x*x^n + a*c^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}n}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[Out] 1/2*ln(a+b*x^n+c*x^(2*n))/c/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c/n
/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1371, 648, 632, 212, 642}

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2cn} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2cn}\\
&= \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{cn}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}n} + \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+x^n(b+cx^n)\right)}{2cn}$$

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
[Out] ((-2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + x^n*(b + c*x^n)])/(2*c*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(62) = 124.

Time = 0.24 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.91

method	result
risch	$\frac{\ln(x)}{c}-\frac{4n^2\ln(x)ac}{4a\,c^2n^2-b^2c\,n^2}+\frac{n^2\ln(x)b^2}{4a\,c^2n^2-b^2c\,n^2}+\frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)a}{(4ac-b^2)n}-\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)b^2}{2c(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2}{}\right.}{}$

```
[In] int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)
*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c
)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1
/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a
-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/c
/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)+(b^2-4\,ac)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c-4\,ac^2)n},\frac{2\sqrt{-b^2+}}{}\right.$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2
- 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)
*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} \, dx = -\frac{2\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}n}$$

```
[Out] -2*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1366, 632, 212}

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} \, dx = -\frac{2\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

```
[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{n}\\
&= -\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{2\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}n}$$

```
[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
[Out] (2*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(35) = 70.

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.90

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{b^2-4ac+b\sqrt{-4ac+b^2}}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}+\frac{\ln\left(x^n+\frac{b\sqrt{-4ac+b^2}+4ac-b^2}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}$	113

```
[In] int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(-4*a
*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)+4*a*
c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.08

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}n}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)}{(b^2-4\,ac)n}\right]$$

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt(-
b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}n}$$

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] (2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx = -\frac{x^{-n}}{an} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}n} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n+cx^{2n})}{2a^2n}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+1/2*b*ln(a+b*x^n+c*x^(2*n))/a^2/n-(-2*a*c+b^2)*arc
tanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx = -\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}} + \frac{b\log(a+bx^n+cx^{2n})}{2a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

```
[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-n}}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-n}}{an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^n\right)}{a^2n}$$

$$= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^2n} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^2n}$$

$$= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n+cx^{2n})}{2a^2n} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{a^2n}$$

$$= -\frac{x^{-n}}{an} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}n} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n+cx^{2n})}{2a^2n}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \frac{-2ax^{-n} + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x^n) + b\log(a+x^n(b+cx^n))}{2a^2n}$$

```
[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a)/x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sq
rt[-b^2 + 4*a*c] - 2*b*Log[x^n] + b*Log[a + x^n*(b + c*x^n)])/(2*a^2*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.71

method	result
risch	$-\frac{x^{-n}}{an} - \frac{4n^2 \ln(x) abc}{4a^3 c n^2 - a^2 b^2 n^2} + \frac{n^2 \ln(x) b^3}{4a^3 c n^2 - a^2 b^2 n^2} + \frac{2 \ln\left(x^n - \frac{-2abc+b^3+\sqrt{-16c^3a^3+20a^2b^2c^2-8a\,b^4c+b^6}}{2c(2ac-b^2)}\right) bc}{a(4ac-b^2)n} - \frac{\ln\left(x^n - \frac{-2abc+b^3+}{}\right.}{}$

[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)

[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^2 *b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3 *c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1/2/a^2/(4*a*c -b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6) ^(1/2))/c/(2*a*c-b^2))*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^ 3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c -b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1 /2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8* a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2* a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))* (-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-n}}{a + bx^n + cx^{2n}}\,dx$$
$$= \left[-\frac{2\,(b^3 - 4\,abc)nx^n \log(x) + (b^2 - 2\,ac)\sqrt{b^2 - 4\,ac}x^n \log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right) + 2\,a}{2\,(a^2b^2 - 4\,a^3c)nx^n}\right.$$
$$-\frac{2\,(b^3 - 4\,abc)nx^n \log(x) + 2\,(b^2 - 2\,ac)\sqrt{-b^2 + 4\,ac}x^n \arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right) + 2\,ab^2 - 8\,a^2}{2\,(a^2b^2 - 4\,a^3c)nx^n}$$

[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] [-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n *log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqr

```
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a
*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*
(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arcta
n(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a
*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2
- 4*a^3*c)*n*x^n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x
)
```

Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}$$

```
[Out] -1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(a+b
*x^n+c*x^(2*n))/a^3/n+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2)
)/a^3/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*1/(a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3
 - ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^n\right)}{a^3n}$$

$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^3n} - \frac{(b^2-ac)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^3n}$$

$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n} + \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{a^3n}$$

$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{ax^{-2n}(-a+2bx^n) - \frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 2(b^2-ac)\log(x^n) - (b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2a^3n}$$

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-a + 2*b*x^n))/x^(2*n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[
-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x^n] - (b^2 - a*c)*L
og[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(120) = 240.

Time = 0.29 (sec) , antiderivative size = 958, normalized size of antiderivative = 7.60

method	result
risch	$\frac{b\,x^{-n}}{a^{2} n}-\frac{x^{-2 n}}{2 a n}-\frac{4 n^{2} \ln \left(x \right) a^{2} c^{2}}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}+\frac{5 n^{2} \ln \left(x \right) a\,b^{2} c}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}-\frac{n^{2} \ln \left(x \right) b^{4}}{4 a^{4} c\,n^{2}-a^{3} b^{2} n^{2}}+\frac{2 \ln \left(x^{n}+\frac{3 a\,b^{2} c-b^{4}+\sqrt{-36 a^{3} b^{2} c^{3}+33 a^{2} b^{4} c^{2}-10 a\,b^{6}}}{2 c b \left(3 a c-b^{2}\right)}\right)}{a \left(4 a c-b^{2}\right) n}$

```
[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b
^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2)
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \left[-\frac{a^{2}b^{2}-4\,a^{3}c-2\,(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2})nx^{2\,n}\log\left(x\right)+(b^{3}-3\,abc)\sqrt{b^{2}-4\,ac}x^{2\,n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,ac+2\,\left(bc-\right.}{cx^{2\,n}}\right.}{2\,(a^{3}b^{2}-4\,a^{4}c)n}\right.$$

$$-\frac{a^{2}b^{2}-4\,a^{3}c-2\,(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2})nx^{2\,n}\log\left(x\right)-2\,(b^{3}-3\,abc)\sqrt{-b^{2}+4\,ac}x^{2\,n}\arctan\left(-\frac{2\,\sqrt{-b^{2}+4\,acc}}{b^{2}-}\right.}{2\,(a^{3}b^{2}-4\,a^{4}c)nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x)
 + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2*a
*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) +
b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x^n +
 a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2
*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n +
 sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*
n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^
4*c)*n*x^(2*n))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=-\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)\,x^{-n}}{a^3n}-\frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}n}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}$$

```
[Out] -1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))+(a*c-b^2)/a^3/n/(x^n)-b*(-2*a*c+b^
2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/a^4/n-(2*a^2*c^2-4*a*
b^2*c+b^4)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^4/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\,(b^2-2ac)}{a^4}-\frac{x^{-n}(b^2-ac)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{(2a^2c^2-4ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-3n}}{3an}$$

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/3*1/(a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^
4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*S
```

```
qrt[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a
 + b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx+cx^2)}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx, x, x^n\right)}{an} \\
&= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx, x, x^n\right)}{an} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} \\
&\quad + \frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx, x, x^n\right)}{a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} \\
&\quad + \frac{(b(b^2-2ac))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^4n} \\
&\quad + \frac{(b^4-4ab^2c+2a^2c^2)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} \\
&\quad + \frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n} \\
&\quad - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}n} \\
&\quad - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{ax^{-3n}(-2a^2-6b^2x^{2n}+3ax^n(b+2cx^n))+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x^n)+3(b^3-}{6a^4n}$$

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-2*a^2 - 6*b^2*x^(2*n) + 3*a*x^n*(b + 2*c*x^n)))/x^(3*n) + (6*(b^4 - 4
*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x^n] + 3*(b^3 - 2*a*b*c)*Log[a + x^n*(b + c
*x^n)])/(6*a^4*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(158) = 316.

Time = 0.37 (sec) , antiderivative size = 1300, normalized size of antiderivative = 7.93

method	result	size
risch	Expression too large to display	1300

```
[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*
a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*
ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n
*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b
^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*
b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^
5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2
*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^
2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-1
2*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)
/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4)
)*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c
/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
```

```
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$

$$= \left[-\frac{2\,a^3b^2 - 8\,a^4c + 6\,(b^5 - 6\,ab^3c + 8\,a^2bc^2)nx^{3\,n}\log\left(x\right) - 3\,(b^4 - 4\,ab^2c + 2\,a^2c^2)\sqrt{b^2 - 4\,ac}x^{3\,n}\log\left(\frac{2\,c^2x}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\,a^3b^2 - 8\,a^4c + 6\,(b^5 - 6\,ab^3c + 8\,a^2bc^2)nx^{3\,n}\log\left(x\right) + 6\,(b^4 - 4\,ab^2c + 2\,a^2c^2)\sqrt{-b^2 + 4\,ac}x^{3\,n}\arctan\left(\cdots\right.}{\cdots}$$

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*lo
g(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c^2
*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a
*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)
*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) -
 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(2*a^3*
b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) + 6*(b^4
 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqrt(-b^2 +
 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x
^(3*n))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

Giac [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 353

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\, 2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\, 2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\, 2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\, 2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

```
[Out] 2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))/n/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2*2^(3/4)*c^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/n/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*
c^(1/4)*x^(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(
-4*a*c+b^2)^(1/2))^(3/4)-2*2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(3/4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1395, 1361, 218, 214, 211}

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{2\,2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2\,2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{4\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, x^{n/4}\right)}{n}$$

$$= \frac{(4c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}n} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}n}$$

$$= \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}n} + \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}n} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}n} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}n}$$

$$= \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}} - \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\sqrt[4]{-b-\sqrt{b^2}}\cdots}{\cdots}\right.}{n}$$

```
[In] Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/
4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4*a*c
])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])
/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))))/n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(256a^7c^4n^8-256a^6b^2c^3n^8+96a^5b^4c^2n^8-16a^4b^6cn^8+a^3b^8n^8\right)\_Z^8+\left(-48a^3bc^3n^4+40a^2b^3c^2n^4-11ab^5cn^4+b^7n^4\right)\_Z^4\cdots\right.}$

```
[In] int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^3
*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*c^
2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8-25
6*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-4
8*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3481 vs. $2(273) = 546$.

Time = 0.40 (sec) , antiderivative size = 3481, normalized size of antiderivative = 9.86

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)
*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)
 - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))
/x) + 1/2*sqrt(2)*sqrt(-sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8
*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*n)*sqrt(-sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^
```

4))))/x) - 1/2*sqrt(2)*sqrt(-sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*
b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*n)*sqrt(-sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n
^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48
*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) + sqrt(2)*((a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5*a*b^2
*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*
n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)*((a^
3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(-sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) + sqrt(2)*((
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*n)*sqrt(-sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*
b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(-sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/4*n - 1) - sqrt(2)
*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1

```
6*a^5*c^2)*n^4))))/x)
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{4}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{4}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 610

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

$$= -\frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

```
[Out] 2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b-(-4*a*c+b^2)^(1/2))^(1/3))/
n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/2*c^(2/3)*ln(2^(2/3)*c^
(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^
2)^(1/2)-2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/(b-(-4*a
*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a
*c+b^2)^(1/2)-2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b+(-4*a*c+b^2)^
(1/2))^(1/3))/n/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/2*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b+(-4*a*c+b^2)^(1/
2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(-4*a*c+b^2)^(1/2)/(b+(-4
```

```
*a*c+b^2)^(1/2))^(2/3)+2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1
/3*n)/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1395, 1361, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$
$$= -\frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
```

```
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{1}{a+bx^3+cx^6}\,dx, x, x^{n/3}\right)}{n}$$

$$= \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}n} - \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}n}$$

$$= \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$- \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$- \frac{\left(2^{2/3}c\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx,x,x^{n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx,x,x^{n/3}\right)}{2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}n} + \frac{c^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx,x,x^{n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{(3c)\text{Subst}\left(\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx,x,x^{n/3}\right)}{2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}n}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$- \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$- \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$+ \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$+ \frac{\left(3\ 2^{2/3}c^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$- \frac{\left(3\ 2^{2/3}c^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$= -\frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{c^{2/3}\left(-2\sqrt{3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)+2\sqrt{3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{}}{\sqrt[3]{b-}}}{}\right.\right.}{}$$

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

[Out] (c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b - Sq
rt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt
[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - (b +
Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3
)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)] + (b
 - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(
1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]))
/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.43

method	result
risch	$\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)} \_R\ln\left(x^{\frac{n}{3}}+\left(-\frac{16}{2}\right.\right.$

[In] int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)

[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n^
4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*c^
2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64*a^
5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c
^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. 2(465) = 930.

Time = 0.35 (sec) , antiderivative size = 2461, normalized size of antiderivative = 4.03

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] -1/2*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b
^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n
^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(4*(b^2*c - 2*a*c^2)*x*x^(1
/3*n - 1) + (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*n + (b^4 -

6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*n^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4)*sqrt((b^4 - 4*a*b^2*c +
4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)))*(
((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*
a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^
3))^(1/3))/x) + 1/2*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
- 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(4*(b^2*c -
2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c
^2)*n - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*n^4 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4)*sqrt((b^4
- 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^
7*c^3)*n^6)))*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/
((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^
2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(sqrt(-3) + 1)*(-((a^2*b^2 -
4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c +
48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*l
og(-(4*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a
*b^2*c + 8*a^2*c^2)*n + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n + (sqrt(-3)*(a^2*b^
5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*n^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6
*b^2*c^2 - 64*a^7*c^3)*n^6)))*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^
6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) + 1/2*(1/2)^(1/3)*(sqrt(-3) -
1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6
- 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3
*c)*n^3))^(1/3)*log(-(4*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*(sq
rt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*n - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n +
(sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4 - (a^2*b^5 - 8*a^3*b^3
*c + 16*a^4*b*c^2)*n^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)))*(-((a^2*b^2 - 4*a^3*c)*n^3*s
qrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
- 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) + (1/2)^(1/3
)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 -
12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)
*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*((a^2*
b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((
a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*n)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((
a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c
- 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48

```
*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2
*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b
^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(
1/3))/x)
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}n}-\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}n}$$

```
[Out] 2*arctan(x^(1/2*n)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
(1/2)/n/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctan(x^(1/2*n)*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/n/(-4*a*c+b^2
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1395, 1107, 211}

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^{n/2}\right)}{n} \\
&= \frac{(2c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}n} - \frac{(2c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}n} \\
&= \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}n} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}n}$$

```
[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*
a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[
b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]*n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(4n^3b\,a^2-\frac{n^3b^3a}{c}\right)\_R^3+\left(\right.\right.$

```
[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R),
_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3
*n^2)*_Z^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(129) = 258.

Time = 0.29 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.74

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{1}{2}\sqrt{2}\sqrt{-\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}+\sqrt{2}\left((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-(b^2-\right.}{x}\right.$$

$$-\frac{1}{2}\sqrt{2}\sqrt{-\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}-\sqrt{2}\left((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-(b^2\right.}{x}\right.$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}+\sqrt{2}\left((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+(b^2-\right.}{x}\right.$$

$$+\frac{1}{2}\sqrt{2}\sqrt{\frac{(ab^2-4a^2c)n^2\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}-b}{(ab^2-4a^2c)n^2}}\log\left(\frac{4cxx^{\frac{1}{2}n-1}-\sqrt{2}\left((ab^3-4a^2bc)n^3\sqrt{\frac{1}{(a^2b^2-4a^3c)n^4}}+(b^2-\right.}{x}\right.$$

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
+ b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 -
4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt(-(
(a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^
2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2
- 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - s
qrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 -
4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) +
b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2*c)*n^2*s
qrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x
^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)
*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2
))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1
/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x)
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(129) = 258.

Time = 0.68 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}a^2c^2+8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}abc^2+\sqrt{2}\cdots}{\cdots}$$

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b
^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2
*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b
^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}n}$$

```
[Out] -2/a/n/(x^(1/2*n))+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*2^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1395, 1354, 1136, 1180, 211}

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

```
[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^4}+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^4}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{n}$$

$$= -\frac{2x^{-n/2}}{an} + \frac{2\text{Subst}\left(\int \frac{c+bx^2}{c+bx^2+ax^4}\,dx, x, x^{-n/2}\right)}{an}$$

$$= -\frac{2x^{-n/2}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx, x, x^{-n/2}\right)}{an} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx, x, x^{-n/2}\right)}{an}$$

$$= -\frac{2x^{-n/2}}{an} + \frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}n} + \frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.62

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{4cx^{-n/2}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
[Out] (4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{2x^{-\frac{n}{2}}}{an} + \left(\sum_{\_R=\text{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2cn^4+a^3b^4n^4\right)\_Z^4+\left(12a^2bc^2n^2-7ab^3cn^2+b^5n^2\right)\_Z^2+c^3\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(-\frac{8}{ac}\right.\right.\right.$

```
[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a
*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-
b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_
R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^
2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(169) = 338.

Time = 0.30 (sec) , antiderivative size = 1229, normalized size of antiderivative = 6.00

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)
)*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)
*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4
*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*
a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 -
 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))
 - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^
3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*(
(a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*
c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*
b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) -
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a
^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3
 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(-n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 699

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=-\frac{3x^{-n/3}}{an}-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}-\frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

```
[Out] -3/a/n/(x^(1/3*n))+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2)
)^(1/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^
2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b-(-4*a
*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/2*a
rctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^
(1/2))*3^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*
a*c+b^2)^(1/2))^(2/3)+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1
/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*
c+b^2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b+(
-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*c+b^2)^(1/2))^(2/3)-
```

```
1/2*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b+(-4*a*c+b^2)^(1/2))^(1/3
))*3^(1/2))*3^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1395, 1354, 1381, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\sqrt{3}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right)}{2\sqrt[3]{2}a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{3x^{-n/3}}{an}$$

```
[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
```

```
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3\text{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{3\text{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an} + \frac{\left(3\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+a^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{a}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+a^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2a^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}x}}{\sqrt[3]{2}}+a^{2/3}x^2}dx, x, x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(3\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}x}}{\sqrt[3]{2}}+a^{2/3}x^2}dx, x, x^{-n/3}\right)}{2\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}n}}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2a^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}x}}{\sqrt[3]{2}}+a^{2/3}x^2}dx, x, x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(3\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}x}}{\sqrt[3]{2}}+a^{2/3}x^2}dx, x, x^{-n/3}\right)}{2\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}n}}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$- \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3}a^{2/3}x^{-2n/3} - \sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$- \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3}a^{2/3}x^{-2n/3} - \sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(3\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$+ \frac{\left(3\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

$$= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.18

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{6cx^{-n/3}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
[Out] (6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*c*
```

```
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{3x^{-\frac{n}{3}}}{an}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4c\,n^6-a^4b^6n^6\right)\_Z^6+\left(-32a^3b\,c^3n^3+32a^2b^3c^2n^3-10a\,b^5c\,n^3+b^7n^3\right)\_Z^3\right.}\right.$

```
[In] int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*
c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a
^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*
a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*
n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^
6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^
7*n^3)*_Z^3+c^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3155 vs. 2(567) = 1134.

Time = 0.37 (sec) , antiderivative size = 3155, normalized size of antiderivative = 4.51

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\text{Too large to display}$$

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^
10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(1/3)*log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2)^(1
/3)*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*
c^3)*n)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 -
16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64
```

*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + 2*
(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2
*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)
log((2(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*((
a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)) + (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n
)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^
3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11
*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) - (1/2)^(
1/3)*(sqrt(-3)*a*n + a*n)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48
*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^
3))^(1/3)*log((4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) + (1/2)
^(1/3)*(sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n + (b^6 -
8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n - (sqrt(-3)*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*n^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)))*(((a^4*b^2 - 4*a^5*c)*n
^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + (1/2)^(1/3)*(sqrt(-3)*a*n - a*n)*
(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b
^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^
3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((4*(b^4*c -
4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)^(1/3)*(sqrt(-3)*(b^6 - 8*
a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2
 - 8*a^3*c^3)*n - (sqrt(-3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4 - (a
^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)))*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48
*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^
3))^(1/3))/x) - (1/2)^(1/3)*(sqrt(-3)*a*n + a*n)*(-((a^4*b^2 - 4*a^5*c)*n^3
*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8
*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)
/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*log((4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
x*x^(-1/3*n - 1) + (1/2)^(1/3)*(sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2
- 8*a^3*c^3)*n + (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n + (sqrt(-
3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4 + (a^4*b^5 - 8*a^5*b^3*c + 16
*a^6*b*c^2)*n^4)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 +
4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)))
*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*

```
c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) + (1/2)^(1
/3)*(sqrt(-3)*a*n - a*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48
*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^
3))^(1/3)*log((4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^(-1/3*n - 1) - (1/2)
^(1/3)*(sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n - (b^6 -
8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n + (sqrt(-3)*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*n^4 - (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)))*(-((a^4*b^2 - 4*a^5*c)*
n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b
*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3))/x) - 6*x*x^(-1/3*n - 1))/(a*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(-n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{\frac{n}{3}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 414

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

```
[Out] -4/a/n/(x^(1/4*n))-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+b
^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*
c+b^2)^(1/2))^(3/4)-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
```

```
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1395, 1354, 1381, 1436, 218, 214, 211}

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{4x^{-n/4}}{an}$$

```
[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^8}+\frac{b}{x^4}}\,dx,x,x^{-n/4}\right)}{n}\\
&= -\frac{4\text{Subst}\left(\int \frac{x^8}{c+bx^4+ax^8}\,dx,x,x^{-n/4}\right)}{n}\\
&= -\frac{4x^{-n/4}}{an}+\frac{4\text{Subst}\left(\int \frac{c+bx^4}{c+bx^4+ax^8}\,dx,x,x^{-n/4}\right)}{an}\\
&= -\frac{4x^{-n/4}}{an}+\frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx,x,x^{-n/4}\right)}{an}\\
&\quad+\frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx,x,x^{-n/4}\right)}{an}\\
&= -\frac{4x^{-n/4}}{an}-\frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}x^2}\,dx,x,x^{-n/4}\right)}{a\sqrt{-b+\sqrt{b^2-4ac}}n}\\
&\quad-\frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{a}x^2}\,dx,x,x^{-n/4}\right)}{a\sqrt{-b+\sqrt{b^2-4ac}}n}\\
&\quad-\frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}x^2}\,dx,x,x^{-n/4}\right)}{a\sqrt{-b-\sqrt{b^2-4ac}}n}\\
&\quad-\frac{\left(2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{a}x^2}\,dx,x,x^{-n/4}\right)}{a\sqrt{-b-\sqrt{b^2-4ac}}n}
\end{aligned}
$$

$$= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n}$$
$$- \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$
$$- \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n}$$
$$- \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{8cx^{-n/4}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
[Out] (8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(
b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n
/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4x^{-\frac{n}{4}}}{an}+\left(\sum_{\_R=\text{RootOf}\left((256a^{9}c^{4}n^{8}-256a^{8}b^{2}c^{3}n^{8}+96a^{7}b^{4}c^{2}n^{8}-16a^{6}b^{6}c\,n^{8}+a^{5}b^{8}n^{8}\right)\_Z^{8}+(80a^{4}b\,c^{4}n^{4}-120a^{3}b^{3}c^{3}n^{4}+61a^{2}}\right.$

```
[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6
-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*
b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b
^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b
*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*
a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96
*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a
^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4375 vs. 2(342) = 684.

Time = 0.44 (sec) , antiderivative size = 4375, normalized size of antiderivative = 10.57

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\text{Too large to display}$$

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n
^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3
*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*
c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3
*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n
^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(
-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6
```

*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(-sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(-sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(-sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(-sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 +

```
 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x)
+ sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*s
qrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c -
 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c -
3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c +
 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8))
 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(sqrt(2)*sqrt(((a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)*n^4))))/x) - sqrt(2)*a*n*sqrt(-sqrt(2)*sqrt(((a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*x*x^(-1/4*n - 1) + sqrt(
2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^
3)*n)*sqrt(-sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*
b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4))))/x) + sqrt(2)*a*n*sqrt(
-sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*log((4*(b^4*c - 3*a*b^2*c^2 + a^2*
c^3)*x*x^(-1/4*n - 1) - sqrt(2)*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*n^5
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^6 - 7*a*b^4*
c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*n)*sqrt(-sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8
)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n
^4))))/x) - 8*x*x^(-1/4*n - 1))/(a*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**(-n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] -4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx=-\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

```
[Out] -2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b
^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1397, 371}

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx=-\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeometric2
```

```
F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c
+ b*Sqrt[b^2 - 4*a*c]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2c)\int \frac{x^2}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^2}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} \\ &= -\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = -\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-8^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
[Out] (-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c
```

```
])/c + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometr
ic2F1[-3/n, -3/n, (-3 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)]/(8^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/n)))
/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

Maple [F]

$$\int \frac{x^2}{a + b\,x^n + c\,x^{2n}}dx$$

```
[In] int(x^2/(a+b*x^n+c*x^(2*n)),x)
[Out] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\,dx = \int \frac{x^2}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\,dx = \int \frac{x^2}{a + bx^n + cx^{2n}}\,dx$$

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\,dx = \int \frac{x^2}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^2}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^2}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^2/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^2/(a + b*x^n + c*x^(2*n)), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=-\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1397, 371}

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=-\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric2F1[1,
```

```
2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(2c)\int \frac{x}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$
$$= -\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.93

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = -cx^2\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-4^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])
```

```
/c + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometric
2F1[-2/n, -2/n, (-2 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(4^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)))/(
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

Maple [F]

$$\int \frac{x}{a + b\,x^{n} + c\,x^{2n}}dx$$

```
[In] int(x/(a+b*x^n+c*x^(2*n)),x)
[Out] int(x/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x}{a + bx^n + cx^{2n}}\,dx = \int \frac{x}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{x}{a + bx^n + cx^{2n}}\,dx = \int \frac{x}{a + bx^n + cx^{2n}}\,dx$$

```
[In] integrate(x/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x}{a + bx^n + cx^{2n}}\,dx = \int \frac{x}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{a + bx^n + cx^{2n}} \, dx = \int \frac{x}{cx^{2n} + bx^n + a} \, dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + bx^n + cx^{2n}} \, dx = \int \frac{x}{a + b\,x^n + c\,x^{2\,n}} \, dx$$

```
[In] int(x/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x/(a + b*x^n + c*x^(2*n)), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1361, 251}

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Hypergeometric2F1[1
```

```
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(124) = 248$.

Time = 0.46 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.10

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$
$$= -2cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-2^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

[Out] -2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))

Maple [F]

$$\int \frac{1}{a + b\,x^{n} + c\,x^{2n}}dx$$

[In] int(1/(a+b*x^n+c*x^(2*n)),x)

[Out] int(1/(a+b*x^n+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n} + bx^n + a}\,dx$$

[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral(1/(c*x^(2*n) + b*x^n + a), x)

Sympy [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{a + bx^n + cx^{2n}}\,dx$$

[In] integrate(1/(a+b*x**n+c*x**(2*n)),x)

[Out] Integral(1/(a + b*x**n + c*x**(2*n)), x)

Maxima [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n} + bx^n + a}\,dx$$

[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)

Giac [F]

$$\int \frac{1}{a + bx^n + cx^{2n}} \, dx = \int \frac{1}{cx^{2\,n} + bx^n + a} \, dx$$

[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + bx^n + cx^{2n}} \, dx = \int \frac{1}{a + b\,x^n + c\,x^{2\,n}} \, dx$$

[In] int(1/(a + b*x^n + c*x^(2*n)),x)

[Out] int(1/(a + b*x^n + c*x^(2*n)), x)

3.565 $\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n} + \frac{\log(x)}{a} - \frac{\log(a+bx^n+cx^{2n})}{2an}$$

```
[Out] ln(x)/a-1/2*ln(a+b*x^n+c*x^(2*n))/a/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1
/2))/a/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}} - \frac{\log(a+bx^n+cx^{2n})}{2an} + \frac{\log(x)}{a}$$

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^n\right)}{an}\\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{2an} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2an}
\end{aligned}$$

$$= \frac{\log(x)}{a} - \frac{\log(a + bx^n + cx^{2n})}{2an} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{an}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}n} + \frac{\log(x)}{a} - \frac{\log(a + bx^n + cx^{2n})}{2an}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a + bx^n + cx^{2n})} dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x^n) + \log(a + x^n(b + cx^n))}{2an}$$

[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]

[Out] -1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2 *Log[x^n] + Log[a + x^n*(b + c*x^n)])/(a*n)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(68) = 136.

Time = 0.19 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.36

method	result
risch	$\frac{4n^2\ln(x)ac}{4a^2cn^2-ab^2n^2} - \frac{n^2\ln(x)b^2}{4a^2cn^2-ab^2n^2} - \frac{2\ln\left(x^n - \frac{-b^2+\sqrt{-4ab^2c+b^4}}{2bc}\right)c}{(4ac-b^2)n} + \frac{\ln\left(x^n - \frac{-b^2+\sqrt{-4ab^2c+b^4}}{2bc}\right)b^2}{2a(4ac-b^2)n} + \frac{\ln\left(x^n - \frac{-b^2+\sqrt{-4a}}{2bc}\right.}{2a(4a}$

[In] int(1/x/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)

[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.50

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \left[\frac{2(b^2-4ac)n\log(x)+\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc+\sqrt{b^2-4ac}\,c\right)x^n+\sqrt{b^2-4ac}\,b}{cx^{2n}+bx^n+a}\right)-(b^2-4ac)\log(cx^{2n}}{2(ab^2-4a^2c)n}\right.$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b
^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^
(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4
*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 -
 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.03

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{\ln\left(-\frac{1}{c\,x}-\frac{(2\,a\,n+b\,n\,x^n)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}\right)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} - \frac{\ln\left(\frac{(2\,a\,n+b\,n\,x^n)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}-\frac{1}{c\,x}\right)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,n}$$

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] (log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/
(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*(a
*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) - 4*a
*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1/2)
- 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

3.566 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

```
[Out] 2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/x/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1397, 371}

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)} + \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeometric2F1
```

```
[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a
*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2c)\int \frac{1}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{2^{1+\frac{1}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1+\frac{1}{n},1+\frac{1}{n},2+\frac{1}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n} + \frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{1+\frac{1}{n}}\text{Hypergeometric}}{c}\right.}{\sqrt{b^2-4ac}(1+n)x}$$

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)*Hype
rgeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b - Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((
c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1 + n^(-1))*Hypergeometric2F1[1
+ n^(-1), 1 + n^(-1), 2 + n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

Maple [F]

$$\int \frac{1}{x^2\left(a+b\,x^n+c\,x^{2n}\right)}dx$$

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{x^2 \, (a + b\,x^n + c\,x^{2\,n})} \, dx$$

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.567 $\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

```
[Out] c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x^2/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))/x^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1397, 371}

$$\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)} + \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hypergeometric2F1[1, -
```

```
2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*S
qrt[b^2 - 4*a*c])*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2c)\int \frac{1}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{2^{\frac{2+n}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{2/n}\text{Hypergeometric2F1}\left(\frac{2+n}{n},\frac{2+n}{n},2+\frac{2}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n} + \frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{2+n}{n}}\text{Hypergeometric}}{c}\right.}{\sqrt{b^2-4ac}(2+n)x^2}$$

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)*Hyperg
eometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^n)/
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((2 + n)/n)*Hypergeometric2F1[(2 + n)/n,
 (2 + n)/n, 2 + 2/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

Maple [F]

$$\int \frac{1}{x^3\,(a+b\,x^n+c\,x^{2n})}dx$$

[In] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)

[Out] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)

Fricas [F]

$$\int \frac{1}{x^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)x^3}\,dx$$

[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\,(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{x^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)x^3}\,dx$$

[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} dx = \int \frac{1}{x^3 (a + b x^n + c x^{2n})} dx$$

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,-1/2,-1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt
```

```
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. $2(148) = 296$.

Time = 0.61 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.47

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{x^4\left(4(4+n)\left(a+x^n(b+cx^n)\right)+an(4+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{}{b+}\right.}{4(4+n)}$$

```
[In] Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
```

```
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x^n*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2
, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int x^3\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^3\,dx$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^3\,dx$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.569 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,-1/2,-1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt
```

```
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 366 vs. $2(148) = 296$.

Time = 0.57 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.47

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{x^3\left(6(3+n)\left(a+x^n(b+cx^n)\right)+2an(3+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{}{b-}\right.\right.}{6(3+n)}$$

```
[In] Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
```

```
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1
/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int x^{2} \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \sqrt{a+bx^n+cx^{2n}}\,dx = \int x^{2} \sqrt{a + bx^{n} + cx^{2n}}\, dx$$

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^2\,dx$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x^2\,dx$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,-1/2,-1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt
```

```
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(148) = 296.

Time = 0.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.46

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{x^2\left(2(2+n)\left(a+x^n(b+cx^n)\right)+an(2+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{}{b+}\right.\right.}{2(2+n)^2}$$

```
[In] Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
```

```
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2,
1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int x\sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int x\sqrt{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x\,dx$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x\,dx$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1362, 440}

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt
```

```
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(139) = 278.

Time = 0.51 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.53

$$\int\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{x\left(bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+n\right.}{2(1+n)^2\sqrt{}}$$

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
```

```
^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{cx^{2\,n} + bx^n + a}\, dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{cx^{2\,n} + bx^n + a}\, dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{a + b\,x^n + c\,x^{2\,n}}\, dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))*a^(1/2)/n+1/2*b
*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/c^(1/2)+(a+b*
x^n+c*x^(2*n))^(1/2)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1371, 748, 857, 635, 212, 738}

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n} + \frac{\sqrt{a+bx^n+cx^{2n}}}{n}$$

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{2\sqrt{a+x^n(b+cx^n)} + 4\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right) - \frac{b\log\left(n\left(b+2cx^n-2\sqrt{c}\sqrt{a+x^n(b+cx^n)}\right)\right)}{\sqrt{c}}}{2n}$$

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (2*Sqrt[a + x^n*(b + c*x^n)] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^
n*(b + c*x^n)])/Sqrt[a]] - (b*Log[n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a + x^n*(
b + c*x^n)])])/Sqrt[c])/(2*n)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\sqrt{a+b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln(x)}}}{n}+\frac{b \ln\left(\frac{\frac{b}{2}+c\,{\mathrm e}^{n \ln(x)}}{\sqrt{c}}+\sqrt{a+b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln(x)}}\right)}{2 n \sqrt{c}}-\frac{\sqrt{a}\, \ln\left(\left(2 a+b\,{\mathrm e}^{n \ln(x)}+2 \sqrt{a}\, \sqrt{a+b\,{\mathrm e}^{n \ln(x)}+c\,{\mathrm e}^{2 n \ln}}\right.\right.}{n}$

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(
x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)
*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)/exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 658, normalized size of antiderivative = 5.53

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^{2\,n}-8\,bcx^n-b^2-4\,ac-4\left(2\,c^{\frac{3}{2}}x^n+b\sqrt{c}\right)\sqrt{cx^{2\,n}+bx^n+a}\right)+2\,\sqrt{a}c\log\left(-\frac{8\,abx^n+8\,a^2}{\cdots}\right.}{4\,cn}\right.$$

$$-\frac{b\sqrt{-c}\arctan\left(\frac{\left(2\,\sqrt{-c}cx^n+b\sqrt{-c}\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,(c^2x^{2\,n}+bcx^n+ac)}\right)-\sqrt{a}c\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\left(\sqrt{a}bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+\cdots}}{x^{2\,n}}\right.}{2\,cn}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)
*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*x^
n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^
(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), -1/2
*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2
 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b
*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(4*sqrt(-
a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)
/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n
```

```
- b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a))
+ 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqr
t(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*
x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*
x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*
x^n + a)*c)/(c*n)]
```

Sympy [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\,dx = \int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\,dx = \int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} \, dx = \int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x} \, dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -AppellF1(-1/n,-1/2,-1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
```

```
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a+bx^n+cx^{2n}}\int\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= -\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. $2(149) = 298$.

Time = 0.48 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.45

$$\int\frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$$

$$=\frac{2(-1+n)\left(a+x^n(b+cx^n)\right)-2a(-1+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},\right.}{2(-1+}$$

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] (2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
```

```
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 + n)/n,
1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}+c\,x^{2n}}}{x^{2}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^2}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*AppellF1(-2/n,-1/2,-1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
```

```
^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 365 vs. $2(151) = 302$.

Time = 0.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$$
$$= \frac{2(-2+n)\left(a+x^n(b+cx^n)\right)-a(-2+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\right.}{2(-2+n}$$

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] (2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
```

```
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2,
1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}+c\,x^{2n}}}{x^{3}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^3}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^3}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^3}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3, x)
```

3.575 $\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/n,-3/2,-3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
```

```
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ax^4\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 469 vs. 2(149) = 298.

Time = 1.10 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.15

$$\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx = \frac{x^4\Big(2(4+n)\left(3b^2n^2+32ac(2+3n+n^2)+2bc(32+36n+7n^2)\,x^n+8c^2(8+6n+n^2)\,x^{2n}\right)}{}$$

```
[In] Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(2*(4 + n)*(3*b^2*n^2 + 32*a*c*(2 + 3*n + n^2) + 2*b*c*(32 + 36*n + 7*
n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^2*(
```

```
4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a*c*(8 +
3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4
+ n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-
b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[a + x^n*(b
 + c*x^n)])
```

Maple [F]

$$\int x^{3}\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int x^{3}\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x**3*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^3 dx$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^3 dx$$

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int x^3 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} dx$$

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.576 $\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/3*a*x^3*AppellF1(3/n,-3/2,-3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
```

```
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ax^3\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(149) = 298.

Time = 1.05 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.19

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}dx = \frac{x^3\left(2(3+n)\left(3b^2n^2+4ac(9+18n+8n^2)+2bc(18+27n+7n^2)x^n+4c^2(9+9n+2n^2)x^{2n}\right.\right.}{}$$

```
[In] Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(2*(3 + n)*(3*b^2*n^2 + 4*a*c*(9 + 18*n + 8*n^2) + 2*b*c*(18 + 27*n +
7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) + 2*a*n
```

```
^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c*(2 + n)
+ b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2*n)*Sqrt[a
 + x^n*(b + c*x^n)])
```

Maple [F]

$$\int x^{2} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}d x$$

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} \, dx = \int x^{2} \left(a + b x^{n} + c x^{2 n}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^2 \, dx$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^2 \, dx$$

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int x^2 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} dx$$

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.577 $\int x(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/n,-3/2,-3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int x(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
```

```
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ax^2\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 471 vs. 2(149) = 298.

Time = 1.05 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.16

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}dx = \frac{x^2\left(2(2+n)\left(3b^2n^2+16ac(1+3n+2n^2)+2bc(8+18n+7n^2)x^n+8c^2(2+3n+n^2)x^{2n}\right)\right.}{}$$

```
[In] Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(2*(2 + n)*(3*b^2*n^2 + 16*a*c*(1 + 3*n + 2*n^2) + 2*b*c*(8 + 18*n + 7
*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^2*
```

```
(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*c*(4 +
 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2
 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt[a + x^n*(
b + c*x^n)])
```

Maple [F]

$$\int x\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\left(a+bx^{n}+cx^{2n}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\left(a+bx^{n}+cx^{2n}\right)^{3/2}\,dx=\int x\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x(a+bx^n+cx^{2n})^{3/2}\,dx=\int (cx^{2n}+bx^n+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^n+cx^{2n})^{3/2}\,dx=\int (cx^{2n}+bx^n+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n+cx^{2n})^{3/2}\,dx=\int x\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.578 $\int (a+bx^n+cx^{2n})^{3/2}\, dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int (a+bx^n+cx^{2n})^{3/2}\, dx = \frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1362, 440}

$$\int (a+bx^n+cx^{2n})^{3/2}\, dx = \frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
```

```
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{ax\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(140) = 280.

Time = 1.06 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.33

$$\int\left(a+bx^n+cx^{2n}\right)^{3/2}dx = \frac{x\left(-3bn^2(b^2(2+n)-4ac(2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\right.\right.}{}$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c
```

```
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 +
 n)*((3*b^2*n^2 + 4*a*c*(1 + 6*n + 8*n^2) + 2*b*c*(2 + 9*n + 7*n^2)*x^n + 4
*c^2*(1 + 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 3*a*n^2*(b^2 - 4*a*
c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*Sqrt[a + x
^n*(b + c*x^n)])
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int {\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.579 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx = \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}$$

```
[Out] 1/3*(a+b*x^n+c*x^(2*n))^(3/2)/n-a^(3/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+
b*x^n+c*x^(2*n))^(1/2))/n-1/16*b*(-12*a*c+b^2)*arctanh(1/2*(b+2*c*x^n)/c^(1
/2)/(a+b*x^n+c*x^(2*n))^(1/2))/c^(3/2)/n+1/8*(b^2+8*a*c+2*b*c*x^n)*(a+b*x^n
+c*x^(2*n))^(1/2)/c/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1371, 748, 828, 857, 635, 212, 738}

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n} + \frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{2n}\\
&= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n}\\
&\quad + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{8cn}\\
&= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n}\\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n} - \frac{(b(b^2-12ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{16cn}\\
&= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n}\\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}\\
&\quad - \frac{(b(b^2-12ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{8cn}
\end{aligned}$$

$$= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{2\sqrt{c}\sqrt{a+x^n\,(b+cx^n)}(3b^2+14bcx^n+8c(4a+cx^{2n}))+96a^{3/2}c^{3/2}\text{arctanh}\Big(\frac{\sqrt{c}}{}}{48c}$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))) + 96*a^(3/2)*c^(3/2)*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)
])/Sqrt[a]] + 3*(b^3 - 12*a*b*c)*Log[c*n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a +
x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(8c^2\mathrm{e}^{2n\ln(x)}+14b\,\mathrm{e}^{n\ln(x)}c+32ac+3b^2)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}}}{24cn}-\frac{b^3\ln\left(\frac{\frac{b}{2}+c\,\mathrm{e}^{n\ln(x)}}{\sqrt{c}}+\sqrt{a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}}\right)}{16c^{\frac{3}{2}}n}+\frac{3ab\ln\Big(\frac{b}{2}}{}$

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*exp(n*ln(x))*c+32*a*c+3*b^2)*(a+b*exp(n*ln(
x))+c*exp(n*ln(x))^2)^(1/2)/c/n-1/16/c^(3/2)/n*b^3*ln((1/2*b+c*exp(n*ln(x))
)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))+3/4/c^(1/2)/n*a*b*ln((
1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/
n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))
^2)^(1/2))/exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.78

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x} dx = \left[\frac{48 a^{\frac{3}{2}} c^2 \log\left(-\frac{8 abx^n + 8 a^2 + (b^2 + 4 ac)x^{2n} - 4\left(\sqrt{a}bx^n + 2 a^{\frac{3}{2}}\right)\sqrt{cx^{2n} + bx^n + a}}{x^{2n}}\right) - 3 (b^3 - 12 ab\dots}{\dots} \right.$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
 a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a
*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
 b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
 + a))/(c^2*n)]
```

Sympy [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x} dx = \int \frac{(a + bx^n + cx^{2n})^{\frac{3}{2}}}{x} dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)
```

3.580 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -a*AppellF1(-1/n,-3/2,-3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
```

```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. $2(150) = 300$.

Time = 0.98 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.18

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \frac{2(-1+n)\left(3b^2n^2+4ac(1-6n+8n^2)+2bc(2-9n+7n^2)\,x^n+4c^2(1-3n+\right.}{}$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (2*(-1 + n)*(3*b^2*n^2 + 4*a*c*(1 - 6*n + 8*n^2) + 2*b*c*(2 - 9*n + 7*n^2)*
x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(-1 + n)
*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
```

```
*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) + b^2*(
-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-1 + 3*n)*x
*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{(a+b\,x^{n}+c\,x^{2n})^{\frac{3}{2}}}{x^{2}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(a+bx^n+cx^{2n})^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x^2}\,dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2, x)
```

3.581 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*a*AppellF1(-2/n,-3/2,-3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
```

```
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$
$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 471 vs. 2(152) = 304.

Time = 0.97 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \frac{2(-2+n)\left(3b^2n^2+16ac(1-3n+2n^2)+2bc(8-18n+7n^2)\,x^n+8c^2(2-3n\right.}{}$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (2*(-2 + n)*(3*b^2*n^2 + 16*a*c*(1 - 3*n + 2*n^2) + 2*b*c*(8 - 18*n + 7*n^2
)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(b^2 + 4
*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
```

```
*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*(-4 +
n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(
-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*x^2*Sqrt[a
 + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{(a+b\,x^{n}+c\,x^{2n})^{\frac{3}{2}}}{x^{3}}dx$$

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x^3} \, dx = \int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x^3} \, dx = \int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x^3} \, dx = \int \frac{(a + b\,x^n + c\,x^{2n})^{3/2}}{x^3} \, dx$$

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,1/2,1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+x^n(b+cx^n)}}$$

```
[In] Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2
, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,1/2,1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{x^2}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$=\frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int\frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

$$=\frac{x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+x^n(b+cx^n)}}$$

```
[In] Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2
, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,1/2,1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+x^n\left(b+cx^n\right)}}$$

```
[In] Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2
, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^{n}+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^{n}+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{a+bx^{n}+cx^{2n}}}\,dx$$

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^{n}+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{cx^{2\,n}+bx^{n}+a}}\,dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

[In] int(x/(a + b*x^n + c*x^(2*n))^(1/2),x)

[Out] int(x/(a + b*x^n + c*x^(2*n))^(1/2), x)

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1362, 440}

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(
2*n)]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int\frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$
$$= \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+x^n\left(b+cx^n\right)}}$$

```
[In] Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/
2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1371, 738, 212}

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n}\\ &= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [F]

$$\int \frac{1}{x\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\,\left(\sqrt{a}bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}n}, \frac{\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}bx^n+2\,\sqrt{-a}a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,(acx^{2\,n}+abx^n+a^2)}\right)}{an}\right]$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2
*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arcta
n(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2
*n) + a*b*x^n + a^2))/(a*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.587 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,1/2,1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^n +
 c*x^(2*n)]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}\\&=-\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16

$$\int\frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx=-\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+x^n\left(b+cx^n\right)}}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1
/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n} + bx^n + a}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{1}{x^2\,\sqrt{a + b\,x^n + c\,x^{2\,n}}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,1/2,1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\mathrm{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.16

$$\int\frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+x^n(b+cx^n)}}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -1/2*(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1
/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.589 $\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,3/2,3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

Time = 0.65 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^4\Big(-8(4+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-8+n)-4ac(-4+n)\right)(4+n)\sqrt{\frac{b-\sqrt{b^2\cdots}}{b-\cdots}}$$

```
[In] Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))*
(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n,
1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*n*(4 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x^3}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

[In] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)

[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)

[Out] Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)

Maxima [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)

Giac [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.590 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}} \, dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,3/2,3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}} \, dx = \frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(151) = 302$.

Time = 0.68 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^3\Big(-6(3+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-6+n)-4ac(-3+n)\right)(3+n)\sqrt{\frac{b-\sqrt{b^2}}{b-}}$$

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))*
(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n,
1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*n*(3 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.591 $\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,3/2,3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1399, 524}

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{x}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(151) = 302$.

Time = 0.63 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int\frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^2\left(-4(2+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-4+n)-4ac(-2+n)\right)(2+n)\sqrt{\frac{b-\sqrt{b^2}}{b-}}\right.}{}$$

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))*
(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n,
1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n*(2 + n)
*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{x}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{{\left(cx^{2\,n}+bx^{n}+a\right)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

```
[In] int(x/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1362, 440}

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}\\ &= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(142) = 284.

Time = 0.70 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x\left(2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2}}\right.\right.}{}$$

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) + (b^2
*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sq
rt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}+c\,x^{2 n}\right)^{\frac{3}{2}}}d x$$

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^{n}+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a + b x^n + c x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(c x^{2\,n} + b x^n + a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a + b x^n + c x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(a + b\,x^n + c\,x^{2\,n}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.593 $\int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(3/2)/n+2*(b^
2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1371, 754, 12, 738, 212}

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2(-2ac+b^2+bcx^n)}{an\left(b^2-4ac\right)\sqrt{a+bx^n+cx^{2n}}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, x^n\right)}{n} \\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{a\left(b^2-4ac\right)n} \\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{an} \\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}} - \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an}
\end{aligned}$$

$$=\frac{2(b^2-2ac+bcx^n)}{a(b^2-4ac)\,n\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int\frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2\left(-\frac{\sqrt{a}(-b^2+2ac-bcx^n)}{(b^2-4ac)\sqrt{a+x^n(b+cx^n)}}+\operatorname{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)\right)}{a^{3/2}n}$$

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(-((Sqrt[a]*(-b^2 + 2*a*c - b*c*x^n))/((b^2 - 4*a*c)*Sqrt[a + x^n*(b + c
*x^n)])) + ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]]))/(a^
(3/2)*n)
```

Maple [F]

$$\int\frac{1}{x\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(88) = 176.

Time = 0.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.58

$$\int\frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\left[\frac{\left((b^2c-4\,ac^2)\sqrt{a}x^{2\,n}+(b^3-4\,abc)\sqrt{a}x^n+(ab^2-4\,a^2c)\sqrt{a}\right)\log\left(-\frac{8\,abx^n+8\,a^2}{\ldots}\right.}{2\left((a^2b^2c-4\,a^3c^2)nx^{2\,n}+(a^2\ldots\right.}\right.$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) -
4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*
b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3
*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((
b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 -
```

```
4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*
n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a
^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^
2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.594 $\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}} dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,3/2,3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}} dx = -\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b - S
```

```
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^n
 + c*x^(2*n)]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. $2(152) = 304$.

Time = 0.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.60

$$\int\frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{(-1+n)\left(-4ac(1+n)+b^2(2+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\text{Appel}$$

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(b^2
- 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sq
```

```
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*(-1 + n)
*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(a +b\,x^{n}+c\,x^{2 n}\right)^{\frac{3}{2}}}d x$$

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x^{2} \left(a + b x^{n} + c x^{2 n}\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{x^2 (a + b\,x^n + c\,x^{2\,n})^{3/2}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.595 $\int \frac{1}{x^3(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,3/2,3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b - S
```

```
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x
^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(154) = 308.

Time = 0.61 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{(-2+n)\left(-4ac(2+n)+b^2(4+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{Appel}}{}$$

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2 - 2
*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
```

```
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-2 + n)*n
*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{\frac{3}{2}}}d x$$

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x^{3} \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{1}{x^3 (a + b x^n + c x^{2n})^{3/2}} dx$$

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.596 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx = \frac{3a^2bx^{1+n}(dx)^m}{1+m+n}+\frac{3a(b^2+ac)\,x^{1+2n}(dx)^m}{1+m+2n} + \frac{b(b^2+6ac)\,x^{1+3n}(dx)^m}{1+m+3n}+\frac{3c(b^2+ac)\,x^{1+4n}(dx)^m}{1+m+4n} + \frac{3bc^2x^{1+5n}(dx)^m}{1+m+5n}+\frac{c^3x^{1+6n}(dx)^m}{1+m+6n}+\frac{a^3(dx)^{1+m}}{d(1+m)}$$

```
[Out] 3*a^2*b*x^(1+n)*(d*x)^m/(1+m+n)+3*a*(a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+b
*(6*a*c+b^2)*x^(1+3*n)*(d*x)^m/(1+m+3*n)+3*c*(a*c+b^2)*x^(1+4*n)*(d*x)^m/(1
+m+4*n)+3*b*c^2*x^(1+5*n)*(d*x)^m/(1+m+5*n)+c^3*x^(1+6*n)*(d*x)^m/(1+m+6*n)
+a^3*(d*x)^(1+m)/d/(1+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1367, 20, 30}

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx = \frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}(ac+b^2)\,(dx)^m}{m+2n+1} + \frac{bx^{3n+1}(6ac+b^2)\,(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}(ac+b^2)\,(dx)^m}{m+4n+1} + \frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \int \left(a^3(dx)^m + 3a^2bx^n(dx)^m + 3ab^2\left(1+\frac{ac}{b^2}\right)x^{2n}(dx)^m + b^3\left(1+\frac{6ac}{b^2}\right)x^{3n}(dx)^m + 3b^2c\left(1+\frac{ac}{b^2}\right)x^{4n}(dx)^m + 3bc^2x^{5n}(dx)^m + c^3x^{6n}(dx)^m\right)dx$$

$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int x^n(dx)^m\,dx + \left(3bc^2\right)\int x^{5n}(dx)^m\,dx + c^3\int x^{6n}(dx)^m\,dx + \left(3a\left(b^2+ac\right)\right)\int x^{2n}(dx)^m\,dx + \left(3c\left(b^2+ac\right)\right)\int x^{4n}(dx)^m\,dx + \left(b\left(b^2+6ac\right)\right)\int x^{3n}(dx)^m\,dx$$

$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \left(3a^2bx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(3bc^2x^{-m}(dx)^m\right)\int x^{m+5n}\,dx + \left(c^3x^{-m}(dx)^m\right)\int x^{m+6n}\,dx + \left(3a\left(b^2+ac\right)x^{-m}(dx)^m\right)\int x^{m+2n}\,dx + \left(3c\left(b^2+ac\right)x^{-m}(dx)^m\right)\int x^{m+4n}\,dx + \left(b\left(b^2+6ac\right)x^{-m}(dx)^m\right)\int x^{m+3n}\,dx$$

$$= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n} + \frac{3a(b^2+ac)\,x^{1+2n}(dx)^m}{1+m+2n} + \frac{b(b^2+6ac)\,x^{1+3n}(dx)^m}{1+m+3n} + \frac{3c(b^2+ac)\,x^{1+4n}(dx)^m}{1+m+4n} + \frac{3bc^2x^{1+5n}(dx)^m}{1+m+5n} + \frac{c^3x^{1+6n}(dx)^m}{1+m+6n} + \frac{a^3(dx)^{1+m}}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = x(dx)^m \left(\frac{a^3}{1+m} + \frac{3a^2bx^n}{1+m+n} + \frac{3a(b^2+ac)\,x^{2n}}{1+m+2n} + \frac{b(b^2+6ac)\,x^{3n}}{1+m+3n} + \frac{3c(b^2+ac)\,x^{4n}}{1+m+4n} + \frac{3bc^2x^{5n}}{1+m+5n} + \frac{c^3x^{6n}}{1+m+6n}\right)$$

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2*
n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 + a
*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6*n)
)/(1 + m + 6*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.56 (sec) , antiderivative size = 3765, normalized size of antiderivative = 20.69

method	result	size
risch	Expression too large to display	3765
parallelrisch	Expression too large to display	5804

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(3*a*c^2*m^6*(x^n)^4+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a
^2*b*m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n
^4*x^n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*
(x^n)^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3
*(x^n)^2+120*a*b*c*m^3*(x^n)^3+2904*a*b*c*m*n^2*(x^n)^3+540*a*b*c*m*n*(x^n)
^3+6*(x^n)^3*c*a*b+3*b^2*c*m^6*(x^n)^4+a^3+3*(x^n)^4*c^2*a+3*(x^n)^4*c*b^2+
3*(x^n)^2*c*a^2+3*(x^n)^5*c^2*b+15*c^3*m^5*n*(x^n)^6+1383*a^2*c*m^3*n^3*(x^
n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+1080*a^2*c*m*n^5*(x^n)^2+57*a*b^2*m^5*n*(x^
n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^
4*(x^n)^2+1080*a*b^2*m*n^5*(x^n)^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*c*n^5*(x^n
```

)^3+2106*a^2*c*n^4*(x^n)^2+45*a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+45*a
*c^2*m^2*(x^n)^4+321*a*c^2*n^2*(x^n)^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5
n(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m
^2*n^4*(x^n)^4+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+1116*b^3*m*n^3
*(x^n)^3+60*b^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+6*b^3*m^5*(x^n)^3+240*b
^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*(x^n)^6+15*b^3*m^4*(x^n)^3+508
*b^3*n^4*(x^n)^3+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+540*a*c^2*n^5*(
x^n)^4+6096*a*b*c*m*n^4*(x^n)^3+1080*a*b*c*m^3*n*(x^n)^3+4356*a*b*c*m^2*n^2
*(x^n)^3+85*c^3*m^4*n^2*(x^n)^6+2763*b^2*c*m^2*n^3*(x^n)^4+2376*b^2*c*m*n^4
*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710*b*c^2*m^2*n^2*(x^n)^5+2340*b*c^2*m*n^
3*(x^n)^5+57*a^2*c*m^5*n*(x^n)^2+411*a^2*c*m^4*n^2*(x^n)^2+6*m*a^3+2232*a*b
*c*n^3*(x^n)^3+255*a*c^2*m*n*(x^n)^4+255*b^2*c*m*n*(x^n)^4+570*a^2*c*m^2*n*
(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x^n)^3+726*a*b*c*n^2*(x^n)^3
+285*a^2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+432*b*c^2*n^5*(x^n)^5+150*c^3*m^3
n(x^n)^6+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(x^n)^3+60*a^2*b*m^3*x^n+18*m*a^
2*b*x^n+60*a^2*b*n*x^n+45*a*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*
m^2*x^n+465*a^2*b*n^2*x^n+18*m*b^2*a*(x^n)^2+180*b^3*m^2*n*(x^n)^3+484*b^3*
m*n^2*(x^n)^3+57*b^2*a*(x^n)^2*n+20*a^3*m^3+15*a^3*m^2+600*a^2*b*m^3*x^n*n+
18*b^2*c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+45*b*c^2*m^4*(x^n)^5+972*b*c^2*n
^4*(x^n)^5+150*c^3*m^2*n*(x^n)^6+340*c^3*m*n^2*(x^n)^6+3*a^2*c*m^6*(x^n)^2+
3*a*b^2*m^6*(x^n)^2+225*c^3*m^3*n^3*(x^n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3
*m*n^5*(x^n)^6+3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+c^3*m^6*(x^n)^6+6*c
^3*m^5*(x^n)^6+120*c^3*n^5*(x^n)^6+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^6+b
^3*m^6*(x^n)^3+57*a^2*c*(x^n)^2*n+18*b^3*m^5*n*(x^n)^3+121*b^3*m^4*n^2*(x^n
)^3+372*b^3*m^3*n^3*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+240*b^3*m*n^5*(x^n)^3+5
10*a*c^2*m^3*n*(x^n)^4+1926*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+
6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+108*a*b*c*m^5*n*(x^n)^3+7
26*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3048*a*b*c*m^2*n^4*(x^n
)^3+1440*a*b*c*m*n^5*(x^n)^3+540*a*b*c*m^4*n*(x^n)^3+2904*a*b*c*m^3*n^2*(x^
n)^3+6696*a*b*c*m^2*n^3*(x^n)^3+108*a*b*c*(x^n)^3*n+48*b*c^2*m^5*n*(x^n)^5+
285*b*c^2*m^4*n^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2*n^4*(x^n)
^5+432*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n^2*(x^n)^4
+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n)
^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(x
^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4+
1284*a*c^2*m^3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+2376*a*c^2*m*n^4*(x^n
)^4+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+15*a^3*m^4+6*m*c^3*(
x^n)^6+15*c^3*(x^n)^6*n+372*b^3*n^3*(x^n)^3+(x^n)^3*b^3+45*a*c^2*m^4*(x^n)^
4+1188*a*c^2*n^4*(x^n)^4+90*b^3*m^4*n*(x^n)^3+484*b^3*m^3*n^2*(x^n)^3+1116*
b^3*m^2*n^3*(x^n)^3+1383*a*b^2*n^3*(x^n)^2+340*c^3*m^3*n^2*(x^n)^6+675*c^3*
m^2*n^3*(x^n)^6+548*c^3*m*n^4*(x^n)^6+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x
^n)^4+18*m*b*c^2*(x^n)^5+48*b*c^2*(x^n)^5*n+3132*a^2*b*n^4*x^n+60*a^2*c*m^3
*(x^n)^2+510*b^2*c*m^3*n*(x^n)^4+1926*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^
3*(x^n)^4+480*b*c^2*m^2*n*(x^n)^5+1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x
^n+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+1860*a^2*b*m*n^2*x^n+285*a*

```
b^2*m*n*(x^n)^2+300*a^2*b*m*n*x^n+45*b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^
5+18*a^2*b*m^5*x^n+2160*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+18*a^2*c*(x^n)^2
*m+1383*a^2*c*n^3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*(x^n)^4*n+18*b^2*c*(x
^n)^4*m+51*b^2*c*(x^n)^4*n+45*a^2*c*m^2*(x^n)^2+411*a^2*c*n^2*(x^n)^2+510*c
^3*m^2*n^2*(x^n)^6+60*a*c^2*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+1016*b^3*m*n^
4*(x^n)^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2*m^3*(x^n)^5+
780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^2*c*m^5*(x^n)
^2+1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*(x^n)^2+700*a
^3*m*n^2+105*a^3*m*n+18*b*c^2*m^5*(x^n)^5+465*a^2*b*m^4*n^2*x^n+1740*a^2*b*
m^3*n^3*x^n+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(x^
n)^2+1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n^4
*(x^n)^2+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b*m*n^3*x^n+210*a^3*m^2*n+285*a*b^
2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^2*n^3*(x^n)^2+4212*
a*b^2*m*n^4*(x^n)^2+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m
^2*n*(x^n)^4+1284*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+a^3*m^6+6*a^3
*m^5+1764*a^3*n^5+1624*a^3*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^
3+720*a^3*n^6+15*b^3*m^2*(x^n)^3+20*b^3*m^3*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m
*b^3*(x^n)^3+18*b^3*(x^n)^3*n+175*a^3*n^2+21*a^3*n+21*a^3*m^5*n+175*a^3*m^4
*n^2+735*a^3*m^3*n^3+1624*a^3*m^2*n^4+1764*a^3*m*n^5+105*a^3*m^4*n+700*a^3*
m^3*n^2+2205*a^3*m^2*n^3+3248*a^3*m*n^4+20*c^3*m^3*(x^n)^6+225*c^3*n^3*(x^n
)^6+3*(x^n)^2*a*b^2+3*x^n*a^2*b+(x^n)^6*c^3+735*a^3*n^3+570*a*b^2*m^2*n*(x^
n)^2+45*x^n*a^2*b*m^4+1740*a^2*b*n^3*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)
/(1+m+4*n)/(1+m+5*n)/(1+m+6*n)*d^m*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d
*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 2303, normalized size of antiderivative = 12.65

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 1
5*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*
c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2 +
4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*
c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*
m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^
2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m^3 +
3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*c^2*m^3
+ 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m^4 + 10*b
*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*l
```

og(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 + (b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 307*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 + 3*(b^2*c + a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 107*((b^2*c + a*c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c + a*c^2)*m^2 + 4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 + 5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c + a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + ((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b^3 + 6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*c + (b^3 + 6*a*b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*a*b*c)*m^3 + 372*((b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 + 6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m)*n^3 + b^3 + 6*a*b*c + 15*(b^3 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 + 4*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6*a*b*c)*m)*n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6*a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6*a*b*c)*m^2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*((a*b^2 + a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 702*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2 + 2*(a*b^2 + a^2*c)*m)*n^4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a*b^2 + a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*b^2 + a^2*c)*m)*n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m^5 + 5*(a*b^2 + a^2*c)*m^4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a*b^2 + a^2*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2*b*m^5 + 15*a^2*b*m^4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b*m^2 + 1044*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4 + 4*a^2*b*m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b*m^5 + 5*a^2*b*m^4 + 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^6 + 720*a^3*n^6 + 6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 1764*(a^3*m + a^3)*n^5 + 15*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6*a^3*m + 735*(a^3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4 + 4*a^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4 + 10*a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^7 + 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 + 3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 35*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68491 vs. 2(170) = 340.

Time = 65.90 (sec) , antiderivative size = 68491, normalized size of antiderivative = 376.32

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Piecewise(((a + b + c)**3*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**3*log(x) +
3*a**2*b*x**n/n + 3*a**2*c*x**(2*n)/(2*n) + 3*a*b**2*x**(2*n)/(2*n) + 2*a*b
*c*x**(3*n)/n + 3*a*c**2*x**(4*n)/(4*n) + b**3*x**(3*n)/(3*n) + 3*b**2*c*x*
*(4*n)/(4*n) + 3*b*c**2*x**(5*n)/(5*n) + c**3*x**(6*n)/(6*n))/d, Eq(m, -1))
, (a**3*Piecewise((0**(-6*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(6*n*(d*x)**(
6*n)), Ne(n, 0)), (log(d*x), True))/d, True)) + 3*a**2*b*Piecewise((-x*x**n
*(d*x)**(-6*n - 1)/(5*n), Ne(n, 0)), (x*x**n*(d*x)**(-6*n - 1)*log(x), True
)) + 3*a**2*c*Piecewise((-x*x**(2*n)*(d*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x
*x**(2*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*b**2*Piecewise((-x*x**(2*n
)*(d*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-6*n - 1)*log(x),
 True)) + 6*a*b*c*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(3*n), Ne(n, 0))
, (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*c**2*Piecewise((-x*x**
(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x)**(-6*n - 1)*log
(x), True)) + b**3*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(3*n), Ne(n, 0)
), (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*b**2*c*Piecewise((-x*x*
*(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x)**(-6*n - 1)*lo
g(x), True)) + 3*b*c**2*Piecewise((-x*x**(5*n)*(d*x)**(-6*n - 1)/n, Ne(n, 0
)), (x*x**(5*n)*(d*x)**(-6*n - 1)*log(x), True)) + c**3*x*x**(6*n)*(d*x)**(
-6*n - 1)*log(x), Eq(m, -6*n - 1)), (a**3*Piecewise((0**(-5*n - 1)*x, Eq(d,
 0)), (Piecewise((-1/(5*n*(d*x)**(5*n)), Ne(n, 0)), (log(d*x), True))/d, Tr
ue)) + 3*a**2*b*Piecewise((-x*x**n*(d*x)**(-5*n - 1)/(4*n), Ne(n, 0)), (x*x
**n*(d*x)**(-5*n - 1)*log(x), True)) + 3*a**2*c*Piecewise((-x*x**(2*n)*(d*x
)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-5*n - 1)*log(x), True)
) + 3*a*b**2*Piecewise((-x*x**(2*n)*(d*x)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*
x**(2*n)*(d*x)**(-5*n - 1)*log(x), True)) + 6*a*b*c*Piecewise((-x*x**(3*n)*
(d*x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-5*n - 1)*log(x), T
rue)) + 3*a*c**2*Piecewise((-x*x**(4*n)*(d*x)**(-5*n - 1)/n, Ne(n, 0)), (x*
x**(4*n)*(d*x)**(-5*n - 1)*log(x), True)) + b**3*Piecewise((-x*x**(3*n)*(d*
x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-5*n - 1)*log(x), True
)) + 3*b**2*c*Piecewise((-x*x**(4*n)*(d*x)**(-5*n - 1)/n, Ne(n, 0)), (x*x**
(4*n)*(d*x)**(-5*n - 1)*log(x), True)) + 3*b*c**2*x*x**(5*n)*(d*x)**(-5*n -
 1)*log(x) + c**3*Piecewise((x*x**(6*n)*(d*x)**(-5*n - 1)/n, Ne(n, 0)), (x*
x**(6*n)*(d*x)**(-5*n - 1)*log(x), True)), Eq(m, -5*n - 1)), (a**3*Piecewis
e((0**(-4*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(4*n*(d*x)**(4*n)), Ne(n, 0))
, (log(d*x), True))/d, True)) + 3*a**2*b*Piecewise((-x*x**n*(d*x)**(-4*n -
```

1)/(3*n), Ne(n, 0)), (x*x**n*(d*x)**(-4*n - 1)*log(x), True)) + 3*a**2*c*Pi
ecewise((-x*x**(2*n)*(d*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(d*x)*
*(-4*n - 1)*log(x), True)) + 3*a*b**2*Piecewise((-x*x**(2*n)*(d*x)**(-4*n -
 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-4*n - 1)*log(x), True)) + 6*a*b*
c*Piecewise((-x*x**(3*n)*(d*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)*
*(-4*n - 1)*log(x), True)) + 3*a*c**2*x*x**(4*n)*(d*x)**(-4*n - 1)*log(x) +
 b**3*Piecewise((-x*x**(3*n)*(d*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d
*x)**(-4*n - 1)*log(x), True)) + 3*b**2*c*x*x**(4*n)*(d*x)**(-4*n - 1)*log(
x) + 3*b*c**2*Piecewise((x*x**(5*n)*(d*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(
5*n)*(d*x)**(-4*n - 1)*log(x), True)) + c**3*Piecewise((x*x**(6*n)*(d*x)**(
-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(6*n)*(d*x)**(-4*n - 1)*log(x), True)), E
q(m, -4*n - 1)), (a**3*Piecewise((0**(-3*n - 1)*x, Eq(d, 0)), (Piecewise((-
1/(3*n*(d*x)**(3*n)), Ne(n, 0)), (log(d*x), True))/d, True)) + 3*a**2*b*Pie
cewise((-x*x**n*(d*x)**(-3*n - 1)/(2*n), Ne(n, 0)), (x*x**n*(d*x)**(-3*n -
1)*log(x), True)) + 3*a**2*c*Piecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n, Ne
(n, 0)), (x*x**(2*n)*(d*x)**(-3*n - 1)*log(x), True)) + 3*a*b**2*Piecewise(
(-x*x**(2*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-3*n - 1)*
log(x), True)) + 6*a*b*c*x*x**(3*n)*(d*x)**(-3*n - 1)*log(x) + 3*a*c**2*Pie
cewise((x*x**(4*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(4*n)*(d*x)**(-3*n
 - 1)*log(x), True)) + b**3*x*x**(3*n)*(d*x)**(-3*n - 1)*log(x) + 3*b**2*c*
Piecewise((x*x**(4*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(4*n)*(d*x)**(-
3*n - 1)*log(x), True)) + 3*b*c**2*Piecewise((x*x**(5*n)*(d*x)**(-3*n - 1)/
(2*n), Ne(n, 0)), (x*x**(5*n)*(d*x)**(-3*n - 1)*log(x), True)) + c**3*Piece
wise((x*x**(6*n)*(d*x)**(-3*n - 1)/(3*n), Ne(n, 0)), (x*x**(6*n)*(d*x)**(-3
*n - 1)*log(x), True)), Eq(m, -3*n - 1)), (a**3*Piecewise((0**(-2*n - 1)*x,
 Eq(d, 0)), (Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(d*x), True))
/d, True)) + 3*a**2*b*Piecewise((-x*x**n*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x
*x**n*(d*x)**(-2*n - 1)*log(x), True)) + 3*a**2*c*x*x**(2*n)*(d*x)**(-2*n -
 1)*log(x) + 3*a*b**2*x*x**(2*n)*(d*x)**(-2*n - 1)*log(x) + 6*a*b*c*Piecewi
se((x*x**(3*n)*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-2*n - 1
)*log(x), True)) + 3*a*c**2*Piecewise((x*x**(4*n)*(d*x)**(-2*n - 1)/(2*n),
Ne(n, 0)), (x*x**(4*n)*(d*x)**(-2*n - 1)*log(x), True)) + b**3*Piecewise((x
*x**(3*n)*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-2*n - 1)*log
(x), True)) + 3*b**2*c*Piecewise((x*x**(4*n)*(d*x)**(-2*n - 1)/(2*n), Ne(n,
 0)), (x*x**(4*n)*(d*x)**(-2*n - 1)*log(x), True)) + 3*b*c**2*Piecewise((x*
x**(5*n)*(d*x)**(-2*n - 1)/(3*n), Ne(n, 0)), (x*x**(5*n)*(d*x)**(-2*n - 1)*
log(x), True)) + c**3*Piecewise((x*x**(6*n)*(d*x)**(-2*n - 1)/(4*n), Ne(n,
0)), (x*x**(6*n)*(d*x)**(-2*n - 1)*log(x), True)), Eq(m, -2*n - 1)), (a**3*
Piecewise((0**(-n - 1)*x, Eq(d, 0)), (Piecewise((-1/(n*(d*x)**n), Ne(n, 0))
, (log(d*x), True))/d, True)) + 3*a**2*b*x*x**n*(d*x)**(-n - 1)*log(x) + 3*
a**2*c*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x
)**(-n - 1)*log(x), True)) + 3*a*b**2*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)
/n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-n - 1)*log(x), True)) + 6*a*b*c*Piecewi
se((x*x**(3*n)*(d*x)**(-n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-n - 1
)*log(x), True)) + 3*a*c**2*Piecewise((x*x**(4*n)*(d*x)**(-n - 1)/(3*n), Ne

(n, 0)), (x*x**(4*n)*(d*x)**(-n - 1)*log(x), True)) + b**3*Piecewise((x*x**
(3*n)*(d*x)**(-n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-n - 1)*log(x),
True)) + 3*b**2*c*Piecewise((x*x**(4*n)*(d*x)**(-n - 1)/(3*n), Ne(n, 0)),
(x*x**(4*n)*(d*x)**(-n - 1)*log(x), True)) + 3*b*c**2*Piecewise((x*x**(5*n)
*(d*x)**(-n - 1)/(4*n), Ne(n, 0)), (x*x**(5*n)*(d*x)**(-n - 1)*log(x), True
)) + c**3*Piecewise((x*x**(6*n)*(d*x)**(-n - 1)/(5*n), Ne(n, 0)), (x*x**(6*
n)*(d*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (a**3*m**6*x*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 21*a**3*m**
5*n*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 6*a**3*m**5*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 175*a**3*m**4*n**2*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**
6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 4
20*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 17
50*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**
4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*
n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 105*a**3*m**4*n*x*(d*x)**m/(m**7 +
21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
+ 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4
410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m
*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 15*a**3*m**4*x*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 7
35*a**3*m**3*n**3*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1

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26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 700*a**3*m**3*n**2*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**
6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 4
20*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 17
50*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**
4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*
n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 210*a**3*m**3*n*x*(d*x)**m/(m**7 +
 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
+ 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4
410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m
*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 20*a**3*m**3*x*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1
624*a**3*m**2*n**4*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87
5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17
5*n**2 + 21*n + 1) + 2205*a**3*m**2*n**3*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m
**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
+ 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n
**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 162
4*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1050*a**3*m**2*n**2*x*(d*x)**m/(
m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4
*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n
**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n*
*4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*
n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 210*a**3*m
**2*n*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*
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n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n
5 + 4872*m2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m*
*2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 12
6*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
 + 1) + 15*a**3*m**2*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
+ 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m*
*4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**
3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315
*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
175*n**2 + 21*n + 1) + 1764*a**3*m*n**5*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m*
*6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1
750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n*
*4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624
*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3248*a**3*m*n**4*x*(d*x)**m/(m**7
 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**
3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528
*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2205*a**3*m*n*
*3*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*
m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**
4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m
*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 700*a**3*m*n**2*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
+ 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m*
*4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**
3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315
*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
175*n**2 + 21*n + 1) + 105*a**3*m*n*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 +
 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315
*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*
m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*
m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 +
 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**
4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6*a**3*m*x*(d*x)**m/(m**7 + 21*m**6*n
 + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4
*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*
n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n

3 + 1750*m2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 48
72*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 720*a**3*n**6*x*(d*x)**m/(
m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4
*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n
3 + 1750*m3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n*
*4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*
n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1764*a**3*
n**5*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 2
1*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n
4 + 2940*m3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n*
*5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**
2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126
*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
+ 1) + 1624*a**3*n**4*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m
4 + 1624*m3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m*
*3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 31
5*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
 175*n**2 + 21*n + 1) + 735*a**3*n**3*x*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 3
15*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 42
0*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 175
0*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n
4 + 735*n3 + 175*n**2 + 21*n + 1) + 175*a**3*n**2*x*(d*x)**m/(m**7 + 21
*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 8
75*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 175
0*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410
*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n*
*5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 17
64*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 21*a**3*n*x*(d*x)**
m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m
4*n3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**
3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2
*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 7
20*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + a**3*x*
(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2
940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 48
72*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720
*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +

7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
3*a**2*b*m**6*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
2*n + 21*m2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87
5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17
5*n**2 + 21*n + 1) + 60*a**2*b*m**5*n*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7
*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**
2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m
*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1
624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a**2*b*m**5*x*x**n*(d*x)**m
/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m*
*4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3
*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*
n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
+ 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 72
0*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 465*a**2
*b*m**4*n**2*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 300*a**2*b*m**4*n*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7
*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**
2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m
*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1
624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*a**2*b*m**4*x*x**n*(d*x)**m
/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m*
*4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3
*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*
n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
+ 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 72
0*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1740*a**
2*b*m**3*n**3*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
2*n + 21*m2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87
5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17

5*n**2 + 21*n + 1) + 1860*a**2*b*m**3*n**2*x*x**n*(d*x)**m/(m**7 + 21*m**6*
n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**
4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3
*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*
n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4
872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**
5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 600*a**2*b*m**3*n*x*x**n*
(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2
940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 48
72*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720
*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
60*a**2*b*m**3*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**
4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*
m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 8
75*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 1
75*n**2 + 21*n + 1) + 3132*a**2*b*m**2*n**4*x*x**n*(d*x)**m/(m**7 + 21*m**6
*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m*
*4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**
3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2
*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 +
4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n*
*5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 5220*a**2*b*m**2*n**3*x*
x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*
m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**
4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m
*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 2790*a**2*b*m**2*n**2*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175
*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**
4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3
*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2
*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 294
0*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
735*n**3 + 175*n**2 + 21*n + 1) + 600*a**2*b*m**2*n*x*x**n*(d*x)**m/(m**7 +
 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
+ 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4
410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m
*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*a**2*b*m**2*x

*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
 1) + 2160*a**2*b*m*n**5*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m
5*n2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*
n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
 + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n
2 + 315*m2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*
m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 73
5*n**3 + 175*n**2 + 21*n + 1) + 6264*a**2*b*m*n**4*x*x**n*(d*x)**m/(m**7 +
21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1
750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 44
10*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*
n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 5220*a**2*b*m*n**
3*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3
*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*
n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m
2 + 720*m*n6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 1
26*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*
n + 1) + 1860*a**2*b*m*n**2*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 17
5*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m*
*4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**
3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**
2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 29
40*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
 735*n**3 + 175*n**2 + 21*n + 1) + 300*a**2*b*m*n*x*x**n*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441
0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a**2*b*m*x*x**n
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
 2160*a**2*b*n**5*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**

2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*
m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m
3 + 1764*m2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 3
15*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3
+ 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3
+ 175*n**2 + 21*n + 1) + 3132*a**2*b*n**4*x*x**n*(d*x)**m/(m**7 + 21*m**6*n
 + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4
*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*
n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n
3 + 1750*m2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 48
72*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1740*a**2*b*n**3*x*x**n*(d
*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 294
0*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872
*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m
*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*
m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 46
5*a**2*b*n**2*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
2*n + 21*m2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87
5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17
5*n**2 + 21*n + 1) + 60*a**2*b*n*x*x**n*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 3
15*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 42
0*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 175
0*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n
4 + 735*n3 + 175*n**2 + 21*n + 1) + 3*a**2*b*x*x**n*(d*x)**m/(m**7 + 21
*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 8
75*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 175
0*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410
*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n*
*5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 17
64*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*a**2*c*m**6*x*x**
(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
 1) + 57*a**2*c*m**5*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175
*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**

4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3
*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2
*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 294
0*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
735*n**3 + 175*n**2 + 21*n + 1) + 18*a**2*c*m**5*x*x**(2*n)*(d*x)**m/(m**7
+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
+ 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*
m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 411*a**2*c*m**4
*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1
624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*
n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 285*a**2*c*m**4*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n*
*2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**
2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*
m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*a**2*c*m**4*x*x**(2*n)*(d*
x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 7
35*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940
*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*
m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*
n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m
+ 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 138
3*a**2*c*m**3*n**3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**
5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
+ 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n +
35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**
2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*
n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*
n**3 + 175*n**2 + 21*n + 1) + 1644*a**2*c*m**3*n**2*x*x**(2*n)*(d*x)**m/(m*
*7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n
3 + 875*m4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**
3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
+ 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 35
28*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n*
*6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 570*a**2*c*m
3*n*x*x(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1

624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*
n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 60*a**2*c*m**3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m
6 + 175*m5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
+ 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n
4 + 2940*m*n3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 162
4*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2106*a**2*c*m**2*n**4*x*x**(2*n)
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
4149*a**2*c*m**2*n**3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175
*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**
4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3
*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2
*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 294
0*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
735*n**3 + 175*n**2 + 21*n + 1) + 2466*a**2*c*m**2*n**2*x*x**(2*n)*(d*x)**m
/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m*
*4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3
*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*
n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
+ 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 72
0*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 570*a**2
*c*m**2*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
2*n + 21*m2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87
5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17
5*n**2 + 21*n + 1) + 45*a**2*c*m**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*
*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1080*a**2*c*m*n**5*x*x**(2*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +

 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 4212*a**2*c*m*n**4*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m
5*n2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*
n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
 + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n
2 + 315*m2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*
m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 73
5*n**3 + 175*n**2 + 21*n + 1) + 4149*a**2*c*m*n**3*x*x**(2*n)*(d*x)**m/(m**
7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n*
*3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
+ 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 352
8*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**
6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1644*a**2*c*m
*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1
624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*
n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 285*a**2*c*m*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m
6 + 175*m5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
+ 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n
4 + 2940*m*n3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 162
4*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a**2*c*m*x*x**(2*n)*(d*x)**m/
(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**
4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*
n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n
4 + 4410*m2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1080*a**2
*c*n**5*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 2106*a**2*c*n**4*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n +
 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*

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*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1383*a**2*c*n**3*x*x**(2*n)*
(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2
940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 48
72*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720
*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
411*a**2*c*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n
**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 3
5*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35
*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 +
 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**
3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**
3 + 175*n**2 + 21*n + 1) + 57*a**2*c*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*
n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**
4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3
*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*
n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4
872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**
5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*a**2*c*x*x**(2*n)*(d*x)
**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735
*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m
**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m*
*2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n*
*6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m +
 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*a*b
**2*m**6*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 57*a*b**2*m**5*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n
+ 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*
n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n
**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n*
*3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 487
2*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
+ 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a*b**2*m**5*x*x**(2*n)*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
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2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 4
11*a*b**2*m**4*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m*
*5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
 + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n*
*2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m
*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 285*a*b**2*m**4*n*x*x**(2*n)*(d*x)**m/(m**7
+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*
m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*a*b**2*m**4*
x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n
 + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m*
*3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**
2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21
*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 2
1*n + 1) + 1383*a*b**2*m**3*n**3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*
m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
+ 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*
n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 16
24*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1644*a*b**2*m**3*n**2*x*x**(2*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 570*a*b**2*m**3*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m*
*5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
 + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n*
*2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m
*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 60*a*b**2*m**3*x*x**(2*n)*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441

0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2106*a*b**2*m**2*n
4*x*x(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m*
*5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 162
4*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764
*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
+ 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n*
*2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
 + 21*n + 1) + 4149*a*b**2*m**2*n**3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n
+ 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*
n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n
2 + 420*m3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n*
*3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 487
2*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
+ 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2466*a*b**2*m**2*n**2*x*x**
(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
 1) + 570*a*b**2*m**2*n*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 17
5*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m*
*4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**
3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**
2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 29
40*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
 735*n**3 + 175*n**2 + 21*n + 1) + 45*a*b**2*m**2*x*x**(2*n)*(d*x)**m/(m**7
 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**
3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528
*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1080*a*b**2*m*
n**5*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m
5*n + 21*m5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 16
24*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 176
4*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
 + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n
2 + 126*m*n + 7*m + 720*n6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**
2 + 21*n + 1) + 4212*a*b**2*m*n**4*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n*
*2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**
2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3

+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*
m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 4149*a*b**2*m*n**3*x*x**(2*n)
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
1644*a*b**2*m*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m*
*5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
+ 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n*
*2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m
*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 285*a*b**2*m*n*x*x**(2*n)*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441
0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a*b**2*m*x*x**(
2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*
m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**
4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
+ 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m
*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 1080*a*b**2*n**5*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*
m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4
*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*
n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*
n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940
*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 7
35*n**3 + 175*n**2 + 21*n + 1) + 2106*a*b**2*n**4*x*x**(2*n)*(d*x)**m/(m**7
+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**
3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528
*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1383*a*b**2*n*
*3*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**
5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624
*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*
m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n +

21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**
2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
+ 21*n + 1) + 411*a*b**2*n**2*x*x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**
6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 4
20*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 17
50*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**
4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*
n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 57*a*b**2*n*x*x**(2*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*a*b**2*x*
x**(2*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3
*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*
n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m
2 + 720*m*n6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 1
26*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*
n + 1) + 6*a*b*c*m**6*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*
m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4
*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*
n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*
n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940
*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 7
35*n**3 + 175*n**2 + 21*n + 1) + 108*a*b*c*m**5*n*x*x**(3*n)*(d*x)**m/(m**7
+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**
3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528
*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 36*a*b*c*m**5*
x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n
+ 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m*
*3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**
2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21
*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 2
1*n + 1) + 726*a*b*c*m**4*n**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m*
*6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1
750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n*

*4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624
*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 540*a*b*c*m**4*n*x*x**(3*n)*(d*x)
m/(m7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735
*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m
3*n3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m*
*2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n*
*6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m +
 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 90*a*
b*c*m**4*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 2232*a*b*c*m**3*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**
6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m
4*n2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m*
*3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**
2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 +
 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n
5 + 1624*n4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2904*a*b*c*m**3*n**2*x*
x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3
*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*
n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m
2 + 720*m*n6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 1
26*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*
n + 1) + 1080*a*b*c*m**3*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 +
 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315
*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*
m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*
m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 +
 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**
4 + 735*n**3 + 175*n**2 + 21*n + 1) + 120*a*b*c*m**3*x*x**(3*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3048*a*b*c*
m**2*n**4*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
+ 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m
2*n + 21*m2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 87

5*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17
5*n**2 + 21*n + 1) + 6696*a*b*c*m**2*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m*
*6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*
m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m
3*n2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m*
*2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5
+ 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*
n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 4356*a*b*c*m**2*n**2*x
*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n
+ 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**
3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2
*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*
m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21
*n + 1) + 1080*a*b*c*m**2*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
+ 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 31
5*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420
*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750
*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
+ 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n*
*4 + 735*n**3 + 175*n**2 + 21*n + 1) + 90*a*b*c*m**2*x*x**(3*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1440*a*b*c*
m*n**5*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126
*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1
764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2
*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m
*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n
2 + 21*n + 1) + 6096*a*b*c*m*n4*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*
*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6696*a*b*c*m*n**3*x*x**(3*n)
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +

7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
2904*a*b*c*m*n**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**
5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
+ 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n +
35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**
2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*
n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*
n**3 + 175*n**2 + 21*n + 1) + 540*a*b*c*m*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*
m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 87
5*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750
*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*
m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**
5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 176
4*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 36*a*b*c*m*x*x**(3*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 1440*a*b*c*n**5*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5
*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n +
35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n +
35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2
+ 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n
3 + 875*m*n2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n
3 + 175*n2 + 21*n + 1) + 3048*a*b*c*n**4*x*x**(3*n)*(d*x)**m/(m**7 + 21
*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 8
75*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 175
0*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410
*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n*
*5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 17
64*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2232*a*b*c*n**3*x*x
(3*n)*(d*x)m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*
n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n
5 + 4872*m2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m*
*2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 12
6*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
+ 1) + 726*a*b*c*n**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175
*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**
4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3
*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2
*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 294
0*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +

735*n**3 + 175*n**2 + 21*n + 1) + 108*a*b*c*n*x*x**(3*n)*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441
0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6*a*b*c*x*x**(3*n)
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
 3*a*c**2*m**6*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n*
*2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35
*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*
m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 +
315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3
 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3
 + 175*n**2 + 21*n + 1) + 51*a*c**2*m**5*n*x*x**(4*n)*(d*x)**m/(m**7 + 21*m
6*n + 7*m6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875
*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*
m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m
2*n3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5
 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764
*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a*c**2*m**5*x*x**(
4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*
m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**
4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m
*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 321*a*c**2*m**4*n**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 +
175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*
m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m
3*n + 35*m3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m
2*n2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 +
2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
 + 735*n**3 + 175*n**2 + 21*n + 1) + 255*a*c**2*m**4*n*x*x**(4*n)*(d*x)**m/
(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**
4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*
n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n
4 + 4410*m2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*a*c**2

*m**4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1
624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*
n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 921*a*c**2*m**3*n**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n
 + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4
*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*
n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n
3 + 1750*m2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 48
72*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1284*a*c**2*m**3*n**2*x*x*
*(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 2
1*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n
4 + 2940*m3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n*
*5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**
2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126
*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
 + 1) + 510*a*c**2*m**3*n*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 1
75*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m
4*n + 35*m4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m*
*3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m*
*2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2
940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
 + 735*n**3 + 175*n**2 + 21*n + 1) + 60*a*c**2*m**3*x*x**(4*n)*(d*x)**m/(m**
7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n*
*3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 352
8*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**
6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1188*a*c**2*m
2*n4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 2763*a*c**2*m**2*n**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m*
*6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*
m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m
3*n2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m*
*2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5
 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*
n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1926*a*c**2*m**2*n**2*

x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n
 + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m*
*3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**
2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21
*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 2
1*n + 1) + 510*a*c**2*m**2*n*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 3
15*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 42
0*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 175
0*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n
4 + 735*n3 + 175*n**2 + 21*n + 1) + 45*a*c**2*m**2*x*x**(4*n)*(d*x)**m/
(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**
4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*
n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n
4 + 4410*m2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 540*a*c**
2*m*n**5*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 1
26*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4
+ 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m*
*2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875
*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175
*n**2 + 21*n + 1) + 2376*a*c**2*m*n**4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*
n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**
4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3
*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*
n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4
872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**
5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2763*a*c**2*m*n**3*x*x**(
4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*
m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**
4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m
*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 1284*a*c**2*m*n**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 17
5*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m*
*4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**
3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**
2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 29
40*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
 735*n**3 + 175*n**2 + 21*n + 1) + 255*a*c**2*m*n*x*x**(4*n)*(d*x)**m/(m**7

+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**
3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528
*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*a*c**2*m*x*
x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n +
21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3
*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*
n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m
2 + 720*m*n6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 1
26*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*
n + 1) + 540*a*c**2*n**5*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 1
75*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m
4*n + 35*m4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m*
*3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m*
*2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2
940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
+ 735*n**3 + 175*n**2 + 21*n + 1) + 1188*a*c**2*n**4*x*x**(4*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 921*a*c**2*
n**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m
5*n + 21*m5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 16
24*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 176
4*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
+ 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n
2 + 126*m*n + 7*m + 720*n6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**
2 + 21*n + 1) + 321*a*c**2*n**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m
6 + 175*m5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
+ 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n
4 + 2940*m*n3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 162
4*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 51*a*c**2*n*x*x**(4*n)*(d*x)**m/
(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**
4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*
n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n
4 + 4410*m2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*a*c**2*
x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n

+ 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m*
*3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**
2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21
*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 2
1*n + 1) + b**3*m**6*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m
5*n2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*
n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n
2 + 315*m2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*
m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 73
5*n**3 + 175*n**2 + 21*n + 1) + 18*b**3*m**5*n*x*x**(3*n)*(d*x)**m/(m**7 +
21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1
750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 44
10*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*
n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6*b**3*m**5*x*x**
(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
+ 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
1) + 121*b**3*m**4*n**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 1
75*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m
4*n + 35*m4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m*
*3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m*
*2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2
940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
+ 735*n**3 + 175*n**2 + 21*n + 1) + 90*b**3*m**4*n*x*x**(3*n)*(d*x)**m/(m**
7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n*
*3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3
+ 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
+ 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 352
8*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**
6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 15*b**3*m**4*
x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n
+ 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m*
*3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**
2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21
*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 +
126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 2
1*n + 1) + 372*b**3*m**3*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**
6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +

315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 4
20*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 17
50*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**
4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*
n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 484*b**3*m**3*n**2*x*x**(3*n)*(d*x
)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 73
5*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*
m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m
2*n4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n
6 + 3528*m*n5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m
+ 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 180*
b**3*m**3*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
+ 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m*
*4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**
3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315
*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
175*n**2 + 21*n + 1) + 20*b**3*m**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n +
 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*
*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 508*b**3*m**2*n**4*x*x**(3*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 1116*b**3*m**2*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*
m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4
*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*
n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*
n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940
*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 7
35*n**3 + 175*n**2 + 21*n + 1) + 726*b**3*m**2*n**2*x*x**(3*n)*(d*x)**m/(m*
*7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n
3 + 875*m4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**
3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 35
28*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n*
*6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 180*b**3*m**
2*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m*
*5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 162

4*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764
*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
+ 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n*
*2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
 + 21*n + 1) + 15*b**3*m**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
+ 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 31
5*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420
*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750
*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
+ 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n*
*4 + 735*n**3 + 175*n**2 + 21*n + 1) + 240*b**3*m*n**5*x*x**(3*n)*(d*x)**m/
(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**
4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*
n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n
4 + 4410*m2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1016*b**3
*m*n**4*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 1116*b**3*m*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n +
 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*
*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 484*b**3*m*n**2*x*x**(3*n)*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 9
0*b**3*m*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**
4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*
m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 8
75*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 1
75*n**2 + 21*n + 1) + 6*b**3*m*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m*
*6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +

420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 240*b**3*n**5*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 508*b**3*n**4*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 372*b**3*n**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 121*b**3*n**2*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 18*b**3*n*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + b**3*x*x**(3*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*b**2*c*m**6*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m

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**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5
 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764
*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 51*b**2*c*m**5*n*x*x*
*(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 2
1*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n
**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n*
*5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**
2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126
*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
+ 1) + 18*b**2*c*m**5*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*
m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4
*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*
n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*
n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940
*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 7
35*n**3 + 175*n**2 + 21*n + 1) + 321*b**2*c*m**4*n**2*x*x**(4*n)*(d*x)**m/(
m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4
*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n
**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n*
*4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 +
3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*
n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 255*b**2*c
*m**4*n*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 45*b**2*c*m**4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7
*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**
2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m
*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1
624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 921*b**2*c*m**3*n**3*x*x**(4*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 1284*b**2*c*m**3*n**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 17
5*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m*
*4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**
3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**
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2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 29
40*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 +
 735*n**3 + 175*n**2 + 21*n + 1) + 510*b**2*c*m**3*n*x*x**(4*n)*(d*x)**m/(m
**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 60*b**2*c*m
**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m*
*5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 162
4*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764
*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
+ 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n*
*2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
 + 21*n + 1) + 1188*b**2*c*m**2*n**4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n
+ 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*
n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n
**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n*
*3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 487
2*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
+ 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2763*b**2*c*m**2*n**3*x*x**
(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21
*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n*
*4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**
5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2
 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*
m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n +
 1) + 1926*b**2*c*m**2*n**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
+ 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 31
5*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420
*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750
*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
+ 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n*
*4 + 735*n**3 + 175*n**2 + 21*n + 1) + 510*b**2*c*m**2*n*x*x**(4*n)*(d*x)**
m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m
**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**
3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2
*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 7
20*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 45*b**2
*c*m**2*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
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2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 540*b**2*c*m*n**5*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n
+ 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*
n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n
2 + 420*m3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n*
*3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 487
2*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
+ 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2376*b**2*c*m*n**4*x*x**(4*
n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m*
*5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4
+ 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 +
720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 2763*b**2*c*m*n**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*
m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4
*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*
n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*
n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940
*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 7
35*n**3 + 175*n**2 + 21*n + 1) + 1284*b**2*c*m*n**2*x*x**(4*n)*(d*x)**m/(m*
*7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n
3 + 875*m4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**
3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4
+ 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 35
28*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n*
*6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 255*b**2*c*m
*n*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**
5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624
*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*
m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n +
21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**
2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
+ 21*n + 1) + 18*b**2*c*m*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 +
175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*
m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m
3*n + 35*m3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m
2*n2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 +
2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
+ 735*n**3 + 175*n**2 + 21*n + 1) + 540*b**2*c*n**5*x*x**(4*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3

528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1188*b**2*c
*n**4*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1
624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*
n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 921*b**2*c*n**3*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*
m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
+ 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*
n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 16
24*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 321*b**2*c*n**2*x*x**(4*n)*(d*x
)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 73
5*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*
m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m
2*n4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n
6 + 3528*m*n5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m
+ 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 51*b
2*c*n*x*x(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 3*b**2*c*x*x**(4*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6
+ 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 31
5*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420
*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750
*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4
+ 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n*
*4 + 735*n**3 + 175*n**2 + 21*n + 1) + 3*b*c**2*m**6*x*x**(5*n)*(d*x)**m/(m
7 + 21*m6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*
n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n*
*3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**
4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3
528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n
6 + 1764*n5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 48*b*c**2*m
5*n*x*x(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*
m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1
624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 17
64*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*
n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*

n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n*
*2 + 21*n + 1) + 18*b*c**2*m**5*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m
6 + 175*m5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
+ 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n
4 + 2940*m*n3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 162
4*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 285*b*c**2*m**4*n**2*x*x**(5*n)*
(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
+ 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2
940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 48
72*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720
*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
240*b*c**2*m**4*n*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5
*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n +
35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n +
35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2
+ 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n
3 + 875*m*n2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n
3 + 175*n2 + 21*n + 1) + 45*b*c**2*m**4*x*x**(5*n)*(d*x)**m/(m**7 + 21*
m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 87
5*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750
*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*
m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**
5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 176
4*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 780*b*c**2*m**3*n**3
*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*
n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m
3*n4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m*
*2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 2
1*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2
+ 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 +
21*n + 1) + 1140*b*c**2*m**3*n**2*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7
*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**
2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
+ 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
+ 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m
*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1
624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 480*b*c**2*m**3*n*x*x**(5*n)*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7

*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6
0*b*c**2*m**3*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**
2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*
m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m
3 + 1764*m2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 3
15*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3
+ 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3
+ 175*n**2 + 21*n + 1) + 972*b*c**2*m**2*n**4*x*x**(5*n)*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441
0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 2340*b*c**2*m**2*n
3*x*x(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m*
*5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 162
4*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764
*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n
+ 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n*
*2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
 + 21*n + 1) + 1710*b*c**2*m**2*n**2*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n
+ 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*
n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n
2 + 420*m3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n*
*3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 487
2*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
+ 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 480*b*c**2*m**2*n*x*x**(5*n
)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**
5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 +
4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 7
20*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n
+ 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1)
+ 45*b*c**2*m**2*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*
n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n +
35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 3
5*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2
+ 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n*
*3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n*
*3 + 175*n**2 + 21*n + 1) + 432*b*c**2*m*n**5*x*x**(5*n)*(d*x)**m/(m**7 + 2
1*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 17
50*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 441
0*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n
5 + 4872*m*n4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1

764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1944*b*c**2*m*n**4
*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*
n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m
3*n4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m*
*2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 2
1*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2
+ 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 +
21*n + 1) + 2340*b*c**2*m*n**3*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m*
*6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1
750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n*
*4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624
*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1140*b*c**2*m*n**2*x*x**(5*n)*(d*
x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 7
35*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940
*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*
m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*
n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m
 + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 240
*b*c**2*m*n*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
+ 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m*
*4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**
3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315
*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
175*n**2 + 21*n + 1) + 18*b*c**2*m*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n*
*2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**
2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*
m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 432*b*c**2*n**5*x*x**(5*n)*(d
*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 294
0*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872
*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m
*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*
m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 97
2*b*c**2*n**4*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**
2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*
m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m
3 + 1764*m2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 3
15*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3
+ 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3

+ 175*n**2 + 21*n + 1) + 780*b*c**2*n**3*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**
6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m
4*n2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m*
*3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**
2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 +
 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n
5 + 1624*n4 + 735*n**3 + 175*n**2 + 21*n + 1) + 285*b*c**2*n**2*x*x**(5
n)(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m
5 + 735*m4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4
 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
+ 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 +
 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*
n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1
) + 48*b*c**2*n*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n
2 + 126*m5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 3
5*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35
*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 +
 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**
3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**
3 + 175*n**2 + 21*n + 1) + 3*b*c**2*x*x**(5*n)*(d*x)**m/(m**7 + 21*m**6*n +
 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n
2 + 315*m4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n*
*2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**
3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872
*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + c**3*m**6*x*x**(6*n)*(d*x)**
m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m
4*n3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**
3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2
*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 7
20*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 15*c**3
*m**5*n*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 12
6*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 +
 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 +
1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**
2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*
m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*
n**2 + 21*n + 1) + 6*c**3*m**5*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m*
*6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 +
 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 +
420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1
750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n*
*4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624
*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 85*c**3*m**4*n**2*x*x**(6*n)*(d*x

)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 73
5*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*
m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m
2*n4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n
6 + 3528*m*n5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m
+ 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 75*c
3*m4*n*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**
4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*
m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 8
75*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 1
75*n**2 + 21*n + 1) + 15*c**3*m**4*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n +
7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n*
*2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**
2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3
 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*
m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 +
1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 225*c**3*m**3*n**3*x*x**(6*n)
*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5
 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 +
2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4
872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 72
0*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n +
 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) +
 340*c**3*m**3*n**2*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m*
*5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
 + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n*
*2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m
*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 150*c**3*m**3*n*x*x**(6*n)*(d*x)**m/(m**7 +
21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 +
 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1
750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 44
10*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*
n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 +
1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 20*c**3*m**3*x*x*
*(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 2
1*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n
4 + 2940*m3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n*
*5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**
2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126
*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
+ 1) + 274*c**3*m**2*n**4*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 +

175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*
m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m
3*n + 35*m3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m
2*n2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 +
2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4
 + 735*n**3 + 175*n**2 + 21*n + 1) + 675*c**3*m**2*n**3*x*x**(6*n)*(d*x)**m
/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m*
*4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3
*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*
n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6
 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 72
0*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 510*c**3
*m**2*n**2*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 +
 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**
4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3
 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*
m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 8
75*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 1
75*n**2 + 21*n + 1) + 150*c**3*m**2*n*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n
 + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4
*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*
n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n
3 + 1750*m2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 48
72*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 15*c**3*m**2*x*x**(6*n)*(d
*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 294
0*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872
*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m
*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*
m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 12
0*c**3*m*n**5*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**
2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*
m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m
3 + 1764*m2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 3
15*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3
 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3
 + 175*n**2 + 21*n + 1) + 548*c**3*m*n**4*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**
6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m
4*n2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m*
*3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**
2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 +
 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n
5 + 1624*n4 + 735*n**3 + 175*n**2 + 21*n + 1) + 675*c**3*m*n**3*x*x**(6
n)(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m

5 + 735*m4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4
 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5
+ 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 +
 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*
n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1
) + 340*c**3*m*n**2*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m*
*5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n
 + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n
+ 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n*
*2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m
*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 75*c**3*m*n*x*x**(6*n)*(d*x)**m/(m**7 + 21*m
6*n + 7*m6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875
*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*
m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m
2*n3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5
 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764
*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 6*c**3*m*x*x**(6*n)*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1
20*c**3*n**5*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m
4 + 1624*m3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m*
*3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 31
5*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
 175*n**2 + 21*n + 1) + 274*c**3*n**4*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n
 + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4
*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*
n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n
3 + 1750*m2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 48
72*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5
 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 225*c**3*n**3*x*x**(6*n)*(
d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 +
 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 29
40*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 487
2*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*
m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7
*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 8
5*c**3*n**2*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*m**6 + 175*m**5*n**2
+ 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2 + 315*m**4*n + 35*m*

```
*4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2 + 420*m**3*n + 35*m**
3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 + 1750*m**2*n**2 + 315
*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*n**4 + 2940*m*n**3 +
875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 +
175*n**2 + 21*n + 1) + 15*c**3*n*x*x**(6*n)*(d*x)**m/(m**7 + 21*m**6*n + 7*
m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3 + 875*m**4*n**2
 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 + 1750*m**3*n**2
+ 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 + 4410*m**2*n**3 +
 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*m*n**5 + 4872*m*
n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6 + 1764*n**5 + 16
24*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + c**3*x*x**(6*n)*(d*x)**m/(m**7
+ 21*m**6*n + 7*m**6 + 175*m**5*n**2 + 126*m**5*n + 21*m**5 + 735*m**4*n**3
 + 875*m**4*n**2 + 315*m**4*n + 35*m**4 + 1624*m**3*n**4 + 2940*m**3*n**3 +
 1750*m**3*n**2 + 420*m**3*n + 35*m**3 + 1764*m**2*n**5 + 4872*m**2*n**4 +
4410*m**2*n**3 + 1750*m**2*n**2 + 315*m**2*n + 21*m**2 + 720*m*n**6 + 3528*
m*n**5 + 4872*m*n**4 + 2940*m*n**3 + 875*m*n**2 + 126*m*n + 7*m + 720*n**6
+ 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.50

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \frac{c^3 d^m x e^{(m\log(x)+6\,n\log(x))}}{m+6\,n+1} + \frac{3\,bc^2 d^m x e^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{3\,b^2 c d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,ac^2 d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{b^3 d^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{6\,abcd^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,ab^2 d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2 c d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2 b d^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(dx)^{m+1} a^3}{d(m+1)}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log(
x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/(m
 + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^3*d
^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*
n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^2*b*d^
m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25656 vs. 2(182) = 364.

Time = 0.49 (sec) , antiderivative size = 25656, normalized size of antiderivative = 140.97

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] (c^3*m^6*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(6*n)*e^(m*lo
g(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 225*c
^3*m^3*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(6*n)*e^
(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3
*b*c^2*m^6*x*x^(5*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(5*n)*e^(m*log(d
) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m
^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(5*n)*e^(m*l
og(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 780*
b*c^2*m^3*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(
x)) + 274*c^3*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2*m*n^5*x
*x^(5*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(5*n)*e^(m*log(d) + m*
log(x)) + 3*b^2*c*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(
4*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + c^3*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*x^(4*n)*e^
(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
48*b*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + 321*a*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2
*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(4*n)*e^(m*log(d) +
 m*log(x)) + 921*b^2*c*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 921*a*c^
2*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3*n^3*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 1188*b^2*c*m^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4
*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 540*b^2
*c*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*x^(4*n)*e^(m
*log(d) + m*log(x)) + 432*b*c^2*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1
20*c^3*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + b^3*m^6*x*x^(3*n)*e^(m*log
(d) + m*log(x)) + 6*a*b*c*m^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*b^2*c*m
^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(3*n)*e^(m*log(d) +
m*log(x)) + 3*b*c^2*m^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 18*b^3*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x))
 + 108*a*b*c*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log
```

(x)) + 48*b*c^2*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 726*a*b*c*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 372*b^3*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2232*a*b*c*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 921*b^2*c*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 921*a*c^2*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 508*b^3*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3048*a*b*c*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 240*b^3*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1440*a*b*c*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*b^2*c*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*a*b^2*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b^3*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a^2*c*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*a*b*c*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*b^2*c*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 57*a*b^2*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*b^3*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 57*a^2*c*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 108*a*b*c*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 411*a*b^2*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 411*a^2*c*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 726*a*b*c*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1383*a*b^2*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 372*b^3*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1383*a^2*c*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2232*a*b*c*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 921*b^2*c*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 921*a*c^2*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2106*a*b^2*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 508*b^3*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2106*a^2*c*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3048*a*b*c*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(2*n)*e^

(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 1080*a*b^2*m*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 240*b^3*m*n^5*x*x^(2*
n)*e^(m*log(d) + m*log(x)) + 1080*a^2*c*m*n^5*x*x^(2*n)*e^(m*log(d) + m*log
(x)) + 1440*a*b*c*m*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 540*b^2*c*m*n^5
*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + 432*b*c^2*m*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m
*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a^2*b*m^6*x*x^n*e^(m*log(d) + m*
log(x)) + 3*a*b^2*m^6*x*x^n*e^(m*log(d) + m*log(x)) + b^3*m^6*x*x^n*e^(m*lo
g(d) + m*log(x)) + 3*a^2*c*m^6*x*x^n*e^(m*log(d) + m*log(x)) + 6*a*b*c*m^6*
x*x^n*e^(m*log(d) + m*log(x)) + 3*b^2*c*m^6*x*x^n*e^(m*log(d) + m*log(x)) +
 3*a*c^2*m^6*x*x^n*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^n*e^(m*log(d)
+ m*log(x)) + c^3*m^6*x*x^n*e^(m*log(d) + m*log(x)) + 60*a^2*b*m^5*n*x*x^n*
e^(m*log(d) + m*log(x)) + 57*a*b^2*m^5*n*x*x^n*e^(m*log(d) + m*log(x)) + 18
*b^3*m^5*n*x*x^n*e^(m*log(d) + m*log(x)) + 57*a^2*c*m^5*n*x*x^n*e^(m*log(d)
 + m*log(x)) + 108*a*b*c*m^5*n*x*x^n*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5
*n*x*x^n*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^n*e^(m*log(d) + m*log
(x)) + 48*b*c^2*m^5*n*x*x^n*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^n*e^
(m*log(d) + m*log(x)) + 465*a^2*b*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 4
11*a*b^2*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 411*a^2*c*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 726
*a*b*c*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 285
*b*c^2*m^4*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^n*e^(m*lo
g(d) + m*log(x)) + 1740*a^2*b*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 1383*
a*b^2*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 372*b^3*m^3*n^3*x*x^n*e^(m*lo
g(d) + m*log(x)) + 1383*a^2*c*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 2232*
a*b*c*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 921*b^2*c*m^3*n^3*x*x^n*e^(m*
log(d) + m*log(x)) + 921*a*c^2*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 780*
b*c^2*m^3*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^n*e^(m*lo
g(d) + m*log(x)) + 3132*a^2*b*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 2106*
a*b^2*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 508*b^3*m^2*n^4*x*x^n*e^(m*lo
g(d) + m*log(x)) + 2106*a^2*c*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 3048*
a*b*c*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 1188*b^2*c*m^2*n^4*x*x^n*e^(m
*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 97
2*b*c^2*m^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^n*e^(m*
log(d) + m*log(x)) + 2160*a^2*b*m*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 1080*
a*b^2*m*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 240*b^3*m*n^5*x*x^n*e^(m*log(d)
 + m*log(x)) + 1080*a^2*c*m*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 1440*a*b*c*
m*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 540*b^2*c*m*n^5*x*x^n*e^(m*log(d) + m
*log(x)) + 540*a*c^2*m*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 432*b*c^2*m*n^5*
x*x^n*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^n*e^(m*log(d) + m*log(x))
 + a^3*m^6*x*e^(m*log(d) + m*log(x)) + 3*a^2*b*m^6*x*e^(m*log(d) + m*log(x)
) + 3*a*b^2*m^6*x*e^(m*log(d) + m*log(x)) + b^3*m^6*x*e^(m*log(d) + m*log(x
)) + 3*a^2*c*m^6*x*e^(m*log(d) + m*log(x)) + 6*a*b*c*m^6*x*e^(m*log(d) + m*
log(x)) + 3*b^2*c*m^6*x*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*e^(m*log(d)

+ m*log(x)) + 3*b*c^2*m^6*x*e^(m*log(d) + m*log(x)) + c^3*m^6*x*e^(m*log(d
) + m*log(x)) + 21*a^3*m^5*n*x*e^(m*log(d) + m*log(x)) + 60*a^2*b*m^5*n*x*e
^(m*log(d) + m*log(x)) + 57*a*b^2*m^5*n*x*e^(m*log(d) + m*log(x)) + 18*b^3*
m^5*n*x*e^(m*log(d) + m*log(x)) + 57*a^2*c*m^5*n*x*e^(m*log(d) + m*log(x))
+ 108*a*b*c*m^5*n*x*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*e^(m*log(d)
+ m*log(x)) + 51*a*c^2*m^5*n*x*e^(m*log(d) + m*log(x)) + 48*b*c^2*m^5*n*x*e
^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*e^(m*log(d) + m*log(x)) + 175*a^3*m
^4*n^2*x*e^(m*log(d) + m*log(x)) + 465*a^2*b*m^4*n^2*x*e^(m*log(d) + m*log(
x)) + 411*a*b^2*m^4*n^2*x*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*e^(m*
log(d) + m*log(x)) + 411*a^2*c*m^4*n^2*x*e^(m*log(d) + m*log(x)) + 726*a*b*
c*m^4*n^2*x*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*e^(m*log(d) + m*l
og(x)) + 321*a*c^2*m^4*n^2*x*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*
e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*e^(m*log(d) + m*log(x)) + 735*a^
3*m^3*n^3*x*e^(m*log(d) + m*log(x)) + 1740*a^2*b*m^3*n^3*x*e^(m*log(d) + m*
log(x)) + 1383*a*b^2*m^3*n^3*x*e^(m*log(d) + m*log(x)) + 372*b^3*m^3*n^3*x*
e^(m*log(d) + m*log(x)) + 1383*a^2*c*m^3*n^3*x*e^(m*log(d) + m*log(x)) + 22
32*a*b*c*m^3*n^3*x*e^(m*log(d) + m*log(x)) + 921*b^2*c*m^3*n^3*x*e^(m*log(d
) + m*log(x)) + 921*a*c^2*m^3*n^3*x*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3
*n^3*x*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*e^(m*log(d) + m*log(x))
+ 1624*a^3*m^2*n^4*x*e^(m*log(d) + m*log(x)) + 3132*a^2*b*m^2*n^4*x*e^(m*lo
g(d) + m*log(x)) + 2106*a*b^2*m^2*n^4*x*e^(m*log(d) + m*log(x)) + 508*b^3*m
^2*n^4*x*e^(m*log(d) + m*log(x)) + 2106*a^2*c*m^2*n^4*x*e^(m*log(d) + m*log
(x)) + 3048*a*b*c*m^2*n^4*x*e^(m*log(d) + m*log(x)) + 1188*b^2*c*m^2*n^4*x*
e^(m*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4*x*e^(m*log(d) + m*log(x)) + 97
2*b*c^2*m^2*n^4*x*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*e^(m*log(d) +
m*log(x)) + 1764*a^3*m*n^5*x*e^(m*log(d) + m*log(x)) + 2160*a^2*b*m*n^5*x*
e^(m*log(d) + m*log(x)) + 1080*a*b^2*m*n^5*x*e^(m*log(d) + m*log(x)) + 240*
b^3*m*n^5*x*e^(m*log(d) + m*log(x)) + 1080*a^2*c*m*n^5*x*e^(m*log(d) + m*lo
g(x)) + 1440*a*b*c*m*n^5*x*e^(m*log(d) + m*log(x)) + 540*b^2*c*m*n^5*x*e^(m
*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*e^(m*log(d) + m*log(x)) + 432*b*c^2
*m*n^5*x*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*e^(m*log(d) + m*log(x))
+ 720*a^3*n^6*x*e^(m*log(d) + m*log(x)) + 6*c^3*m^5*x*x^(6*n)*e^(m*log(d) +
m*log(x)) + 75*c^3*m^4*n*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m^3*n
^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m^2*n^3*x*x^(6*n)*e^(m*log(d
) + m*log(x)) + 548*c^3*m*n^4*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 120*c^3*n
^5*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 18*b*c^2*m^5*x*x^(5*n)*e^(m*log(d) +
m*log(x)) + 6*c^3*m^5*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*
x*x^(5*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m^4*n*x*x^(5*n)*e^(m*log(d) + m*
log(x)) + 1140*b*c^2*m^3*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m^
3*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^3*x*x^(5*n)*e^(m
*log(d) + m*log(x)) + 675*c^3*m^2*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 1
944*b*c^2*m*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 548*c^3*m*n^4*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 432*b*c^2*n^5*x*x^(5*n)*e^(m*log(d) + m*log(x))
+ 120*c^3*n^5*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 18*b^2*c*m^5*x*x^(4*n)*e^
(m*log(d) + m*log(x)) + 18*a*c^2*m^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18

*b*c^2*m^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^3*m^5*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 255*b^2*c*m^4*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 255*a
*c^2*m^4*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*x*x^(4*n)*e^
(m*log(d) + m*log(x)) + 75*c^3*m^4*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 12
84*b^2*c*m^3*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m^3*n^2*x*x
^(4*n)*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m^3*n^2*x*x^(4*n)*e^(m*log(d) +
m*log(x)) + 340*c^3*m^3*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c
*m^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m^2*n^3*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x
)) + 675*c^3*m^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2376*b^2*c*m*n^4*x
*x^(4*n)*e^(m*log(d) + m*log(x)) + 2376*a*c^2*m*n^4*x*x^(4*n)*e^(m*log(d) +
m*log(x)) + 1944*b*c^2*m*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 548*c^3*m
*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 540*b^2*c*n^5*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 540*a*c^2*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2
*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 120*c^3*n^5*x*x^(4*n)*e^(m*log(d)
+ m*log(x)) + 6*b^3*m^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 36*a*b*c*m^5*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 18*b^2*c*m^5*x*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 18*a*c^2*m^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*b*c^2*m^5*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^3*m^5*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ 90*b^3*m^4*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*a*b*c*m^4*n*x*x^(3*n
)*e^(m*log(d) + m*log(x)) + 255*b^2*c*m^4*n*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 255*a*c^2*m^4*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m^4*n*x*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 484*b^3*m^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m^3*
n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m^3*n^2*x*x^(3*n)*e^(m*l
og(d) + m*log(x)) + 1284*a*c^2*m^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
1140*b*c^2*m^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m^3*n^2*x*x^
(3*n)*e^(m*log(d) + m*log(x)) + 1116*b^3*m^2*n^3*x*x^(3*n)*e^(m*log(d) + m*
log(x)) + 6696*a*b*c*m^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c
*m^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m^2*n^3*x*x^(3*n)*e
^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 675*c^3*m^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1016*b^3*m*n^4*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 6096*a*b*c*m*n^4*x*x^(3*n)*e^(m*log(d) + m
*log(x)) + 2376*b^2*c*m*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2376*a*c^2*
m*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1944*b*c^2*m*n^4*x*x^(3*n)*e^(m*l
og(d) + m*log(x)) + 548*c^3*m*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 240*b
^3*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1440*a*b*c*n^5*x*x^(3*n)*e^(m*lo
g(d) + m*log(x)) + 540*b^2*c*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*a*
c^2*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2*n^5*x*x^(3*n)*e^(m*lo
g(d) + m*log(x)) + 120*c^3*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*a*b^2
*m^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*b^3*m^5*x*x^(2*n)*e^(m*log(d) +
m*log(x)) + 18*a^2*c*m^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 36*a*b*c*m^5*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*b^2*c*m^5*x*x^(2*n)*e^(m*log(d) + m*l
og(x)) + 18*a*c^2*m^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*b*c^2*m^5*x*x^
(2*n)*e^(m*log(d) + m*log(x)) + 6*c^3*m^5*x*x^(2*n)*e^(m*log(d) + m*log(x))

+ 285*a*b^2*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 90*b^3*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 285*a^2*c*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 540*a*b*c*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 255*b^2*c*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 255*a*c^2*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m^4*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 484*b^3*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1644*a^2*c*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4149*a*b^2*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1116*b^3*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4149*a^2*c*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4212*a*b^2*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1016*b^3*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4212*a^2*c*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6096*a*b*c*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2376*b^2*c*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2376*a*c^2*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1944*b*c^2*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 548*c^3*m*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1080*a*b^2*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 240*b^3*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1080*a^2*c*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1440*a*b*c*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 540*b^2*c*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 540*a*c^2*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 120*c^3*n^5*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*a^2*b*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b^2*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 6*b^3*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 18*a^2*c*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 36*a*b*c*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 18*b^2*c*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*c^2*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 18*b*c^2*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^3*m^5*x*x^n*e^(m*log(d) + m*log(x)) + 300*a^2*b*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 285*a*b^2*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 90*b^3*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 285*a^2*c*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 540*a*b*c*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 255*b^2*c*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 255*a*c^2*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 75*c^3*m^4*n*x*x^n*e^(m*log(d) + m*log(x)) + 1860*a^2*b*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 484*b^3*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1644*a^2*c*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 340*c^3*m^3*n^2*x*x^n*e^(m*log(d) + m*log(x))

+ 5220*a^2*b*m^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4149*a*b^2*m^2*n^3*x*
x^n*e^(m*log(d) + m*log(x)) + 1116*b^3*m^2*n^3*x*x^n*e^(m*log(d) + m*log(x)
) + 4149*a^2*c*m^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m^2*n^3*x
*x^n*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m^2*n^3*x*x^n*e^(m*log(d) + m*log
(x)) + 2763*a*c^2*m^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^
3*x*x^n*e^(m*log(d) + m*log(x)) + 675*c^3*m^2*n^3*x*x^n*e^(m*log(d) + m*log
(x)) + 6264*a^2*b*m*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 4212*a*b^2*m*n^4*x*
x^n*e^(m*log(d) + m*log(x)) + 1016*b^3*m*n^4*x*x^n*e^(m*log(d) + m*log(x))
+ 4212*a^2*c*m*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 6096*a*b*c*m*n^4*x*x^n*e
^(m*log(d) + m*log(x)) + 2376*b^2*c*m*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 2
376*a*c^2*m*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 1944*b*c^2*m*n^4*x*x^n*e^(m
*log(d) + m*log(x)) + 548*c^3*m*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 2160*a^
2*b*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 1080*a*b^2*n^5*x*x^n*e^(m*log(d) +
m*log(x)) + 240*b^3*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 1080*a^2*c*n^5*x*x^
n*e^(m*log(d) + m*log(x)) + 1440*a*b*c*n^5*x*x^n*e^(m*log(d) + m*log(x)) +
540*b^2*c*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 540*a*c^2*n^5*x*x^n*e^(m*log(
d) + m*log(x)) + 432*b*c^2*n^5*x*x^n*e^(m*log(d) + m*log(x)) + 120*c^3*n^5*
x*x^n*e^(m*log(d) + m*log(x)) + 6*a^3*m^5*x*e^(m*log(d) + m*log(x)) + 18*a^
2*b*m^5*x*e^(m*log(d) + m*log(x)) + 18*a*b^2*m^5*x*e^(m*log(d) + m*log(x))
+ 6*b^3*m^5*x*e^(m*log(d) + m*log(x)) + 18*a^2*c*m^5*x*e^(m*log(d) + m*log(
x)) + 36*a*b*c*m^5*x*e^(m*log(d) + m*log(x)) + 18*b^2*c*m^5*x*e^(m*log(d) +
m*log(x)) + 18*a*c^2*m^5*x*e^(m*log(d) + m*log(x)) + 18*b*c^2*m^5*x*e^(m*l
og(d) + m*log(x)) + 6*c^3*m^5*x*e^(m*log(d) + m*log(x)) + 105*a^3*m^4*n*x*e
^(m*log(d) + m*log(x)) + 300*a^2*b*m^4*n*x*e^(m*log(d) + m*log(x)) + 285*a*
b^2*m^4*n*x*e^(m*log(d) + m*log(x)) + 90*b^3*m^4*n*x*e^(m*log(d) + m*log(x)
) + 285*a^2*c*m^4*n*x*e^(m*log(d) + m*log(x)) + 540*a*b*c*m^4*n*x*e^(m*log(
d) + m*log(x)) + 255*b^2*c*m^4*n*x*e^(m*log(d) + m*log(x)) + 255*a*c^2*m^4*
n*x*e^(m*log(d) + m*log(x)) + 240*b*c^2*m^4*n*x*e^(m*log(d) + m*log(x)) + 7
5*c^3*m^4*n*x*e^(m*log(d) + m*log(x)) + 700*a^3*m^3*n^2*x*e^(m*log(d) + m*l
og(x)) + 1860*a^2*b*m^3*n^2*x*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m^3*n^2*
x*e^(m*log(d) + m*log(x)) + 484*b^3*m^3*n^2*x*e^(m*log(d) + m*log(x)) + 164
4*a^2*c*m^3*n^2*x*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m^3*n^2*x*e^(m*log(d
) + m*log(x)) + 1284*b^2*c*m^3*n^2*x*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m
^3*n^2*x*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m^3*n^2*x*e^(m*log(d) + m*log
(x)) + 340*c^3*m^3*n^2*x*e^(m*log(d) + m*log(x)) + 2205*a^3*m^2*n^3*x*e^(m*
log(d) + m*log(x)) + 5220*a^2*b*m^2*n^3*x*e^(m*log(d) + m*log(x)) + 4149*a*
b^2*m^2*n^3*x*e^(m*log(d) + m*log(x)) + 1116*b^3*m^2*n^3*x*e^(m*log(d) + m*
log(x)) + 4149*a^2*c*m^2*n^3*x*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m^2*n^3
*x*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m^2*n^3*x*e^(m*log(d) + m*log(x)) +
2763*a*c^2*m^2*n^3*x*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m^2*n^3*x*e^(m*l
og(d) + m*log(x)) + 675*c^3*m^2*n^3*x*e^(m*log(d) + m*log(x)) + 3248*a^3*m*
n^4*x*e^(m*log(d) + m*log(x)) + 6264*a^2*b*m*n^4*x*e^(m*log(d) + m*log(x))
+ 4212*a*b^2*m*n^4*x*e^(m*log(d) + m*log(x)) + 1016*b^3*m*n^4*x*e^(m*log(d)
+ m*log(x)) + 4212*a^2*c*m*n^4*x*e^(m*log(d) + m*log(x)) + 6096*a*b*c*m*n^
4*x*e^(m*log(d) + m*log(x)) + 2376*b^2*c*m*n^4*x*e^(m*log(d) + m*log(x)) +

2376*a*c^2*m*n^4*x*e^(m*log(d) + m*log(x)) + 1944*b*c^2*m*n^4*x*e^(m*log(d) + m*log(x)) + 548*c^3*m*n^4*x*e^(m*log(d) + m*log(x)) + 1764*a^3*n^5*x*e^(m*log(d) + m*log(x)) + 2160*a^2*b*n^5*x*e^(m*log(d) + m*log(x)) + 1080*a*b^2*n^5*x*e^(m*log(d) + m*log(x)) + 240*b^3*n^5*x*e^(m*log(d) + m*log(x)) + 1080*a^2*c*n^5*x*e^(m*log(d) + m*log(x)) + 1440*a*b*c*n^5*x*e^(m*log(d) + m*log(x)) + 540*b^2*c*n^5*x*e^(m*log(d) + m*log(x)) + 540*a*c^2*n^5*x*e^(m*log(d) + m*log(x)) + 432*b*c^2*n^5*x*e^(m*log(d) + m*log(x)) + 120*c^3*n^5*x*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 45*a*c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 510*b^2*c*m^3*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^3*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1926*b^2*c*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1926*a*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*b^3*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 90*a*b*c*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*a*c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 180*b^3*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 510*b^2*c*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 726*b^3*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4356*a*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1926*b^2*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1926*a*c^2*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1116*b^3*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m*n^3*x *x^(3*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 508*b^3*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3048*a*b*c*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*a*b^2*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*b^3*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*a^2*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 90*a*b*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*a*c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 570*a*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 180*b^3*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 570*a^2*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 510*b^2*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2466*a*b^2*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 726*b^3*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2466*a^2*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4356*a*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1926*b^2*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1926*a*c^2*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4149*a*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1116*b^3*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4149*a^2*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2763*b^2*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2763*a*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 675*c^3*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2106*a*b^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 508*b^3*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2106*a^2*c*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3048*a*b*c*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*a^2*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 45*a*b^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 15*b^3*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 45*a^2*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 90*a*b*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 45*a*c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 15*c^3*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 600*a^2*b*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 570*a*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 180*b^3*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 570*a^2*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 510*b^2*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 150*c^3*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) +

2790*a^2*b*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 2466*a*b^2*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 726*b^3*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) +
2466*a^2*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 4356*a*b*c*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 1926*b^2*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x))
 + 1926*a*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*
x^n*e^(m*log(d) + m*log(x)) + 510*c^3*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x))
 + 5220*a^2*b*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4149*a*b^2*m*n^3*x*x^n*
e^(m*log(d) + m*log(x)) + 1116*b^3*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 41
49*a^2*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m*n^3*x*x^n*e^(m*
log(d) + m*log(x)) + 2763*b^2*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 2763*
a*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*x^n*e^(m*log
(d) + m*log(x)) + 675*c^3*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 3132*a^2*b*
n^4*x*x^n*e^(m*log(d) + m*log(x)) + 2106*a*b^2*n^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 508*b^3*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 2106*a^2*c*n^4*x*x^n*e^
(m*log(d) + m*log(x)) + 3048*a*b*c*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 1188
*b^2*c*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 1188*a*c^2*n^4*x*x^n*e^(m*log(d)
 + m*log(x)) + 972*b*c^2*n^4*x*x^n*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*
x^n*e^(m*log(d) + m*log(x)) + 15*a^3*m^4*x*e^(m*log(d) + m*log(x)) + 45*a^2
*b*m^4*x*e^(m*log(d) + m*log(x)) + 45*a*b^2*m^4*x*e^(m*log(d) + m*log(x)) +
 15*b^3*m^4*x*e^(m*log(d) + m*log(x)) + 45*a^2*c*m^4*x*e^(m*log(d) + m*log(
x)) + 90*a*b*c*m^4*x*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^4*x*e^(m*log(d) +
 m*log(x)) + 45*a*c^2*m^4*x*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^4*x*e^(m*l
og(d) + m*log(x)) + 15*c^3*m^4*x*e^(m*log(d) + m*log(x)) + 210*a^3*m^3*n*x*
e^(m*log(d) + m*log(x)) + 600*a^2*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 570*a
*b^2*m^3*n*x*e^(m*log(d) + m*log(x)) + 180*b^3*m^3*n*x*e^(m*log(d) + m*log(
x)) + 570*a^2*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^3*n*x*e^(m*l
og(d) + m*log(x)) + 510*b^2*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 510*a*c^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^3*n*x*e^(m*log(d) + m*log(x))
+ 150*c^3*m^3*n*x*e^(m*log(d) + m*log(x)) + 1050*a^3*m^2*n^2*x*e^(m*log(d)
+ m*log(x)) + 2790*a^2*b*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 2466*a*b^2*m^2
*n^2*x*e^(m*log(d) + m*log(x)) + 726*b^3*m^2*n^2*x*e^(m*log(d) + m*log(x))
+ 2466*a^2*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 4356*a*b*c*m^2*n^2*x*e^(m*
log(d) + m*log(x)) + 1926*b^2*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 1926*a*
c^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 1710*b*c^2*m^2*n^2*x*e^(m*log(d) +
m*log(x)) + 510*c^3*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 2205*a^3*m*n^3*x*e^
(m*log(d) + m*log(x)) + 5220*a^2*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 4149*a
*b^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 1116*b^3*m*n^3*x*e^(m*log(d) + m*log
(x)) + 4149*a^2*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 6696*a*b*c*m*n^3*x*e^(m
*log(d) + m*log(x)) + 2763*b^2*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 2763*a*c
^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 2340*b*c^2*m*n^3*x*e^(m*log(d) + m*log
(x)) + 675*c^3*m*n^3*x*e^(m*log(d) + m*log(x)) + 1624*a^3*n^4*x*e^(m*log(d)
 + m*log(x)) + 3132*a^2*b*n^4*x*e^(m*log(d) + m*log(x)) + 2106*a*b^2*n^4*x*
e^(m*log(d) + m*log(x)) + 508*b^3*n^4*x*e^(m*log(d) + m*log(x)) + 2106*a^2*
c*n^4*x*e^(m*log(d) + m*log(x)) + 3048*a*b*c*n^4*x*e^(m*log(d) + m*log(x))
+ 1188*b^2*c*n^4*x*e^(m*log(d) + m*log(x)) + 1188*a*c^2*n^4*x*e^(m*log(d) +

m*log(x)) + 972*b*c^2*n^4*x*e^(m*log(d) + m*log(x)) + 274*c^3*n^4*x*e^(m*l
og(d) + m*log(x)) + 20*c^3*m^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 150*c^3*
m^2*n*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m*n^2*x*x^(6*n)*e^(m*log(
d) + m*log(x)) + 225*c^3*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 60*b*c^2*m
^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 20*c^3*m^3*x*x^(5*n)*e^(m*log(d) + m
*log(x)) + 480*b*c^2*m^2*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^2*
n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m*n^2*x*x^(5*n)*e^(m*log(d
) + m*log(x)) + 340*c^3*m*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2
*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 225*c^3*n^3*x*x^(5*n)*e^(m*log(d)
+ m*log(x)) + 60*b^2*c*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 60*a*c^2*m^3
*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 60*b*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m
*log(x)) + 20*c^3*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 510*b^2*c*m^2*n*x
*x^(4*n)*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^2*n*x*x^(4*n)*e^(m*log(d) +
m*log(x)) + 480*b*c^2*m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 150*c^3*m^2
*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m*n^2*x*x^(4*n)*e^(m*log(
d) + m*log(x)) + 1284*a*c^2*m*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1140*
b*c^2*m*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m*n^2*x*x^(4*n)*e^(
m*log(d) + m*log(x)) + 921*b^2*c*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 92
1*a*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*n^3*x*x^(4*n)*e^(
m*log(d) + m*log(x)) + 225*c^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 20*b
^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 120*a*b*c*m^3*x*x^(3*n)*e^(m*log
(d) + m*log(x)) + 60*b^2*c*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 60*a*c^2
*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 60*b*c^2*m^3*x*x^(3*n)*e^(m*log(d)
+ m*log(x)) + 20*c^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 180*b^3*m^2*n
*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^2*n*x*x^(3*n)*e^(m*log(d)
+ m*log(x)) + 510*b^2*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 510*a*c^
2*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^2*n*x*x^(3*n)*e^(m*
log(d) + m*log(x)) + 150*c^3*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 484*
b^3*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m*n^2*x*x^(3*n)*e^
(m*log(d) + m*log(x)) + 1284*b^2*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ 1284*a*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1140*b*c^2*m*n^2*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m*n^2*x*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 372*b^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 2232*a*b*c*n^3*x*x^
(3*n)*e^(m*log(d) + m*log(x)) + 921*b^2*c*n^3*x*x^(3*n)*e^(m*log(d) + m*log
(x)) + 921*a*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*n^3*x*x^
(3*n)*e^(m*log(d) + m*log(x)) + 225*c^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 60*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 20*b^3*m^3*x*x^(2*n)*
e^(m*log(d) + m*log(x)) + 60*a^2*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
120*a*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 60*b^2*c*m^3*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 60*a*c^2*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 60
*b*c^2*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 20*c^3*m^3*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 570*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 180*
b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 570*a^2*c*m^2*n*x*x^(2*n)*e^(
m*log(d) + m*log(x)) + 1080*a*b*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
510*b^2*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^2*n*x*x^(2

*n)*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log
(x)) + 150*c^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m*n^2*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 484*b^3*m*n^2*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 1644*a^2*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2904*a*b*c*m
*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m*n^2*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 1284*a*c^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 114
0*b*c^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 340*c^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x)) + 1383*a*b^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 372*b^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 1383*a^2*c*n^3*x*x^(2*n)*e
^(m*log(d) + m*log(x)) + 2232*a*b*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 921*b^2*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 921*a*c^2*n^3*x*x^(2*n)*
e^(m*log(d) + m*log(x)) + 780*b*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 225*c^3*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 60*a^2*b*m^3*x*x^n*e^(m*lo
g(d) + m*log(x)) + 60*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 20*b^3*m^3*
x*x^n*e^(m*log(d) + m*log(x)) + 60*a^2*c*m^3*x*x^n*e^(m*log(d) + m*log(x))
 + 120*a*b*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 60*b^2*c*m^3*x*x^n*e^(m*log
(d) + m*log(x)) + 60*a*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 60*b*c^2*m^3
*x*x^n*e^(m*log(d) + m*log(x)) + 20*c^3*m^3*x*x^n*e^(m*log(d) + m*log(x)) +
 600*a^2*b*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 570*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x)) + 180*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 570*a^2
*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 1080*a*b*c*m^2*n*x*x^n*e^(m*log(d)
 + m*log(x)) + 510*b^2*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 480*b*c^2*m^2*n*x*x^n*e^(m*log(d) + m*l
og(x)) + 150*c^3*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 1860*a^2*b*m*n^2*x*x
^n*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))
 + 484*b^3*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1644*a^2*c*m*n^2*x*x^n*e^(
m*log(d) + m*log(x)) + 2904*a*b*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 128
4*b^2*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m*n^2*x*x^n*e^(m*l
og(d) + m*log(x)) + 1140*b*c^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 340*c^
3*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 1740*a^2*b*n^3*x*x^n*e^(m*log(d) +
m*log(x)) + 1383*a*b^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 372*b^3*n^3*x*x^
n*e^(m*log(d) + m*log(x)) + 1383*a^2*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) +
2232*a*b*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 921*b^2*c*n^3*x*x^n*e^(m*log
(d) + m*log(x)) + 921*a*c^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 780*b*c^2*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 225*c^3*n^3*x*x^n*e^(m*log(d) + m*log(x)
) + 20*a^3*m^3*x*e^(m*log(d) + m*log(x)) + 60*a^2*b*m^3*x*e^(m*log(d) + m*l
og(x)) + 60*a*b^2*m^3*x*e^(m*log(d) + m*log(x)) + 20*b^3*m^3*x*e^(m*log(d)
+ m*log(x)) + 60*a^2*c*m^3*x*e^(m*log(d) + m*log(x)) + 120*a*b*c*m^3*x*e^(m
*log(d) + m*log(x)) + 60*b^2*c*m^3*x*e^(m*log(d) + m*log(x)) + 60*a*c^2*m^3
*x*e^(m*log(d) + m*log(x)) + 60*b*c^2*m^3*x*e^(m*log(d) + m*log(x)) + 20*c^
3*m^3*x*e^(m*log(d) + m*log(x)) + 210*a^3*m^2*n*x*e^(m*log(d) + m*log(x)) +
 600*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x)) + 570*a*b^2*m^2*n*x*e^(m*log(d)
+ m*log(x)) + 180*b^3*m^2*n*x*e^(m*log(d) + m*log(x)) + 570*a^2*c*m^2*n*x*e
^(m*log(d) + m*log(x)) + 1080*a*b*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 510*b
^2*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 510*a*c^2*m^2*n*x*e^(m*log(d) + m*lo

g(x)) + 480*b*c^2*m^2*n*x*e^(m*log(d) + m*log(x)) + 150*c^3*m^2*n*x*e^(m*lo
g(d) + m*log(x)) + 700*a^3*m*n^2*x*e^(m*log(d) + m*log(x)) + 1860*a^2*b*m*n
^2*x*e^(m*log(d) + m*log(x)) + 1644*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x)) +
484*b^3*m*n^2*x*e^(m*log(d) + m*log(x)) + 1644*a^2*c*m*n^2*x*e^(m*log(d) +
m*log(x)) + 2904*a*b*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 1284*b^2*c*m*n^2*
x*e^(m*log(d) + m*log(x)) + 1284*a*c^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 11
40*b*c^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 340*c^3*m*n^2*x*e^(m*log(d) + m*
log(x)) + 735*a^3*n^3*x*e^(m*log(d) + m*log(x)) + 1740*a^2*b*n^3*x*e^(m*log
(d) + m*log(x)) + 1383*a*b^2*n^3*x*e^(m*log(d) + m*log(x)) + 372*b^3*n^3*x*
e^(m*log(d) + m*log(x)) + 1383*a^2*c*n^3*x*e^(m*log(d) + m*log(x)) + 2232*a
*b*c*n^3*x*e^(m*log(d) + m*log(x)) + 921*b^2*c*n^3*x*e^(m*log(d) + m*log(x)
) + 921*a*c^2*n^3*x*e^(m*log(d) + m*log(x)) + 780*b*c^2*n^3*x*e^(m*log(d) +
m*log(x)) + 225*c^3*n^3*x*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*x^(6*n)*e
^(m*log(d) + m*log(x)) + 75*c^3*m*n*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 85*
c^3*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^2*x*x^(5*n)*e^(m*log
(d) + m*log(x)) + 15*c^3*m^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*
m*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m*n*x*x^(5*n)*e^(m*log(d) +
m*log(x)) + 285*b*c^2*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 85*c^3*n^2*x*
x^(5*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^2*x*x^(4*n)*e^(m*log(d) + m*lo
g(x)) + 45*a*c^2*m^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*m^2*x*x^(
4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 255*b^2*c*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 255*a*c^2*m*n*x*x^(4*n
)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 75*c^3*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*n^2*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 321*a*c^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
285*b*c^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 85*c^3*n^2*x*x^(4*n)*e^(m
*log(d) + m*log(x)) + 15*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 90*a*b
*c*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^2*x*x^(3*n)*e^(m*log(
d) + m*log(x)) + 45*a*c^2*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 45*b*c^2*
m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*x^(3*n)*e^(m*log(d) +
m*log(x)) + 90*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*a*b*c*m*n*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 255*b^2*c*m*n*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 255*a*c^2*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 240*b*c^2*m*n*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(
x)) + 121*b^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 726*a*b*c*n^2*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 321*a*c^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*n^2*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 85*c^3*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
45*a*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*b^3*m^2*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 45*a^2*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 90*a
*b*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^2*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 45*a*c^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*b*c^
2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + 285*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 90*b^3*m*n*
x*x^(2*n)*e^(m*log(d) + m*log(x)) + 285*a^2*c*m*n*x*x^(2*n)*e^(m*log(d) + m

*log(x)) + 540*a*b*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 255*b^2*c*m*n*
x*x^(2*n)*e^(m*log(d) + m*log(x)) + 255*a*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m
*log(x)) + 240*b*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 75*c^3*m*n*x*x
^(2*n)*e^(m*log(d) + m*log(x)) + 411*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 121*b^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 411*a^2*c*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 726*a*b*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 321*b^2*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 321*a*c^2*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 85*c^3*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 45*a^2*b*m^2*x*x^n*e^(
m*log(d) + m*log(x)) + 45*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 15*b^3*
m^2*x*x^n*e^(m*log(d) + m*log(x)) + 45*a^2*c*m^2*x*x^n*e^(m*log(d) + m*log(
x)) + 90*a*b*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 45*b^2*c*m^2*x*x^n*e^(m*
log(d) + m*log(x)) + 45*a*c^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 45*b*c^2*
m^2*x*x^n*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*x^n*e^(m*log(d) + m*log(x)
) + 300*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 285*a*b^2*m*n*x*x^n*e^(m*
log(d) + m*log(x)) + 90*b^3*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 285*a^2*c*m
*n*x*x^n*e^(m*log(d) + m*log(x)) + 540*a*b*c*m*n*x*x^n*e^(m*log(d) + m*log(
x)) + 255*b^2*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 255*a*c^2*m*n*x*x^n*e^(
m*log(d) + m*log(x)) + 240*b*c^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 75*c^3
*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 465*a^2*b*n^2*x*x^n*e^(m*log(d) + m*lo
g(x)) + 411*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 121*b^3*n^2*x*x^n*e^(
m*log(d) + m*log(x)) + 411*a^2*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 726*a*
b*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 321*b^2*c*n^2*x*x^n*e^(m*log(d) + m
*log(x)) + 321*a*c^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 285*b*c^2*n^2*x*x^
n*e^(m*log(d) + m*log(x)) + 85*c^3*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 15*a
^3*m^2*x*e^(m*log(d) + m*log(x)) + 45*a^2*b*m^2*x*e^(m*log(d) + m*log(x)) +
 45*a*b^2*m^2*x*e^(m*log(d) + m*log(x)) + 15*b^3*m^2*x*e^(m*log(d) + m*log(
x)) + 45*a^2*c*m^2*x*e^(m*log(d) + m*log(x)) + 90*a*b*c*m^2*x*e^(m*log(d) +
 m*log(x)) + 45*b^2*c*m^2*x*e^(m*log(d) + m*log(x)) + 45*a*c^2*m^2*x*e^(m*l
og(d) + m*log(x)) + 45*b*c^2*m^2*x*e^(m*log(d) + m*log(x)) + 15*c^3*m^2*x*e
^(m*log(d) + m*log(x)) + 105*a^3*m*n*x*e^(m*log(d) + m*log(x)) + 300*a^2*b*
m*n*x*e^(m*log(d) + m*log(x)) + 285*a*b^2*m*n*x*e^(m*log(d) + m*log(x)) + 9
0*b^3*m*n*x*e^(m*log(d) + m*log(x)) + 285*a^2*c*m*n*x*e^(m*log(d) + m*log(x
)) + 540*a*b*c*m*n*x*e^(m*log(d) + m*log(x)) + 255*b^2*c*m*n*x*e^(m*log(d)
+ m*log(x)) + 255*a*c^2*m*n*x*e^(m*log(d) + m*log(x)) + 240*b*c^2*m*n*x*e^(
m*log(d) + m*log(x)) + 75*c^3*m*n*x*e^(m*log(d) + m*log(x)) + 175*a^3*n^2*x
*e^(m*log(d) + m*log(x)) + 465*a^2*b*n^2*x*e^(m*log(d) + m*log(x)) + 411*a*
b^2*n^2*x*e^(m*log(d) + m*log(x)) + 121*b^3*n^2*x*e^(m*log(d) + m*log(x)) +
 411*a^2*c*n^2*x*e^(m*log(d) + m*log(x)) + 726*a*b*c*n^2*x*e^(m*log(d) + m*
log(x)) + 321*b^2*c*n^2*x*e^(m*log(d) + m*log(x)) + 321*a*c^2*n^2*x*e^(m*lo
g(d) + m*log(x)) + 285*b*c^2*n^2*x*e^(m*log(d) + m*log(x)) + 85*c^3*n^2*x*e
^(m*log(d) + m*log(x)) + 6*c^3*m*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 15*c^3
*n*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 18*b*c^2*m*x*x^(5*n)*e^(m*log(d) + m
*log(x)) + 6*c^3*m*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 48*b*c^2*n*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 15*c^3*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 18*

b^2*c*m*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*a*c^2*m*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 18*b*c^2*m*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^3*m*x*x^
(4*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*n*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + 51*a*c^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 48*b*c^2*n*x*x^(4*n)*e^(
m*log(d) + m*log(x)) + 15*c^3*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*b^3*m
*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 36*a*b*c*m*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 18*b^2*c*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*a*c^2*m*x*x^(3*n
)*e^(m*log(d) + m*log(x)) + 18*b*c^2*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
6*c^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*b^3*n*x*x^(3*n)*e^(m*log(d)
+ m*log(x)) + 108*a*b*c*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*n*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(
x)) + 48*b*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 15*c^3*n*x*x^(3*n)*e^(
m*log(d) + m*log(x)) + 18*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*b^3
*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*a^2*c*m*x*x^(2*n)*e^(m*log(d) + m
*log(x)) + 36*a*b*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*b^2*c*m*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 18*a*c^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))
+ 18*b*c^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^3*m*x*x^(2*n)*e^(m*log
(d) + m*log(x)) + 57*a*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 18*b^3*n*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 57*a^2*c*n*x*x^(2*n)*e^(m*log(d) + m*log
(x)) + 108*a*b*c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*n*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 51*a*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4
8*b*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 15*c^3*n*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 18*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b^2*m*x*x^n
*e^(m*log(d) + m*log(x)) + 6*b^3*m*x*x^n*e^(m*log(d) + m*log(x)) + 18*a^2*c
*m*x*x^n*e^(m*log(d) + m*log(x)) + 36*a*b*c*m*x*x^n*e^(m*log(d) + m*log(x))
 + 18*b^2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*c^2*m*x*x^n*e^(m*log(d)
+ m*log(x)) + 18*b*c^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^3*m*x*x^n*e^(m
*log(d) + m*log(x)) + 60*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x)) + 57*a*b^2*n
*x*x^n*e^(m*log(d) + m*log(x)) + 18*b^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 5
7*a^2*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 108*a*b*c*n*x*x^n*e^(m*log(d) + m
*log(x)) + 51*b^2*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 51*a*c^2*n*x*x^n*e^(m
*log(d) + m*log(x)) + 48*b*c^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 15*c^3*n*x
*x^n*e^(m*log(d) + m*log(x)) + 6*a^3*m*x*e^(m*log(d) + m*log(x)) + 18*a^2*b
*m*x*e^(m*log(d) + m*log(x)) + 18*a*b^2*m*x*e^(m*log(d) + m*log(x)) + 6*b^3
*m*x*e^(m*log(d) + m*log(x)) + 18*a^2*c*m*x*e^(m*log(d) + m*log(x)) + 36*a*
b*c*m*x*e^(m*log(d) + m*log(x)) + 18*b^2*c*m*x*e^(m*log(d) + m*log(x)) + 18
*a*c^2*m*x*e^(m*log(d) + m*log(x)) + 18*b*c^2*m*x*e^(m*log(d) + m*log(x)) +
 6*c^3*m*x*e^(m*log(d) + m*log(x)) + 21*a^3*n*x*e^(m*log(d) + m*log(x)) + 6
0*a^2*b*n*x*e^(m*log(d) + m*log(x)) + 57*a*b^2*n*x*e^(m*log(d) + m*log(x))
+ 18*b^3*n*x*e^(m*log(d) + m*log(x)) + 57*a^2*c*n*x*e^(m*log(d) + m*log(x))
 + 108*a*b*c*n*x*e^(m*log(d) + m*log(x)) + 51*b^2*c*n*x*e^(m*log(d) + m*log
(x)) + 51*a*c^2*n*x*e^(m*log(d) + m*log(x)) + 48*b*c^2*n*x*e^(m*log(d) + m*
log(x)) + 15*c^3*n*x*e^(m*log(d) + m*log(x)) + c^3*x*x^(6*n)*e^(m*log(d) +
m*log(x)) + 3*b*c^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + c^3*x*x^(5*n)*e^(m*
log(d) + m*log(x)) + 3*b^2*c*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*x*

```
x^(4*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*x*x^(4*n)*e^(m*log(d) + m*log(x))
 + c^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + b^3*x*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 6*a*b*c*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*b^2*c*x*x^(3*n)*e^(m*
log(d) + m*log(x)) + 3*a*c^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + c^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3
*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b^3*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 3*a^2*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*a*b*c*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 3*b^2*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*x*
x^(2*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*x*x^(2*n)*e^(m*log(d) + m*log(x))
 + c^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x)) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x)) + b^3*x*x^n*e^(m*log(d) + m*l
og(x)) + 3*a^2*c*x*x^n*e^(m*log(d) + m*log(x)) + 6*a*b*c*x*x^n*e^(m*log(d)
+ m*log(x)) + 3*b^2*c*x*x^n*e^(m*log(d) + m*log(x)) + 3*a*c^2*x*x^n*e^(m*lo
g(d) + m*log(x)) + 3*b*c^2*x*x^n*e^(m*log(d) + m*log(x)) + c^3*x*x^n*e^(m*l
og(d) + m*log(x)) + a^3*x*e^(m*log(d) + m*log(x)) + 3*a^2*b*x*e^(m*log(d) +
 m*log(x)) + 3*a*b^2*x*e^(m*log(d) + m*log(x)) + b^3*x*e^(m*log(d) + m*log(
x)) + 3*a^2*c*x*e^(m*log(d) + m*log(x)) + 6*a*b*c*x*e^(m*log(d) + m*log(x))
 + 3*b^2*c*x*e^(m*log(d) + m*log(x)) + 3*a*c^2*x*e^(m*log(d) + m*log(x)) +
3*b*c^2*x*e^(m*log(d) + m*log(x)) + c^3*x*e^(m*log(d) + m*log(x)))/(m^7 + 2
1*m^6*n + 175*m^5*n^2 + 735*m^4*n^3 + 1624*m^3*n^4 + 1764*m^2*n^5 + 720*m*n
^6 + 7*m^6 + 126*m^5*n + 875*m^4*n^2 + 2940*m^3*n^3 + 4872*m^2*n^4 + 3528*m
*n^5 + 720*n^6 + 21*m^5 + 315*m^4*n + 1750*m^3*n^2 + 4410*m^2*n^3 + 4872*m*
n^4 + 1764*n^5 + 35*m^4 + 420*m^3*n + 1750*m^2*n^2 + 2940*m*n^3 + 1624*n^4
+ 35*m^3 + 315*m^2*n + 875*m*n^2 + 735*n^3 + 21*m^2 + 126*m*n + 175*n^2 + 7
*m + 21*n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 1734, normalized size of antiderivative = 9.53

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] (a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 255
*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 +
10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 +
 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*
n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 +
 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*
n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c + b^2)*(
5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n + 702*m*
n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3 + 702*n^
```

4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (b*x*x^(
3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^2*n + 7
44*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5
+ 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^3 + 121*m
^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 2
10*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3
+ 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6
+ 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 +
175*m^4*n^2 + 1) + (3*c*x*x^(4*n)*(d*x)^m*(a*c + b^2)*(5*m + 17*n + 68*m*n
+ 321*m*n^2 + 102*m^2*n + 614*m*n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10
*m^2 + 10*m^3 + 5*m^4 + m^5 + 107*n^2 + 307*n^3 + 396*n^4 + 180*n^5 + 321*m
^2*n^2 + 307*m^2*n^3 + 107*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2
+ 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5
+ 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1
624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 +
1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a^2*b*x*x^n*(d*x)^m*(5*m
+ 20*n + 80*m*n + 465*m*n^2 + 120*m^2*n + 1160*m*n^3 + 80*m^3*n + 1044*m*n
^4 + 20*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 155*n^2 + 580*n^3 + 1044*n^
4 + 720*n^5 + 465*m^2*n^2 + 580*m^2*n^3 + 155*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*b*c^2
*x*x^(5*n)*(d*x)^m*(5*m + 16*n + 64*m*n + 285*m*n^2 + 96*m^2*n + 520*m*n^3
+ 64*m^3*n + 324*m*n^4 + 16*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 95*n^2
+ 260*n^3 + 324*n^4 + 144*n^5 + 285*m^2*n^2 + 260*m^2*n^3 + 95*m^3*n^2 + 1)
)/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n +
3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 +
6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2
*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^
2 + 1)

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{2abx^{1+n}(dx)^m}{1+m+n} + \frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n} + \frac{2bcx^{1+3n}(dx)^m}{1+m+3n} + \frac{c^2x^{1+4n}(dx)^m}{1+m+4n} + \frac{a^2(dx)^{1+m}}{d(1+m)}$$

```
[Out] 2*a*b*x^(1+n)*(d*x)^m/(1+m+n)+(2*a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+2*b*c
*x^(1+3*n)*(d*x)^m/(1+m+3*n)+c^2*x^(1+4*n)*(d*x)^m/(1+m+4*n)+a^2*(d*x)^(1+m
)/d/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1367, 20, 30}

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{a^2(dx)^{m+1}}{d(m+1)} + \frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1} + \frac{2abx^{n+1}(dx)^m}{m+n+1} + \frac{2bcx^{3n+1}(dx)^m}{m+3n+1} + \frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \int \left(a^2(dx)^m + 2abx^n(dx)^m + b^2\left(1+\frac{2ac}{b^2}\right)x^{2n}(dx)^m + 2bcx^{3n}(dx)^m + c^2x^{4n}(dx)^m\right) dx$$

$$= \frac{a^2(dx)^{1+m}}{d(1+m)} + (2ab)\int x^n(dx)^m\,dx + (2bc)\int x^{3n}(dx)^m\,dx + c^2\int x^{4n}(dx)^m\,dx + \left(b^2+2ac\right)\int x^{2n}(dx)^m\,dx$$

$$= \frac{a^2(dx)^{1+m}}{d(1+m)} + \left(2abx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(2bcx^{-m}(dx)^m\right)\int x^{m+3n}\,dx + \left(c^2x^{-m}(dx)^m\right)\int x^{m+4n}\,dx + \left(\left(b^2+2ac\right)x^{-m}(dx)^m\right)\int x^{m+2n}\,dx$$

$$= \frac{2abx^{1+n}(dx)^m}{1+m+n} + \frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n} + \frac{2bcx^{1+3n}(dx)^m}{1+m+3n} + \frac{c^2x^{1+4n}(dx)^m}{1+m+4n} + \frac{a^2(dx)^{1+m}}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = x(dx)^m \left(\frac{a^2}{1+m}+\frac{2abx^n}{1+m+n}+\frac{(b^2+2ac)\, x^{2n}}{1+m+2n} + \frac{2bcx^{3n}}{1+m+3n}+\frac{c^2x^{4n}}{1+m+4n}\right)$$

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))/
(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*n)
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 1032, normalized size of antiderivative = 8.82

method	result	size
risch	Expression too large to display	1032
parallelrisch	Expression too large to display	1566

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(a^2+54*a*b*m*n*x^n+18*a*b*m^3*n*x^n+24*a*c*m*n^3*(x^n)^2+2*(x^n)^3*b*c+2
*(x^n)^2*a*c+8*m*b*c*(x^n)^3+14*b*c*(x^n)^3*n+12*a*b*m^2*x^n+52*a*b*n^2*x^n
+8*a*c*(x^n)^2*m+16*a*c*(x^n)^2*n+24*b^2*m^2*n*(x^n)^2+18*a*b*x^n*n+8*a*b*x
^n*m+38*a*c*n^2*(x^n)^2+24*b^2*m*n*(x^n)^2+14*b*c*m^3*n*(x^n)^3+28*b*c*m^2*
n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+42*b*c*m^2*n*(x^n)^3+56*b*c*m*n^2*(x^n)^3+
19*b^2*m^2*n^2*(x^n)^2+22*c^2*m*n^2*(x^n)^4+2*a*c*m^4*(x^n)^2+8*b^2*m^3*n*(
x^n)^2+30*a^2*m*n+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*a^2*n^2+10*a^2*
m^3*n+52*a*b*m^2*n^2*x^n+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a*c*m*n^2
*(x^n)^2+42*b*c*m*n*(x^n)^3+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(
x^n)^2+18*c^2*m^2*n*(x^n)^4+38*b^2*m*n^2*(x^n)^2+12*b*c*m^2*(x^n)^3+28*b*c*
n^2*(x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+12*a*c*m^2*(x^n)^2+12*b^2*m*n^3*(x
^n)^2+8*b*c*m^3*(x^n)^3+16*b*c*n^3*(x^n)^3+35*a^2*m^2*n^2+50*a^2*m*n^3+11*c
^2*m^2*n^2*(x^n)^4+6*c^2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+6*c^2*m^3*n*(x^n)^
4+6*c^2*(x^n)^4*n+6*b^2*m^2*(x^n)^2+19*b^2*n^2*(x^n)^2+12*b^2*n^3*(x^n)^2+4
*m*c^2*(x^n)^4+16*a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+30*a^2*m^2*n+70*
a^2*m*n^2+24*a^2*n^4+b^2*(x^n)^2+24*a*c*n^3*(x^n)^2+4*a^2*m+10*a^2*n+6*c^2*
n^3*(x^n)^4+b^2*m^4*(x^n)^2+6*c^2*m^2*(x^n)^4+c^2*m^4*(x^n)^4+4*c^2*m^3*(x^
n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+4*b^2*(x^n)^2*m+18*c^2*m*n*(x^n)^4+c^2
```

```
*(x^n)^4+8*b^2*(x^n)^2*n+11*c^2*n^2*(x^n)^4+4*b^2*m^3*(x^n)^2+2*a*b*x^n)/(1
+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*x^m*d^m*exp(1/2*I*Pi*csgn(I*d*x)*
m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(117) = 234.

Time = 0.31 (sec) , antiderivative size = 706, normalized size of antiderivative = 6.03

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx = \frac{(c^2m^4 + 4\,c^2m^3 + 6\,c^2m^2 + 6\,(c^2m + c^2)n^3 + 4\,c^2m + 11\,(c^2m^2 + 2\,c^2m + c^2)n^2 + c^2 + 6\,(c^2m^3 + 3\,c^2m^2}{}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2
*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n
)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 +
8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*
(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c
)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b
^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3
 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)
*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*
b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2
*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n)*x
*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1
)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. 2(107) = 214.

Time = 16.76 (sec) , antiderivative size = 12323, normalized size of antiderivative = 105.32

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise(((a + b + c)**2*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) +
2*a*b*x**n/n + a*c*x**(2*n)/n + b**2*x**(2*n)/(2*n) + 2*b*c*x**(3*n)/(3*n)
+ c**2*x**(4*n)/(4*n))/d, Eq(m, -1)), (a**2*Piecewise((0**(-4*n - 1)*x, Eq(
d, 0)), (Piecewise((-1/(4*n*(d*x)**(4*n)), Ne(n, 0)), (log(d*x), True))/d,
True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x*
*n*(d*x)**(-4*n - 1)*log(x), True)) + 2*a*c*Piecewise((-x*x**(2*n)*(d*x)**(
-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-4*n - 1)*log(x), True)) +
b**2*Piecewise((-x*x**(2*n)*(d*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)
*(d*x)**(-4*n - 1)*log(x), True)) + 2*b*c*Piecewise((-x*x**(3*n)*(d*x)**(-4
*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-4*n - 1)*log(x), True)) + c**2*x
*x**(4*n)*(d*x)**(-4*n - 1)*log(x), Eq(m, -4*n - 1)), (a**2*Piecewise((0**(
-3*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(3*n*(d*x)**(3*n)), Ne(n, 0)), (log(
d*x), True))/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-3*n - 1)/(2*n),
Ne(n, 0)), (x*x**n*(d*x)**(-3*n - 1)*log(x), True)) + 2*a*c*Piecewise((-x*x
**(2*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-3*n - 1)*log(x
), True)) + b**2*Piecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*
x**(2*n)*(d*x)**(-3*n - 1)*log(x), True)) + 2*b*c*x*x**(3*n)*(d*x)**(-3*n -
 1)*log(x) + c**2*Piecewise((x*x**(4*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*
x**(4*n)*(d*x)**(-3*n - 1)*log(x), True)), Eq(m, -3*n - 1)), (a**2*Piecewis
e((0**(-2*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0))
, (log(d*x), True))/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-2*n - 1)/
n, Ne(n, 0)), (x*x**n*(d*x)**(-2*n - 1)*log(x), True)) + 2*a*c*x*x**(2*n)*(
d*x)**(-2*n - 1)*log(x) + b**2*x*x**(2*n)*(d*x)**(-2*n - 1)*log(x) + 2*b*c*
Piecewise((x*x**(3*n)*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-
2*n - 1)*log(x), True)) + c**2*Piecewise((x*x**(4*n)*(d*x)**(-2*n - 1)/(2*n
), Ne(n, 0)), (x*x**(4*n)*(d*x)**(-2*n - 1)*log(x), True)), Eq(m, -2*n - 1)
), (a**2*Piecewise((0**(-n - 1)*x, Eq(d, 0)), (Piecewise((-1/(n*(d*x)**n),
Ne(n, 0)), (log(d*x), True))/d, True)) + 2*a*b*x*x**n*(d*x)**(-n - 1)*log(x
) + 2*a*c*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(
d*x)**(-n - 1)*log(x), True)) + b**2*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/
n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-n - 1)*log(x), True)) + 2*b*c*Piecewise(
(x*x**(3*n)*(d*x)**(-n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-n - 1)*l
og(x), True)) + c**2*Piecewise((x*x**(4*n)*(d*x)**(-n - 1)/(3*n), Ne(n, 0))
, (x*x**(4*n)*(d*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (a**2*m**4*x*
(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 +
50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3
 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10
*a**2*m**3*n*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3
*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n*
*4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
 10*n + 1) + 4*a**2*m**3*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n*
*2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m*
*2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a**2*m**2*n**2*x*(d*x)**m/(m**5 + 10*m**4*n + 5
*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 +
```

```
60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m +
 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 30*a**2*m**2*n*x*(d*x)**m/(m**5
+ 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 +
105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 +
 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*a**2*m**2*x*(d*
x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*
m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 +
105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*
*2*m*n**3*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n
+ 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4
+ 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 70*a**2*m*n**2*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n*
*2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m*
*2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 30*a**2*m*n*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4
+ 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m*
*2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n*
*4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*a**2*m*x*(d*x)**m/(m**5 + 10*m**4*n
+ 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**
2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*
m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a**2*n**4*x*(d*x)**m/(m**5
 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 +
 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2
+ 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*n**3*x*(
d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 5
0*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3
+ 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*
a**2*n**2*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n
+ 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4
+ 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a**2*n*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 +
40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 +
24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35
*n**2 + 10*n + 1) + a**2*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n*
*2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m*
*2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 2*a*b*m**4*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m
**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 6
0*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*m**3*n*x*x**n*(d*x)**m/(m**
5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3
+ 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2
 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*b*m**3*x*x*
*n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3
 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n
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**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
 52*a*b*m**2*n**2*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2
 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2
 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 +
 35*n**2 + 10*n + 1) + 54*a*b*m**2*n*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*
m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 +
60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m +
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*a*b*m**2*x*x**n*(d*x)**m/(m**5
 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 +
 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2
+ 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*m*n**3*x*
x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m*
*3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m
*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 104*a*b*m*n**2*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2
 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2
 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 +
 35*n**2 + 10*n + 1) + 54*a*b*m*n*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**
4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*
m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*b*m*x*x**n*(d*x)**m/(m**5 + 10*m
**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m*
*2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*
n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*n**3*x*x**n*(d*x
)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m
**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 1
05*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b
*n**2*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n
 + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4
 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 18*a*b*n*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n*
*2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m*
*2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 2*a*b*x*x**n*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 +
 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**
2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*c*m**4*x*x**(2*n)*(d*x)**m/(m**5 +
10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 10
5*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 4
0*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16*a*c*m**3*n*x*x**
(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m
**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*
m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 8*a*c*m**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**
2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**
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2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 38*a*c*m**2*n**2*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**
4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2
*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n
+ 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*c*m**2*n*x*x**(2*n)*
(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 +
50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3
 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12
*a*c*m**2*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 4
0*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 2
4*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*
n**2 + 10*n + 1) + 24*a*c*m*n**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*
m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 +
60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m +
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 76*a*c*m*n**2*x*x**(2*n)*(d*x)**m
/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*
n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m
*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*c*m*n
*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n
+ 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4
+ 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 8*a*c*m*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*
n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*
m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 24*a*c*n**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*
n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n
2 + 60*m2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n +
5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 38*a*c*n**2*x*x**(2*n)*(d*x
)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m
2*n3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 1
05*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16*a*c
*n*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*
n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**
4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
10*n + 1) + 2*a*c*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*
n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*
m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + b**2*m**4*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n
+ 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**
2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*
m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*m**3*n*x*x**(2*n)*(d*x
)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m
2*n3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 1
05*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*b**2
*m**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m*

*3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*
n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2
 + 10*n + 1) + 19*b**2*m**2*n**2*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*
m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 +
60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m +
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b**2*m**2*n*x*x**(2*n)*(d*x)**
m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2
*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*
m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*m*
*2*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*
n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**
4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
10*n + 1) + 12*b**2*m*n**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 +
 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**
2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 38*b**2*m*n**2*x*x**(2*n)*(d*x)**m/(m**
5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3
+ 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2
 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b**2*m*n*x*x
(2*n)*(d*x)m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10
*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 10
0*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 4*b**2*m*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**
2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**
2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 12*b**2*n**3*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n
+ 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**
2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*
m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*n**2*x*x**(2*n)*(d*x)
m/(m5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m*
*2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 10
5*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*
n*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n
 + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4
 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + b**2*x*x**(2*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n*
*2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m*
*2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 2*b*c*m**4*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n +
 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2
 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m
 + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*b*c*m**3*n*x*x**(3*n)*(d*x)
m/(m5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m*
*2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 10
5*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b*c*m

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**3*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3
*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n*
*4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
 10*n + 1) + 28*b*c*m**2*n**2*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**
4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*
m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 42*b*c*m**2*n*x*x**(3*n)*(d*x)**m/(m
**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**
3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n*
*2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b*c*m**2*x
*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n +
10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 +
100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n
 + 1) + 16*b*c*m*n**3*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m
**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n +
 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 56*b*c*m*n**2*x*x**(3*n)*(d*x)**m/(m**5 + 10
*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*
m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*
m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 42*b*c*m*n*x*x**(3*n)
*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 +
 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**
3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8
*b*c*m*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m
**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m
*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 16*b*c*n**3*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4
+ 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m*
*2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n*
*4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*b*c*n**2*x*x**(3*n)*(d*x)**m/(m**5
+ 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 +
105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 +
 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*b*c*n*x*x**(3*
n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3
 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n
**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
 2*b*c*x*x**(3*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m
**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m
*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + c**2*m**4*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 +
35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2
*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*c**2*m**3*n*x*x**(4*n)*(d*x)**m/(m**5
+ 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 +
105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 +
```

```
 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*c**2*m**3*x*x**
(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m
**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*
m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 11*c**2*m**2*n**2*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m
**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n +
 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 18*c**2*m**2*n*x*x**(4*n)*(d*x)**m/(m**5 + 1
0*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105
*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40
*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*c**2*m**2*x*x**(4*
n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3
 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n
**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
 6*c**2*m*n**3*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**
2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**
2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 22*c**2*m*n**2*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*
n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n
**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n +
5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*c**2*m*n*x*x**(4*n)*(d*x
)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m
**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 1
05*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*c**2
*m*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*
n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**
4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
10*n + 1) + 6*c**2*n**3*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35
*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n
 + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 +
 50*n**3 + 35*n**2 + 10*n + 1) + 11*c**2*n**2*x*x**(4*n)*(d*x)**m/(m**5 + 1
0*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105
*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40
*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*c**2*n*x*x**(4*n)*
(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 +
50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3
 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + c*
*2*x*x**(4*n)*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*
n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**
4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 +
10*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{c^2 d^m x e^{(m \log(x)+4\,n \log(x))}}{m+4\,n+1} + \frac{2\,bcd^m x e^{(m \log(x)+3\,n \log(x))}}{m+3\,n+1} + \frac{b^2 d^m x e^{(m \log(x)+2\,n \log(x))}}{m+2\,n+1} + \frac{2\,acd^m x e^{(m \log(x)+2\,n \log(x))}}{m+2\,n+1} + \frac{2\,abd^m x e^{(m \log(x)+n \log(x))}}{m+n+1} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x)
 + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n
 + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*x*e
^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. 2(117) = 234.

Time = 0.33 (sec) , antiderivative size = 5454, normalized size of antiderivative = 46.62

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*
m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d) +
 m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m
^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
```

2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^n*e^(m*log(
d) + m*log(x)) + c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*x^n
*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a
*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^n*e^(m*log(d) + m
*log(x)) + 6*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 3
8*a*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^n*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*m*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^n*e^(m*l
og(d) + m*log(x)) + 6*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + a^2*m^4*x*e
^(m*log(d) + m*log(x)) + 2*a*b*m^4*x*e^(m*log(d) + m*log(x)) + b^2*m^4*x*e^
(m*log(d) + m*log(x)) + 2*a*c*m^4*x*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*e
^(m*log(d) + m*log(x)) + c^2*m^4*x*e^(m*log(d) + m*log(x)) + 10*a^2*m^3*n*x
*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1
4*b*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*e^(m*log(d) + m*log(x
)) + 35*a^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*e^(m*log(d
) + m*log(x)) + 19*b^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x
*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*m*n^3*x*e^(m*log(d) + m*log(x)
) + 48*a*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*e^(m*log(d) + m
*log(x)) + 24*a*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*e^(m*log
(d) + m*log(x)) + 6*c^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 24*a^2*n^4*x*e^(m
*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*
m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 42*b*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(3*n)
*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
 22*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^(3*n)*e^(m
*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m
^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 8*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 48*a*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38
*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 22*c^
2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 24*a*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x

*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 8*a*b*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*x^n*e^(m*log(d)
 + m*log(x)) + 8*a*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x^n*e^
(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(
x)) + 48*a*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^n*e^(m*
log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 104*a*b*m
*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 38*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log
(x)) + 76*a*c*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^n*e^(m
*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^n*e^(m*log(d) + m*log(x))
 + 24*a*c*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^n*e^(m*log(d)
+ m*log(x)) + 6*c^2*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*a^2*m^3*x*e^(m*lo
g(d) + m*log(x)) + 8*a*b*m^3*x*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*e^(m*l
og(d) + m*log(x)) + 8*a*c*m^3*x*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*e^(m*
log(d) + m*log(x)) + 4*c^2*m^3*x*e^(m*log(d) + m*log(x)) + 30*a^2*m^2*n*x*e
^(m*log(d) + m*log(x)) + 54*a*b*m^2*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m^
2*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 42
*b*c*m^2*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*e^(m*log(d) + m*log(x
)) + 70*a^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 104*a*b*m*n^2*x*e^(m*log(d) +
 m*log(x)) + 38*b^2*m*n^2*x*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*e^(m*l
og(d) + m*log(x)) + 56*b*c*m*n^2*x*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x
*e^(m*log(d) + m*log(x)) + 50*a^2*n^3*x*e^(m*log(d) + m*log(x)) + 48*a*b*n^
3*x*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*e^(m*log(d) + m*log(x)) + 24*a*c
*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*e^(m*log(d) + m*log(x)) + 6*c
^2*n^3*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*x^(4*n)*e^(m*log(d) + m*log(
x)) + 18*c^2*m*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c
^2*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x^(3*n)*e^(m*log(d)
 + m*log(x)) + 18*c^2*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*
x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(
x)) + 6*b^2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 12*b*c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^
2*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*x^(2*n)*e^(m*log(d)
+ m*log(x)) + 48*a*c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*x
^(2*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 19*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c
^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 6*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*x^n*e^(m*
log(d) + m*log(x)) + 12*b*c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x
*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 2
4*b^2*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*x^n*e^(m*log(d) + m*
log(x)) + 42*b*c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*x^n*e^(m*
log(d) + m*log(x)) + 52*a*b*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*

x*x^n*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 6*a^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*b*m^2*x*e^(m*log(d) + m*log(x)) + 6*b^2*m^2*x*e^(m*log(d) + m*log(x)) + 12*a*c*m^2*x*e^(m*log(d) + m*log(x)) + 12*b*c*m^2*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^2*x*e^(m*log(d) + m*log(x)) + 30*a^2*m*n*x*e^(m*log(d) + m*log(x)) + 54*a*b*m*n*x*e^(m*log(d) + m*log(x)) + 24*b^2*m*n*x*e^(m*log(d) + m*log(x)) + 48*a*c*m*n*x*e^(m*log(d) + m*log(x)) + 42*b*c*m*n*x*e^(m*log(d) + m*log(x)) + 18*c^2*m*n*x*e^(m*log(d) + m*log(x)) + 35*a^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*n^2*x*e^(m*log(d) + m*log(x)) + 19*b^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*n^2*x*e^(m*log(d) + m*log(x)) + 28*b*c*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^2*n^2*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*x^n*e^(m*log(d) + m*log(x)) + 8*b^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*x^n*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 4*a^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*b*m*x*e^(m*log(d) + m*log(x)) + 4*b^2*m*x*e^(m*log(d) + m*log(x)) + 8*a*c*m*x*e^(m*log(d) + m*log(x)) + 8*b*c*m*x*e^(m*log(d) + m*log(x)) + 4*c^2*m*x*e^(m*log(d) + m*log(x)) + 10*a^2*n*x*e^(m*log(d) + m*log(x)) + 18*a*b*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*n*x*e^(m*log(d) + m*log(x)) + 14*b*c*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*n*x*e^(m*log(d) + m*log(x)) + c^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(3*n)*e^(m*log(d) + m*log(x)) + c^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*a*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(d) + m*log(x)) + b^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*c*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*x*x^n*e^(m*log(d) + m*log(x)) + c^2*x*x^n*e^(m*log(d) + m*log(x)) + a^2*x*e^(m*log(d) + m*log(x)) + 2*a*b*x*e^(m*log(d) + m*log(x)) + b^2*x*e^(m*log(d) + m*log(x)) + 2*a*c*x*e^(m*log(d) + m*log(x)) + 2*b*c*x*e^(m*log(d) + m*log(x)) + c^2*x*e^(m*log(d) + m*log(x)))/(m^5 + 10*m^4*n + 35*m^3*n^2 + 50*m^2*n^3 + 24*m*n^4 + 5*m^4 + 40*m^3*n + 105*m^2*n^2 + 100*m*n^3 + 24*n^4 + 10*m^3 + 60*m^2*n + 105*m*n^2 + 50*n^3 + 10*m^2 + 40*m*n + 35*n^2 + 5*m + 10*n + 1)

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 543, normalized size of antiderivative = 4.64

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2\,dx = \frac{a^2\,x\,(d\,x)^m}{m+1} + \frac{x\,x^{2\,n}\,(d\,x)^m\,(b^2+2\,a\,c)\,(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n^2+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n} + \frac{c^2\,x\,x^{4\,n}\,(d\,x)^m\,(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n} + \frac{2\,a\,b\,x\,x^n\,(d\,x)^m\,(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2+9\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n} + \frac{2\,b\,c\,x\,x^{3\,n}\,(d\,x)^m\,(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2+7\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] (a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16*
m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
+ 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(3*m
+ 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4
*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*
m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^n*(d*x)
^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3
 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6
*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*b*c*x*
x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14
*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

3.598 $\int (dx)^m (a + bx^n + cx^{2n})\, dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx = \frac{bx^{1+n}(dx)^m}{1+m+n} + \frac{cx^{1+2n}(dx)^m}{1+m+2n} + \frac{a(dx)^{1+m}}{d(1+m)}$$

```
[Out] b*x^(1+n)*(d*x)^m/(1+m+n)+c*x^(1+2*n)*(d*x)^m/(1+m+2*n)+a*(d*x)^(1+m)/d/(1+
m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 20, 30}

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx = \frac{a(dx)^{m+1}}{d(m+1)} + \frac{bx^{n+1}(dx)^m}{m+n+1} + \frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(dx)^m + bx^n(dx)^m + cx^{2n}(dx)^m\right)\,dx \\
&= \frac{a(dx)^{1+m}}{d(1+m)} + b\int x^n(dx)^m\,dx + c\int x^{2n}(dx)^m\,dx \\
&= \frac{a(dx)^{1+m}}{d(1+m)} + \left(bx^{-m}(dx)^m\right)\int x^{m+n}\,dx + \left(cx^{-m}(dx)^m\right)\int x^{m+2n}\,dx \\
&= \frac{bx^{1+n}(dx)^m}{1+m+n} + \frac{cx^{1+2n}(dx)^m}{1+m+2n} + \frac{a(dx)^{1+m}}{d(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx = x(dx)^m\left(\frac{a}{1+m} + x^n\left(\frac{b}{1+m+n} + \frac{cx^n}{1+m+2n}\right)\right)$$

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

method	result
risch	$\frac{x\left(c\,m^2x^{2n}+cmn\,x^{2n}+x^nb\,m^2+2mb\,x^nn+2x^{2n}cm+c\,x^{2n}n+a\,m^2+3amn+2a\,n^2+2mb\,x^n+2b\,x^nn+c\,x^{2n}+2am+3an+b\,x^n+\right.}{(1+m)(1+m+n)(1+m+2n)}$
parallelrisch	$\frac{x\,x^{2n}(dx)^mc+3x(dx)^man+x(dx)^ma+2x(dx)^ma\,n^2+x\,x^n(dx)^mb+2x\,x^n(dx)^mbmn+x\,x^{2n}(dx)^mcmn+x\,x^n(dx)^mb\,m^2+x\,x^{2}}{(1+m)(1+m+}$

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+x^n*b*m^2+2*m*b*x^n*n+2*m*c*(x^n)^2+c*(x^n)^
2*n+a*m^2+3*a*m*n+2*a*n^2+2*m*b*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a
)/(1+m)/(1+m+n)/(1+m+2*n)*d^m*x^m*exp(1/2*I*Pi*csgn(I*d*x)*m*(csgn(I*d*x)-c
sgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.45

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx = \frac{(cm^2 + 2\,cm + (cm + c)n + c)xx^{2\,n}e^{(m\log(d) + m\log(x))} + (bm^2 + 2\,bm + 2\,(bm + b)n + b)xx^n e^{(m\log(d) + m\log(x))}}{m^3 + 2\,(m+1)n^2 + 3\,m^2 + 3\,(m^2 + 2\,m + 1)n + 3}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m
^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*
a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m +
 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(49) = 98$.

Time = 3.36 (sec) , antiderivative size = 1096, normalized size of antiderivative = 18.90

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**n
/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((0**(-2*n - 1)*x, Eq(d,
0)), (Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(d*x), True))/d, Tru
e)) + b*Piecewise((-x*x**n*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(d*x)**(
-2*n - 1)*log(x), True)) + c*x*x**(2*n)*(d*x)**(-2*n - 1)*log(x), Eq(m, -2*
n - 1)), (a*Piecewise((0**(-n - 1)*x, Eq(d, 0)), (Piecewise((-1/(n*(d*x)**n
), Ne(n, 0)), (log(d*x), True))/d, True)) + b*x*x**n*(d*x)**(-n - 1)*log(x)
 + c*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x)*
*(-n - 1)*log(x), True)), Eq(m, -n - 1)), (a*m**2*x*(d*x)**m/(m**3 + 3*m**2
*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*m*n*x*(d*x)*
*m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) +
 2*a*m*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n*
```

```
*2 + 3*n + 1) + 2*a*n**2*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 +
6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*n*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*x*(d*x)**m/(m**3 + 3*m**
2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b*m**2*x*x**n*(
d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n +
 1) + 2*b*m*n*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
+ 3*m + 2*n**2 + 3*n + 1) + 2*b*m*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2
 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*n*x*x**n*(d*x)**m/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b*x*x**
n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*
n + 1) + c*m**2*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 +
6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*m*n*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*
n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*c*m*x*x**(2*n)*
(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n
+ 1) + c*n*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
 + 3*m + 2*n**2 + 3*n + 1) + c*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx = \frac{cd^m x e^{(m\log(x) + 2n\log(x))}}{m + 2n + 1} + \frac{bd^m x e^{(m\log(x) + n\log(x))}}{m + n + 1} + \frac{(dx)^{m+1} a}{d(m+1)}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*l
og(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 557, normalized size of antiderivative = 9.60

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right) dx$$
$$= \frac{cm^2 x x^{2n} e^{(m\log(d) + m\log(x))} + cmnx x^{2n} e^{(m\log(d) + m\log(x))} + bm^2 x x^n e^{(m\log(d) + m\log(x))} + cm^2 x x^n e^{(m\log(d) + m\log}}{}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m
```

```
*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d)
+ m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log
(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x))
+ 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a
*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x))
 + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)
) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2
*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*
e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(
d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d)
+ m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x))
 + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*
n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx=(d\,x)^m\left(\frac{a\,x}{m+1}+\frac{b\,x\,x^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{c\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] (d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 +
 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 +
1))
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m)*
hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1397, 371}

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1
 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(2c)\int \frac{(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.75

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = -\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{1+m}{n}}\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}\text{Hypergeometric2F1}\left(-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2c\left(1-2^{-\frac{1+m}{n}}\cdots\right.}{\cdots}\right.}{1+m}$$

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (
1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^((1
 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(b^2 -
```

```
4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -(
(1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1
+ m)/n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

Maple [F]

$$\int \frac{(dx)^m}{a + b\,x^n + c\,x^{2n}}dx$$

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx = \int \frac{(dx)^m}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx = \int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx$$

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx = \int \frac{(dx)^m}{cx^{2\,n} + bx^n + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d\,x)^m}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n)), x)
```

3.600 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 328

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2} dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} + \frac{c\left(\frac{4ac(1+m-2n)-b^2(1+m-n)}{\sqrt{b^2-4ac}}-b(1+m-n)\right)(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)d(1+m)n} - \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)+b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\right.}{a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)n}$$

```
[Out] (d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))+c*(d
*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
))*(-b*(1+m-n)+(4*a*c*(1+m-2*n)-b^2*(1+m-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+
b^2)/d/(1+m)/n/(b-(-4*a*c+b^2)^(1/2))-c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],
[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1+m-2*n)-b^2*(1+m-n)+b
*(1+m-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/n/(b+(-4*a*c+b^2)
^(1/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{1398, 1574, 371\}$

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$$
$$= \frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4ac(m-2n+1)-\left(b^2(m-n+1)\right)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\right.}{ad(m+1)n\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c
*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c]
- b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*d*(1 + m)*n) - (c*(4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2
 - 4*a*c]*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1398

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
```

```
[q, 0])
```

Rubi steps

$$\text{integral} = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-2n)+b^2(1+m-n)+bc(1+m-n)x^n\right)}{a+bx^n+cx^{2n}}\,dx}{a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \left(\frac{\left(bc(1+m-n)+\frac{c\left(b^2-4ac+b^2m-4acm-b^2n+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n} + \frac{\left(bc(1+m-n)-\frac{c\left(b^2-4ac+b^2m-4acm-b^2n+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\right) d}{a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{\left(c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4ac}(1+m-n)\right)\right)\int \frac{(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{a\left(b^2-4ac\right)^{3/2}n} - \frac{\left(c\left(4ac(1+m-2n)-b^2(1+m-n)+b\sqrt{b^2-4ac}(1+m-n)\right)\right)\int \frac{(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{a\left(b^2-4ac\right)^{3/2}n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\right.}{a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)n} - \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)+b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\right.}{a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1890 vs. 2(328) = 656.

Time = 4.41 (sec) , antiderivative size = 1890, normalized size of antiderivative = 5.76

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^2}\,dx$$

$$= \frac{x(dx)^m\left(\frac{2b^2c}{a\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)} - \frac{8c^2}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)} + \frac{2b^2c}{a\left(b^2-4ac-b\sqrt{b^2-4ac}\right)(1+m)} + \frac{8c^2}{\left(-b^2+4ac+b\sqrt{b^2-4ac}\right.}\right.}{}$$

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
[Out] (x*(d*x)^m*((2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
 - (8*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)) + (2*b^2*c)/
(a*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)) + (8*c^2)/((-b^2 + 4*a*c +
b*Sqrt[b^2 - 4*a*c])*(1 + m)) - (2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])*(1 + m)*n) + (4*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])
*(1 + m)*n) + (4*c^2)/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (2*
b^2*c)/(a*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) - (2*b^2*c*m)/(a*
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/(Sqrt[b^2
 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/((b^2 - 4*a*c - b*S
qrt[b^2 - 4*a*c])*(1 + m)*n) + (2*b^2*c*m)/(a*(-b^2 + 4*a*c + b*Sqrt[b^2 -
4*a*c])*(1 + m)*n) - b^2/(a*n*(a + x^n*(b + c*x^n))) + (2*c)/(n*(a + x^n*(b
 + c*x^n))) - (b*c*x^n)/(a*n*(a + x^n*(b + c*x^n))) + (c*(4*a*c*Sqrt[b^2 -
4*a*c]*(1 + m - 2*n) + 4*a*b*c*(1 + m - n) - b^2*Sqrt[b^2 - 4*a*c]*(1 + m -
 n) + b^3*(-1 - m + n))*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (
1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^((
1 + m)/n)*a*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + m)*
n*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (b*c*(-1 - m +
 n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, (b + Sqrt[
b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^((1 + m)/n)*a*Sqrt[b^2
 - 4*a*c]*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)
) - (2^((-1 - m + n)/n)*b^2*c*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n),
 (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]
)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*((c*x^n)/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (c^2*Hypergeometric2F1[-((1 + m)/n),
 -((1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^n)])/(2^((1 + m - 3*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*
c])*(1 + m)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (2^(
(-1 - m + n)/n)*b^2*c*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), (-1 - m
 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(a*Sqr
t[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n))^((1 + m)/n)) - (c^2*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^((1 + m - 2*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1
 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (2^((-1
- m + n)/n)*b^2*c*m*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), (-1 - m +
 n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(a*Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n))^((1 + m)/n)) - (c^2*m*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^((1 + m - 2*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1
 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(-b^2 +
4*a*c)
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^2}dx$$

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
+ a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
[Out] (b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2*
n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)
*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d^m*
(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2
*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2\,n}+bx^n+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2,x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2, x)
```

3.601 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 615

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3} dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)dn(a+bx^n+cx^{2n})^2}$$
$$-\frac{(dx)^{1+m}(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+\ldots}{2a^2(b^2-4ac)^2dn^2(a+bx^n+cx^{2n})}$$
$$-\frac{c\left(b\sqrt{b^2-4ac}(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)-b^4(1+m^2+m(2-3n)-3n+2n^2\ldots\right.}{2\ldots}$$
$$-\frac{c\left(b\sqrt{b^2-4ac}(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)+b^4(1+m^2+m(2-3n)-3n+2n^2\ldots\right.}{2\ldots}$$

```
[Out] 1/2*(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))^
2-1/2*(d*x)^(1+m)*(4*a^2*c^2*(1+m-4*n)-5*a*b^2*c*(1+m-3*n)+b^4*(1+m-2*n)-b*
c*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/d/n^2/(a+b*x^n+
c*x^(2*n))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*(-b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2*c*(1+m^2+m*
(2-4*n)-4*n+3*n^2)-8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b*(2*a*c*(2+2*m-7*
n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/d/(1+m
)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m
+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*
a*b^2*c*(1+m^2+m*(2-4*n)-4*n+3*n^2)+8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b
*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(5/2)/d/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 6.73 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1398, 1572, 1574, 371}

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx =$$

$$-\frac{c(dx)^{m+1}\left(-8a^2c^2(m^2+m(2-6n)+8n^2-6n+1)+6ab^2c(m^2+m(2-4n)+3n^2-4n+1)+b(m-\right.}{}$$

$$-\frac{c(dx)^{m+1}\left(8a^2c^2(m^2+m(2-6n)+8n^2-6n+1)-6ab^2c(m^2+m(2-4n)+3n^2-4n+1)+b(m-n\right.}{2a}$$

$$-\frac{(dx)^{m+1}\left(4a^2c^2(m-4n+1)-bcx^n(2ac(2m-7n+2)-b^2(m-2n+1))-5ab^2c(m-3n+1)+b^4(m\right.}{2a^2dn^2\left(b^2-4ac\right)^2(a+bx^n+cx^{2n})}$$

$$+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^n\right)}{2adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
 c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(1 + m
 - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n
))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt
[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
 4*n + 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)^(1 +
 m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n
^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(
1 + m - n) + b^4*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2
 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*
n^2))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1398

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[p + 1, 0]
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

integral

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(dx)^m\left(-2ac(1+m-4n)+b^2(1+m-2n)+bc(1+m-3n)x^n\right)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2} - \frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+\right.}{2a^2\left(b^2-4ac\right)^2dn^2\left(a+bx^n+cx^{2n}\right)} + \frac{\int \frac{(dx)^m\left(\left(2ac(1+m-4n)-b^2(1+m-2n)\right)\left(2ac(1+m-2n)-b^2(1+m-n)\right)-ab^2c(1+m)(1+m-3n)-bc\left(2ac(2+2m-7n)-b^2(1+m-2n)\right)(1+m\right.}{a+bx^n+cx^{2n}}}{2a^2\left(b^2-4ac\right)^2n^2}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2}$$
$$-\frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+m\right.}{2a^2\left(b^2-4ac\right)^2dn^2\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{\int\left(\frac{\left(-bc(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)+\frac{c\left(b^4-6ab^2c+8a^2c^2+2b^4m-12ab^2cm+16a^2c^2m+b^4m^2-6ab^2cm^2+8a^2c^2m^2-3b^4n+24ab^2cn-\right.}{\sqrt{b^2-4ac}}\right.}{b-\sqrt{b^2-4ac}+2cx^n}\right.}{}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2}$$
$$-\frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+m\right.}{2a^2\left(b^2-4ac\right)^2dn^2\left(a+bx^n+cx^{2n}\right)}$$
$$-\frac{\left(c\left(b(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)-\frac{b^4(1+m^2+m(2-3n)-3n+2n^2)-6ab^2c(1+m^2+m(2-4n)-4n\right.}{\sqrt{b^2-4ac}}\right.\right.}{2a^2\left(b^2-4ac\right)^2n^2}$$
$$-\frac{\left(c\left(b(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)+\frac{b^4(1+m^2+m(2-3n)-3n+2n^2)-6ab^2c(1+m^2+m(2-4n)-4n\right.}{\sqrt{b^2-4ac}}\right.\right.}{2a^2\left(b^2-4ac\right)^2n^2}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2}$$
$$-\frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-bc(2ac(2+2m-7n)-b^2(1+m\right.}{2a^2\left(b^2-4ac\right)^2dn^2\left(a+bx^n+cx^{2n}\right)}$$
$$-\frac{c\left(b(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)-\frac{b^4(1+m^2+m(2-3n)-3n+2n^2)-6ab^2c(1+m^2+m(2-4n)-4n+\right.}{\sqrt{b^2-4ac}}\right.}{2a^2\left(b^2-4ac\right)^2\left(b-\sqrt{b^2-4ac}\right)d(1-}$$
$$-\frac{c\left(b(2ac(2+2m-7n)-b^2(1+m-2n))(1+m-n)+\frac{b^4(1+m^2+m(2-3n)-3n+2n^2)-6ab^2c(1+m^2+m(2-4n)-4n+\right.}{\sqrt{b^2-4ac}}\right.}{2a^2\left(b^2-4ac\right)^2\left(b+\sqrt{b^2-4ac}\right)d(1-}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12289 vs. 2(615) = 1230.

Time = 7.24 (sec) , antiderivative size = 12289, normalized size of antiderivative = 19.98

$$\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^3}dx$$

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
 + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a
^2*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) - 4*a^3*c^2*
d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n + 2) - b^3*c^2*d^m*(m
- 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*b^2*c^2*d^m*(9*m - 29*n + 9) -
 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x) + 2
*n*log(x)) - (b^5*d^m*(m - 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*a^2*b
*c^2*d^m*n)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*
```

```
a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n
) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2
*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a
^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*(((m^2 - m*(3*n - 2)
 + 2*n^2 - 3*n + 1)*b^4*d^m - (5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a
*b^2*c*d^m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d^m)*x^m + (
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m*(9*n - 4) +
7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4
*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c
^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n),
x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3, x)
```

3.602 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,\mathrm{AppellF1}\left(\frac{1+m}{n},-\frac{3}{2},-\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1((1+m)/n,-3/2,-3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/
(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1399, 524}

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\mathrm{AppellF1}\left(\frac{m+1}{n},-\frac{3}{2},-\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/2
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
```

```
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int (dx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1+m}{n};-\frac{3}{2},-\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 545 vs. 2(161) = 322.

Time = 2.56 (sec) , antiderivative size = 545, normalized size of antiderivative = 3.39

$$\int (dx)^m\left(a+bx^n+cx^{2n}\right)^{3/2}dx = \frac{x(dx)^m\left(-6an^2(1+m+n)\left(b^2(1+m)-4ac(1+m+2n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\right.}{}$$

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
```

```
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2
, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(3*b^2*n^2 + 4*a*c*(1 + m^2 + 6*n +
8*n^2 + m*(2 + 6*n)) + 2*b*c*(2 + 4*m + 2*m^2 + 9*n + 9*m*n + 7*n^2)*x^n +
4*c^2*(1 + 2*m + m^2 + 3*n + 3*m*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n))
- 3*b*n^2*(b^2*(2 + 2*m + n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 +
m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
4*a*c])])))/(8*c*(1 + m)*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3*n)*Sqrt[a +
 x^n*(b + c*x^n)])
```

Maple [F]

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}} dx$$

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int (dx)^m \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int (d\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1+m}{n},-\frac{1}{2},-\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,-1/2,-1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/(1
+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1399, 524}

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{m+1}{n},-\frac{1}{2},-\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
(1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
```

```
- 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}F_1\left(\frac{1+m}{n};-\frac{1}{2},-\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(160) = 320.

Time = 0.71 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.42

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{x(dx)^m\left(2an(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2}}\right.\right.}{}$$

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
```

```
*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*
(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1 + m)*(1
+ m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int (dx)^{m} \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

[In] `int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)`

[Out] `int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)`

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

[In] `integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)

Sympy [F]

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \int (dx)^{m}\sqrt{a+bx^{n}+cx^{2n}}\,dx$$

[In] `integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)`

[Out] `Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)`

Maxima [F]

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2\,n}+bx^n+a}(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2\,n}+bx^n+a}(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \int (d\,x)^m\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,1/2,1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(a+b*x^n+c*x^(2*n))^(1/
2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1399, 524}

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-
```

```
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(
1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+x^n(b+cx^n)}}$$

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(d\,x)^m}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.605 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{3}{2},\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,3/2,3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(a+b*x^n+c*x^(2*n))^(
1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1399, 524}

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{3}{2},\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*d
*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{3}{2},\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 1.45 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x(dx)^m\left((-4ac(1+m-n)+b^2(2+2m-n))(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{b-\cdots}\right.}{}$$

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1
 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 + m)*x^
n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)
/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m + n)*Sqrt
[a + x^n*(b + c*x^n)])
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)

[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)

[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.606 $\int (dx)^m (a + bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m (a+bx^n+cx^{2n})^p \, dx = \frac{(dx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^n+cx^{2n})^p \text{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\ldots\right)}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int (dx)^m (a+bx^n+cx^{2n})^p \, dx = \frac{(dx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} (a+bx^n+cx^{2n})^p \text{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\ldots\right)}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (dx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$=\frac{(dx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4}}\right.}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$=\frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\,\text{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx}{b+\sqrt{b^2}}\right.}{1+m}$$

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+bx^n+a\right)^p (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (d\,x)^m \left(a+b\,x^n+c\,x^{2\,n}\right)^p \, dx$$

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal result

Integrand size = 28, antiderivative size = 46

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx = \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[Out] 1/4*a*(e*x+d)^4/e+1/6*b*(e*x+d)^6/e+1/8*c*(e*x+d)^8/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1156, 14}

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx = \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^3(a+bx^2+cx^4)\,dx,x,d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int (ax^3+bx^5+cx^7)\,dx,x,d+ex\right)}{e}\\ &= \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.26

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx = d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2 + \frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3 + \frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4 + d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6 + cde^6x^7+\frac{1}{8}ce^7x^8$$

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3
*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/
4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c
*e^7*x^8)/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(40) = 80.

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

method	result
norman	$\frac{e^7 c\,x^8}{8}+d\,e^6 c\,x^7+\left(\frac{7}{2}d^2 e^5 c+\frac{1}{6}b\,e^5\right)x^6+\left(7d^3 c\,e^4+bd\,e^4\right)x^5+\left(\frac{35}{4}d^4 c\,e^3+\frac{5}{2}b\,d^2 e^3+\frac{1}{4}a\,e^3\right)x$
gosper	$\frac{x\left(3e^7 c\,x^7+24d\,e^6 c\,x^6+84x^5 d^2 e^5 c+168c\,d^3 e^4 x^4+4x^5 b\,e^5+210x^3 d^4 c\,e^3+24bd\,e^4 x^4+168x^2 c\,d^5 e^2+60x^3 b\,d^2 e^3+84xc\,d^6 e+80x^2\right.}{24}$
risch	$\frac{1}{8}e^7 c\,x^8+d\,e^6 c\,x^7+\frac{7}{2}x^6 d^2 e^5 c+\frac{1}{6}x^6 b\,e^5+7c\,d^3 e^4 x^5+bd\,e^4 x^5+\frac{35}{4}x^4 d^4 c\,e^3+\frac{5}{2}x^4 b\,d^2 e^3+\frac{1}{4}x$
parallelrisch	$\frac{1}{8}e^7 c\,x^8+d\,e^6 c\,x^7+\frac{7}{2}x^6 d^2 e^5 c+\frac{1}{6}x^6 b\,e^5+7c\,d^3 e^4 x^5+bd\,e^4 x^5+\frac{35}{4}x^4 d^4 c\,e^3+\frac{5}{2}x^4 b\,d^2 e^3+\frac{1}{4}x$
default	$\frac{e^7 c\,x^8}{8}+d\,e^6 c\,x^7+\frac{\left(15d^2 e^5 c+e^3\left(6c\,d^2 e^2+b\,e^2\right)\right)x^6}{6}+\frac{\left(13d^3 c\,e^4+3d\,e^2\left(6c\,d^2 e^2+b\,e^2\right)+e^3\left(4d^3 ec+2bde\right)\right)x^5}{5}+\frac{(4d^4 c\,e}{}$

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+(7/2*d^2*e^5*c+1/6*b*e^5)*x^6+(7*c*d^3*e^4+b*d*e^
4)*x^5+(35/4*d^4*c*e^3+5/2*b*d^2*e^3+1/4*a*e^3)*x^4+(7*c*d^5*e^2+10/3*b*d^3
*e^2+d*e^2*a)*x^3+(7/2*c*d^6*e+5/2*b*d^4*e+3/2*a*d^2*e)*x^2+(c*d^7+b*d^5+a*
d^3)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{1}{8}ce^7x^8+cde^6x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5x^6+\left(7\,cd^3+bd\right)e^4x^5+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3x^4$$
$$+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2x^3+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ex^2+\left(cd^7+bd^5+ad^3\right)x$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*
e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5
 + a*d^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.87

$$\int (d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=cde^6x^7+\frac{ce^7x^8}{8}+x^6\left(\frac{be^5}{6}+\frac{7cd^2e^5}{2}\right)+x^5\left(bde^4+7cd^3e^4\right)+x^4\left(\frac{ae^3}{4}+\frac{5bd^2e^3}{2}+\frac{35cd^4e^3}{4}\right)$$
$$+x^3\left(ade^2+\frac{10bd^3e^2}{3}+7cd^5e^2\right)+x^2\cdot\left(\frac{3ad^2e}{2}+\frac{5bd^4e}{2}+\frac{7cd^6e}{2}\right)+x\left(ad^3+bd^5+cd^7\right)$$

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b
*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**
3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*
e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{1}{8} ce^7x^8 + cde^6x^7 + \frac{1}{6} \left(21 cd^2+b\right)e^5x^6 + \left(7 cd^3+bd\right)e^4x^5 + \frac{1}{4} \left(35 cd^4+10 bd^2+a\right)e^3x^4 + \frac{1}{3} \left(21 cd^5+10 bd^3+3 ad\right)e^2x^3 + \frac{1}{2} \left(7 cd^6+5 bd^4+3 ad^2\right)ex^2 + \left(cd^7+bd^5+ad^3\right)x$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*
e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5
 + a*d^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.48

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{1}{2} \left(ex^2+2 dx\right)cd^6 + \frac{3}{4} \left(ex^2+2 dx\right)^2cd^4e + \frac{1}{2} \left(ex^2+2 dx\right)^3cd^2e^2 + \frac{1}{8} \left(ex^2+2 dx\right)^4ce^3 + \frac{1}{2} \left(ex^2+2 dx\right)bd^4 + \frac{1}{2} \left(ex^2+2 dx\right)^2bd^2e + \frac{1}{6} \left(ex^2+2 dx\right)^3be^2 + \frac{1}{2} \left(ex^2+2 dx\right)ad^2 + \frac{1}{4} \left(ex^2+2 dx\right)^2ae$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*c*d^6 + 3/4*(e*x^2 + 2*d*x)^2*c*d^4*e + 1/2*(e*x^2 + 2*
d*x)^3*c*d^2*e^2 + 1/8*(e*x^2 + 2*d*x)^4*c*e^3 + 1/2*(e*x^2 + 2*d*x)*b*d^4
+ 1/2*(e*x^2 + 2*d*x)^2*b*d^2*e + 1/6*(e*x^2 + 2*d*x)^3*b*e^2 + 1/2*(e*x^2
+ 2*d*x)*a*d^2 + 1/4*(e*x^2 + 2*d*x)^2*a*e
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx = x \left(c\,d^7+b\,d^5+a\,d^3\right)+\frac{e^5\,x^6\,\left(21\,c\,d^2+b\right)}{6} + \frac{c\,e^7\,x^8}{8}+\frac{e^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4} + \frac{d^2\,e\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2} + \frac{d\,e^2\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3} + d\,e^4\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,x^7$$

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (e
^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4))
/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2) +
c*d*e^6*x^7
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 89

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{(b^2+2ac)\,(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*(e*x+d)^4/e+1/3*a*b*(e*x+d)^6/e+1/8*(2*a*c+b^2)*(e*x+d)^8/e+1/5*b*c
*(e*x+d)^10/e+1/12*c^2*(e*x+d)^12/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1156, 1128, 645}

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2(d+ex)^4}{4e}+\frac{(2ac+b^2)\,(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

<u>Rule 645</u>

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
```

```
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^3(a+bx^2+cx^4)^2\,dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int x(a+bx+cx^2)^2\,dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+(b^2+2ac)\,x^3+2bcx^4+c^2x^5\right)\,dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{(b^2+2ac)\,(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(89) = 178$.

Time = 0.08 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.51

$$\begin{aligned}
&\int (d+ex)^3\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx \\
&= d^3\left(a+bd^2+cd^4\right)^2x+\frac{1}{2}d^2\left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right)ex^2 \\
&+\frac{1}{3}d\left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right)e^2x^3 \\
&+\frac{1}{4}\left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right)e^3x^4 \\
&+\frac{1}{5}d\left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right)e^4x^5 \\
&+\frac{1}{6}\left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right)e^5x^6 \\
&+d\left(b^2+2ac+24bcd^2+66c^2d^4\right)e^6x^7+\frac{1}{8}\left(b^2+2ac+72bcd^2+330c^2d^4\right)e^7x^8 \\
&+\frac{1}{3}cd\left(6b+55cd^2\right)e^8x^9+\frac{1}{10}c\left(2b+55cd^2\right)e^9x^{10}+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}
\end{aligned}$$

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c
*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*
d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2
 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10
*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((
2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d
^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(2*b +
 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(79) = 158$.

Time = 0.63 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.55

method	result
norman	$\frac{e^{11}c^2x^{12}}{12} + d\,e^{10}c^2x^{11} + \left(\frac{11}{2}d^2e^9c^2 + \frac{1}{5}bc\,e^9\right)x^{10} + \left(\frac{55}{3}d^3c^2e^8 + 2bcd\,e^8\right)x^9 + \left(\frac{165}{4}c^2d^4e^7 + 9bc\,d^2\right.$
gosper	$\frac{x(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3c^2e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080x^7bc}{}$
risch	$24bc\,d^3e^6x^7 + 2x^9bcd\,e^8 + 9x^8bc\,d^2e^7 + 42x^6bc\,d^4e^5 + 7x^6ac\,d^2e^5 + \frac{11}{2}x^{10}d^2e^9c^2 + \frac{1}{5}x^{10}bc\,e^9 +$
parallelrisch	$24bc\,d^3e^6x^7 + 2x^9bcd\,e^8 + 9x^8bc\,d^2e^7 + 42x^6bc\,d^4e^5 + 7x^6ac\,d^2e^5 + \frac{11}{2}x^{10}d^2e^9c^2 + \frac{1}{5}x^{10}bc\,e^9 +$
default	Expression too large to display

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+(11/2*d^2*e^9*c^2+1/5*b*c*e^9)*x^10+(55/
3*d^3*c^2*e^8+2*b*c*d*e^8)*x^9+(165/4*c^2*d^4*e^7+9*b*c*d^2*e^7+1/4*a*c*e^7
+1/8*b^2*e^7)*x^8+(66*c^2*d^5*e^6+24*b*c*d^3*e^6+2*a*c*d*e^6+b^2*d*e^6)*x^7
+(77*c^2*d^6*e^5+42*b*c*d^4*e^5+7*a*c*d^2*e^5+7/2*b^2*d^2*e^5+1/3*a*b*e^5)*
x^6+(66*c^2*d^7*e^4+252/5*b*c*d^5*e^4+14*a*c*d^3*e^4+7*b^2*d^3*e^4+2*a*b*d*
e^4)*x^5+(165/4*c^2*d^8*e^3+42*b*c*d^6*e^3+35/2*a*c*d^4*e^3+35/4*b^2*d^4*e^
3+5*e^3*a*b*d^2+1/4*e^3*a^2)*x^4+(55/3*c^2*d^9*e^2+24*b*c*d^7*e^2+14*a*c*d^
5*e^2+7*b^2*d^5*e^2+20/3*a*b*d^3*e^2+d*e^2*a^2)*x^3+(11/2*c^2*d^10*e+9*b*c*
d^8*e+7*a*c*d^6*e+7/2*b^2*d^6*e+5*a*b*d^4*e+3/2*a^2*d^2*e)*x^2+(c^2*d^11+2*
b*c*d^9+2*a*c*d^7+b^2*d^7+2*a*b*d^5+a^2*d^3)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(79) = 158.

Time = 0.25 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.53

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx \\
&= \frac{1}{12}c^2e^{11}x^{12}+c^2de^{10}x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9x^{10}+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8x^9 \\
&+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7x^8+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6x^7 \\
&+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5x^6 \\
&+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4x^5 \\
&+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3x^4 \\
&+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2x^3 \\
&+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ex^2 \\
&+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)x
\end{aligned}$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x
^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(71) = 142.

Time = 0.07 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.28

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$$
$$= c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9 \cdot \left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right)$$
$$+ x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7 \cdot \left(2acde^6 + b^2de^6 + 24bcd^3e^6 + 66c^2d^5e^6\right)$$
$$+ x^6\left(\frac{abe^5}{3} + 7acd^2e^5 + \frac{7b^2d^2e^5}{2} + 42bcd^4e^5 + 77c^2d^6e^5\right) + x^5$$
$$\cdot \left(2abde^4 + 14acd^3e^4 + 7b^2d^3e^4 + \frac{252bcd^5e^4}{5} + 66c^2d^7e^4\right)$$
$$+ x^4\left(\frac{a^2e^3}{4} + 5abd^2e^3 + \frac{35acd^4e^3}{2} + \frac{35b^2d^4e^3}{4} + 42bcd^6e^3 + \frac{165c^2d^8e^3}{4}\right)$$
$$+ x^3\left(a^2de^2 + \frac{20abd^3e^2}{3} + 14acd^5e^2 + 7b^2d^5e^2 + 24bcd^7e^2 + \frac{55c^2d^9e^2}{3}\right)$$
$$+ x^2 \cdot \left(\frac{3a^2d^2e}{2} + 5abd^4e + 7acd^6e + \frac{7b^2d^6e}{2} + 9bcd^8e + \frac{11c^2d^{10}e}{2}\right)$$
$$+ x\left(a^2d^3 + 2abd^5 + 2acd^7 + b^2d^7 + 2bcd^9 + c^2d^{11}\right)$$

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2
*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b
**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 +
 b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7
*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5)
 + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*
e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*
d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4
) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5
*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5
*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10
*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c
**2*d**11)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(79) = 158$.

Time = 0.21 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.53

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx = \frac{1}{12}c^2e^{11}x^{12} + c^2de^{10}x^{11} + \frac{1}{10}\left(55c^2d^2+2bc\right)e^9x^{10} + \frac{1}{3}\left(55c^2d^3+6bcd\right)e^8x^9 + \frac{1}{8}\left(330c^2d^4+72bcd^2+b^2+2ac\right)e^7x^8 + \left(66c^2d^5+24bcd^3+\left(b^2+2ac\right)d\right)e^6x^7 + \frac{1}{6}\left(462c^2d^6+252bcd^4+21\left(b^2+2ac\right)d^2+2ab\right)e^5x^6 + \frac{1}{5}\left(330c^2d^7+252bcd^5+35\left(b^2+2ac\right)d^3+10abd\right)e^4x^5 + \frac{1}{4}\left(165c^2d^8+168bcd^6+35\left(b^2+2ac\right)d^4+20abd^2+a^2\right)e^3x^4 + \frac{1}{3}\left(55c^2d^9+72bcd^7+21\left(b^2+2ac\right)d^5+20abd^3+3a^2d\right)e^2x^3 + \frac{1}{2}\left(11c^2d^{10}+18bcd^8+7\left(b^2+2ac\right)d^6+10abd^4+3a^2d^2\right)ex^2 + \left(c^2d^{11}+2bcd^9+\left(b^2+2ac\right)d^7+2abd^5+a^2d^3\right)x$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x
^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.34

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$$
$$= \frac{1}{2} \left(ex^2+2\,dx\right)c^2d^{10} + \frac{5}{4} \left(ex^2+2\,dx\right)^2c^2d^8e + \frac{5}{3} \left(ex^2+2\,dx\right)^3c^2d^6e^2 + \frac{5}{4} \left(ex^2+2\,dx\right)^4c^2d^4e^3 + \frac{1}{2} \left(ex^2+2\,dx\right)^5c^2d^2e^4 + \frac{1}{12} \left(ex^2+2\,dx\right)^6c^2e^5 + \left(ex^2+2\,dx\right)bcd^8 + 2\left(ex^2+2\,dx\right)^2bcd^6e + 2\left(ex^2+2\,dx\right)^3bcd^4e^2 + \left(ex^2+2\,dx\right)^4bcd^2e^3 + \frac{1}{5} \left(ex^2+2\,dx\right)^5bce^4 + \frac{1}{2} \left(ex^2+2\,dx\right)b^2d^6 + \left(ex^2+2\,dx\right)acd^6 + \frac{3}{4} \left(ex^2+2\,dx\right)^2b^2d^4e + \frac{3}{2} \left(ex^2+2\,dx\right)^2acd^4e + \frac{1}{2} \left(ex^2+2\,dx\right)^3b^2d^2e^2 + \left(ex^2+2\,dx\right)^3acd^2e^2 + \frac{1}{8} \left(ex^2+2\,dx\right)^4b^2e^3 + \frac{1}{4} \left(ex^2+2\,dx\right)^4ace^3 + \left(ex^2+2\,dx\right)abd^4 + \left(ex^2+2\,dx\right)^2abd^2e + \frac{1}{3} \left(ex^2+2\,dx\right)^3abe^2 + \frac{1}{2} \left(ex^2+2\,dx\right)a^2d^2 + \frac{1}{4} \left(ex^2+2\,dx\right)^2a^2e$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*c^2*d^10 + 5/4*(e*x^2 + 2*d*x)^2*c^2*d^8*e + 5/3*(e*x^2
 + 2*d*x)^3*c^2*d^6*e^2 + 5/4*(e*x^2 + 2*d*x)^4*c^2*d^4*e^3 + 1/2*(e*x^2 +
2*d*x)^5*c^2*d^2*e^4 + 1/12*(e*x^2 + 2*d*x)^6*c^2*e^5 + (e*x^2 + 2*d*x)*b*c
*d^8 + 2*(e*x^2 + 2*d*x)^2*b*c*d^6*e + 2*(e*x^2 + 2*d*x)^3*b*c*d^4*e^2 + (e
*x^2 + 2*d*x)^4*b*c*d^2*e^3 + 1/5*(e*x^2 + 2*d*x)^5*b*c*e^4 + 1/2*(e*x^2 +
2*d*x)*b^2*d^6 + (e*x^2 + 2*d*x)*a*c*d^6 + 3/4*(e*x^2 + 2*d*x)^2*b^2*d^4*e
+ 3/2*(e*x^2 + 2*d*x)^2*a*c*d^4*e + 1/2*(e*x^2 + 2*d*x)^3*b^2*d^2*e^2 + (e*
x^2 + 2*d*x)^3*a*c*d^2*e^2 + 1/8*(e*x^2 + 2*d*x)^4*b^2*e^3 + 1/4*(e*x^2 + 2
*d*x)^4*a*c*e^3 + (e*x^2 + 2*d*x)*a*b*d^4 + (e*x^2 + 2*d*x)^2*a*b*d^2*e + 1
/3*(e*x^2 + 2*d*x)^3*a*b*e^2 + 1/2*(e*x^2 + 2*d*x)*a^2*d^2 + 1/4*(e*x^2 + 2
*d*x)^2*a^2*e
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.30

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{e^7\,x^8\,\left(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c\right)}{8} + \frac{e^5\,x^6\,\left(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2\right)}{6} + \frac{e^3\,x^4\,\left(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8\right)}{4} + \frac{c^2\,e^{11}\,x^{12}}{12} + d^3\,x\,\left(c\,d^4+b\,d^2+a\right)^2 + \frac{c\,e^9\,x^{10}\,\left(55\,c\,d^2+2\,b\right)}{10} + c^2\,d\,e^{10}\,x^{11} + \frac{d^2\,e\,x^2\,\left(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2\,d^4+18\,b\,c\,d^6+11\,c^2\,d^8\right)}{2} + \frac{d\,e^2\,x^3\,\left(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2\,d^4+72\,b\,c\,d^6+55\,c^2\,d^8\right)}{3} + d\,e^6\,x^7\,\left(b^2+24\,b\,c\,d^2+66\,c^2\,d^4+2\,a\,c\right) + \frac{d\,e^4\,x^5\,\left(35\,b^2\,d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2\,d^6+70\,a\,c\,d^2\right)}{5} + \frac{c\,d\,e^8\,x^9\,\left(55\,c\,d^2+6\,b\right)}{3}$$

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 21
*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 35*
b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2*e^
11*x^12)/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^10*(2*b + 55*c*d^2))/1
0 + c^2*d*e^10*x^11 + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d
^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*
d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c + b^2 + 6
6*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 7
0*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 138

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,(d+ex)^{12}}{4e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

```
[Out] 1/4*a^3*(e*x+d)^4/e+1/2*a^2*b*(e*x+d)^6/e+3/8*a*(a*c+b^2)*(e*x+d)^8/e+1/10*
b*(6*a*c+b^2)*(e*x+d)^10/e+1/4*c*(a*c+b^2)*(e*x+d)^12/e+3/14*b*c^2*(e*x+d)^
14/e+1/16*c^3*(e*x+d)^16/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1156, 1128, 645}

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c(ac+b^2)\,(d+ex)^{12}}{4e}+\frac{b(6ac+b^2)\,(d+ex)^{10}}{10e}+\frac{3a(ac+b^2)\,(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
```

```
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right)^3\,dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int x\left(a+bx+cx^2\right)^3\,dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a\left(b^2+ac\right)x^3+b\left(b^2+6ac\right)x^4+3c\left(b^2+ac\right)x^5+3bc^2x^6+c^3x^7\right)\,dx,x,\right.}{2e}\\
&= \frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a\left(b^2+ac\right)(d+ex)^8}{8e}+\frac{b\left(b^2+6ac\right)(d+ex)^{10}}{10e}\\
&\quad+\frac{c\left(b^2+ac\right)(d+ex)^{12}}{4e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 797 vs. 2(138) = 276.

Time = 0.20 (sec) , antiderivative size = 797, normalized size of antiderivative = 5.78

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$
$$= d^3\left(a+bd^2+cd^4\right)^3 x+\frac{3}{2}d^2\left(a+bd^2+cd^4\right)^2\left(a+3bd^2+5cd^4\right)ex^2+d\left(a^3+10a^2bd^2+21ab^2d^4+21a^2cd^4+12b^3d^6+72abcd^6+55b^2cd^8+55ac^2d^8+78bc^2d^{10}+35c^3d^{12}\right)e^2x^3+\frac{1}{4}\left(a^3+30a^2bd^2+105ab^2d^4+105a^2cd^4+84b^3d^6+504abcd^6+495b^2cd^8+495ac^2d^8+858bc^2d^{10}+455c^3d^{12}\right)e^3x^4+\frac{3}{5}d\left(5a^2b+35ab^2d^2+35a^2cd^2+42b^3d^4+252abcd^4+330b^2cd^6+330ac^2d^6+715bc^2d^8+455c^3d^{10}\right)e^4x^5+\frac{1}{2}\left(a^2b+21ab^2d^2+21a^2cd^2+42b^3d^4+252abcd^4+462b^2cd^6+462ac^2d^6+1287bc^2d^8+1001c^3d^{10}\right)e^5x^6+\frac{1}{7}d\left(21ab^2+21a^2c+84b^3d^2+504abcd^2+1386b^2cd^4+1386ac^2d^4+5148bc^2d^6+5005c^3d^8\right)e^6x^7+\frac{3}{8}\left(ab^2+a^2c+12b^3d^2+72abcd^2+330b^2cd^4+330ac^2d^4+1716bc^2d^6+2145c^3d^8\right)e^7x^8+d\left(b^3+6abc+55b^2cd^2+55ac^2d^2+429bc^2d^4+715c^3d^6\right)e^8x^9+\frac{1}{10}\left(b^3+6abc+165b^2cd^2+165ac^2d^2+2145bc^2d^4+5005c^3d^6\right)e^9x^{10}+3cd\left(b^2+ac+26bcd^2+91c^2d^4\right)e^{10}x^{11}+\frac{1}{4}c\left(b^2+ac+78bcd^2+455c^2d^4\right)e^{11}x^{12}+c^2d\left(3b+35cd^2\right)e^{12}x^{13}+\frac{3}{14}c^2\left(b+35cd^2\right)e^{13}x^{14}+c^3de^{14}x^{15}+\frac{1}{16}c^3e^{15}x^{16}$$

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5
*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12
*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*
c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 +
84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10
 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8
 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^
3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 10
01*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*
d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*
x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 33
0*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^
3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6
```

```
)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c
*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^
2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3
*e^15*x^16)/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(124) = 248.

Time = 0.64 (sec) , antiderivative size = 1130, normalized size of antiderivative = 8.19

method	result	size
norman	Expression too large to display	1130
gosper	Expression too large to display	1315
risch	Expression too large to display	1336
parallelrisch	Expression too large to display	1336
default	Expression too large to display	7550

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c^3*d^15+3*b*c^2*d^13+3*a*c^2*d^11+3*b^2*c*d^11+6*a*b*c*d^9+b^3*d^9+3*a^2*
c*d^7+3*a*b^2*d^7+3*a^2*b*d^5+a^3*d^3)*x+(15/2*c^3*d^14*e+39/2*b*c^2*d^12*e
+33/2*a*c^2*d^10*e+33/2*b^2*c*d^10*e+27*a*b*c*d^8*e+9/2*b^3*d^8*e+21/2*a^2*
c*d^6*e+21/2*a*b^2*d^6*e+15/2*a^2*b*d^4*e+3/2*a^3*d^2*e)*x^2+(35*c^3*d^13*e
^2+78*b*c^2*d^11*e^2+55*a*c^2*d^9*e^2+55*b^2*c*d^9*e^2+72*a*b*c*d^7*e^2+12*
b^3*d^7*e^2+21*a^2*c*d^5*e^2+21*a*b^2*d^5*e^2+10*a^2*b*d^3*e^2+a^3*d*e^2)*x
^3+(455/4*c^3*d^12*e^3+429/2*b*c^2*d^10*e^3+495/4*a*c^2*d^8*e^3+495/4*b^2*c
*d^8*e^3+126*a*b*c*d^6*e^3+21*b^3*d^6*e^3+105/4*a^2*c*d^4*e^3+105/4*a*b^2*d
^4*e^3+15/2*a^2*b*d^2*e^3+1/4*a^3*e^3)*x^4+(273*c^3*d^11*e^4+429*b*c^2*d^9*
e^4+198*a*c^2*d^7*e^4+198*b^2*c*d^7*e^4+756/5*a*b*c*d^5*e^4+126/5*b^3*d^5*e
^4+21*a^2*c*d^3*e^4+21*a*b^2*d^3*e^4+3*a^2*b*d*e^4)*x^5+(1001/2*c^3*d^10*e^
5+1287/2*b*c^2*d^8*e^5+231*a*c^2*d^6*e^5+231*b^2*c*d^6*e^5+126*a*b*c*d^4*e^
5+21*b^3*d^4*e^5+21/2*a^2*c*d^2*e^5+21/2*a*b^2*d^2*e^5+1/2*a^2*b*e^5)*x^6+(
715*c^3*d^9*e^6+5148/7*b*c^2*d^7*e^6+198*a*c^2*d^5*e^6+198*b^2*c*d^5*e^6+72
*a*b*c*d^3*e^6+12*b^3*d^3*e^6+3*a^2*c*d*e^6+3*a*b^2*d*e^6)*x^7+(6435/8*c^3*
d^8*e^7+1287/2*b*c^2*d^6*e^7+495/4*a*c^2*d^4*e^7+495/4*b^2*c*d^4*e^7+27*a*b
*c*d^2*e^7+9/2*b^3*d^2*e^7+3/8*a^2*c*e^7+3/8*a*b^2*e^7)*x^8+(715*c^3*d^7*e^
8+429*b*c^2*d^5*e^8+55*a*c^2*d^3*e^8+55*b^2*c*d^3*e^8+6*a*b*c*d*e^8+b^3*d*e
^8)*x^9+(1001/2*c^3*d^6*e^9+429/2*b*c^2*d^4*e^9+33/2*a*c^2*d^2*e^9+33/2*b^2
*c*d^2*e^9+3/5*a*b*c*e^9+1/10*b^3*e^9)*x^10+(273*c^3*d^5*e^10+78*b*c^2*d^3*
e^10+3*a*c^2*d*e^10+3*b^2*c*d*e^10)*x^11+(455/4*d^4*c^3*e^11+39/2*b*c^2*d^2
*e^11+1/4*a*c^2*e^11+1/4*b^2*c*e^11)*x^12+(35*c^3*d^3*e^12+3*b*c^2*d*e^12)*
x^13+(15/2*d^2*e^13*c^3+3/14*b*c^2*e^13)*x^14+d*e^14*c^3*x^15+1/16*e^15*c^3
*x^16
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(124) = 248$.

Time = 0.26 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.32

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$
$$= \frac{1}{16}c^3e^{15}x^{16} + c^3de^{14}x^{15} + \frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}x^{14} + \left(35\,c^3d^3+3\,bc^2d\right)e^{12}x^{13}$$
$$+ \frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}x^{12} + 3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}x^{11}$$
$$+ \frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9x^{10}$$
$$+ \left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8x^9$$
$$+ \frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7x^8$$
$$+ \frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6x^7$$
$$+ \frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5x^6$$
$$+ \frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4x^5$$
$$+ \frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right)e^{3}$$
$$+ \left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2x^3$$
$$+ \frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ex^2$$
$$+ \left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)x$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
 a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
```

```
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 1314, normalized size of antiderivative = 9.52

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c*
*3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*(a
*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*e**
11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 +
 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**
3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6
*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 + 55*b*
*2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*(3*a**2*
c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*e**7/4 +
9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e**7/2 + 643
5*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d*
*3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 +
 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 + 231*a*c**
2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 1287*b*c**2*d**8*e
**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 21*a**2*c*d**3*e*
*4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 1
26*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**
3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4
*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d**8*e*
*3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c**2*d**10*e**3/2
 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10*a**2*b*d**3*e**2 + 21*a*
*2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 72*a*b*c*d**7*e**2 + 55*a*c**2*d**9*
e**2 + 12*b**3*d**7*e**2 + 55*b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*
c**3*d**13*e**2) + x**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d
**6*e/2 + 21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3*d**14*e/2
) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*b**2*d**7 + 6*a*b*c*
```

```
d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*d**11 + 3*b*c**2*d**13 + c**3*
d**15)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(124) = 248$.

Time = 0.22 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.32

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx \\
&= \frac{1}{16}\,c^3e^{15}x^{16} + c^3de^{14}x^{15} + \frac{3}{14}\left(35\,c^3d^2 + bc^2\right)e^{13}x^{14} + \left(35\,c^3d^3 + 3\,bc^2d\right)e^{12}x^{13} \\
&\quad + \frac{1}{4}\left(455\,c^3d^4 + 78\,bc^2d^2 + b^2c + ac^2\right)e^{11}x^{12} + 3\left(91\,c^3d^5 + 26\,bc^2d^3 + \left(b^2c + ac^2\right)d\right)e^{10}x^{11} \\
&\quad + \frac{1}{10}\left(5005\,c^3d^6 + 2145\,bc^2d^4 + b^3 + 6\,abc + 165\left(b^2c + ac^2\right)d^2\right)e^9x^{10} \\
&\quad + \left(715\,c^3d^7 + 429\,bc^2d^5 + 55\left(b^2c + ac^2\right)d^3 + \left(b^3 + 6\,abc\right)d\right)e^8x^9 \\
&\quad + \frac{3}{8}\left(2145\,c^3d^8 + 1716\,bc^2d^6 + 330\left(b^2c + ac^2\right)d^4 + ab^2 + a^2c + 12\left(b^3 + 6\,abc\right)d^2\right)e^7x^8 \\
&\quad + \frac{1}{7}\left(5005\,c^3d^9 + 5148\,bc^2d^7 + 1386\left(b^2c + ac^2\right)d^5 + 84\left(b^3 + 6\,abc\right)d^3 + 21\left(ab^2 + a^2c\right)d\right)e^6x^7 \\
&\quad + \frac{1}{2}\left(1001\,c^3d^{10} + 1287\,bc^2d^8 + 462\left(b^2c + ac^2\right)d^6 + 42\left(b^3 + 6\,abc\right)d^4 + a^2b + 21\left(ab^2 + a^2c\right)d^2\right)e^5x^6 \\
&\quad + \frac{3}{5}\left(455\,c^3d^{11} + 715\,bc^2d^9 + 330\left(b^2c + ac^2\right)d^7 + 42\left(b^3 + 6\,abc\right)d^5 + 5\,a^2bd + 35\left(ab^2 + a^2c\right)d^3\right)e^4x^5 \\
&\quad + \frac{1}{4}\left(455\,c^3d^{12} + 858\,bc^2d^{10} + 495\left(b^2c + ac^2\right)d^8 + 84\left(b^3 + 6\,abc\right)d^6 + 30\,a^2bd^2 + 105\left(ab^2 + a^2c\right)d^4 + a^3\right)e^{3} \\
&\quad + \left(35\,c^3d^{13} + 78\,bc^2d^{11} + 55\left(b^2c + ac^2\right)d^9 + 12\left(b^3 + 6\,abc\right)d^7 + 10\,a^2bd^3 + 21\left(ab^2 + a^2c\right)d^5 + a^3d\right)e^2x^3 \\
&\quad + \frac{3}{2}\left(5\,c^3d^{14} + 13\,bc^2d^{12} + 11\left(b^2c + ac^2\right)d^{10} + 3\left(b^3 + 6\,abc\right)d^8 + 5\,a^2bd^4 + 7\left(ab^2 + a^2c\right)d^6 + a^3d^2\right)ex^2 \\
&\quad + \left(c^3d^{15} + 3\,bc^2d^{13} + 3\left(b^2c + ac^2\right)d^{11} + \left(b^3 + 6\,abc\right)d^9 + 3\,a^2bd^5 + 3\left(ab^2 + a^2c\right)d^7 + a^3d^3\right)x
\end{aligned}
$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
```

```
a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 1079, normalized size of antiderivative = 7.82

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*c^3*d^14 + 7/4*(e*x^2 + 2*d*x)^2*c^3*d^12*e + 7/2*(e*x^
2 + 2*d*x)^3*c^3*d^10*e^2 + 35/8*(e*x^2 + 2*d*x)^4*c^3*d^8*e^3 + 7/2*(e*x^2
 + 2*d*x)^5*c^3*d^6*e^4 + 7/4*(e*x^2 + 2*d*x)^6*c^3*d^4*e^5 + 1/2*(e*x^2 +
2*d*x)^7*c^3*d^2*e^6 + 1/16*(e*x^2 + 2*d*x)^8*c^3*e^7 + 3/2*(e*x^2 + 2*d*x)
*b*c^2*d^12 + 9/2*(e*x^2 + 2*d*x)^2*b*c^2*d^10*e + 15/2*(e*x^2 + 2*d*x)^3*b
*c^2*d^8*e^2 + 15/2*(e*x^2 + 2*d*x)^4*b*c^2*d^6*e^3 + 9/2*(e*x^2 + 2*d*x)^5
*b*c^2*d^4*e^4 + 3/2*(e*x^2 + 2*d*x)^6*b*c^2*d^2*e^5 + 3/14*(e*x^2 + 2*d*x)
^7*b*c^2*e^6 + 3/2*(e*x^2 + 2*d*x)*b^2*c*d^10 + 3/2*(e*x^2 + 2*d*x)*a*c^2*d
^10 + 15/4*(e*x^2 + 2*d*x)^2*b^2*c*d^8*e + 15/4*(e*x^2 + 2*d*x)^2*a*c^2*d^8
*e + 5*(e*x^2 + 2*d*x)^3*b^2*c*d^6*e^2 + 5*(e*x^2 + 2*d*x)^3*a*c^2*d^6*e^2
+ 15/4*(e*x^2 + 2*d*x)^4*b^2*c*d^4*e^3 + 15/4*(e*x^2 + 2*d*x)^4*a*c^2*d^4*e
^3 + 3/2*(e*x^2 + 2*d*x)^5*b^2*c*d^2*e^4 + 3/2*(e*x^2 + 2*d*x)^5*a*c^2*d^2*
e^4 + 1/4*(e*x^2 + 2*d*x)^6*b^2*c*e^5 + 1/4*(e*x^2 + 2*d*x)^6*a*c^2*e^5 + 1
/2*(e*x^2 + 2*d*x)*b^3*d^8 + 3*(e*x^2 + 2*d*x)*a*b*c*d^8 + (e*x^2 + 2*d*x)^
2*b^3*d^6*e + 6*(e*x^2 + 2*d*x)^2*a*b*c*d^6*e + (e*x^2 + 2*d*x)^3*b^3*d^4*e
^2 + 6*(e*x^2 + 2*d*x)^3*a*b*c*d^4*e^2 + 1/2*(e*x^2 + 2*d*x)^4*b^3*d^2*e^3
+ 3*(e*x^2 + 2*d*x)^4*a*b*c*d^2*e^3 + 1/10*(e*x^2 + 2*d*x)^5*b^3*e^4 + 3/5*
(e*x^2 + 2*d*x)^5*a*b*c*e^4 + 3/2*(e*x^2 + 2*d*x)*a*b^2*d^6 + 3/2*(e*x^2 +
2*d*x)*a^2*c*d^6 + 9/4*(e*x^2 + 2*d*x)^2*a*b^2*d^4*e + 9/4*(e*x^2 + 2*d*x)^
2*a^2*c*d^4*e + 3/2*(e*x^2 + 2*d*x)^3*a*b^2*d^2*e^2 + 3/2*(e*x^2 + 2*d*x)^3
*a^2*c*d^2*e^2 + 3/8*(e*x^2 + 2*d*x)^4*a*b^2*e^3 + 3/8*(e*x^2 + 2*d*x)^4*a^
2*c*e^3 + 3/2*(e*x^2 + 2*d*x)*a^2*b*d^4 + 3/2*(e*x^2 + 2*d*x)^2*a^2*b*d^2*e
 + 1/2*(e*x^2 + 2*d*x)^3*a^2*b*e^2 + 1/2*(e*x^2 + 2*d*x)*a^3*d^2 + 1/4*(e*x
^2 + 2*d*x)^2*a^3*e
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.63

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$
$$= \frac{3\,e^7\,x^8\,\left(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8\right)}{8}$$
$$+ \frac{e^5\,x^6\,\left(a^2\,b+21\,a^2\,c\,d^2+21\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+462\,a\,c^2\,d^6+42\,b^3\,d^4+462\,b^2\,c\,d^6+1287\,b\,c^2\,d^8+1001\right.}{2}$$
$$+ \frac{e^9\,x^{10}\,\left(b^3+165\,b^2\,c\,d^2+2145\,b\,c^2\,d^4+6\,a\,b\,c+5005\,c^3\,d^6+165\,a\,c^2\,d^2\right)}{10}$$
$$+ \frac{c^3\,e^{15}\,x^{16}}{16}+d^3\,x\,\left(c\,d^4+b\,d^2+a\right)^3$$
$$+ \frac{e^3\,x^4\,\left(a^3+30\,a^2\,b\,d^2+105\,a^2\,c\,d^4+105\,a\,b^2\,d^4+504\,a\,b\,c\,d^6+495\,a\,c^2\,d^8+84\,b^3\,d^6+495\,b^2\,c\,d^8+858\,b\right.}{4}$$
$$+ \frac{3\,c^2\,e^{13}\,x^{14}\,\left(35\,c\,d^2+b\right)}{14}+c^3\,d\,e^{14}\,x^{15}+d\,e^2\,x^3\,\left(a^3+10\,a^2\,b\,d^2+21\,a^2\,c\,d^4+21\,a\,b^2\,d^4\right.$$
$$\left.+72\,a\,b\,c\,d^6+55\,a\,c^2\,d^8+12\,b^3\,d^6+55\,b^2\,c\,d^8+78\,b\,c^2\,d^{10}+35\,c^3\,d^{12}\right)$$
$$+ \frac{c\,e^{11}\,x^{12}\,\left(b^2+78\,b\,c\,d^2+455\,c^2\,d^4+a\,c\right)}{4}$$
$$+ \frac{d\,e^6\,x^7\,\left(21\,a^2\,c+21\,a\,b^2+504\,a\,b\,c\,d^2+1386\,a\,c^2\,d^4+84\,b^3\,d^2+1386\,b^2\,c\,d^4+5148\,b\,c^2\,d^6+5005\,c^3\,d^8\right)}{7}$$
$$+ \frac{3\,d\,e^4\,x^5\,\left(5\,a^2\,b+35\,a^2\,c\,d^2+35\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+330\,a\,c^2\,d^6+42\,b^3\,d^4+330\,b^2\,c\,d^6+715\,b\,c^2\,d^8+4\right.}{5}$$
$$+ d\,e^8\,x^9\,\left(b^3+55\,b^2\,c\,d^2+429\,b\,c^2\,d^4+6\,a\,b\,c+715\,c^3\,d^6+55\,a\,c^2\,d^2\right)$$
$$+ \frac{3\,d^2\,e\,x^2\,\left(c\,d^4+b\,d^2+a\right)^2\,\left(5\,c\,d^4+3\,b\,d^2+a\right)}{2}$$
$$+ c^2\,d\,e^{12}\,x^{13}\,\left(35\,c\,d^2+3\,b\right)+3\,c\,d\,e^{10}\,x^{11}\,\left(b^2+26\,b\,c\,d^2+91\,c^2\,d^4+a\,c\right)$$

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330
*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3*d
^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b^2*
c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*d^6
+ 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3*e^15
*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6 + 455*
c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*d^8 + 4
95*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^14*(b + 3
5*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12
+ 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8
+ 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*c^2*d^4 + 7
8*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8
 + 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (
```

3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^12*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*x^11*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d^2)

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 55

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[Out] 1/4*a*f^3*(e*x+d)^4/e+1/6*b*f^3*(e*x+d)^6/e+1/8*c*f^3*(e*x+d)^8/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1156, 14}

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f^3\text{Subst}\left(\int x^3(a+bx^2+cx^4)\ dx, x, d+ex\right)}{e}\\ &= \frac{f^3\text{Subst}\left(\int (ax^3+bx^5+cx^7)\ dx, x, d+ex\right)}{e}\\ &= \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(55) = 110.

Time = 0.02 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\begin{aligned}&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\ dx\\ &= f^3\left(d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2+\frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3\right.\\ &\quad\left.+\frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4+d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6+cde^6x^7+\frac{1}{8}ce^7x^8\right)\end{aligned}$$

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
(d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*
x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7
 + (c*e^7*x^8)/8)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(49) = 98.

Time = 0.63 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.25

method	result
gosper	$\frac{f^3x(3e^7c\,x^7+24d\,e^6c\,x^6+84x^5d^2e^5c+168c\,d^3e^4x^4+4x^5b\,e^5+210x^3d^4c\,e^3+24bd\,e^4x^4+168x^2c\,d^5e^2+60x^3b\,d^2e^3+84xc\,d^6e+80x}{24}$
norman	$\left(\frac{7}{2}d^2f^3e^5c+\frac{1}{6}b\,e^5f^3\right)x^6+\left(7c\,d^5e^2f^3+\frac{10}{3}b\,d^3e^2f^3+ad\,e^2f^3\right)x^3+\left(\frac{7}{2}c\,d^6e\,f^3+\frac{5}{2}b\,d^4e\,f^3+\frac{3}{2}\right.$
risch	$\frac{1}{8}e^7f^3c\,x^8+d\,f^3e^6c\,x^7+\frac{7}{2}f^3x^6d^2e^5c+\frac{1}{6}f^3x^6b\,e^5+7f^3c\,d^3e^4x^5+f^3bd\,e^4x^5+\frac{35}{4}f^3x^4d^4c\,e^3+$
parallelrisch	$\frac{1}{8}e^7f^3c\,x^8+d\,f^3e^6c\,x^7+\frac{7}{2}f^3x^6d^2e^5c+\frac{1}{6}f^3x^6b\,e^5+7f^3c\,d^3e^4x^5+f^3bd\,e^4x^5+\frac{35}{4}f^3x^4d^4c\,e^3+$
default	$\frac{e^7f^3c\,x^8}{8}+d\,f^3e^6c\,x^7+\frac{(15d^2f^3e^5c+e^3f^3(6c\,d^2e^2+b\,e^2))x^6}{6}+\frac{(13d^3f^3c\,e^4+3d\,f^3e^2(6c\,d^2e^2+b\,e^2)+e^3f^3(4d^3ec+2bde}{5}$

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*f^3*x*(3*c*e^7*x^7+24*c*d*e^6*x^6+84*c*d^2*e^5*x^5+168*c*d^3*e^4*x^4+4
*b*e^5*x^5+210*c*d^4*e^3*x^3+24*b*d*e^4*x^4+168*c*d^5*e^2*x^2+60*b*d^2*e^3*
x^3+84*c*d^6*e*x+80*b*d^3*e^2*x^2+24*c*d^7+6*a*e^3*x^3+60*b*d^4*e*x+24*a*d*
e^2*x^2+24*b*d^5+36*a*d^2*e*x+24*a*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{1}{8}\,ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5$$
$$+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2f^3x^3$$
$$+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ef^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*c
*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(44) = 88$.

Time = 0.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\begin{aligned}&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx\\&=cde^6f^3x^7+\frac{ce^7f^3x^8}{8}+x^6\left(\frac{be^5f^3}{6}+\frac{7cd^2e^5f^3}{2}\right)+x^5\left(bde^4f^3+7cd^3e^4f^3\right)\\&\quad+x^4\left(\frac{ae^3f^3}{4}+\frac{5bd^2e^3f^3}{2}+\frac{35cd^4e^3f^3}{4}\right)+x^3\left(ade^2f^3+\frac{10bd^3e^2f^3}{3}+7cd^5e^2f^3\right)\\&\quad+x^2\cdot\left(\frac{3ad^2ef^3}{2}+\frac{5bd^4ef^3}{2}+\frac{7cd^6ef^3}{2}\right)+x\left(ad^3f^3+bd^5f^3+cd^7f^3\right)\end{aligned}$$

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\begin{aligned}&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx\\&=\frac{1}{8}ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5\\&\quad+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2f^3x^3\\&\quad+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ef^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x\end{aligned}$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*c
*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.71

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{1}{2}\left(efx^2+2\,dfx\right)cd^6f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)bd^4f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)ad^2f^2$$
$$+\frac{18\left(efx^2+2\,dfx\right)^2cd^4ef^2+12\left(efx^2+2\,dfx\right)^3cd^2e^2f+3\left(efx^2+2\,dfx\right)^4ce^3+12\left(efx^2+2\,dfx\right)^2bd^2ef^2}{24\,f}$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(e*f*x^2 + 2*d*f*x)*c*d^6*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b*d^4*f^2 + 1/2
*(e*f*x^2 + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(e*f*x^2 + 2*d*f*x)^2*c*d^4*e*f^2
 + 12*(e*f*x^2 + 2*d*f*x)^3*c*d^2*e^2*f + 3*(e*f*x^2 + 2*d*f*x)^4*c*e^3 + 1
2*(e*f*x^2 + 2*d*f*x)^2*b*d^2*e*f^2 + 4*(e*f*x^2 + 2*d*f*x)^3*b*e^2*f + 6*(
e*f*x^2 + 2*d*f*x)^2*a*e*f^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.98

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{e^5\,f^3\,x^6\,(21\,c\,d^2+b)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)$$
$$+\frac{e^3\,f^3\,x^4\,(35\,c\,d^4+10\,b\,d^2+a)}{4}+\frac{d^2\,e\,f^3\,x^2\,(7\,c\,d^4+5\,b\,d^2+3\,a)}{2}$$
$$+\frac{d\,e^2\,f^3\,x^3\,(21\,c\,d^4+10\,b\,d^2+3\,a)}{3}+d\,e^4\,f^3\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,f^3\,x^7$$

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2 +
 c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a +
 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*
e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 104

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{(b^2+2ac)\,f^3(d+ex)^8}{8e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*f^3*(e*x+d)^4/e+1/3*a*b*f^3*(e*x+d)^6/e+1/8*(2*a*c+b^2)*f^3*(e*x+d)
^8/e+1/5*b*c*f^3*(e*x+d)^10/e+1/12*c^2*f^3*(e*x+d)^12/e
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1156, 1128, 645}

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3(2ac+b^2)\,(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f^3\text{Subst}\left(\int x^3(a+bx^2+cx^4)^2\,dx,x,d+ex\right)}{e}\\
&= \frac{f^3\text{Subst}\left(\int x(a+bx+cx^2)^2\,dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{f^3\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right)dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{\left(b^2+2ac\right)f^3(d+ex)^8}{8e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 405 vs. $2(104) = 208$.

Time = 0.06 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.89

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$
$$=f^3\left(d^3\left(a+bd^2+cd^4\right)^2x+\frac{1}{2}d^2\left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right)ex^2\right.$$
$$+\frac{1}{3}d\left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right)e^2x^3$$
$$+\frac{1}{4}\left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right)e^3x^4$$
$$+\frac{1}{5}d\left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right)e^4x^5$$
$$+\frac{1}{6}\left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right)e^5x^6$$
$$+d\left(b^2+2ac+24bcd^2+66c^2d^4\right)e^6x^7+\frac{1}{8}\left(b^2+2ac+72bcd^2+330c^2d^4\right)e^7x^8$$
$$\left.+\frac{1}{3}cd\left(6b+55cd^2\right)e^8x^9+\frac{1}{10}c\left(2b+55cd^2\right)e^9x^{10}+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}\right)$$

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 1
4*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21
*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*
b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (
d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/
6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(
2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(94) = 188$.

Time = 0.60 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.44

method	result
gosper	$\frac{f^3x\left(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3c^2e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080\right.}{}$
norman	$\left(\frac{11}{2}d^2f^3e^9c^2+\frac{1}{5}bc\,e^9f^3\right)x^{10}+\left(\frac{55}{3}d^3f^3c^2e^8+2bcd\,e^8f^3\right)x^9+\left(\frac{165}{4}d^4f^3c^2e^7+9bc\,d^2e^7f^3+\frac{1}{4}\right.$
risch	$\frac{11}{2}f^3x^{10}d^2e^9c^2+\frac{1}{5}f^3x^{10}bc\,e^9+d\,f^3e^{10}c^2x^{11}+\frac{55}{3}f^3x^9d^3c^2e^8+\frac{165}{4}f^3x^8c^2d^4e^7+\frac{1}{4}f^3x^8ac\,e^7+$
parallelrisch	$\frac{11}{2}f^3x^{10}d^2e^9c^2+\frac{1}{5}f^3x^{10}bc\,e^9+d\,f^3e^{10}c^2x^{11}+\frac{55}{3}f^3x^9d^3c^2e^8+\frac{165}{4}f^3x^8c^2d^4e^7+\frac{1}{4}f^3x^8ac\,e^7+$
default	Expression too large to display

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*f^3*x*(10*c^2*e^11*x^11+120*c^2*d*e^10*x^10+660*c^2*d^2*e^9*x^9+2200*
c^2*d^3*e^8*x^8+24*b*c*e^9*x^9+4950*c^2*d^4*e^7*x^7+240*b*c*d*e^8*x^8+7920*
c^2*d^5*e^6*x^6+1080*b*c*d^2*e^7*x^7+9240*c^2*d^6*e^5*x^5+2880*b*c*d^3*e^6*
x^6+7920*c^2*d^7*e^4*x^4+30*a*c*e^7*x^7+15*b^2*e^7*x^7+5040*b*c*d^4*e^5*x^5
+4950*c^2*d^8*e^3*x^3+240*a*c*d*e^6*x^6+120*b^2*d*e^6*x^6+6048*b*c*d^5*e^4*
x^4+2200*c^2*d^9*e^2*x^2+840*a*c*d^2*e^5*x^5+420*b^2*d^2*e^5*x^5+5040*b*c*d
^6*e^3*x^3+660*c^2*d^10*e*x+1680*a*c*d^3*e^4*x^4+840*b^2*d^3*e^4*x^4+2880*b
*c*d^7*e^2*x^2+120*c^2*d^11+40*a*b*e^5*x^5+2100*a*c*d^4*e^3*x^3+1050*b^2*d^
4*e^3*x^3+1080*b*c*d^8*e*x+240*a*b*d*e^4*x^4+1680*a*c*d^5*e^2*x^2+840*b^2*d
^5*e^2*x^2+240*b*c*d^9+600*a*b*d^2*e^3*x^3+840*a*c*d^6*e*x+420*b^2*d^6*e*x+
800*a*b*d^3*e^2*x^2+240*a*c*d^7+120*b^2*d^7+30*a^2*e^3*x^3+600*a*b*d^4*e*x+
120*a^2*d*e^2*x^2+240*a*b*d^5+180*a^2*d^2*e*x+120*a^2*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\begin{aligned}
&\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx \\
&= \frac{1}{12}c^2e^{11}f^3x^{12} + c^2de^{10}f^3x^{11} + \frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10} + \frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9 \\
&\quad + \frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8 + \left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7 \\
&\quad + \frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6 \\
&\quad + \frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5 \\
&\quad + \frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4 \\
&\quad + \frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3 \\
&\quad + \frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2 \\
&\quad + \left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x
\end{aligned}$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^
9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
```

```
3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 +
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(88) = 176$.

Time = 0.07 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.94

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^2 dx$$
$$= c^2de^{10}f^3x^{11} + \frac{c^2e^{11}f^3x^{12}}{12} + x^{10}\left(\frac{bce^9f^3}{5} + \frac{11c^2d^2e^9f^3}{2}\right) + x^9 \cdot \left(2bcde^8f^3 + \frac{55c^2d^3e^8f^3}{3}\right) + x^8\left(\frac{ace^7f^3}{4} + \frac{b^2e^7f^3}{8} + 9bcd^2e^7f^3 + \frac{165c^2d^4e^7f^3}{4}\right) + x^7 \cdot \left(2acde^6f^3 + b^2de^6f^3 + 24bcd^3e^6f^3 + 66c^2d^5e^6f^3\right) + x^6\left(\frac{abe^5f^3}{3} + 7acd^2e^5f^3 + \frac{7b^2d^2e^5f^3}{2} + 42bcd^4e^5f^3 + 77c^2d^6e^5f^3\right) + x^5 \cdot \left(2abde^4f^3 + 14acd^3e^4f^3 + 7b^2d^3e^4f^3 + \frac{252bcd^5e^4f^3}{5} + 66c^2d^7e^4f^3\right) + x^4\left(\frac{a^2e^3f^3}{4} + 5abd^2e^3f^3 + \frac{35acd^4e^3f^3}{2} + \frac{35b^2d^4e^3f^3}{4} + 42bcd^6e^3f^3 + \frac{165c^2d^8e^3f^3}{4}\right) + x^3\left(a^2de^2f^3 + \frac{20abd^3e^2f^3}{3} + 14acd^5e^2f^3 + 7b^2d^5e^2f^3 + 24bcd^7e^2f^3 + \frac{55c^2d^9e^2f^3}{3}\right) + x^2 \cdot \left(\frac{3a^2d^2ef^3}{2} + 5abd^4ef^3 + 7acd^6ef^3 + \frac{7b^2d^6ef^3}{2} + 9bcd^8ef^3 + \frac{11c^2d^{10}ef^3}{2}\right) + x\left(a^2d^3f^3 + 2abd^5f^3 + 2acd^7f^3 + b^2d^7f^3 + 2bcd^9f^3 + c^2d^{11}f^3\right)$$

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5
 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*
f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 +
 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 +
24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7
*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*
c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*
b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) +
x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3
5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4
) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**
```

```
3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/
3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2
*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**
9*f**3 + c**2*d**11*f**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.22 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^2 \, dx$$
$$= \frac{1}{12} c^2 e^{11} f^3 x^{12} + c^2 d e^{10} f^3 x^{11} + \frac{1}{10} \left(55 \, c^2 d^2 + 2 \, bc\right) e^9 f^3 x^{10} + \frac{1}{3} \left(55 \, c^2 d^3 + 6 \, bcd\right) e^8 f^3 x^9$$
$$+ \frac{1}{8} \left(330 \, c^2 d^4 + 72 \, bcd^2 + b^2 + 2 \, ac\right) e^7 f^3 x^8 + \left(66 \, c^2 d^5 + 24 \, bcd^3 + \left(b^2 + 2 \, ac\right) d\right) e^6 f^3 x^7$$
$$+ \frac{1}{6} \left(462 \, c^2 d^6 + 252 \, bcd^4 + 21 \left(b^2 + 2 \, ac\right) d^2 + 2 \, ab\right) e^5 f^3 x^6$$
$$+ \frac{1}{5} \left(330 \, c^2 d^7 + 252 \, bcd^5 + 35 \left(b^2 + 2 \, ac\right) d^3 + 10 \, abd\right) e^4 f^3 x^5$$
$$+ \frac{1}{4} \left(165 \, c^2 d^8 + 168 \, bcd^6 + 35 \left(b^2 + 2 \, ac\right) d^4 + 20 \, abd^2 + a^2\right) e^3 f^3 x^4$$
$$+ \frac{1}{3} \left(55 \, c^2 d^9 + 72 \, bcd^7 + 21 \left(b^2 + 2 \, ac\right) d^5 + 20 \, abd^3 + 3 \, a^2 d\right) e^2 f^3 x^3$$
$$+ \frac{1}{2} \left(11 \, c^2 d^{10} + 18 \, bcd^8 + 7 \left(b^2 + 2 \, ac\right) d^6 + 10 \, abd^4 + 3 \, a^2 d^2\right) e f^3 x^2$$
$$+ \left(c^2 d^{11} + 2 \, bcd^9 + \left(b^2 + 2 \, ac\right) d^7 + 2 \, abd^5 + a^2 d^3\right) f^3 x$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^
9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 +
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(94) = 188$.

Time = 0.30 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.74

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$
$$=\frac{1}{2}\left(efx^2+2\,dfx\right)c^2d^{10}f^2+\left(efx^2+2\,dfx\right)bcd^8f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)b^2d^6f^2$$
$$+\left(efx^2+2\,dfx\right)acd^6f^2+\left(efx^2+2\,dfx\right)abd^4f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)a^2d^2f^2$$
$$+\frac{150\left(efx^2+2\,dfx\right)^2c^2d^8ef^4+200\left(efx^2+2\,dfx\right)^3c^2d^6e^2f^3+150\left(efx^2+2\,dfx\right)^4c^2d^4e^3f^2+240\left(efx^2\right.}{}$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(e*f*x^2 + 2*d*f*x)*c^2*d^10*f^2 + (e*f*x^2 + 2*d*f*x)*b*c*d^8*f^2 + 1/
2*(e*f*x^2 + 2*d*f*x)*b^2*d^6*f^2 + (e*f*x^2 + 2*d*f*x)*a*c*d^6*f^2 + (e*f*
x^2 + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*a^2*d^2*f^2 + 1/120*(1
50*(e*f*x^2 + 2*d*f*x)^2*c^2*d^8*e*f^4 + 200*(e*f*x^2 + 2*d*f*x)^3*c^2*d^6*
e^2*f^3 + 150*(e*f*x^2 + 2*d*f*x)^4*c^2*d^4*e^3*f^2 + 240*(e*f*x^2 + 2*d*f*
x)^2*b*c*d^6*e*f^4 + 60*(e*f*x^2 + 2*d*f*x)^5*c^2*d^2*e^4*f + 240*(e*f*x^2
+ 2*d*f*x)^3*b*c*d^4*e^2*f^3 + 10*(e*f*x^2 + 2*d*f*x)^6*c^2*e^5 + 120*(e*f*
x^2 + 2*d*f*x)^4*b*c*d^2*e^3*f^2 + 90*(e*f*x^2 + 2*d*f*x)^2*b^2*d^4*e*f^4 +
 180*(e*f*x^2 + 2*d*f*x)^2*a*c*d^4*e*f^4 + 24*(e*f*x^2 + 2*d*f*x)^5*b*c*e^4
*f + 60*(e*f*x^2 + 2*d*f*x)^3*b^2*d^2*e^2*f^3 + 120*(e*f*x^2 + 2*d*f*x)^3*a
*c*d^2*e^2*f^3 + 15*(e*f*x^2 + 2*d*f*x)^4*b^2*e^3*f^2 + 30*(e*f*x^2 + 2*d*f
*x)^4*a*c*e^3*f^2 + 120*(e*f*x^2 + 2*d*f*x)^2*a*b*d^2*e*f^4 + 40*(e*f*x^2 +
 2*d*f*x)^3*a*b*e^2*f^3 + 30*(e*f*x^2 + 2*d*f*x)^2*a^2*e*f^4)/f^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.03

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$
$$=\frac{e^3\,f^3\,x^4\,(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8)}{4}+\frac{c^2\,e^{11}\,f^3\,x^{12}}{12}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)^2+\frac{e^7\,f^3\,x^8\,(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c)}{8}+\frac{e^5\,f^3\,x^6\,(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2)}{6}+\frac{d^2\,e\,f^3\,x^2\,(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2\,d^4+18\,b\,c\,d^6+11\,c^2\,d^8)}{2}+\frac{d\,e^2\,f^3\,x^3\,(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2\,d^4+72\,b\,c\,d^6+55\,c^2\,d^8)}{3}+d\,e^6\,f^3\,x^7\,\left(b^2+24\,b\,c\,d^2+66\,c^2\,d^4+2\,a\,c\right)+\frac{d\,e^4\,f^3\,x^5\,(35\,b^2\,d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2\,d^6+70\,a\,c\,d^2)}{5}+\frac{c\,e^9\,f^3\,x^{10}\,(55\,c\,d^2+2\,b)}{10}+c^2\,d\,e^{10}\,f^3\,x^{11}+\frac{c\,d\,e^8\,f^3\,x^9\,(55\,c\,d^2+6\,b)}{3}$$

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 16
8*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2 +
(e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*(2*
a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*f^3*
x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6)
)/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c
*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^
2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b
*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3*x^11 + (
c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,f^3(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,f^3(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,f^3(d+ex)^{12}}{4e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

```
[Out] 1/4*a^3*f^3*(e*x+d)^4/e+1/2*a^2*b*f^3*(e*x+d)^6/e+3/8*a*(a*c+b^2)*f^3*(e*x+
d)^8/e+1/10*b*(6*a*c+b^2)*f^3*(e*x+d)^10/e+1/4*c*(a*c+b^2)*f^3*(e*x+d)^12/e
+3/14*b*c^2*f^3*(e*x+d)^14/e+1/16*c^3*f^3*(e*x+d)^16/e
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1156, 1128, 645}

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3(ac+b^2)\,(d+ex)^{12}}{4e}+\frac{bf^3(6ac+b^2)\,(d+ex)^{10}}{10e}+\frac{3af^3(ac+b^2)\,(d+ex)^8}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
```

```
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{f^3\text{Subst}\left(\int x^3(a+bx^2+cx^4)^3\ dx, x, d+ex\right)}{e}$$

$$= \frac{f^3\text{Subst}\left(\int x(a+bx+cx^2)^3\ dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{f^3\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a(b^2+ac)\,x^3+b(b^2+6ac)\,x^4+3c(b^2+ac)\,x^5+3bc^2x^6+c^3x^7\right)\ dx, x\right.}{2e}$$

$$= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,f^3(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,f^3(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,f^3(d+ex)^{12}}{4e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 801 vs. $2(159) = 318$.

Time = 0.03 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.04

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$= f^3\Bigg(d^3\left(a+bd^2+cd^4\right)^3x+\frac{3}{2}d^2\left(a+bd^2+cd^4\right)^2\left(a+3bd^2+5cd^4\right)ex^2+d\big(a^3+10a^2bd^2+21ab^2d^4+21a^2cd^4+12b^3d^6+72abcd^6+55b^2cd^8+55ac^2d^8+78bc^2d^{10}+35c^3d^{12}\big)e^2x^3+\frac{1}{4}\big(a^3+30a^2bd^2+105ab^2d^4+105a^2cd^4+84b^3d^6+504abcd^6+495b^2cd^8+495ac^2d^8+858bc^2d^{10}+455c^3d^{12}\big)e^3x^4+\frac{3}{5}d\big(5a^2b+35ab^2d^2+35a^2cd^2+42b^3d^4+252abcd^4+330b^2cd^6+330ac^2d^6+715bc^2d^8+455c^3d^{10}\big)e^4x^5+\frac{1}{2}\big(a^2b+21ab^2d^2+21a^2cd^2+42b^3d^4+252abcd^4+462b^2cd^6+462ac^2d^6+1287bc^2d^8+1001c^3d^{10}\big)e^5x^6+\frac{1}{7}d\big(21ab^2+21a^2c+84b^3d^2+504abcd^2+1386b^2cd^4+1386ac^2d^4+5148bc^2d^6+5005c^3d^8\big)e^6x^7+\frac{3}{8}\big(ab^2+a^2c+12b^3d^2+72abcd^2+330b^2cd^4+330ac^2d^4+1716bc^2d^6+2145c^3d^8\big)e^7x^8+d\big(b^3+6abc+55b^2cd^2+55ac^2d^2+429bc^2d^4+715c^3d^6\big)e^8x^9+\frac{1}{10}\big(b^3+6abc+165b^2cd^2+165ac^2d^2+2145bc^2d^4+5005c^3d^6\big)e^9x^{10}+3cd\big(b^2+ac+26bcd^2+91c^2d^4\big)e^{10}x^{11}+\frac{1}{4}c\big(b^2+ac+78bcd^2+455c^2d^4\big)e^{11}x^{12}+c^2d\big(3b+35cd^2\big)e^{12}x^{13}+\frac{3}{14}c^2\big(b+35cd^2\big)e^{13}x^{14}+c^3de^{14}x^{15}+\frac{1}{16}c^3e^{15}x^{16}\Bigg)$$

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10
+ 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d
^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 +
42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8
 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a
*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)
*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4
 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 +
```

```
((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^
3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11
 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35
*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 +
 (c^3*e^15*x^16)/16)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. 2(145) = 290.

Time = 0.64 (sec) , antiderivative size = 1318, normalized size of antiderivative = 8.29

method	result	size
gosper	Expression too large to display	1318
norman	Expression too large to display	1430
risch	Expression too large to display	1636
parallelrisch	Expression too large to display	1636
default	Expression too large to display	7697

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/560*f^3*x*(35*c^3*e^15*x^15+560*c^3*d*e^14*x^14+4200*c^3*d^2*e^13*x^13+19
600*c^3*d^3*e^12*x^12+120*b*c^2*e^13*x^13+63700*c^3*d^4*e^11*x^11+1680*b*c^
2*d*e^12*x^12+152880*c^3*d^5*e^10*x^10+10920*b*c^2*d^2*e^11*x^11+280280*c^3
*d^6*e^9*x^9+43680*b*c^2*d^3*e^10*x^10+400400*c^3*d^7*e^8*x^8+140*a*c^2*e^1
1*x^11+140*b^2*c*e^11*x^11+120120*b*c^2*d^4*e^9*x^9+450450*c^3*d^8*e^7*x^7+
1680*a*c^2*d*e^10*x^10+1680*b^2*c*d*e^10*x^10+240240*b*c^2*d^5*e^8*x^8+4004
00*c^3*d^9*e^6*x^6+9240*a*c^2*d^2*e^9*x^9+9240*b^2*c*d^2*e^9*x^9+360360*b*c
^2*d^6*e^7*x^7+280280*c^3*d^10*e^5*x^5+30800*a*c^2*d^3*e^8*x^8+30800*b^2*c*
d^3*e^8*x^8+411840*b*c^2*d^7*e^6*x^6+152880*c^3*d^11*e^4*x^4+336*a*b*c*e^9*
x^9+69300*a*c^2*d^4*e^7*x^7+56*b^3*e^9*x^9+69300*b^2*c*d^4*e^7*x^7+360360*b
*c^2*d^8*e^5*x^5+63700*c^3*d^12*e^3*x^3+3360*a*b*c*d*e^8*x^8+110880*a*c^2*d
^5*e^6*x^6+560*b^3*d*e^8*x^8+110880*b^2*c*d^5*e^6*x^6+240240*b*c^2*d^9*e^4*
x^4+19600*c^3*d^13*e^2*x^2+15120*a*b*c*d^2*e^7*x^7+129360*a*c^2*d^6*e^5*x^5
+2520*b^3*d^2*e^7*x^7+129360*b^2*c*d^6*e^5*x^5+120120*b*c^2*d^10*e^3*x^3+42
00*c^3*d^14*e*x+40320*a*b*c*d^3*e^6*x^6+110880*a*c^2*d^7*e^4*x^4+6720*b^3*d
^3*e^6*x^6+110880*b^2*c*d^7*e^4*x^4+43680*b*c^2*d^11*e^2*x^2+560*c^3*d^15+2
10*a^2*c*e^7*x^7+210*a*b^2*e^7*x^7+70560*a*b*c*d^4*e^5*x^5+69300*a*c^2*d^8*
e^3*x^3+11760*b^3*d^4*e^5*x^5+69300*b^2*c*d^8*e^3*x^3+10920*b*c^2*d^12*e*x+
1680*a^2*c*d*e^6*x^6+1680*a*b^2*d*e^6*x^6+84672*a*b*c*d^5*e^4*x^4+30800*a*c
^2*d^9*e^2*x^2+14112*b^3*d^5*e^4*x^4+30800*b^2*c*d^9*e^2*x^2+1680*b*c^2*d^1
3+5880*a^2*c*d^2*e^5*x^5+5880*a*b^2*d^2*e^5*x^5+70560*a*b*c*d^6*e^3*x^3+924
0*a*c^2*d^10*e*x+11760*b^3*d^6*e^3*x^3+9240*b^2*c*d^10*e*x+11760*a^2*c*d^3*
e^4*x^4+11760*a*b^2*d^3*e^4*x^4+40320*a*b*c*d^7*e^2*x^2+1680*a*c^2*d^11+672
0*b^3*d^7*e^2*x^2+1680*b^2*c*d^11+280*a^2*b*e^5*x^5+14700*a^2*c*d^4*e^3*x^3
```

```
+14700*a*b^2*d^4*e^3*x^3+15120*a*b*c*d^8*e*x+2520*b^3*d^8*e*x+1680*a^2*b*d*
e^4*x^4+11760*a^2*c*d^5*e^2*x^2+11760*a*b^2*d^5*e^2*x^2+3360*a*b*c*d^9+560*
b^3*d^9+4200*a^2*b*d^2*e^3*x^3+5880*a^2*c*d^6*e*x+5880*a*b^2*d^6*e*x+5600*a
^2*b*d^3*e^2*x^2+1680*a^2*c*d^7+1680*a*b^2*d^7+140*a^3*e^3*x^3+4200*a^2*b*d
^4*e*x+560*a^3*d*e^2*x^2+1680*a^2*b*d^5+840*a^3*d^2*e*x+560*a^3*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.26 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$
\begin{aligned}
&\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx \\
&= \frac{1}{16}c^3e^{15}f^3x^{16} + c^3de^{14}f^3x^{15} + \frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}f^3x^{14} \\
&\quad + \left(35\,c^3d^3+3\,bc^2d\right)e^{12}f^3x^{13} + \frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}f^3x^{12} \\
&\quad + 3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}f^3x^{11} \\
&\quad + \frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9f^3x^{10} \\
&\quad + \left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8f^3x^9 \\
&\quad + \frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7f^3x^8 \\
&\quad + \frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6f^3x^7 \\
&\quad + \frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5f^3x^6 \\
&\quad + \frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4f^3x^5 \\
&\quad + \frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right) \\
&\quad + \left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2f^3x \\
&\quad + \frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ef^3x^2 \\
&\quad + \left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)f^3x
\end{aligned}
$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^
13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2
```

```
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6
*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^13 + 78*b
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(141) = 282$.

Time = 0.15 (sec) , antiderivative size = 1654, normalized size of antiderivative = 10.40

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*
f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c*
*3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39
*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d
*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*
d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2
+ b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**
3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d
**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d*
*5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*
b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 +
9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e*
*7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12
*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f*
*3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2
*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*
a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3
+ 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**
```

```
2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3 + 756
*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b**3*d**5*e**4*f*
*3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9*e**4*f**3 + 273*c**3*d**
11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 + 15*a**2*b*d**2*e**3*f**3/2 + 105*a
**2*c*d**4*e**3*f**3/4 + 105*a*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*
f**3 + 495*a*c**2*d**8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d*
*8*e**3*f**3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*d**5*e**2*
f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f**3 + 55*a*c**2*d**9*
e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*c*d**9*e**2*f**3 + 78*b*c**2*d
**11*e**2*f**3 + 35*c**3*d**13*e**2*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15
*a**2*b*d**4*e*f**3/2 + 21*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 +
 27*a*b*c*d**8*e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 3
3*b**2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f**3/2
) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f**3 + 3*a*b**2*
d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**3 + b**3*d**9*f**3 + 3*b*
*2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 + c**3*d**15*f**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.22 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$= \frac{1}{16}c^3e^{15}f^3x^{16}+c^3de^{14}f^3x^{15}+\frac{3}{14}\left(35\,c^3d^2+bc^2\right)e^{13}f^3x^{14}$$
$$+\left(35\,c^3d^3+3\,bc^2d\right)e^{12}f^3x^{13}+\frac{1}{4}\left(455\,c^3d^4+78\,bc^2d^2+b^2c+ac^2\right)e^{11}f^3x^{12}$$
$$+3\left(91\,c^3d^5+26\,bc^2d^3+\left(b^2c+ac^2\right)d\right)e^{10}f^3x^{11}$$
$$+\frac{1}{10}\left(5005\,c^3d^6+2145\,bc^2d^4+b^3+6\,abc+165\left(b^2c+ac^2\right)d^2\right)e^9f^3x^{10}$$
$$+\left(715\,c^3d^7+429\,bc^2d^5+55\left(b^2c+ac^2\right)d^3+\left(b^3+6\,abc\right)d\right)e^8f^3x^9$$
$$+\frac{3}{8}\left(2145\,c^3d^8+1716\,bc^2d^6+330\left(b^2c+ac^2\right)d^4+ab^2+a^2c+12\left(b^3+6\,abc\right)d^2\right)e^7f^3x^8$$
$$+\frac{1}{7}\left(5005\,c^3d^9+5148\,bc^2d^7+1386\left(b^2c+ac^2\right)d^5+84\left(b^3+6\,abc\right)d^3+21\left(ab^2+a^2c\right)d\right)e^6f^3x^7$$
$$+\frac{1}{2}\left(1001\,c^3d^{10}+1287\,bc^2d^8+462\left(b^2c+ac^2\right)d^6+42\left(b^3+6\,abc\right)d^4+a^2b+21\left(ab^2+a^2c\right)d^2\right)e^5f^3x^6$$
$$+\frac{3}{5}\left(455\,c^3d^{11}+715\,bc^2d^9+330\left(b^2c+ac^2\right)d^7+42\left(b^3+6\,abc\right)d^5+5\,a^2bd+35\left(ab^2+a^2c\right)d^3\right)e^4f^3x^5$$
$$+\frac{1}{4}\left(455\,c^3d^{12}+858\,bc^2d^{10}+495\left(b^2c+ac^2\right)d^8+84\left(b^3+6\,abc\right)d^6+30\,a^2bd^2+105\left(ab^2+a^2c\right)d^4+a^3\right)e^3$$
$$+\left(35\,c^3d^{13}+78\,bc^2d^{11}+55\left(b^2c+ac^2\right)d^9+12\left(b^3+6\,abc\right)d^7+10\,a^2bd^3+21\left(ab^2+a^2c\right)d^5+a^3d\right)e^2f^3x^3$$
$$+\frac{3}{2}\left(5\,c^3d^{14}+13\,bc^2d^{12}+11\left(b^2c+ac^2\right)d^{10}+3\left(b^3+6\,abc\right)d^8+5\,a^2bd^4+7\left(ab^2+a^2c\right)d^6+a^3d^2\right)ef^3x^2$$
$$+\left(c^3d^{15}+3\,bc^2d^{13}+3\left(b^2c+ac^2\right)d^{11}+\left(b^3+6\,abc\right)d^9+3\,a^2bd^5+3\left(ab^2+a^2c\right)d^7+a^3d^3\right)f^3x$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^
13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6
*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^13 + 78*b
```

```
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 1330, normalized size of antiderivative = 8.36

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e*f*x^2 + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b*c^2*d^12*f
^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*c^2
*d^10*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b^3*d^8*f^2 + 3*(e*f*x^2 + 2*d*f*x)*a*b
*c*d^8*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(e*f*x^2 + 2*d*f*x
)*a^2*c*d^6*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(e*f*x^2 + 2*
d*f*x)*a^3*d^2*f^2 + 1/560*(980*(e*f*x^2 + 2*d*f*x)^2*c^3*d^12*e*f^6 + 1960
*(e*f*x^2 + 2*d*f*x)^3*c^3*d^10*e^2*f^5 + 2450*(e*f*x^2 + 2*d*f*x)^4*c^3*d^
8*e^3*f^4 + 2520*(e*f*x^2 + 2*d*f*x)^2*b*c^2*d^10*e*f^6 + 1960*(e*f*x^2 + 2
*d*f*x)^5*c^3*d^6*e^4*f^3 + 4200*(e*f*x^2 + 2*d*f*x)^3*b*c^2*d^8*e^2*f^5 +
980*(e*f*x^2 + 2*d*f*x)^6*c^3*d^4*e^5*f^2 + 4200*(e*f*x^2 + 2*d*f*x)^4*b*c^
2*d^6*e^3*f^4 + 2100*(e*f*x^2 + 2*d*f*x)^2*b^2*c*d^8*e*f^6 + 2100*(e*f*x^2
+ 2*d*f*x)^2*a*c^2*d^8*e*f^6 + 280*(e*f*x^2 + 2*d*f*x)^7*c^3*d^2*e^6*f + 25
20*(e*f*x^2 + 2*d*f*x)^5*b*c^2*d^4*e^4*f^3 + 2800*(e*f*x^2 + 2*d*f*x)^3*b^2
*c*d^6*e^2*f^5 + 2800*(e*f*x^2 + 2*d*f*x)^3*a*c^2*d^6*e^2*f^5 + 35*(e*f*x^2
 + 2*d*f*x)^8*c^3*e^7 + 840*(e*f*x^2 + 2*d*f*x)^6*b*c^2*d^2*e^5*f^2 + 2100*
(e*f*x^2 + 2*d*f*x)^4*b^2*c*d^4*e^3*f^4 + 2100*(e*f*x^2 + 2*d*f*x)^4*a*c^2*
d^4*e^3*f^4 + 560*(e*f*x^2 + 2*d*f*x)^2*b^3*d^6*e*f^6 + 3360*(e*f*x^2 + 2*d
*f*x)^2*a*b*c*d^6*e*f^6 + 120*(e*f*x^2 + 2*d*f*x)^7*b*c^2*e^6*f + 840*(e*f*
x^2 + 2*d*f*x)^5*b^2*c*d^2*e^4*f^3 + 840*(e*f*x^2 + 2*d*f*x)^5*a*c^2*d^2*e^
4*f^3 + 560*(e*f*x^2 + 2*d*f*x)^3*b^3*d^4*e^2*f^5 + 3360*(e*f*x^2 + 2*d*f*x
)^3*a*b*c*d^4*e^2*f^5 + 140*(e*f*x^2 + 2*d*f*x)^6*b^2*c*e^5*f^2 + 140*(e*f*
x^2 + 2*d*f*x)^6*a*c^2*e^5*f^2 + 280*(e*f*x^2 + 2*d*f*x)^4*b^3*d^2*e^3*f^4
+ 1680*(e*f*x^2 + 2*d*f*x)^4*a*b*c*d^2*e^3*f^4 + 1260*(e*f*x^2 + 2*d*f*x)^2
*a*b^2*d^4*e*f^6 + 1260*(e*f*x^2 + 2*d*f*x)^2*a^2*c*d^4*e*f^6 + 56*(e*f*x^2
 + 2*d*f*x)^5*b^3*e^4*f^3 + 336*(e*f*x^2 + 2*d*f*x)^5*a*b*c*e^4*f^3 + 840*(
e*f*x^2 + 2*d*f*x)^3*a*b^2*d^2*e^2*f^5 + 840*(e*f*x^2 + 2*d*f*x)^3*a^2*c*d^
2*e^2*f^5 + 210*(e*f*x^2 + 2*d*f*x)^4*a*b^2*e^3*f^4 + 210*(e*f*x^2 + 2*d*f*
x)^4*a^2*c*e^3*f^4 + 840*(e*f*x^2 + 2*d*f*x)^2*a^2*b*d^2*e*f^6 + 280*(e*f*x
^2 + 2*d*f*x)^3*a^2*b*e^2*f^5 + 140*(e*f*x^2 + 2*d*f*x)^2*a^3*e*f^6)/f^5
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.19

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$=\frac{3\,e^7\,f^3\,x^8\,(a^2\,c+a\,b^2+72\,a\,b\,c\,d^2+330\,a\,c^2\,d^4+12\,b^3\,d^2+330\,b^2\,c\,d^4+1716\,b\,c^2\,d^6+2145\,c^3\,d^8)}{8}$$
$$+\frac{e^5\,f^3\,x^6\,(a^2\,b+21\,a^2\,c\,d^2+21\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+462\,a\,c^2\,d^6+42\,b^3\,d^4+462\,b^2\,c\,d^6+1287\,b\,c^2\,d^8+10}{2}$$
$$+\frac{e^9\,f^3\,x^{10}\,(b^3+165\,b^2\,c\,d^2+2145\,b\,c^2\,d^4+6\,a\,b\,c+5005\,c^3\,d^6+165\,a\,c^2\,d^2)}{10}$$
$$+\frac{c^3\,e^{15}\,f^3\,x^{16}}{16}+d^3\,f^3\,x\left(c\,d^4+b\,d^2+a\right)^3$$
$$+\frac{e^3\,f^3\,x^4\,(a^3+30\,a^2\,b\,d^2+105\,a^2\,c\,d^4+105\,a\,b^2\,d^4+504\,a\,b\,c\,d^6+495\,a\,c^2\,d^8+84\,b^3\,d^6+495\,b^2\,c\,d^8+858}{4}$$
$$+\frac{c\,e^{11}\,f^3\,x^{12}\,(b^2+78\,b\,c\,d^2+455\,c^2\,d^4+a\,c)}{4}$$
$$+\frac{d\,e^6\,f^3\,x^7\,(21\,a^2\,c+21\,a\,b^2+504\,a\,b\,c\,d^2+1386\,a\,c^2\,d^4+84\,b^3\,d^2+1386\,b^2\,c\,d^4+5148\,b\,c^2\,d^6+5005\,c^3\,d^8}{7}$$
$$+\frac{3\,d\,e^4\,f^3\,x^5\,(5\,a^2\,b+35\,a^2\,c\,d^2+35\,a\,b^2\,d^2+252\,a\,b\,c\,d^4+330\,a\,c^2\,d^6+42\,b^3\,d^4+330\,b^2\,c\,d^6+715\,b\,c^2\,d^8}{5}$$
$$+d\,e^8\,f^3\,x^9\left(b^3+55\,b^2\,c\,d^2+429\,b\,c^2\,d^4+6\,a\,b\,c+715\,c^3\,d^6+55\,a\,c^2\,d^2\right)$$
$$+\frac{3\,c^2\,e^{13}\,f^3\,x^{14}\,(35\,c\,d^2+b)}{14}+c^3\,d\,e^{14}\,f^3\,x^{15}+d\,e^2\,f^3\,x^3\left(a^3+10\,a^2\,b\,d^2+21\,a^2\,c\,d^4\right.$$
$$\left.+21\,a\,b^2\,d^4+72\,a\,b\,c\,d^6+55\,a\,c^2\,d^8+12\,b^3\,d^6+55\,b^2\,c\,d^8+78\,b\,c^2\,d^{10}+35\,c^3\,d^{12}\right)$$
$$+\frac{3\,d^2\,e\,f^3\,x^2\,(c\,d^4+b\,d^2+a)^2\,(5\,c\,d^4+3\,b\,d^2+a)}{2}$$
$$+c^2\,d\,e^{12}\,f^3\,x^{13}\left(35\,c\,d^2+3\,b\right)+3\,c\,d\,e^{10}\,f^3\,x^{11}\left(b^2+26\,b\,c\,d^2+91\,c^2\,d^4+a\,c\right)$$

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 +
 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b +
42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 +
462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3 + 5
005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10
 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^3*x^4*
(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c
*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 +
 (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*f^3*x^
7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386*
b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^5*(5*a^2*b
+ 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 +
 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3*x^9*(b^3 + 7
```

```
15*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*c^
2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d*e^2*f^3*x^3*(a
^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
+ 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d^6) + (3*d^2*e*f^
3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^12*f^3*x^1
3*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d
^2)
```

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] x/c-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*
a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1156, 1136, 1180, 211}

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$

$$= \frac{x}{c} - \frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{2\sqrt{c}(d+ex) - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2c^{3/2}e}$$

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.82

method	result
default	$\frac{x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2e^2+b e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b d^2+a\right)} \frac{\left(-\_R^2 b e^2-2\_R bde-b d^2-a\right)\ln\left(x-\_\right.}{2e^3c\_R^3+6cd e^2\_R^2+6c d^2e\_R+2d^3c+be}}{2ce}$
risch	$\frac{x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2e^2+b e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b d^2+a\right)} \frac{\left(-\_R^2 b e^2-2\_R bde-b d^2-a\right)\ln\left(x-\_\right.}{2e^3c\_R^3+6cd e^2\_R^2+6c d^2e\_R+2d^3c+be}}{2ce}$

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(157) = 314$.

Time = 0.28 (sec) , antiderivative size = 1231, normalized size of antiderivative = 6.38

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2)
)*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 -
4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4))
- (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(-((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3 - 3*a*b*c)/((b^2*c
^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2 - a^2*c)*d - sqrt
(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6
 - 4*a*c^7)*e^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-((b^2*c^3 - 4*a*
c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + b^3
- 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(((b^2*c^3 - 4*a*c
^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) - b^3 +
 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*x - 2*(a*b^2
- a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(
((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c
^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt((
(b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^
7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)
*e*x - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*e^3*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((b^2*c^6 - 4*a*c^7)*e^4)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*sqrt(((b^2*c^3 - 4*a*c^4)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
(b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*
x)/c
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\cdot\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+a^3,\left(t\mapsto t\log\right.\right.$$
$$+\frac{x}{c}$$

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, L
ambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_
t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*
c*e - a*b**2*e)))) + x/c
```

Maxima [F]

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(ex+d)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x
)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(157) = 314.

Time = 0.30 (sec) , antiderivative size = 1315, normalized size of antiderivative = 6.81

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] x/c + 1/2*((b*e^6*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 2*b*d*e^5*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e) + b*d^2*e^4 + a*e^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
```

```
 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e
^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*
e^6*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*
b*d*e^5*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) +
b*d^2*e^4 + a*e^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^6*(sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*d*e^5*(sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4 + a*
e^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e
)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*e^6*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e^5*(sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2*e^4 + a*e^4)*log(x - s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3
*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d
^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)))/(c*e^4)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 3988, normalized size of antiderivative = 20.66

$$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*
e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13
- 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)
))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b
^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
```

8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3* c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + x/c

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ace}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*arctanh((b+2*c*(e*x+d)^2)/(-4*a
*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1156, 1128, 648, 632, 212, 642}

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce}
\end{aligned}$$

$$= \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2ce}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}e} + \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cde^2\_R^2+6cd^2e\_R+2d^3c+be\_R+bd}}{2e}$
risch	Expression too large to display

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(73) = 146$.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.36

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\,(b^2c-4\,ac}\right.$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2
)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a))/((b^2*c - 4*a*c^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(68) = 136$.

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.46

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)$$

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
 + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

Maxima [F]

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{(ex+d)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{b\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}ce} + \frac{\log\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)}{4\,ce}$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c*e) + 1/4*log(c*d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^2
+ 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e + a)/(c*e)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.43

$$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{4\,a\,c\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$
$$-\frac{b^2\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^
2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b
^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^
2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*ata
n(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*
a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(
1/2))
```

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

```
[Out] -1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)
/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1156, 1144, 211}

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx = \frac{\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e}\\
&\quad+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e}\\
&= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}e}$$

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[
b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_R-}}{2e}$
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_R-}}{2e}$

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.29

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b}{(b^2c-4\,ac^2)e^2}}\log\left(\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b}{(b^2c-4\,ac^2)e^2}} + ex+d\right)$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}+b}{(b^2c-4\,ac^2)e^2}}\log\left(-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{-\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}}{(b^2c-4\,ac^2)e^2}} + ex+d\right)$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}}\log\left(\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}} + ex+d\right)$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-b}{(b^2c-4\,ac^2)e^2}}\log\left(-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)e^3\sqrt{\frac{(b^2c-4\,ac^2)e^2\sqrt{\frac{1}{(b^2c^2-4\,ac^3)e^4}}-}{(b^2c-4\,ac^2)e^2}} + ex+d\right)$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)
) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-(
```

```
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*
c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqr
t(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c -
4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2
)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(
1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*
c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*lo
g(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b
^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x +
d)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2+4b^3e^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3ac^2e\right.\right.\right.$$

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3
*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

Maxima [F]

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(ex+d)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1403 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 1403, normalized size of antiderivative = 8.55

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
[Out] -1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
+ d^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/
e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c*d^3*e + b*e^2*(sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d*e) - 1/2*(e^2
*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*d*e
*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + d^2)*lo
g(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*
e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*
c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2
+ 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e) - b*d*e) + 1/2*(e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) - d/e) + d^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d*e)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.60

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3+\sqrt{-(4\,a\,c-b^2)^3}-4\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\left(x\,(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12})+\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2}\right.}{a\,c\,e^{10}}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{\sqrt{-(4\,a\,c-b^2)^3}-b^3+4\,a\,b\,c}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\left(x\,(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12})-\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2}\right.}{a\,c\,e^{10}}\right.$$

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*
a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) + (
(x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^1
3)*(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e
^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(-(b
^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8
*a*b^2*c^2*e^2)))^(1/2) - 2*atanh(((((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e
^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b
^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*
e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^
2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 43

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ace}}$$

```
[Out] -arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1156, 1121, 632, 212}

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{e}\\
&= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,e}$$

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+be\_R}}{2e}$
risch	$-\frac{\ln\left(\left(e^{2}\sqrt{-4ac+b^{2}}-b\,e^{2}\right)x^{2}+\left(2ed\sqrt{-4ac+b^{2}}-2bde\right)x+d^{2}\sqrt{-4ac+b^{2}}-b\,d^{2}-2a\right)}{2\sqrt{-4ac+b^{2}}\,e}+\frac{\ln\left(\left(e^{2}\sqrt{-4ac+b^{2}}+b\,e^{2}\right)x^{2}+\left(2ed\sqrt{-4ac+\right.\right.}{2\sqrt{-4ac+}}$

[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.33

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left[\frac{\log\left(\frac{2\,c^{2}e^{4}x^{4}+8\,c^{2}de^{3}x^{3}+2\,c^{2}d^{4}+2\,(6\,c^{2}d^{2}+bc)e^{2}x^{2}+2\,bcd^{2}+4\,(2\,c^{2}d^{3}+bcd)ex+b^{2}-2\,ac-(2\,ce^{2}x^{2}+4\,cdex+2\,cd^{2}+b)\sqrt{b^{2}-4\,ac}}{ce^{4}x^{4}+4\,cde^{3}x^{3}+cd^{4}+(6\,cd^{2}+b)e^{2}x^{2}+bd^{2}+2\,(2\,cd^{3}+bd)ex+a}\right)}{2\,\sqrt{b^{2}-4\,ac}e}, -\frac{\sqrt{-b^{2}+4\,ac}\arctan\left(-\frac{(2\,ce^{2}x^{2}+4\,cdex+2\,cd^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{(b^{2}-4\,ac)e}\right]$$

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/2*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(39) = 78$.

Time = 0.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.91

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(
-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b*
*2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

Maxima [F]

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{ex+d}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}e}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*e)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{\operatorname{atan}\left(\frac{2\,a\,c\,d^2+4\,a\,c\,d\,e\,x+2\,a\,c\,e^2\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{e\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2)
))/(e*(4*a*c - b^2)^(1/2))
```

3.617 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx$

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx = \frac{b\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}$$

```
[Out] ln(e*x+d)/a/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e+1/2*b*arctanh((b+2*c*(e
*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1156, 1128, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} dx = \frac{b\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{\log(d+ex)}{ae}$$

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2ae}$$
$$= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ae} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ae}$$
$$= \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2ae}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$= \frac{4\sqrt{b^2-4ac}\log(d+ex) - \left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(b+\right.}{4a\sqrt{b^2-4ac}e}$$

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
risch	$\frac{\ln(ex+d)}{ae}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2ce^2-ab^2e^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+c\right)}\_R\ln\left(\left(\left(10e^3ac-3b^2e^3\right)\_R+5ce^2\right)x^2+\left(\left(20acde^2-6b^2de\ldots\right.\right.\right.\right.}{2}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+bd^2+a\right)}\frac{\left(-e^3c\_R^3-3cde^2\_R^2+e\left(-3cd^2-b\right)\_R-d^3\ldots\right.}{2e^3c\_R^3+6cde^2\_R^2+6cd^2e\_R+2d^3c\ldots}}{2ae}$

[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] ln(e*x+d)/a/e+1/2*sum(_R*ln(((10*a*c*e^3-3*b^2*e^3)*_R+5*c*e^2)*x^2+((20*a*c*d*e^2-6*b^2*d*e^2)*_R+10*d*c*e)*x+(10*a*c*d^2*e-3*b^2*d^2*e-a*b*e)*_R+5*c*d^2+2*b),_R=RootOf((4*a^2*c*e^2-a*b^2*e^2)*_Z^2+(4*a*c*e-b^2*e)*_Z+c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.98

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-\ldots}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.\ldots}{\ldots}\right.$$

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*b^2 - 4*a^2*c)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*b^2 - 4*a^2*c)*e)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(78) = 156$.

Time = 13.83 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.40

$$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)} - \frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)} - \frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right) + \frac{\log\left(\frac{d}{e}+x\right)}{ae}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x
**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*
e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4
*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c
 + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c +
b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + log(d/e + x)/(a*e)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x + b*d)/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a), x)/a + log(e*x + d)/(a*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(86) = 172$.

Time = 0.36 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.98

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,ae} + \frac{\log\left(\left|ex+d\right|\right)}{ae} - \frac{\frac{abce^3\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{abce^3\log\left(\left|-be^2x^2+\sqrt{b^2-4\,ac}e^2x^2-2\,bdex+2\,\sqrt{b^2-\right.\right.}{\sqrt{b^2-4\,ac}}}{4\,a^2ce^4}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*
d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a*e) + log(abs(e*x + d))/(a*e) -
 1/4*(a*b*c*e^3*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x +
 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(b^2
 - 4*a*c) - a*b*c*e^3*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*
d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/s
qrt(b^2 - 4*a*c))/(a^2*c*e^4)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 2173, normalized size of antiderivative = 23.12

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d
*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e))/
(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*
((b^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e
^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b
^2*e - 8*a*c*e)^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 -
40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*(4*a*b^2*e^2 - 16*a
^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19)
)/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*
a*c - 6*b^2)) - (((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^
19))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) - (b^3*(12
```

*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*e^14) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) + (16*a^3*x*(((3*b^3 - 8*a*b*c)*((b^2*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*e^14)))/(2*a*e*(4*a*c - b^2)^(1/2))

3.618 $\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx = -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] -1/a/e/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2
)*(1-b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1156, 1137, 1180, 211}

$$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx = -\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}ae\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}ae\sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ae(d+ex)}$$

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e} \\
&= -\frac{1}{ae(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ae} \\
&= -\frac{1}{ae(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ae} \\
&\quad - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ae}
\end{aligned}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{\frac{2}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2ae}$$

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.86

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_Z+d^4c+b d^2+a\right)} \frac{\left(-\_R^2 c e^2-2\_R cde-c d^2-b\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd e^2\_R^2+6c d^2 e\_R+2d^3c+be\_R}}{2ae}$
risch	$-\frac{1}{ae(ex+d)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2e^4-8b^2e^4ca^4+b^4e^4a^3\right)\_Z^4+\left(12a^2bc^2e^2-7ab^3ce^2+b^5e^2\right)\_Z^2+c^3\right)} \_R\ln\left(\left(\left(40a^5c^2e^5-22a^4b\right.\right.\right.\right.}{}$

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^
3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a/e/(e*
x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(158) = 316$.

Time = 0.28 (sec) , antiderivative size = 1339, normalized size of antiderivative = 6.87

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2
 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sq
rt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt
(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a
^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2
*x + a*d*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log
(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*
a^7*c)*e^4)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-((a^3*b^2 - 4*a^4*c
)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + b^3 - 3
*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^
3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*
e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*
e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*e^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 -
5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a
^4*c)*e^2))) + sqrt(1/2)*(a*e^2*x + a*d*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/
((a^3*b^2 - 4*a^4*c)*e^2))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^
3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2
)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - 2)/(a*e^
2*x + a*d*e)
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\cdot\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\,\mathrm{lo}\right.\right.$$
$$-\frac{1}{ade+ae^2x}$$

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, L
ambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3
- 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b*
*5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e))) - 1/(a*d*e + a*e
**2*x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 + b)/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
 c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a -
 1/(a*e^2*x + a*d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(158) = 316$.

Time = 0.31 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.78

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx =$$

$$-\frac{\left(\left(b^8-9\,ab^6c+25\,a^2b^4c^2-20\,a^3b^2c^3+\left(b^7-7\,ab^5c+13\,a^2b^3c^2-4\,a^3bc^3\right)\sqrt{b^2-4\,ac}\right)\sqrt{2\,ab+2\,\sqrt{b^2-4}}\right.}{}$$

$$-\frac{\left(\left(b^8-9\,ab^6c+25\,a^2b^4c^2-20\,a^3b^2c^3-\left(b^7-7\,ab^5c+13\,a^2b^3c^2-4\,a^3bc^3\right)\sqrt{b^2-4\,ac}\right)\sqrt{2\,ab-2\,\sqrt{b^2-4}}\right.}{}$$

$$-\frac{1}{(ex+d)ae}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/8*((b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 + (b^7 - 7*a*b^5*c
 + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b + 2*sqrt(b^2
 - 4*a*c)*a)*a^2 - 2*(a^2*b^6*c - 7*a^3*b^4*c^2 + 13*a^4*b^2*c^3 - 4*a^5*c^
4 + (a^2*b^5*c - 5*a^3*b^3*c^2 + 5*a^4*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b
 + 2*sqrt(b^2 - 4*a*c)*a)*abs(a) - (a^2*b^8 - 7*a^3*b^6*c + 15*a^4*b^4*c^2
- 10*a^5*b^2*c^3 + (a^2*b^7 - 5*a^3*b^5*c + 7*a^4*b^3*c^2 - 2*a^5*b*c^3)*sq
rt(b^2 - 4*a*c))*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((
e*x + d)*e*sqrt((a*b*e^6 + sqrt(a^2*b^2*e^12 - 4*a^3*c*e^12))/(a^2*e^8)))))/
((a^3*b^6*c - 7*a^4*b^4*c^2 + 13*a^5*b^2*c^3 - 4*a^6*c^4 + (a^3*b^5*c - 5*a
^4*b^3*c^2 + 5*a^5*b*c^3)*sqrt(b^2 - 4*a*c))*a^2*e) - 1/8*((b^8 - 9*a*b^6*c
 + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*
a^3*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*a^2 - 2*(
a^2*b^6*c - 7*a^3*b^4*c^2 + 13*a^4*b^2*c^3 - 4*a^5*c^4 - (a^2*b^5*c - 5*a^3
*b^3*c^2 + 5*a^4*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)
*a)*abs(a) - (a^2*b^8 - 7*a^3*b^6*c + 15*a^4*b^4*c^2 - 10*a^5*b^2*c^3 - (a^
2*b^7 - 5*a^3*b^5*c + 7*a^4*b^3*c^2 - 2*a^5*b*c^3)*sqrt(b^2 - 4*a*c))*sqrt(
2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x + d)*e*sqrt((a*b*e
^6 - sqrt(a^2*b^2*e^12 - 4*a^3*c*e^12))/(a^2*e^8)))))/((a^3*b^6*c - 7*a^4*b^
4*c^2 + 13*a^5*b^2*c^3 - 4*a^6*c^4 - (a^3*b^5*c - 5*a^4*b^3*c^2 + 5*a^5*b*c
^3)*sqrt(b^2 - 4*a*c))*a^2*e) - 1/((e*x + d)*a*e)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 3844, normalized size of antiderivative = 19.71

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*
c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3
*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^
(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*
c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12)
 + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a
^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 -
 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)
+ 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 -
 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^
6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b
^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4
*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) - (-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*
(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*
c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c
^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^
```

5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e
^11) + 2*a^3*c^4*e^10))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3
*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^
3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16
*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2
e^12)(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
c(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*
a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2
*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*
e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c
^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11
)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
c(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c
e^2)))^(1/2)(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*
e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-
(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a
c - b^2)^3)^(1/2))/(8(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(
1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
2*a^3*b^2*c^3*d*e^11) - (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*
e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + (
(x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b
^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a
^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*
a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) + 2*a^3*c^4*e^10))*(-(b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - 1/(a*
e*(d + e*x))

3.619 $\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{1}{2ae(d+ex)^2}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}e} - \frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

```
[Out] -1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^
2/e-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/e/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1156, 1128, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e} - \frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*1/(a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (
b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2e}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, (d+ex)^2\right)}{2ae}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2e}\\
&\quad+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}\\
&\quad-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2a^2e}\\
&= -\frac{1}{2ae(d+ex)^2}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e}\\
&\quad-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{-\frac{2a}{(d+ex)^2}-4b\log(d+ex)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+\right.}{\sqrt{b^2-4ac}}}{4a^2e}$$

[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.76

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(bc\,e^3\_R^3+3bcd\,e^2\_R^2+e\left(3bc\,d^2-ac+b^2\right)\_R+\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2}}{2a^2e}$
risch	$-\frac{1}{2ae(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2-a^2b^2e^2\right)\_Z^2+\left(-4abce+b^3e\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4-3a^2b^2e^4\right)\_R^2-4\right.\right.\right.}{}$

[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] 1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d
^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*
d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4
*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(111) = 222$.

Time = 0.29 (sec) , antiderivative size = 810, normalized size of antiderivative = 6.69

$$\int \frac{1}{(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \left[-\frac{2\,ab^2-8\,a^2c+((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{2\,ab^2-8\,a^2c+2\,((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,ce^2x^2+4\,cde\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3
*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2
*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 +
4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 +
 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*
a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3
*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^3} dx$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -1/2/(a*e^3*x^2 + 2*a*d*e^2*x + a*d^2*e) + integrate((b*c*e^3*x^3 + 3*b*c*d
*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2 - a*c)*d)/(c*e^4*x^
4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*
d)*e*x + a), x)/a^2 - b*log(e*x + d)/(a^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = \frac{b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{4\,a^2e} + \frac{(b^2-2\,ac)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2e} - \frac{1}{2\,(ex+d)^2ae}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c + b/(e*x + d)^2 + a/(e*x + d)^4)/(a^2*e) + 1/2*(b^2 - 2*a*c)*ar
ctan(-(b + 2*a/(e*x + d)^2)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*e)
- 1/2/((e*x + d)^2*a*e)
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 4950, normalized size of antiderivative = 40.91

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(4*a*c - b^2)^(3/2)*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((
(20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4
*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))
```

*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e^17)
/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (c^5*e^16
)/a^3 - (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*
c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^
2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*
(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(
16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(4*a^2*e
*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^
3*e^19 - 12*a^3*b^3*c^2*e^19))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(
4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((((((20*a^3*
c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e
^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c
- b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e
)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2
*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e
 - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) + (6*b*c^4*e^17)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - (
(2*a*c - b^2)^3*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(64*a^9*e^3*(4*a
c - b^2)^(3/2)))(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c -
b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))))/(4*a^2*c^4*e^14 + b^4*c^2*e^14
 - 4*a*b^2*c^3*e^14) + (16*a^6*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3
*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*
a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a
^3*c*e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4
*d*e^16)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (
2*c^5*d*e^15)/a^3 - ((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3
*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8
*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1
/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*
e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)
))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 1
2*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e))/(16*a^7*e^2*(1
6*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
 24*a*b^2*c)) + ((((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d
*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a
*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1/2
)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e
- 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + ((((2*b^3*e -
8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b
*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^
16)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^

18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^3)/(32*a^9*e^3*(4*a*c - b^2)^(3/2
)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^
2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^
2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 -
9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/a^3 + (((4*a*b^2*c^3*e^15 - a^2*c^4
*e^15 + 6*a*b*c^4*d^2*e^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16
+ 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e
)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(
2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*
e^2)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c
^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b
^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*
a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) -
((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*
d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*
(4*a*c - b^2)^(1/2))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^2*(2*
b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b
*c^3*d^2*e^17))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))
/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b
^2*c^3*e^14)) + (2*a^3*(4*a*c - b^2)*(((2*b^3*e - 8*a*b*c*e)*((((4*a^2*b^3*
c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)
/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17
- 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c -
b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)
*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8
*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2))))/(2*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)) + (((4*a*b^2*c^3*e^15 - a^2*c^4*e^15 + 6*a*b*c^4*d^2*e
^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 +
2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 +
12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*
(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*
c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(64*a^9*e^3*(4
*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2*a
*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(1/2)) - (log(((c^5*e^16*x^2)/a^3 - ((b +
a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2
- a*c + 6*b*c*d^2))/a^2 - ((b + a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b
^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/a +
(2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b + a^2*e*(-(2*a*c - b^2)
^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*
c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (4*c^3*d*e^17*x
*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)/a^2 + (12*b*c^4*d*e^16*
x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (2*c^5*d*e^15*x)/a^3)*((c

```
^5*e^16*x^2)/a^3 - ((b - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(
1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/a^2 - ((b - a^2*e*(-(2*a*c - b^
2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b
^2*c*d^2 - 2*a*b*c))/a + (2*c^3*e^18*x^2*(10*a*c + b^2))/a + (b*c^2*e^16*(b
 - a^2*e*(-(2*a*c - b^2)^2/(a^4*e^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2
 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x
))/a^2 + (4*c^3*d*e^17*x*(10*a*c + b^2))/a))/(4*a^2*e) + (6*b*c^4*e^17*x^2)
/a^2 + (12*b*c^4*d*e^16*x)/a^2))/(4*a^2*e) + (c^4*e^14*(b + c*d^2))/a^3 + (
2*c^5*d*e^15*x)/a^3))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) - (b*log(d + e*x))/(a^2*e) - 1/(2*a*e*(d^2 + e^2*x^2 + 2*d*e*x))
```

3.620 $\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 224

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx=-\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}e}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] -1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/e*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/e
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1156, 1137, 1295, 1180, 211\}$

$$\int \frac{1}{(d+ex)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2e\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2e\sqrt{\sqrt{b^2-4ac}+b}} + \frac{b}{a^2e(d+ex)} - \frac{1}{3ae(d+ex)^3}$$

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/3*1/(a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e}$$
$$= -\frac{1}{3ae(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{3ae}$$
$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{3a^2e}$$
$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2a^2e} + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2a^2e}$$
$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

$$\int \frac{1}{(d+ex)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2e}$$

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(\_R^{2}bc\,e^{2}+2\_Rbcde+bc\,d^{2}-ac+b^{2}\right)\ln\left(x-\_\right.}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+be\_R}}{2a^{2}e}$
risch	$\frac{\frac{be\,x^{2}}{a^{2}}+\frac{2bdx}{a^{2}}-\frac{-3b\,d^{2}+a}{3e\,a^{2}}}{\left(ex+d\right)^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16e^{4}c^{2}a^{7}-8a^{6}b^{2}c\,e^{4}+a^{5}b^{4}e^{4}\right)\_Z^{4}+\left(-20b\,e^{2}c^{3}a^{3}+25b^{3}e^{2}c^{2}a^{2}-9b^{5}e^{2}ca+b^{7}e^{2}\right)\_Z^{2}+c^{5}\right)}\_R\right.}{}$

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+
4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+
a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2044 vs. 2(188) = 376.

Time = 0.33 (sec) , antiderivative size = 2044, normalized size of antiderivative = 9.12

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*
d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2)
)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 +
a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^
```

8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))) - 2*a)/(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)

Sympy [A] (verification not implemented)

Time = 104.66 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3} + \mathrm{RootSum}\left(t^4\cdot\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2b^3c^2e^2-36ab^5ce^2+4b^7e\right.\right.$$

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e*
*2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c**
2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3
*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, L
ambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**
3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40
*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b
**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

Maxima [F]

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(ex+d)^4}\,dx$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2
+ 3*a^2*d^2*e^2*x + a^2*d^3*e) + integrate((b*c*e^2*x^2 + 2*b*c*d*e*x + b*c
*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 1347, normalized size of antiderivative = 6.01

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
```

```
) + d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*c
*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2
*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) +
 b*c*d^2 + b^2 - a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*c*e^2*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*c*d*e*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*c*d^2 +
 b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*c*e^2*(sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*c*d*e*(sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*c*d^2 + b^2 - a*c
)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(
2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3
+ 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/a^2 + 1/3*(3*b*e^2*x^2 + 6*b*d*e*x
+ 3*b*d^2 - a)/((e*x + d)^3*a^2*e)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 5214, normalized size of antiderivative = 23.28

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)} dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 +
3*d*e^2*x^2 + 3*d^2*e*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a
^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c
^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*
```

((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a
^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 2
5*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)
))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4
*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + ((b^4*(
-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12
 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*
c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2
*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2
)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2
+ 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*
c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 -
 8*a^7*b^2*c^4*d*e^11)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b
*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^
2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^
12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(
32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b
^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1
/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*
c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - ((b^4*(-(4*a*c
- b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6
*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
+ 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) +
 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/
2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7
*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12
 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b
^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*

c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i)/((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 1

6*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*2i

3.621 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] 1/2*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/
4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(
b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used

$= \{1156, 1134, 1180, 211\}$

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\,(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)^{3/2}e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)^{3/2}e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4e}$$

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.20

method	result
default	$\frac{-\frac{b e^2 x^3}{2(4ac-b^2)}-\frac{3bde x^2}{2(4ac-b^2)}-\frac{(3b d^2+2a)x}{2(4ac-b^2)}-\frac{d(b d^2+2a)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{\sum_{\_R=\text{RootOf}(c e^4 \_Z^4+4cd e^3 \_Z^3+(6c d^2 e^2+b e^2)\_Z^2+(4d^3 ec-$
risch	$\frac{-\frac{b e^2 x^3}{2(4ac-b^2)}-\frac{3bde x^2}{2(4ac-b^2)}-\frac{(3b d^2+2a)x}{2(4ac-b^2)}-\frac{d(b d^2+2a)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{\sum_{\_R=\text{RootOf}(c e^4 \_Z^4+4cd e^3 \_Z^3+(6c d^2 e^2+b e^2)\_Z^2+(4d^3 ec-$

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*a)/(4*
a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*
sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2454 vs. 2(226) = 452.

Time = 0.32 (sec) , antiderivative size = 2454, normalized size of antiderivative = 9.09

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*s
qrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6
```

*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*sqrt(((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) - b^3 - 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + 4*a*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(243) = 486$.

Time = 11.78 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{-2ad-bd^3-3bde^2x^2-be^3}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde}$$
$$+\mathrm{RootSum}\left(t^4\cdot(1048576a^6c^7e^4-1572864a^5b^2c^6e^4+983040a^4b^4c^5e^4-327680a^3b^6c^4e^4+61440a^2b^8c^3e\right.$$

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d - b*d**3 - 3*b*d*e**2*x**2 - b*e**3*x**3 + x*(-2*a*e - 3*b*d**2*e))
/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*
e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**
2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*
b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2
 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6*c**7*
e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 327680*a*
*3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e**4 + 25
6*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2
 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t
**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e*
*3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a*c*d + 3*b**2
*d)/(4*a*c*e + 3*b**2*e))))
```

Maxima [F]

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + b*d^3 + (3*b*d^2 + 2*a)*e*x + 2*a*d)/((b^2
*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*
(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*
c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^
2)*e) - 1/2*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*
c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. $2(226) = 452$.

Time = 0.37 (sec) , antiderivative size = 1408, normalized size of antiderivative = 5.21

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/
e)^2 - 2*b*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e) + b*d^2 - 2*a)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*e^2*(sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2 - 2*a)*lo
g(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*
e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*
c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2
+ 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2 - 2*a)*log(x + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)) - (b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) - d/e)^2 + 2*b*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) - d/e) + b*d^2 - 2*a)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*
(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(b^2 - 4
*a*c) + 1/2*(b*e^3*x^3 + 3*b*d*e^2*x^2 + 3*b*d^2*e*x + b*d^3 + 2*a*e*x + 2*
a*d)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 +
b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 7327, normalized size of antiderivative = 27.14

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out] atan(-(((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i + ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c

^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/(((((64*b
^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b
^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b
*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12
 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 24
0*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*
b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d
e^11)/(8(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^1
2 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^
2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*
c^6*e^2)))^(1/2) - ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^
4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*
a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 -
96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a
*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c
^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*
e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4
*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e
^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*
a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b
^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 -
 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*
b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (4*a^2*b*c^2*e^10 + 3*a*b^3*c
e^10)/(4(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(-(b^9 + (-(4
*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(3
2*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/
2)*2i - ((2*a*d + b*d^3)/(2*e*(4*a*c - b^2)) + (x*(2*a + 3*b*d^2))/(2*(4*a*
c - b^2)) + (b*e^2*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*x^2)/(2*(4*a*c - b^2))
)/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e)

+ c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768
*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c
^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3
840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 -
32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*
b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a
^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3
*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*
c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^1
2 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*1i - (((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*
a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((
64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a
^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a
^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*
b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^
6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b
^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^
2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -
6144*a^5*b^2*c^6*e^2)))^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a
*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*1i)/((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b
^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^
2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 -
6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 3
84*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 1638
4*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 -

192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*(((2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) - (4*a^2*b*c^2*e^10 + 3*a*b^3*c*e^10)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i

3.622 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx = \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1156, 1128, 652, 632, 212}

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx = \frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{2a+b(d+ex)^2}{2\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2\,(b^2-4ac)\,e}\\
&= \frac{2a+b(d+ex)^2}{2\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{(b^2-4ac)\,e}\\
&= \frac{2a+b(d+ex)^2}{2\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}\,e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}}{2e}$$

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))
) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2))/(2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.85

method	result
default	$\frac{-\frac{x^2eb}{2\left(4ac-b^2\right)}-\frac{xbd}{4ac-b^2}-\frac{b\,d^2+2a}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3\right.\right.}\right.}{}$
risch	$\frac{-\frac{x^2eb}{2\left(4ac-b^2\right)}-\frac{xbd}{4ac-b^2}-\frac{b\,d^2+2a}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{b\ln\left(\left(\left(-\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4ab\,e^2c-b^3e^2\right)x^2+\left(-2\left(-4ac+b^2\right.\right.\right.\right.}{2e}$

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/(4*a*c-b^2)*x^2*e*b-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*c-b^2)
)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(91) = 182$.

Time = 0.31 (sec) , antiderivative size = 1021, normalized size of antiderivative = 10.53

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \left[\frac{(b^3-4\,abc)e^2x^2+2\,(b^3-4\,abc)dex+2\,ab^2-8\,a^2c+(b^3-4\,abc)d^2-(bce^4x^4+4\,bcde^3x^3+b}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-8\,a}\right.$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c
 + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*
d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2 -
4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*
c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2
*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x
^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*
b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*
c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a
*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4
+ 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^
4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(80) = 160.

Time = 2.65 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.10

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e} + \frac{-2a-bd^2-2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c}$$

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(
4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(4*a
*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) + (-2*a - b*d**2 - 2*b*d*e*x -
b*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e -
2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x*
*3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d*
*2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c*
*2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*
e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a
```

```
)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b
*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 -
 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{b\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}e} + \frac{bd^2+(ex^2+2\,dx)be+2\,a}{2\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)(b^2e-4\,ace)}$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b^2 -
4*a*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*(b*d^2 + (e*x^2 + 2*d*x)*b*e + 2*a)/((c*
d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2
+ 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.40

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{b\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x\left(\frac{b^3\,(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9)}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7}{a\,(4\,a\,c-b^2)^{7/2}}\right)+x^2\left(\frac{b^3\,(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10})}{2\,a\,e^2\,(4\,a\,c-b^2)^{11/2}}-\frac{b^2\,c^2\,e^8}{a\,(4\,a\,c-b^2)^{7/2}}\right)-\frac{b^3\,(16\,a^2\,c^3\,e^8-4\,a}{2}\right.}{2\,b^2\,c^2\,e^6}\right.}{e\,(4\,a\,c-b^2)^{3/2}} - \frac{\frac{b\,d^2+2\,a}{2\,e\,(4\,a\,c-b^2)}+\frac{b\,e\,x^2}{2\,(4\,a\,c-b^2)}+\frac{b\,d\,x}{4\,a\,c-b^2}}{a+x^2\,(6\,c\,d^2\,e^2+b\,e^2)+b\,d^2+c\,d^4+x\,(4\,c\,e\,d^3+2\,b\,e\,d)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*atan(((4*a*c - b^2)^4*(x*((b^3*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*e
^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7)/(a*(4*a*c - b^2)^(7/2))) + x^2
*((b^3*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2)) -
(b^2*c^2*e^8)/(a*(4*a*c - b^2)^(7/2))) - (b^3*(16*a^2*c^3*e^8 - 4*a*b^2*c^2
```

```
*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(11/2
)) - (b^2*c^2*d^2*e^6)/(a*(4*a*c - b^2)^(7/2))))/(2*b^2*c^2*e^6)))/(e*(4*a*
c - b^2)^(3/2)) - ((2*a + b*d^2)/(2*e*(4*a*c - b^2)) + (b*e*x^2)/(2*(4*a*c
- b^2)) + (b*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c
*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.623 $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 254

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} \, dx = -\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] -1/2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1
/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*
b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/
2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{1156, 1133, 1180, 211\}$

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
 - 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)^{3/2}e} - \frac{\left(c\left(2b+\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)^{3/2}e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{2e}$$

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{\left(6c d^2+b\right)x}{8ac-2b^2}+\frac{d\left(2c d^2+b\right)}{2e\left(4ac-b^2\right)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a} + \sum \_R=\text{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec-\right.$
risch	$\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{\left(6c d^2+b\right)x}{8ac-2b^2}+\frac{d\left(2c d^2+b\right)}{2e\left(4ac-b^2\right)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a} + \sum \_R=\text{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec-\right.$

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c-b^2)*
x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2*_R
^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2
+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. 2(213) = 426.

Time = 0.35 (sec) , antiderivative size = 2474, normalized size of antiderivative = 9.74

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((a*b^
8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*s
```

qrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 +
 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
 (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*d^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*
c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4
)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*(
(a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*
e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) +
b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))
) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/
2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))
 - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^
2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^
4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt
(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a
^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3
*c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^
4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))) + 2*b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3
 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^
2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*d^2)*e)

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
[Out] -1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b^
2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6
*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b
*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d
^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4
*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(213) = 426$.

Time = 0.32 (sec) , antiderivative size = 1416, normalized size of antiderivative = 5.57

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
[Out] 1/4*((2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)^2 - 4*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e) + 2*c*d^2 - b)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (2*c*e^2*(sqrt(
```

```
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 4*c*d*e*(sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c*d^2 - b)
*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2
*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 +
 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 4*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 2*c*d^2 - b)*log(x + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sq
rt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)) - (2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e)^2 + 4*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) - d/e) + 2*c*d^2 - b)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 +
b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e))/(
b^2 - 4*a*c) - 1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 6*c*d^2*e*x + 2*c*d^3 + b
*e*x + b*d)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c
*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 7200, normalized size of antiderivative = 28.35

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c
^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4
*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
+ 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (
```

8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((6

4*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (4*a*c^4*e^10 + 3*b^2*c^3*e^10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i - atan(-(((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*1i + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12

+ 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240
*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b
^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4
*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*
b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 128
0*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*1i
)/((4*a*c^4*e^10 + 3*b^2*c^3*e^10)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) - ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^
6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3
*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2
+ 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2
+ 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*
a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*
c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*
c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e
^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 61
44*a^6*b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a
*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (
-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)
/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^
(1/2) + ((((32*b^9*c^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^1
3 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 -
512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*
a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a
^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2
*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^1
2 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2) + (64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^
4*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^
4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a

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*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)))
*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a
^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^
3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*
c^5*e^2)))^(1/2)*2i + ((x*(b + 6*c*d^2))/(2*(4*a*c - b^2)) + (b*d + 2*c*d^3
)/(2*e*(4*a*c - b^2)) + (c*e^2*x^3)/(4*a*c - b^2) + (3*c*d*e*x^2)/(4*a*c -
b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3
*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.624 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx = \frac{-b-2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*arcta
nh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1156, 1121, 628, 632, 212}

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx = \frac{2c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}} - \frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*
c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{c\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{\left(b^2-4ac\right)e}\\
&= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{(2c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)e}\\
&= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=-\frac{\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)e}$$

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcT
an[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4
*a*c)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.76

method	result
default	$\frac{\frac{c\,x^2e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c\,d^2+b}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3\right.\right.}\right.}{}$
risch	$\frac{\frac{c\,x^2e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c\,d^2+b}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{c\ln\left(\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4ab\,e^2c-b^3e^2\right)x^2+\left(2\left(-4ac+b^2\right)^{\frac{3}{2}}c\right.\right.\right.}{e(-}$

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 1042, normalized size of antiderivative = 10.63

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \left[-\frac{2\,(b^2c-4\,ac^2)e^2x^2+4\,(b^2c-4\,ac^2)dex+b^3-4\,abc+2\,(b^2c-4\,ac^2)d^2+2\,(c^2e^4x^4+4\,c^2de^3x^3}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-8}\right.$$
$$-\frac{2\,(b^2c-4\,ac^2)e^2x^2+4\,(b^2c-4\,ac^2)dex+b^3-4\,abc+2\,(b^2c-4\,ac^2}{2\,((b^4c-8\,ab^2c^2+16\,a^2c^3)e^5x^4+4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)de^4x^3+(b^5-8\,ab^3c+16\,a^2bc^2+6\,(b^4c-8}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4
+ (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2
*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c -
 (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c
^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*e^4*x^4 + 4*c^
2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3
+ b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2
 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(83) = 166.

Time = 2.52 (sec) , antiderivative size = 495, normalized size of antiderivative = 5.05

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e} + \frac{c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e} + \frac{b+2cd^2+4cd}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(
-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/(4*
a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**
3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**
2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x +
2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e
- 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) +
x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*
d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*
c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Maxima [F]

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2
)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
```

```
+ b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^
3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.66

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{2\,c\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}e} - \frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{2\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2+bd^2+(ex^2+2\,dx)be+a\right)(b^2e-4\,ace)}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b^2
 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) - 1/2*(2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)
/((c*d^4 + 2*(e*x^2 + 2*d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e
*x^2 + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.26

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{\frac{2\,c\,d^2+b}{2\,e\,(4\,a\,c-b^2)}+\frac{c\,e\,x^2}{4\,a\,c-b^2}+\frac{2\,c\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3} + \frac{2\,c\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7}{a\,(4\,a\,c-b^2)^{7/2}}-\frac{8\,b\,c^2\,(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9)}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8}{a\,(4\,a\,c-b^2)^{7/2}}-\frac{4\,b\,c^2\,(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10})}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}\right)+\frac{4\,c^4\,d^2}{a\,(4\,a\,c-b^2}\cdots}{8\,c^4\,e^6}\right.}{e\,(4\,a\,c-b^2)^{3/2}}$$

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(4
*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*atan(((4*a*c - b^2)^4*(x*((8
*c^4*d*e^7)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d
```

```
*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^2)^(7
/2)) - (4*b*c^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/(a*e^2*(4*a*c - b^2)^(11/2
))) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(7/2)) + (4*b*c^2*(8*a^2*c^3*e^8 - 2
*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)
^(11/2))))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^(3/2))
```

3.625 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] 1/2*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/e*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

= {1120, 1106, 1180, 211}

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}ae\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}$$

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(a+be^2x^2+ce^4x^4)^2}\,dx, x, \frac{d}{e}+x\right)\\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-2(b^2e^4-4ace^4)-bce^6x^2}{a+be^2x^2+ce^4x^4}\,dx, x, \frac{d}{e}+x\right)}{2a\,(b^2-4ac)\,e^4}\\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)e^2\right)\text{Subst}\left(\int \frac{1}{\frac{be^2}{2}+\frac{1}{2}\sqrt{b^2-4ac}e^2+ce^4x^2}\,dx, x, \frac{d}{e}+x\right)}{4a\,(b^2-4ac)^{3/2}}\\
&\quad + \frac{\left(c\left(b^2-12ac+b\sqrt{b^2-4ac}\right)e^2\right)\text{Subst}\left(\int \frac{1}{\frac{be^2}{2}-\frac{1}{2}\sqrt{b^2-4ac}e^2+ce^4x^2}\,dx, x, \frac{d}{e}+x\right)}{4a\,(b^2-4ac)^{3/2}}\\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e}\\
&\quad - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\frac{\frac{2(d+ex)(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4ae}$$

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x
)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*
e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.22

method	result
default	$\frac{-\frac{bc\,e^2x^3}{2a(4ac-b^2)}-\frac{3dbce\,x^2}{2a(4ac-b^2)}+\frac{(-3bc\,d^2+2ac-b^2)x}{2a(4ac-b^2)}+\frac{d(-bc\,d^2+2ac-b^2)}{2ea(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3e\ldots\right.\right.}}{}$
risch	$\frac{-\frac{bc\,e^2x^3}{2a(4ac-b^2)}-\frac{3dbce\,x^2}{2a(4ac-b^2)}+\frac{(-3bc\,d^2+2ac-b^2)x}{2a(4ac-b^2)}+\frac{d(-bc\,d^2+2ac-b^2)}{2ea(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3e\ldots\right.\right.}}{}$

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d
^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(253) = 506$.

Time = 0.35 (sec) , antiderivative size = 3228, normalized size of antiderivative = 10.80

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a
*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2
)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2
*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5
 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6
4*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x
 + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 2
0*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 +
 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 1
8*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*l
og((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^
2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 2
3*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5
- 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3)*e^2))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(
a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^
2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e
```

```
^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)
*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3
 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt
(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^
7*b*c^4)*e^3*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2
- 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
- (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x
^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^
2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2
)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b
^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4
*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3*sqrt((b^4 - 18*
a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^
3)*e^4)) + (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*
c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*
c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2
*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^
3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4
*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e*
x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^
2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 +
2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c -
 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) - 1/2*int
egrate(-(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(253) = 506$.

Time = 0.30 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e) + b*c*d^2 + b^2 - 6*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*
e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b
*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 +
 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
 + b*c*d^2 + b^2 - 6*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*c*e^2*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*c*d
*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*c*d
```

```
^2 + b^2 - 6*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*c*e^2*(sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*c*d*e*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*c*d^2 + b^2
 - 6*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
 d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(a*b^2 - 4*a^2*c) + 1/2*(b*
c*e^3*x^3 + 3*b*c*d*e^2*x^2 + 3*b*c*d^2*e*x + b*c*d^3 + b^2*e*x - 2*a*c*e*x
 + b^2*d - 2*a*c*d)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3
*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(a*b^2*e - 4*a^2*c*e))
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 9056, normalized size of antiderivative = 30.29

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2
40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8
*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^
6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 6
4*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64
*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 1
6384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*
e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288
*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c
*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 -
6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 7
```

2*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^
2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 + b^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c
^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 -
1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)
*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3
*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 -
1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^
13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a
^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^
4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840
*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e
^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*
b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a
^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^
2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)
))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 -
512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/
2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4
- 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^
9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2
+ 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12
+ 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*
a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)
) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e
^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^
2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b
^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2
- 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^
7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 +
4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*
c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*

c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^1
1)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*
c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*
a^3*b^2*c))) + (-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 28
8*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*
c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 -
 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 2
88*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^
2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*
e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d
*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3
*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
c)))(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2
40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8
*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*
c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c -
9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a
^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c
^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d
*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*
c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12
- 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^
4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a
^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e
^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 61
44*a^8*b^2*c^5*e^2)))^(1/2)*2i + atan(((-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/
2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4
 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^
9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2
+ 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12
 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*
a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)
) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e
^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^
2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b
^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2

- 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32 *(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) + (5*b^3*c^4*e^10 - 36*a*b*c^5*e^10)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*2i - ((b^2*d - 2*a*c*d + b*c*d^3)/(2*a*e*(4*a*c - b^2)) + (x*(b^2 - 2*a*c + 3*b*c*d^2))/(2*a*(4*a*c - b^2)) + (b*c*e^2*x^3)/(2*a*(4*a*c - b^2)) + (3*b*c*d*e*x^2)/(2*a*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)

3.626 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}e} + \frac{\log(d+ex)}{a^2e} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+
1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)^(3/2)/e+ln(e*x+d)/a^2/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/e
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used

= {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$
$$+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
```

```
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2a\left(b^2-4ac\right)e}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a(a+bx+cx^2)}\right)dx, x, (d+ex)^2\right)}{2a\left(b^2-4ac\right)e}
\end{aligned}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\log(d+ex)}{a^2e} - \frac{\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2a^2\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4a^2e} - \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4a^2\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^2e}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e} + \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{2a^2\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}e} + \frac{\log(d+ex)}{a^2e}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(b^3-6\right.}{}}{4a^2e}$$

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.46

method	result
default	$-\frac{\frac{\frac{aebc\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum}{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\ln(ex+d)}{a^2e}+\frac{\Big(\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2-48a^4b^2c^2e^2+12a^3b^4c\,e^2\right.\right.}{}$

[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)

[Out] -1/a^2*((1/2*a/(4*a*c-b^2)*e*b*c*x^2+b*c*d*a/(4*a*c-b^2)*x-1/2/e*a*(-b*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum((e^3*c*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))+ln(e*x+d)/a^2/e

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. 2(152) = 304.

Time = 0.45 (sec) , antiderivative size = 2476, normalized size of antiderivative = 15.28

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +

2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
[Out] 1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c
^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a
*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3
- 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a
*b^3 - 4*a^2*b*c)*d^2)*e) - integrate(((b^2*c - 4*a*c^2)*e^3*x^3 + 3*(b^2*c
 - 4*a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*b*c + 3*(b^2*c -
 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2
*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c
 - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^
2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^2 + log(e*x + d)/(a^2*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(152) = 304$.

Time = 0.40 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.88

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=$$
$$-\frac{(a^2b^3ce^3-6\,a^3bc^2e^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-\right.\right.}{4}$$
$$-\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,a^2e}$$
$$+\frac{\log\left(\left|ex+d\right|\right)}{a^2e}$$
$$+\frac{abce^2x^2+2\,abcdex+abcd^2+ab^2-2\,a^2c}{2\,(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a)(b^2-4\,ac)a^2e}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2
 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(
b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2
*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^4*b^4*
c*e^4 - 8*a^5*b^2*c^2*e^4 + 16*a^6*c^3*e^4) - 1/4*log(abs(c*e^4*x^4 + 4*c*d
*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x +
b*d^2 + a))/(a^2*e) + log(abs(e*x + d))/(a^2*e) + 1/2*(a*b*c*e^2*x^2 + 2*a*
b*c*d*e*x + a*b*c*d^2 + a*b^2 - 2*a^2*c)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*
d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2
 - 4*a*c)*a^2*e)
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 11072, normalized size of antiderivative = 68.35

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b
*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + log(d
 + e*x)/(a^2*e) - (log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b
^2)^3))^(1/2) - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^
3))^(1/2) - 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c
- b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2
*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (2*b*c^2*e^16*(2*b^3 - 10*a*
c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) - (2*b*c^3*e^18*x^2*(10*
a*c - b^2))/(a*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c
 - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17
*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(
4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)
))/(4*a^2*e) + (b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^
2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*
a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) - ((a^2*e*(-(b^2*(
6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*(-(b^2*(6*a*
c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*(-(b^
2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 +
 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))
/a^2 - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a
*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) + (4*b*c^3
```

```
*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^
3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^
4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*
a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)))/(4*a^2*e) + (b^2*c^4*e^14*(b^2 - 4*a*
c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*a*c - b^
2)^3)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*
a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) +
(b*atan(((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2)
 - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))
*(x^2*((((((b*((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^
18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 19
2*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a
*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560
*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5
*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 9
6*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a
^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17
+ 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^
3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 +
1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 -
12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^
4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*
e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 +
 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)
^3*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056
*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c
^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 -
400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a
*b^2*c)*((((6*a*b^5*c^4*e^17 + 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18
 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^
6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e
 + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e
^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2
```

- 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 12
8*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5
*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3
*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (b^3*c^5*e^16)/(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*((b*((320*a^5*b*c^6*e^18 - 2*a
^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(
a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*
a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a
^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 -
184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a
^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*
e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*
(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6
*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19
- 2688*a^6*b^3*c^5*e^19))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3
- 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3
*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 40
0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))) + x*((((((b*((2*(320*a^5*b*c^6*
d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e
^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e -
128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 1
2*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2
688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2
c^2)(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2
e^2)))(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*
b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*
e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*
e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a
^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*
b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((2*(6*a*b^5*c^4*d*e^1
6 + 80*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*
d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*
b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 -
184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18)
)/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 -

256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*
a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c
^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^
(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^1
8 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e
^18))/(32*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*
a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b
^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16 + 80
*a^3*b*c^6*d*e^16 - 44*a^2*b^3*c^5*d*e^16))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17
 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*d*e^17))/(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^
2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^
4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/((a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^
5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^
3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*
a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3
*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^6 - 64*a^6*c
^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((2*(320*a^5*b*c
^6*d*e^17 - 2*a^2*b^7*c^3*d*e^17 + 36*a^3*b^5*c^4*d*e^17 - 192*a^4*b^3*c^5*
d*e^17))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e
 - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18
+ 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4*d*e^18
- 2688*a^6*b^3*c^5*d*e^18))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*
b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*
c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*
(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6
*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3*d*e^18 + 1056*a^5*b^5*c^4
*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(4*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2
)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*
a*b^4*c*e)*(2560*a^7*b*c^6*d*e^18 + 12*a^3*b^9*c^2*d*e^18 - 184*a^4*b^7*c^3
*d*e^18 + 1056*a^5*b^5*c^4*d*e^18 - 2688*a^6*b^3*c^5*d*e^18))/(16*a^4*e^2*(
4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(8
*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*
c))) + (((b*((4*a*b^6*c^3*e^15 - 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15
- 44*a^2*b^3*c^5*d^2*e^15 + 6*a*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(
a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^
16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*
a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 -

320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*
b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5
*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c
^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3
*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5
*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3
*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2
- 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c
- b^2)^(3/2)) - (((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^
16 + 224*a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 -
36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^1
6)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128
*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*
b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2
*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6
*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*
c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c -
b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^
8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e
^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4
*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*
(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*
(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))
)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^
6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b^3*(
6*a*c - b^2)^3*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*
e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2
*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*
c^6*d^2*e^17))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^
4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2
- 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((4*a*b^6*c^3*e^15
- 33*a^2*b^4*c^4*e^15 + 68*a^3*b^2*c^5*e^15 - 44*a^2*b^3*c^5*d^2*e^15 + 6*a
*b^5*c^4*d^2*e^15 + 80*a^3*b*c^6*d^2*e^15)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) - (((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*
a^4*b^4*c^4*e^16 - 320*a^5*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b
^5*c^4*d^2*e^16 + 192*a^4*b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*
e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e
^17 + 192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17
- 184*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*
d^2*e^17 + 2560*a^7*b*c^6*d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 1

```
92*a^4*b^2*c^2*e^2))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^
2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(
2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2
)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*
((4*a^2*b^8*c^2*e^16 - 52*a^3*b^6*c^3*e^16 + 224*a^4*b^4*c^4*e^16 - 320*a^5
*b^2*c^5*e^16 + 2*a^2*b^7*c^3*d^2*e^16 - 36*a^3*b^5*c^4*d^2*e^16 + 192*a^4*
b^3*c^5*d^2*e^16 - 320*a^5*b*c^6*d^2*e^16)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*
a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17
 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^1
7 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*
d^2*e^17))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2
*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*
a^2*e*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96
*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 +
192*a^6*b^4*c^4*e^17 - 256*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184
*a^4*b^7*c^3*d^2*e^17 + 1056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^
17 + 2560*a^7*b*c^6*d^2*e^17))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a
^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 -
48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) +
(b^2*(6*a*c - b^2)^2*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4
*c*e)*(4*a^4*b^8*c^2*e^17 - 48*a^5*b^6*c^3*e^17 + 192*a^6*b^4*c^4*e^17 - 25
6*a^7*b^2*c^5*e^17 + 12*a^3*b^9*c^2*d^2*e^17 - 184*a^4*b^7*c^3*d^2*e^17 + 1
056*a^5*b^5*c^4*d^2*e^17 - 2688*a^6*b^3*c^5*d^2*e^17 + 2560*a^7*b*c^6*d^2*e
^17))/(32*a^4*e^2*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4
*b^2*c^2*e^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*
b^2*c^2 - 72*a*b^4*c))))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4
*e^14))*(6*a*c - b^2))/(2*a^2*e*(4*a*c - b^2)^(3/2))
```

3.627 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 348

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad-\frac{\sqrt{c}\left(3b^3-16abc+\left(3b^2-10ac\right)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}\\ &\quad+\frac{\sqrt{c}\left(3b^3-16abc-\left(3b^2-10ac\right)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}\end{aligned}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)
/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e*x+d)*2^
(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*
c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1156, 1135, 1295, 1180, 211}

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= -\frac{\sqrt{c}\left((3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-(3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2e\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{3b^2-10ac}{2a^2e\,(b^2-4ac)\,(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\,(b^2-4ac)\,(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*
(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a
^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 1
6*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-3bcx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{2a\left(b^2-4ac\right)e}\\
&= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-b(3b^2-13ac)-c(3b^2-10ac)x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2a^2\left(b^2-4ac\right)e}\\
&= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4a^2\left(b^2-4ac\right)e}\\
&\quad - \frac{\left(c\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4a^2\left(b^2-4ac\right)e}
\end{aligned}$$

$$= -\frac{3b^2-10ac}{2a^2(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \frac{-\frac{4}{d+ex}+\frac{2(d+ex)(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+ \cdots}{4a^2e}$$

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\frac{ce^2(2ac-b^2)x^3}{8ac-2b^2}+\frac{3dce(2ac-b^2)x^2}{2(4ac-b^2)}+\frac{(6ac^2d^2-3b^2cd^2+3abc-b^3)x}{8ac-2b^2}+\frac{d(2ac^2d^2-b^2cd^2+3abc-b^3)}{2e(4ac-b^2)}}{cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(ce^4\_Z^4+4cde^3\_Z^3+(6c\cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2*a*c-b^2)/(4*a*c
-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2)*x+1/2*d/e*(
2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*
c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2
)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*d*c*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3
*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^
3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e
^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a^2/e/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4330 vs. $2(300) = 600$.

Time = 0.45 (sec) , antiderivative size = 4330, normalized size of antiderivative = 12.44

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c
+ 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b
^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5
- 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 25
00*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*
b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5
*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*
```

c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*

```
a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2
)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7
- 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4
*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3)*e^2))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2
500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*
a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 -
1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*sqrt((81*b^8 - 918
*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) + (27*b^11 - 486*a*b^9
*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*
c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5
*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12
*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))))/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 +
 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c
 - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 -
 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2
*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + (3*
b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e^2
*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2 + 2*(2*(3*b^2*c - 10*a*c^
2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a
^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*
a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*
(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3
- 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e) - 1/2*integrate(((3*b^2*c - 10
*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)*d*e*x + 3*b^3 - 13*a*b*c + (3*b^2*
c - 10*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x
^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*
x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e*x), x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(300) = 600.

Time = 0.35 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= -\frac{\frac{b^2c}{(ex+d)e}-\frac{2\,ac^2}{(ex+d)e}+\frac{b^3}{(ex+d)^3e}-\frac{3\,abc}{(ex+d)^3e}}{2\,(a^2b^2-4\,a^3c)\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}-\frac{1}{(ex+d)a^2e}$$
$$+\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}a}e^4+2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab+2\,\sqrt{b^2}}\right.}{}$$
$$-\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}a}e^4-2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab-2\,\sqrt{b}}\right.}{}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c/((e*x + d)*e) - 2*a*c^2/((e*x + d)*e) + b^3/((e*x + d)^3*e) - 3
*a*b*c/((e*x + d)^3*e))/((a^2*b^2 - 4*a^3*c)*(c + b/(e*x + d)^2 + a/(e*x +
d)^4)) - 1/((e*x + d)*a^2*e) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3
*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4 + 2*(3*a^3*b^2
*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*
abs(a^2*b^2*e^2 - 4*a^3*c*e^2) - (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*b^3 - 13*
```

```
a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x + d)*e
*sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 -
 4*(a^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a
^4*c*e^4))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*e^3*abs(a^2*b^2*e^2
 - 4*a^3*c*e^2)*abs(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2
- 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4 - 2*(3*a^3*b^2*c -
10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*abs(a
^2*b^2*e^2 - 4*a^3*c*e^2) - (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*b^3 - 13*a*b*c
)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x + d)*e*sqrt
((a^2*b^3*e^2 - 4*a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a
^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*
e^4))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*e^3*abs(a^2*b^2*e^2 - 4*
a^3*c*e^2)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 10556, normalized size of antiderivative = 30.33

$$\int \frac{1}{(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 +
240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^
10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 -
3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 +
 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*
a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-
(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c
^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^
8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5
*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a
^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 98
3040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*
e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^
```

9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 15
72864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12
 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e
^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c
^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*
b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 4
58752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
+ 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^
4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
+ 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^
2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a
^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 +
144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 1
43360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^1
2) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
c(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6
*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4
*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^1
0*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 32768
0*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14)
 + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^
3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*
a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12
 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12
 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c
^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10
*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568
*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*
a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*
(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30
112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 -
 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 5

1*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11) - (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^

5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*e^10 - 8112*a^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*e^10))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*

c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*
b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^1
0*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2
 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/
2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5
*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11
*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6
*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 384
0*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e
^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4
*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*
b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144
*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e
^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^
14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10
*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 12615
68*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 -
3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*
e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^
2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2
)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*
c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b
^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)
^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*
a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213
*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a
^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^
2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-(9*b^13 + 9*b^
4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3
*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^
2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*
b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 12
80*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*
(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e
^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*
c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*e^13 + 256*a^1
0*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 -
 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3
*c^7*d*e^13) + 851968*a^14*b*c^8*e^12 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^
11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12 - 256000*a^11*b^7*c^5*e^12 + 778240*a

^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c^9*d*e^11 + 14
4*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11
 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2
*c^8*d*e^11) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c
*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 -
6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*
e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6
*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 +
9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656
*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2
 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1
/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 -
 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^
14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b
*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*
a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^
13 - 1572864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c
^2*e^12 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^
7*c^5*e^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800
*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 301
12*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e
^11 - 458752*a^11*b^2*c^8*d*e^11) + 128000*a^10*c^9*e^10 + 504*a^6*b^8*c^5*
e^10 - 8112*a^7*b^6*c^6*e^10 + 48704*a^8*b^4*c^7*e^10 - 129280*a^9*b^2*c^8*
e^10))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 2
40*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^1
0*b^2*c^5*e^2)))^(1/2)*2i - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*
a*b*c*d))/(a*(a*b^2 - 4*a^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(
a*b^2 - 4*a^2*c)) - (2*x^3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*
a^2*c)) + (2*a*b^2 - 8*a^2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*
a*b*c*d^2)/(2*a*e*(a*b^2 - 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*
b^2*c*d^2*e - 11*a*b*c*e))/(2*a*(a*b^2 - 4*a^2*c)))/(a*d + x*(a*e + 3*b*d^2
*e + 5*c*d^4*e) + x^3*(b*e^3 + 10*c*d^2*e^3) + b*d^3 + c*d^5 + x^2*(10*c*d^

```
3*e^2 + 3*b*d*e^2) + c*e^5*x^5 + 5*c*d*e^4*x^4)
```

3.628 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 213

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{b^2-3ac}{a^2(b^2-4ac)\,e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad-\frac{(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}\,e} \\ &\quad-\frac{2b\log(d+ex)}{a^3e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}\end{aligned}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-
4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e-2*
b*ln(e*x+d)/a^3/e+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used

$= \{1156, 1128, 754, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e} - \frac{2b\log(d+ex)}{a^3e}$$
$$- \frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2} - \frac{\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}$$
$$+ \frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2a\left(b^2-4ac\right)e}
\end{aligned}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int\left(\frac{2(-b^2+3ac)}{ax^2}-\frac{2b(-b^2+4ac)}{a^2x}+\frac{2(-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x)}{a^2(a+bx+cx^2)}\right)dx,x,(d+ex)^2\right)}{2a(b^2-4ac)e}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{2b\log(d+ex)}{a^3e} - \frac{\text{Subst}\left(\int\frac{-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{a^3(b^2-4ac)e}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{2b\log(d+ex)}{a^3e} + \frac{b\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{2a^3e} + \frac{(b^4-6ab^2c+6a^2c^2)\text{Subst}\left(\int\frac{1}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{2a^3(b^2-4ac)e}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{2b\log(d+ex)}{a^3e} + \frac{b\log(a+b(d+ex)^2+c(d+ex)^4)}{2a^3e} - \frac{(b^4-6ab^2c+6a^2c^2)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}dx,x,b+2c(d+ex)^2\right)}{a^3(b^2-4ac)e}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}e} - \frac{2b\log(d+ex)}{a^3e} + \frac{b\log(a+b(d+ex)^2+c(d+ex)^4)}{2a^3e}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.33

$$\int\frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2}dx$$

$$= \frac{-\frac{a}{(d+ex)^2}+\frac{a(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2\cdots}\right.}{(b^2-4ac)^{3/2}}\cdots}{2a^3e}$$

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\frac{\frac{eac(2ac-b^2)x^2}{8ac-2b^2}+\frac{cda(2ac-b^2)x}{4ac-b^2}+\frac{a(2ac^2d^2-b^2cd^2+3abc-b^3)}{2e(4ac-b^2)}}{cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_}}{}}{}$
risch	$\frac{-\frac{(3ac-b^2)ce^3x^4}{(4ac-b^2)a^2}-\frac{4(3ac-b^2)cde^2x^3}{(4ac-b^2)a^2}-\frac{(36ac^2d^2-12b^2cd^2+7abc-2b^3)ex^2}{2a^2(4ac-b^2)}-\frac{d(12ac^2d^2-4b^2cd^2+7abc-2b^3)x}{a^2(4ac-b^2)}-\frac{6ac^2d^4-2b^2cd^4+7bd^2ca-2b}{2ea^2(4ac-b^2}}{(ex+d)^2(cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a)}$

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((1/2*e*a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4*a*c-b^2
)*x+1/2/e*a*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1
/(4*a*c-b^2)/e*sum((e^3*b*c*(-4*a*c+b^2)*_R^3+3*d*e^2*b*c*(-4*a*c+b^2)*_R^2
+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a*b*c^2*d^3+b
^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+d*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a^2/e/(e*
x+d)^2-2*b*ln(e*x+d)/a^3/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. $2(205) = 410$.

Time = 0.61 (sec) , antiderivative size = 4562, normalized size of antiderivative = 21.42

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)

```
*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*
c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2)*d^2)*e), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a
^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^
2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2
*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x
 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2
*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*
(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*
d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b
^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3
 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16
*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4
+ 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d
^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e
^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*
c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 +
 (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2
)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8
*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*
a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3
)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b
```

```
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^3}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3 + 2*(b^2*
c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c^2)*d^2)*e^2*x^2 + a
*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4*(b^2*c - 3*a*c^2)*d^3 + (2*b^
3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^7*x^6 + 6*(a^2*b^2*c - 4*a^
3*c^2)*d*e^6*x^5 + (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e
^5*x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*c)*d)*e^4*x^
3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 6*(a^2*b^3 - 4*a^
3*b*c)*d^2)*e^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c^2)*d^5 + 2*(a^2*b^3 - 4*a^3
*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^6 + (
a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*a^4*c)*d^2)*e) + 2*integrate(((b^3*
c - 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c - 4*a*b*c^2)*d*e^2*x^2 + (b^3*c - 4*a*b*c
^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2 + 3*(b^3*c - 4*a*b*c^2)*d^2)*e*x + (
b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c
^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*
a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^3 - 2*b*log(e*x + d)/(a^3*e)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}e}+\frac{b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{2\,a^3e}+\frac{\frac{b^3c-3\,abc^2}{a}+\frac{b^4e-4\,ab^2ce+2\,a^2c^2e}{(ex+d)^2ae}}{2\,(b^2-4\,ac)a^2\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)e}-\frac{1}{2\,(ex+d)^2a^2e}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(e*x + d)^2)/sqrt(-b^2 + 4*a
*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*b*log(c + b/(e*x + d)
^2 + a/(e*x + d)^4)/(a^3*e) + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e - 4*a*b^2
*c*e + 2*a^2*c^2*e)/((e*x + d)^2*a*e))/((b^2 - 4*a*c)*a^2*(c + b/(e*x + d)^
2 + a/(e*x + d)^4)*e) - 1/2/((e*x + d)^2*a^2*e)
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 12436, normalized size of antiderivative = 58.38

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e + 4*b*
d^3*e + 6*c*d^5*e) + x^2*(a*e^2 + 6*b*d^2*e^2 + 15*c*d^4*e^2) + x^3*(20*c*d
^3*e^3 + 4*b*d*e^3) + c*e^6*x^6 + 6*c*d*e^5*x^5) + (log((((b + a^3*e*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*
(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2
*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*
b^2*c^2*d^2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a
*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*(-(b^4 + 6*a^2*c^2
```

- 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e ^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3 *e) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2 *c - 23*a*b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69 *a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x ^2*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^ 3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e ^15*x*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3))*(((b - a^3*e*(-(b^4 + 6*a^2*c ^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*(-(b^4 + 6 *a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b ^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^ 2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2* c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 1 0*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (8*c^3*d*e^ 17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3*e) - (4*c ^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a* b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^ 2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a *c - b^2)^3)/(a^6*(4*a*c - b^2)^3)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c *e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (2*b*log(d + e*x))/(a^3*e) - (atan(((2*a^9*b^6*(4*a *c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e ^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6 *b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(276*a^5*b*c^7*d* e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^1 6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8* c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e ^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^ 8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)* (640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264 *a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12* a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^ 2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3 *b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2 *c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2* (a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30* a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^

7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c
^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*
a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a
^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^
6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9
*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2
)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*
a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 -
 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/
(a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)
))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b
^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))
 - (b*((((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^
17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5
*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a
^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^
3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2
*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*
e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^1
8 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 -
 672*a^9*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4
*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^
5*b^2*c^2*e^2)) - (((8*(276*a^5*b*c^7*d*e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^
3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6
*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3
*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*
c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c
^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^
6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e +
48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6
*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*
b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(6
40*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a

^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/(a^9*e^3*(4*a*c - b^2)^(9/2)*(
a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3
 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + x^2*((((4*(54
*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^6*e^16 - 54*a^2*b^2*c^7*e^16))/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^
7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*b^3*c^6*e^17))/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(480*a^8*c^7*
e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^
7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (
2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^
6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 -
672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2))
)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e
^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*
a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^
3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((((4*(480*a^8*c^7*e^18
- a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2
*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^
7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^1
9 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a
^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(
a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^
4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c
^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*
(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b
^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 -
64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2
*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*
a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6
*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^
5*e^19))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c +
48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b
^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2
*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 7
2*a*b^6*c)) - (b*((((((4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c
^4*e^18 + 30*a^6*b^4*c^5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^
3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^
5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*
b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64
*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 1
2*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a
^3*e*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*

b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*e^19 + 3*a^6*b^9
*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^
19))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c
^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a
^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (((
4*(276*a^5*b*c^7*e^17 - 6*a^2*b^7*c^4*e^17 + 65*a^3*b^5*c^5*e^17 - 233*a^4*
b^3*c^6*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((
4*(480*a^8*c^7*e^18 - a^4*b^8*c^3*e^18 + 6*a^5*b^6*c^4*e^18 + 30*a^6*b^4*c^
5*e^18 - 272*a^7*b^2*c^6*e^18))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)
*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*b^7*c^3*e^19 + 264*a^8*
b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^
5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)
)/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)
))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6
*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b*c^6*e^19 + 3*a^6*b^9*c^2*e^19 - 46*a^7*
b^7*c^3*e^19 + 264*a^8*b^5*c^4*e^19 - 672*a^9*b^3*c^5*e^19))/(2*a^9*e^3*(4*
a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(
3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)
^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)
)) + (((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 - 45*a^3*b^6*c^4*e^15 + 1
70*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*b^7*c^4*d^2*e^15 - 65*a^
3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276*a^5*b*c^7*d^2*e^15))/(a
^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(96*a^8*b*c^6*e
^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208
*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6
*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b
^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c
^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^
17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 -
46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^
17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c
^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a
^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7
*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 6
4*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (4*(2*b^7*c^4*e^1
4 - 20*a*b^5*c^5*e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^
8*d^2*e^14 + 2*b^6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^
2*e^14))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((((4*(9
6*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c
^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^1
6 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*

e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(2*a^6*e^2*(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15 + 4*a^2*b^8*c^3*e^15 - 45*a^3*b^6*c^4*e^15 + 170*a^4*b^4*c^5*e^15 - 225*a^5*b^2*c^6*e^15 + 6*a^2*b^7*c^4*d^2*e^15 - 65*a^3*b^5*c^5*d^2*e^15 + 233*a^4*b^3*c^6*d^2*e^15 - 276*a^5*b*c^7*d^2*e^15))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(3/2)) - (((((4*(96*a^8*b*c^6*e^16 + 2*a^4*b^9*c^2*e^16 - 26*a^5*b^7*c^3*e^16 + 118*a^6*b^5*c^4*e^16 - 208*a^7*b^3*c^5*e^16 - 480*a^8*c^7*d^2*e^16 + a^4*b^8*c^3*d^2*e^16 - 6*a^5*b^6*c^4*d^2*e^16 - 30*a^6*b^4*c^5*d^2*e^16 + 272*a^7*b^2*c^6*d^2*e^16))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (2*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2

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*e^17 - 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/((a^6*b^6 - 64
*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 1
2*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a
^3*e*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a^2*
b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(a^7*b^8*c^2*e^17 - 12*a^8*b^6*c
^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*a^6*b^9*c^2*d^2*e^
17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 - 672*a^9*b^3*c^5*d
^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(a^3*e*(4*a*c - b^2)^(3/2)*(a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b
^5*c*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e
^2 + 48*a^5*b^2*c^2*e^2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e
^17 - 12*a^8*b^6*c^3*e^17 + 48*a^9*b^4*c^4*e^17 - 64*a^10*b^2*c^5*e^17 + 3*
a^6*b^9*c^2*d^2*e^17 - 46*a^7*b^7*c^3*d^2*e^17 + 264*a^8*b^5*c^4*d^2*e^17 -
 672*a^9*b^3*c^5*d^2*e^17 + 640*a^10*b*c^6*d^2*e^17))/(2*a^9*e^3*(4*a*c - b
^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 -
 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*
(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36
*a^4*c^6*e^14 + b^8*c^2*e^14 - 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72
*a^3*b^2*c^5*e^14))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(a^3*e*(4*a*c - b^2)^(3/
2))
```

3.629 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 408

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{5b^2-14ac}{6a^2(b^2-4ac)\,e(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3(b^2-4ac)\,e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a^3/(
-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d
)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-19*a*c+5*b^2
)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a
^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1156, 1135, 1295, 1180, 211}

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$= \frac{b(5b^2-19ac)}{2a^3e\left(b^2-4ac\right)(d+ex)} - \frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}$$
$$+ \frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+ \frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*
c))/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*
a*(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[
c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4
- 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2
 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,e(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-5b^2+14ac-5bcx^2}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{2a\,(b^2-4ac)\,e}\\
&= -\frac{5b^2-14ac}{6a^2\,(b^2-4ac)\,e(d+ex)^3}+\frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,e(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-3b(5b^2-19ac)-3c(5b^2-14ac)x^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{6a^2\,(b^2-4ac)\,e}\\
&= -\frac{5b^2-14ac}{6a^2\,(b^2-4ac)\,e(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3\,(b^2-4ac)\,e(d+ex)}\\
&\quad+\frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,e(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-3(5b^4-24ab^2c+14a^2c^2)-3bc(5b^2-19ac)x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{6a^3\,(b^2-4ac)\,e}
\end{aligned}
$$

$$= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) e(d + ex)^3} + \frac{b(5b^2 - 19ac)}{2a^3 (b^2 - 4ac) e(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a (b^2 - 4ac) e(d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\left(c\left(5b^4 - 29ab^2c + 28a^2c^2 - b(5b^2 - 19ac)\sqrt{b^2 - 4ac}\right)\right) \text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, d + ex\right)}{4a^3 (b^2 - 4ac)^{3/2} e} + \frac{\left(c\left(5b^4 - 29ab^2c + 28a^2c^2 + b(5b^2 - 19ac)\sqrt{b^2 - 4ac}\right)\right) \text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, d + ex\right)}{4a^3 (b^2 - 4ac)^{3/2} e}$$

$$= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) e(d + ex)^3} + \frac{b(5b^2 - 19ac)}{2a^3 (b^2 - 4ac) e(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a (b^2 - 4ac) e(d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)} + \frac{\sqrt{c}\left(5b^4 - 29ab^2c + 28a^2c^2 + b(5b^2 - 19ac)\sqrt{b^2 - 4ac}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}e} - \frac{\sqrt{c}\left(5b^4 - 29ab^2c + 28a^2c^2 - b(5b^2 - 19ac)\sqrt{b^2 - 4ac}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}e}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d + ex)^4 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

$$= \frac{-\frac{4a}{(d+ex)^3} + \frac{24b}{d+ex} + \frac{6(d+ex)(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + \frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4}\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3e}$$

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\frac{-\frac{bc\,e^2\left(3ac-b^2\right)x^3}{2\left(4ac-b^2\right)}-\frac{3dbce\left(3ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(-9b\,c^2d^2a+3b^3c\,d^2+2a^2c^2-4a\,b^2c+b^4\right)x}{8ac-2b^2}+\frac{d\left(-3b\,c^2d^2a+b^3c\,d^2+2a^2c^2-4a\,b^2c+b^4\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\_R=\text{Root}}{}}{}$
risch	Expression too large to display

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*c-b^2)/(
4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*
a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*
c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^
4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*b^2)*_R^2+2*
b*c*d*e*(-19*a*c+5*b^2)*_R-19*b*c^2*d^2*a+5*b^3*c*d^2+14*a^2*c^2-24*a*b^2*c
+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5734 vs. 2(358) = 716.

Time = 0.64 (sec) , antiderivative size = 5734, normalized size of antiderivative = 14.05

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] Too large to display
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(ex+d)^4}\,dx$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^5
 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4*x
^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (15*
b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c
^2)*d^4 + (45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^
4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3
 - 4*a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a*b^2*
c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b^2*c - 4*a^4*c^
2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 2
1*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 +
 (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*
a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a
^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x
^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*
b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4
*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e) + 1/2*integrate(((5*b^3*c - 19*a*b*
c^2)*e^2*x^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + 2*(5*b^3*c - 19*a*b*c^2)*d
*e*x + (5*b^3*c - 19*a*b*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c -
4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*
a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c -
 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. 2(358) = 716.

Time = 0.36 (sec) , antiderivative size = 2122, normalized size of antiderivative = 5.20

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)^2} dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24
*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (5*b^3*c*e^2*
(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 - 19*a*b
*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2
 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
 d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c
^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^
3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*
d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^
3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e -
 b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)) - (5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^
2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - s
```

```
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e
^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)))/(a^3*b^2 - 4*a^4*c) + 1/2*(b^3*c*e^3*x^3 - 3*a*b*c^2*e^3*x^3 + 3*b^3*c*
d*e^2*x^2 - 9*a*b*c^2*d*e^2*x^2 + 3*b^3*c*d^2*e*x - 9*a*b*c^2*d^2*e*x + b^3
*c*d^3 - 3*a*b*c^2*d^3 + b^4*e*x - 4*a*b^2*c*e*x + 2*a^2*c^2*e*x + b^4*d -
4*a*b^2*c*d + 2*a^2*c^2*d)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 +
4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(a^3*b^2*e - 4*a^4
*c*e)) + 1/3*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 - a)/((e*x + d)^3*a^3*e)
```

Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 12239, normalized size of antiderivative = 30.00

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9
*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^
2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^
7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 2197
44*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c
*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2
- 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 16
5*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 -
 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^1
1*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14
+ 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^1
4 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*
c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^1
6*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13
+ 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c
^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^1
```

0*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200
*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12
- 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^1
2 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4
*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*
b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 -
488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4
*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*
a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*
e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 -
25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 357
67*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c
^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^
12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280
*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*
((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*
b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 21
5040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246
*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*
c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5
e^2)))^(1/2)(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a
^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 98
3040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*
e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^
9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 15
72864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12
+ 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*
e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18
*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^
10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 14
58688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*
e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*
c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458
688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^
9*d*e^11)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2

+ 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 614
4*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) -
 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c
^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a
^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4
*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b
^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11
6928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(
4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*
c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 +
 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*
c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^
9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*
a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 -
 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^
5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 9175
04*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816
*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12
- 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c
^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^1
0*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280
*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 +
 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*
d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280
*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 - 25*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b
^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 +
 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^
6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b
^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6
*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2
c^2(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2
- 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^
(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^
5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^

9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11) + 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a^10*b^7*c^7*e^10 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5*b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2)) + (2*x^3*(15*b^4*d*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*d*e^2 - 285*a*b*c^2*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 + 45*b^3*c*d^5 + 28*a^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3 - 171*a*b*c^2*d^5))/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b*c^2*e^5))/(2*a*(4*a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84*a^2*c^2*d^2*e - 40*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a*b*c^2*d^4*e))/(6*a*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^4 + 10*a*b^3*d^2 + 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^2*c*d^4 - 57*a*b*c^2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x^2*(10*b*d^3*e^2 + 21*c*d^5*e^2 + 3*a*d*e^2) + x^5*(b*e^5 + 21*c*d^2*e^5) + a*d^3 + b*d^5 + c*d^7 + x^3*(a*e^3 + 10*b*d^2*e^3 + 35*c*d^4*e^3) + x^4*(35*c*d^3*e^4 + 5*b*d*e^4) + x*(3*a*d^2*e + 5*b*d^4*e + 7*c*d^6*e) + c*e^7*x^7 + 7*c*d*e^6*x^6) + atan(((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a

^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 +
 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b
^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*
c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a
c - b^2)^9)^(1/2))/(32(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e
^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 -
6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*
c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*
b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048
576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^1
3 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^
5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^
12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468
480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^
12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c
^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*
b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871
872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11
- 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*
c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 18
71872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2
) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^
7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2
)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*
a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 2
4*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*
b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c
- b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 +
 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^
13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^
2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25
b^6(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6
 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a
^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x
*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^1

4 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i)/((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*

e^11 + 1871872*a^15*b^2*c^9*d*e^11) - (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
- 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e
^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840
*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 + 25*b^6*(-(
4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9
*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^
3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4
096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*
c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^1
5 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 -
 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b
^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c
^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 -
1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1
/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c
^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*
b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256
*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^
13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20
*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^1
3*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 153
6000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e
^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^
4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13
*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 40
1408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11
 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b
^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)
+ 476672*a^13*b*c^10*e^10 + 1800*a^9*b^9*c^6*e^10 - 29080*a^10*b^7*c^7*e^10
 + 176032*a^11*b^5*c^8*e^10 - 473216*a^12*b^3*c^9*e^10))*(-(25*b^15 + 25*b^
6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^
3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 -
49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10
*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*2i

3.630 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\ &\quad - \frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} \\ &\quad - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}\end{aligned}$$

```
[Out] 1/4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-
1/8*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+
c*(e*x+d)^4)+3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1134, 1192, 1180, 211}

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$
$$= \frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\,(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{(d+ex)\,(2a+b(d+ex)^2)}{4e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)\,(-4ac+7b^2+12bc(d+ex)^2)}{8e\,(b^2-4ac)^2\,(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2
 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*
a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}\\
&= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)e}\\
&= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\
&\quad - \frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{3a(b^2+4ac)-12abcx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{8a\left(b^2-4ac\right)^2e}\\
&= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad + \frac{\left(3c\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{8\left(b^2-4ac\right)^{5/2}e}\\
&\quad - \frac{\left(3c\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{8\left(b^2-4ac\right)^{5/2}e}
\end{aligned}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)\left(-7b^2+4ac-12bc(d+ex)^2\right)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \cdots}{8e}$$

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2
 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^
2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.06

method	result
default	$\dfrac{-\frac{3c^2e^6bx^7}{2(16a^2c^2-8ab^2c+b^4)}-\frac{21c^2de^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}+\frac{(-252bcd^2+4ac-19b^2)ce^4x^5}{128a^2c^2-64ab^2c+8b^4}+\frac{5cde^3(-84bcd^2+4ac-19b^2)x^4}{8(16a^2c^2-8ab^2c+b^4)}-\frac{e^2(420bc^2d^4-40ac^2d^2+}{8(16a^2c^2-}}{(cx}$
risch	$\dfrac{-\frac{3c^2e^6bx^7}{2(16a^2c^2-8ab^2c+b^4)}-\frac{21c^2de^5bx^6}{2(16a^2c^2-8ab^2c+b^4)}+\frac{(-252bcd^2+4ac-19b^2)ce^4x^5}{128a^2c^2-64ab^2c+8b^4}+\frac{5cde^3(-84bcd^2+4ac-19b^2)x^4}{8(16a^2c^2-8ab^2c+b^4)}-\frac{e^2(420bc^2d^4-40ac^2d^2+}{8(16a^2c^2-}}{(cx}$

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*
c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*
a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c*
d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d
^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6633 vs. $2(293) = 586$.

Time = 0.45 (sec) , antiderivative size = 6633, normalized size of antiderivative = 19.45

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{(ex+d)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] -1/8*(12*b*c^2*e^7*x^7 + 84*b*c^2*d*e^6*x^6 + (252*b*c^2*d^2 + 19*b^2*c - 4
*a*c^2)*e^5*x^5 + 12*b*c^2*d^7 + 5*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d)*
e^4*x^4 + (420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*
e^3*x^3 + (19*b^2*c - 4*a*c^2)*d^5 + (252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^
2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + (5*b^3 + 16*a*b*c)*d^3 + (84*b*c
^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*
b*c)*d^2)*e*x + 3*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^
7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*
x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a
*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e) - 3/8*integrate((4*b*c*e^2
*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2 - 4*a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4
 - 8*a*b^2*c + 16*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(293) = 586.

Time = 0.37 (sec) , antiderivative size = 1802, normalized size of antiderivative = 5.28

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
[Out] 3/16*((4*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)^2 - 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
- (4*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/
e)^2 + 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
- d/e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*
e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (4
*b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2
 - 8*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/
e) + 4*b*c*d^2 - b^2 - 4*a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)
*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (4*b*c
*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 8
*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) +
 4*b*c*d^2 - b^2 - 4*a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(b^4 - 8*a*
b^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*e^7*x^7 + 84*b*c^2*d*e^6*x^6 + 252*b*c^
2*d^2*e^5*x^5 + 420*b*c^2*d^3*e^4*x^4 + 420*b*c^2*d^4*e^3*x^3 + 19*b^2*c*e^
5*x^5 - 4*a*c^2*e^5*x^5 + 252*b*c^2*d^5*e^2*x^2 + 95*b^2*c*d*e^4*x^4 - 20*a
*c^2*d*e^4*x^4 + 84*b*c^2*d^6*e*x + 190*b^2*c*d^2*e^3*x^3 - 40*a*c^2*d^2*e^
3*x^3 + 12*b*c^2*d^7 + 190*b^2*c*d^3*e^2*x^2 - 40*a*c^2*d^3*e^2*x^2 + 95*b^
2*c*d^4*e*x - 20*a*c^2*d^4*e*x + 5*b^3*e^3*x^3 + 16*a*b*c*e^3*x^3 + 19*b^2*
c*d^5 - 4*a*c^2*d^5 + 15*b^3*d*e^2*x^2 + 48*a*b*c*d*e^2*x^2 + 15*b^3*d^2*e*
```

x + 48*a*b*c*d^2*e*x + 5*b^3*d^3 + 16*a*b*c*d^3 + 3*a*b^2*e*x + 12*a^2*c*e*
x + 3*a*b^2*d + 12*a^2*c*d)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 +
 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)^2*(b^4*e - 8*a*b^
2*c*e + 16*a^2*c^2*e))

Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 12677, normalized size of antiderivative = 37.18

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out] atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 1
5360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28672*
a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^
5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7
*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^
6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b
^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b
^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a
^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*
((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
- 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c
^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c
*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^
2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*
e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*
a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4
*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^
12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) -
 b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b
^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76

80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3
*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 40
96*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a
^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 30
72*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786
432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^
15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 34406
4*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*
e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e
^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^
14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 25
6*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920
*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024
*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576
*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c
^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*
c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*
b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c
^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5
*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12)
)/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))
((9((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
- 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*
c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*
c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e
^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7
*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i)/((13
5*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(64*(b^12 + 4096
*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5
*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 307
2*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 7864
32*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^1
5*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064
*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e
^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12

+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6
144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^
14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^1
4 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*
a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*
a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*
a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^
3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c
^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b
^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^
7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*
c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^1
0*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 -
1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e
^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d
*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
+ (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a
*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*
b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b
^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + ((18432*a^4*c^7*d*
e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^1
1)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^
4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*
b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*
b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^
10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*
c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175
040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8
*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14
- 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81
920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 9
6*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1
/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*
a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*
b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2
- 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2

+ 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e
^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^
15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*
c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2
 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*
a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86016
0*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262
1440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 +
72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^
5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*
e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53
760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1
966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2
)))^(1/2)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a
^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 614
40*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 4
0*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5
*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*
a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/
2)*2i + atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*
e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^
7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13
- 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4
*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 2202009
6*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c
^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a
^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^
4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
- 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 -
560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5
+ 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^
2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 5376
0*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196
6080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2))
)^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2
*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440
*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*
a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b
^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^
8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)
 + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 115

20*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a
^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*
a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^
4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c
- b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 -
11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 +
936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128
*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c
^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^
2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6
*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^
2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
- ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e
^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4
*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13
)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4
*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*
a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3
*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b
^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(
1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^
7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e
^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 768
0*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860
160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2
621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) -
81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 -
1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 10
48576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b
^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7
*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*
a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*
b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 56
0*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 +
61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2
- 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*
a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 19660
80*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^
(1/2)*1i)/((135*b^5*c^3*e^10 + 1080*a*b^3*c^4*e^10 + 432*a^2*b*c^5*e^10)/(6
4*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*

c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (((786432*a^6*c^8*e^12 - 192*b^12*
c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c
^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*
c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)
) + ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d
*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a
^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^
13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a
^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 256
0*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a
^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)
^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*
b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20
*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7
680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 8
60160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2)
- 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4
- 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4
*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a
^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144
0*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 -
6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6
+ 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
- 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e
^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*
c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^1
1*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6
*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*
b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*
c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^
6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (
(18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*
a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*
b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^
6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e
^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13
- 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*
c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 293
60128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 128
0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(1

28*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^2*(15*b^3*d*e - 40*a*c^2*d^3*e + 190*b^2*c*d^3*e + 252*b*c^2*d^5*e + 48*a*b*c*d*e))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(5*b^3*e^2 - 40*a*c^2*d^2*e^2 + 190*b^2*c*d^2*e^2 + 420*b*c^2*d^4*e^2 + 16*a*b*c*e^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(84*b*c^2*d^3*e^3 - 4*a*c^2*d*e^3 + 19*b^2*c*d*e^3))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(19*b^2*c*e^4 - 4*a*c^2*e^4 + 252*b*c^2*d^2*e^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2 + 12*a^2*c + 15*b^3*d^2 - 20*a*c^2*d^4 + 95*b^2*c*d^4 + 84*b*c^2*d^6 + 48*a*b*c*d^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3 - 4*a*c^2*d^5 + 19*b^2*c*d^5 + 12*b*c^2*d^7 + 3*a*b^2*d + 12*a^2*c*d + 16*a*b*c*d^3)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d

```
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

3.631 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3bc\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2} e}$$

```
[Out] 1/4*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*b*(b
+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1156, 1128, 652, 628, 632, 212}

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{3bc\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}} + \frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{4(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{4(b^2-4ac)\,e}$$

$$= \frac{2a+b(d+ex)^2}{4(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(3bc)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2(b^2-4ac)^2\,e}$$

$$= \frac{2a+b(d+ex)^2}{4(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(3bc)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{(b^2-4ac)^2\,e}$$

$$= \frac{2a+b(d+ex)^2}{4(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}\,e}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2} - \frac{12bc\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2\,e}$$

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 -
 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (1
2*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])
/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.63

method	result
default	$\frac{-\frac{3c^2e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3(10c\,d^2+b)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}-\frac{3cd\,e^2b(10c\,d^2+3b)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be(45c^2d^4+27bc\,d^2+5ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{db(9c^2d^4+\cdots}{16a^2c^2\cdots}}{(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a)^2}$
risch	$\frac{-\frac{3c^2e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3(10c\,d^2+b)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}-\frac{3cd\,e^2b(10c\,d^2+3b)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be(45c^2d^4+27bc\,d^2+5ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{db(9c^2d^4+\cdots}{16a^2c^2\cdots}}{(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a)^2}$

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*
c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4
+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*
d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+
10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4
*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. 2(142) = 284.

Time = 0.46 (sec) , antiderivative size = 3739, normalized size of antiderivative = 24.93

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5
 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*
```

c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4

```
*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 12*(b*c^3*e^8*x^8
 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(
14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^
2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 +
 (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 +
 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^
2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a
*b*c^2)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. 2(134) = 268.

Time = 7.15 (sec) , antiderivative size = 1671, normalized size of antiderivative = 11.14

$$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3
```

```
6*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)*
*5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/(4*a
*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)
**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*sqrt(
-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*
b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*c*d**2 -
 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x**5 - 6*b*
c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x**3*(-36*b*
*2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b**3*e**2 -
54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e - 4*b**3*d*e
- 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e
+ 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b
**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2
*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4
*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e*
*9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7
 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e*
*7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**
6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d*
*3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3
*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c
*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**
5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d
*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c
*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6
*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*
b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c
**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**
2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d*
*2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c
**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*
c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d
**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*
d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{(ex+d)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] -3*b*c*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a
^2*c^2) - 1/4*(6*b*c^2*e^6*x^6 + 36*b*c^2*d*e^5*x^5 + 6*b*c^2*d^6 + 9*(10*b
*c^2*d^2 + b^2*c)*e^4*x^4 + 9*b^2*c*d^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3
*x^3 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*x^2 + a*b^2 + 8*
a^2*c + 2*(b^3 + 5*a*b*c)*d^2 + 4*(9*b*c^2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b
*c)*d)*e*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 +
 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6
*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e
^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(142) = 284$.

Time = 0.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.33

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3\,bc\arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}} - \frac{6\,bc^2d^6+18\,(ex^2+2\,dx)bc^2d^4e+18\,(ex^2+2\,dx)^2bc^2d^2e^2+6\,(ex^2+2\,dx)^3bc^2e^3+9\,b^2cd^4+18\,(ex^2+2}{4\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2-\right.}$$

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b
^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) - 1/4*(6*b*c^2*d^6 + 18*
(e*x^2 + 2*d*x)*b*c^2*d^4*e + 18*(e*x^2 + 2*d*x)^2*b*c^2*d^2*e^2 + 6*(e*x^2
 + 2*d*x)^3*b*c^2*e^3 + 9*b^2*c*d^4 + 18*(e*x^2 + 2*d*x)*b^2*c*d^2*e + 9*(e
*x^2 + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(e*x^2 + 2*d*x)*b^
3*e + 10*(e*x^2 + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/((c*d^4 + 2*(e*x^2 + 2*
d*x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e + a)^2
*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 1182, normalized size of antiderivative = 7.88

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$

$$-\frac{\frac{9\,x^4\,\left(b^2\,c\,e^3+10\,b\,c^2\,d^2\,e^3\right)}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\cdots}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3\cdots\right.}$$

$$-\frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\,\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\,\left(4\,a\,c-b^2\right)^5\right)\,\left(x^2\,\left(\frac{9\,b^2\,c^4\,e^8}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^3\,c^2\,\left(32\,a^2\,b\,c^4\,e^{10}-16\cdots\right.}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(\cdots\right.}\right.\right.}{\cdots}\right)}{\cdots}$$

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d^2
)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e + 45*
b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*
b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3 +
 9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*
b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x^5)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8)/
(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*
c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((18*b^2*c^4*d*e^7)/(a*(4*a*c -
b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*c^2*d*e^9 -
16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^3*c^2*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8
- 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*
c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (9*b^2*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
))))/(18*b^2*c^4*e^6)))/(e*(4*a*c - b^2)^(5/2))
```

3.632 $\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 363

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx = -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2)}{8a(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] -1/4*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2
+1/8*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2)^2/e/(a
+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/
(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan((e*x+d)*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b*(-52*a*c
+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1133, 1192, 1180, 211}

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\sqrt{c}\left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}} - \frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(c(20ac+b^2)(d+ex)^2+b(8ac+b^2)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e
*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt
[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*
c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e} \\
&= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{\text{Subst}\left(\int \frac{b-10cx^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)e} \\
&= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad-\frac{\text{Subst}\left(\int \frac{-b(b^2-16ac)-c(b^2+20ac)x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{8a\left(b^2-4ac\right)^2e} \\
&= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad+\frac{\left(c\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{16a\left(b^2-4ac\right)^2e} \\
&\quad+\frac{\left(c\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{16a\left(b^2-4ac\right)^2e}
\end{aligned}
$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2(d+ex)\left(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2\right)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\text{ar}}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16e}$$

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c
^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
(Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 885, normalized size of antiderivative = 2.44

method	result
default	$\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)x^4}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{e^2(}{}$
risch	$\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)x^4}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{e^2(}{}$

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(2
0*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d^2
+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140*a*
c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^
2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*
c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*d^4+1
0*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4
+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2
*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*s
um((c*e^2*(20*a*c+b^2)*_R^2+2*d*c*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-
16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7701 vs. $2(319) = 638$.

Time = 0.62 (sec) , antiderivative size = 7701, normalized size of antiderivative = 21.21

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(ex+d)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*e^7*x^7 + 7*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 + (2*b
^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 5*(7*(b^2*c^2 +
20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + (b^2*c^2 + 20*a*c^3)*d^
7 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c
 + 14*a*b*c^2)*d^2)*e^3*x^3 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (21*(b^2*c^2 + 2
0*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^
2)*d)*e^2*x^2 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + (7*(b^2*c^2 + 20*a*c^3
)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2
*c + 36*a^2*c^2)*d^2)*e*x - (a*b^3 - 16*a^2*b*c)*d)/((a*b^4*c^2 - 8*a^2*b^2
*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e
^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*
a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^
4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x
 + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e
) + 1/8*integrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c^2)*d*e*x
+ b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d
```

```
^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(319) = 638$.

Time = 0.34 (sec) , antiderivative size = 2447, normalized size of antiderivative = 6.74

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*((b^2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 + 20*a*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)^2 - 2*b^2*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e) - 40*a*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e) + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(
x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^
4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*
d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 -
2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 20*a*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^
2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d*e*(sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c*d^2 + 20*a*c^
2*d^2 + b^3 - 16*a*b*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b^2*c*e^2*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 + 20*a*c^
2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 -
2*b^2*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e
) - 40*a*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e) + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(x + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sq
rt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e +
 (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e)) - (b^2*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) - d/e)^2 + 20*a*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
```

```
- 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*
b*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e
)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2) + 1/8*(b^2*c^2*e^7*x^7 + 20*a*c^3*e^7*x^7 + 7*b^2*c^2*d*e^6*x^6 + 140*a*
c^3*d*e^6*x^6 + 21*b^2*c^2*d^2*e^5*x^5 + 420*a*c^3*d^2*e^5*x^5 + 35*b^2*c^2
*d^3*e^4*x^4 + 700*a*c^3*d^3*e^4*x^4 + 35*b^2*c^2*d^4*e^3*x^3 + 700*a*c^3*d
^4*e^3*x^3 + 2*b^3*c*e^5*x^5 + 28*a*b*c^2*e^5*x^5 + 21*b^2*c^2*d^5*e^2*x^2
+ 420*a*c^3*d^5*e^2*x^2 + 10*b^3*c*d*e^4*x^4 + 140*a*b*c^2*d*e^4*x^4 + 7*b^
2*c^2*d^6*e*x + 140*a*c^3*d^6*e*x + 20*b^3*c*d^2*e^3*x^3 + 280*a*b*c^2*d^2*
e^3*x^3 + b^2*c^2*d^7 + 20*a*c^3*d^7 + 20*b^3*c*d^3*e^2*x^2 + 280*a*b*c^2*d
^3*e^2*x^2 + 10*b^3*c*d^4*e*x + 140*a*b*c^2*d^4*e*x + b^4*e^3*x^3 + 5*a*b^2
*c*e^3*x^3 + 36*a^2*c^2*e^3*x^3 + 2*b^3*c*d^5 + 28*a*b*c^2*d^5 + 3*b^4*d*e^
2*x^2 + 15*a*b^2*c*d*e^2*x^2 + 108*a^2*c^2*d*e^2*x^2 + 3*b^4*d^2*e*x + 15*a
*b^2*c*d^2*e*x + 108*a^2*c^2*d^2*e*x + b^4*d^3 + 5*a*b^2*c*d^3 + 36*a^2*c^2
*d^3 - a*b^3*e*x + 16*a^2*b*c*e*x - a*b^3*d + 16*a^2*b*c*d)/((c*e^4*x^4 + 4
*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*
x + b*d^2 + a)^2*(a*b^4*e - 8*a^2*b^2*c*e + 16*a^3*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 14584, normalized size of antiderivative = 40.18

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^5*(2*b^3*c*e^4 + 420*a*c^3*d^2*e^4 + 21*b^2*c^2*d^2*e^4 + 28*a*b*c^2*e^
4))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^4*d*e + 21*b^2*c^2*d^5
*e + 108*a^2*c^2*d*e + 420*a*c^3*d^5*e + 20*b^3*c*d^3*e + 280*a*b*c^2*d^3*e
 + 15*a*b^2*c*d*e))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(b^2*c^2*
d*e^5 + 20*a*c^3*d*e^5))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(20*a*
c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*b^4*d^
2 - a*b^3 + 140*a*c^3*d^6 + 10*b^3*c*d^4 + 108*a^2*c^2*d^2 + 7*b^2*c^2*d^6
+ 16*a^2*b*c + 15*a*b^2*c*d^2 + 140*a*b*c^2*d^4))/(8*a*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^3*(b^4*e^2 + 36*a^2*c^2*e^2 + 700*a*c^3*d^4*e^2 + 20*b^3*c
*d^2*e^2 + 35*b^2*c^2*d^4*e^2 + 5*a*b^2*c*e^2 + 280*a*b*c^2*d^2*e^2))/(8*a*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3 + 20*a*c^3*d^7 + 2*b^3*c*d^5 + 3
6*a^2*c^2*d^3 + b^2*c^2*d^7 - a*b^3*d + 16*a^2*b*c*d + 5*a*b^2*c*d^3 + 28*a
*b*c^2*d^5)/(8*a*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(140*a*c^3*d^3*
e^3 + 7*b^2*c^2*d^3*e^3 + 2*b^3*c*d*e^3 + 28*a*b*c^2*d*e^3))/(8*a*(b^4 + 16
```

*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c

^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d
*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^
12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 38
40*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b
*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b
^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^
2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3
+ 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 409
6*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d
*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384
*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a
^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*
c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^1
4 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^1
4 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 9
6*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2)
- 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^
9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^
13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*
e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6
*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b
^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*
b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*
a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a
c(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 -
40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^
7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080
*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^
(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*
b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 -
256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b
*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a
^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*
c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 4
0*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7
*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*
a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(
1/2)*1i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*
a^2*b^2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*
a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*
a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 550
5024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*
a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((

67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*
d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 3670016
0*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*
d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1
280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*
c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^
4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 +
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10
160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c
^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/
(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^
16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^
10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^
11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c
^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a
^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20
*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7
680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 8
60160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2
- 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3
*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*
c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*
a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a
^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 14
72*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^
4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 172032
0*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 +
43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
- 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*
e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53
760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1
966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e
^2)))^(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*
d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*
b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b
^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3
*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 550502
4*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4
*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((671
08864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e
^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a
^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e

^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280 *a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7 *e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e ^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 25 6*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^ 2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160 *a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(51 2*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16* c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10* c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11* b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b ^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7* b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^ 2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680 *a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 8601 60*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 3 4*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16* a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^ 2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b ^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680* a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 86016 0*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2 621440*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^ 9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a *b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^ 13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3* e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6 *e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b ^2*c^9*e^2)))^(1/2)*2i + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e ^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5 *e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840 *a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2* b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b ^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51 20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32

7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
e^2)))^(1/2))(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
+ 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^
7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)
+ (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 2816
0*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^1
1)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a
^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 -
 b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 +
256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 -
b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 101
60*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^
6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(
512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^1
6*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^1
0*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^1
1*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e
^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11
+ 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4
096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6
*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e
^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5
*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12
 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840
*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*
b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13
+ 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b
^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6
 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 -
6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 51
20*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 32
7680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*
b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4

+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
e^2)))^(1/2))(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
+ 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^
7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)
+ (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^
5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a
^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 +
 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7
*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b
^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/((8000*a^3*c^7*e^10 - 35*b^6*c^4*e^10 - 84*a*b^4*c^5*e^10 + 12720*a^2*b^
2*c^6*e^10)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256*a*b^13
*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^
9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a
^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((671088
64*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13
 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*
b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13
)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^
5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^
14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14
 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a
^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(
-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^
3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1
863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(
a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2
*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5
*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3

*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9

```
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^
4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^
6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)
^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880
*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7
 - 55*a*b^15*c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104
8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^
14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*
b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440
*a^12*b^2*c^9*e^2)))^(1/2)))*(-(b^17 - b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c + 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2)*2i
```

3.633 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx = \frac{-b-2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{6c^2\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2} e}$$

```
[Out] 1/4*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c*(
b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*arctanh
((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1156, 1121, 628, 632, 212}

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx = -\frac{6c^2\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}} + \frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*
c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}\end{aligned}$$

$$= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(3c)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}dx, x, (d+ex)^2\right)}{2(b^2-4ac)e}$$

$$= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(3c^2)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{(b^2-4ac)^2e}$$

$$= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(6c^2)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b+2c(d+ex)^2\right)}{(b^2-4ac)^2e}$$

$$= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}e}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx = \frac{\frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{24c^2\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2e}$$

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*
c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
4*(b^2 - 4*a*c)^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.56

method	result
default	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d^2-\right.}{16a^2c^2-8a\,b^2}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)^2}$
risch	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d^2-\right.}{16a^2c^2-8a\,b^2}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)^2}$

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*d*
e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c*d^
2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*
c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2
*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. $2(142) = 284$.

Time = 0.43 (sec) , antiderivative size = 3708, normalized size of antiderivative = 24.39

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 1
8*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3
- 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d
)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4
+ 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*
c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*
(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c
^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2
```

+ b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*
d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d
^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c
^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2
*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (
b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d
^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2
+ b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
c^4 + 14(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x
^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^
3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2
*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d
^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2
+ 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*
c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 +
(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2
*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b
^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 18*(b^3*c^2
- 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3 - 4*a*c^4)*
d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*x^3 -
b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3
- 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2
*c^3)*d)*e*x - 24*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*e
^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*d^6 + (70*c
^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d^5 + 10*b*c
^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^
3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2
)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2

```
*a*c^3)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 +
3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e
^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d
^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(136) = 272$.

Time = 6.93 (sec) , antiderivative size = 1646, normalized size of antiderivative = 10.83

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a*c -
 b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) -
 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*
a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**
3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b**2*c*d**
2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*e**6*x**6
```

```
 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d*e**3 + 24
0*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b*c**2*d**2*
e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e + 72*b*c**2*d*
*3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**
2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 1
28*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c
*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8
*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c
**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3
*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**
4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e*
*7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c*
*4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**
5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**
2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b*
*3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d
**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**
2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a
*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b*
*5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 76
8*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 +
1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b
**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b*
*5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512
*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 +
 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a
*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b
**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Maxima [F]

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{ex+d}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b
)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^
2*c^2) + 1/4*(12*c^3*e^6*x^6 + 72*c^3*d*e^5*x^5 + 12*c^3*d^6 + 18*(10*c^3*d
^2 + b*c^2)*e^4*x^4 + 18*b*c^2*d^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*x^3 +
4*(45*c^3*d^4 + 27*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*x^2 - b^3 + 10*a*b*c +
4*(b^2*c + 5*a*c^2)*d^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2*c + 5*a*c^2)*d)
*e*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b
```

```
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^
4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2
 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(142) = 284.

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.30

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{6\,c^2 \arctan\left(\frac{2\,cd^2+2\,(ex^2+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}e} + \frac{12\,c^3d^6+36\,(ex^2+2\,dx)c^3d^4e+36\,(ex^2+2\,dx)^2c^3d^2e^2+12\,(ex^2+2\,dx)^3c^3e^3+18\,bc^2d^4+36\,(ex^2+2\,c}{4\,\left(cd^4+2\,(ex^2+2\,dx)cd^2e+(ex^2+2\,dx)^2ce^2-\right.}$$

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*d^2 + 2*(e*x^2 + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))/((b^
4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) + 1/4*(12*c^3*d^6 + 36*(e
*x^2 + 2*d*x)*c^3*d^4*e + 36*(e*x^2 + 2*d*x)^2*c^3*d^2*e^2 + 12*(e*x^2 + 2*
d*x)^3*c^3*e^3 + 18*b*c^2*d^4 + 36*(e*x^2 + 2*d*x)*b*c^2*d^2*e + 18*(e*x^2
+ 2*d*x)^2*b*c^2*e^2 + 4*b^2*c*d^2 + 20*a*c^2*d^2 + 4*(e*x^2 + 2*d*x)*b^2*c
*e + 20*(e*x^2 + 2*d*x)*a*c^2*e - b^3 + 10*a*b*c)/((c*d^4 + 2*(e*x^2 + 2*d*
x)*c*d^2*e + (e*x^2 + 2*d*x)^2*c*e^2 + b*d^2 + (e*x^2 + 2*d*x)*b*e + a)^2*(
b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 1157, normalized size of antiderivative = 7.61

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{-b^3+4\,b^2\,c\,d^2+18\,b\,c^2\,d^4+10\,a \cdots}{4\,e\,(16\,a^2\,c^2-8\,a \cdots}}{x^2\,\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\,\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\,\left(4\,e\,b^2\,d^3+\cdots\right.}$$

$$+\frac{6\,c^2\,\mathrm{atan}\left(\frac{\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\,\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\,\left(4\,a\,c-b^2\right)^5\right)\,\left(x\,\left(\frac{72\,c^6\,d\,e^7}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{72\,b\,c^4\,\left(16\,d\,a^2\,b\,c^4\,e^9-8 \cdots\right.}{a\,e^2\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(1 \cdots\right.}\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)/
(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b^2
*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e^3
+ 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*b*c^2
*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^3*d^2*
e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a*(4*a*c - b^2)^
(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*d*e^9 - 8*a*b^3*
c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*
c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (
36*c^6*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
6*b*c^4*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^
2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6)))/(e*(4*a*c - b^2)^(5/
2))
```

3.634 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 437

$$\begin{aligned}
&\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx \\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} \\
&\quad+\frac{\left(\frac{d}{e}+x\right)\left((b^2-7ac)\left(3b^2-4ac\right)+3bc(b^2-8ac)\,e^2\left(\frac{d}{e}+x\right)^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} \\
&\quad+\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} \\
&\quad-\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}
\end{aligned}$$

```
[Out] 1/4*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)^2+1/8*(d/e+x)*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+
b^2)*e^2*(d/e+x)^2)/a^2/(-4*a*c+b^2)^2/(a+b*e^2*(d/e+x)^2+c*e^4*(d/e+x)^4)+
3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^2*c^2-b
*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1120, 1106, 1192, 1180, 211}

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{\left(\frac{d}{e}+x\right)\left(3bce^2(b^2-8ac)\left(\frac{d}{e}+x\right)^2+(b^2-7ac)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2}$$

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(a+be^2x^2+ce^4x^4)^3}\,dx, x, \frac{d}{e}+x\right) \\
&= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace^4-4\left(b^2e^4-4ace^4\right)-5bce^6x^2}{(a+be^2x^2+ce^4x^4)^2}\,dx, x, \frac{d}{e}+x\right)}{4a\left(b^2-4ac\right)e^4}
\end{aligned}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)(d+ex)^2)}{8a^2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\text{Subst}\left(\int \frac{3(b^4-9ab^2c+28a^2c^2)e^8+3bc(b^2-8ac)e^{10}x^2}{a+be^2x^2+ce^4x^4}dx, x, \frac{d}{e}+x\right)}{8a^2(b^2-4ac)^2 e^8}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)(d+ex)^2)}{8a^2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\left(3c\left(b^4-10ab^2c+56a^2c^2-b(b^2-8ac)\sqrt{b^2-4ac}\right)e^2\right)\text{Subst}\left(\int \frac{1}{\frac{be^2}{2}+\frac{1}{2}\sqrt{b^2-4ac}e^2+ce^4x^2}dx, x, \frac{d}{e}+\right.}{16a^2(b^2-4ac)^{5/2}} + \frac{\left(3c\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)e^2\right)\text{Subst}\left(\int \frac{1}{\frac{be^2}{2}-\frac{1}{2}\sqrt{b^2-4ac}e^2+ce^4x^2}dx, x, \frac{d}{e}+\right.}{16a^2(b^2-4ac)^{5/2}}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)(d+ex)^2)}{8a^2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b(b^2-8ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx$$

$$= \frac{\frac{4a(d+ex)(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2(d+ex)(3b^4-25ab^2c+28a^2c^2+3b^3c(d+ex)^2-24abc^2(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{\right.}{(b^2-4}}{16a^2e}$$

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((4*a*(d + e*x)*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(3*b^4 - 25*a*b^2*c + 28*a^2*c
^2 + 3*b^3*c*(d + e*x)^2 - 24*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*
a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c
^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*a^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 1010, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1010
risch	Expression too large to display	1059

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e
^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6
3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a
^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2
*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-
3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c
^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3
*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c
^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a
*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*
c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-8*a
*c+b^2)*_R-8*b*c^2*d^2*a+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+
6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z
^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d
^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8554 vs. $2(389) = 778$.

Time = 0.77 (sec) , antiderivative size = 8554, normalized size of antiderivative = 19.57

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6 +
 (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5*x
^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3
)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*
c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c
^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2
 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5
 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2
)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2
 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2
```

```
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b
*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 +
10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c
+ 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2
+ 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
- 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*
d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e) - 3/8*integrate(-((b
^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2 + 2*(b^3*c - 8*a*b
*c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2641 vs. $2(389) = 778$.

Time = 0.31 (sec) , antiderivative size = 2641, normalized size of antiderivative = 6.04

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3/16*((b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 8*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e)^2 - 2*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) + d/e) + 16*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e) + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c
+ 28*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^3*c*e^2*(sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 - 8*a*b*c^2*e^2*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b^3*c*d
*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) - 16*a*
b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
+ b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(x - sqrt(1/
```

2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e +
b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) - d/e)) + (b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)^2 - 8*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 16*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b^3*c*d^2 - 8*a*b*c^2*d^2 +
b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2
)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^3*
c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 -
8*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)^2 + 2*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e) - 16*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) - d/e) + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2
)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(
2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3
+ 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
) + 1/8*(3*b^3*c^2*e^7*x^7 - 24*a*b*c^3*e^7*x^7 + 21*b^3*c^2*d*e^6*x^6 - 16
8*a*b*c^3*d*e^6*x^6 + 63*b^3*c^2*d^2*e^5*x^5 - 504*a*b*c^3*d^2*e^5*x^5 + 10
5*b^3*c^2*d^3*e^4*x^4 - 840*a*b*c^3*d^3*e^4*x^4 + 105*b^3*c^2*d^4*e^3*x^3 -
840*a*b*c^3*d^4*e^3*x^3 + 6*b^4*c*e^5*x^5 - 49*a*b^2*c^2*e^5*x^5 + 28*a^2*
c^3*e^5*x^5 + 63*b^3*c^2*d^5*e^2*x^2 - 504*a*b*c^3*d^5*e^2*x^2 + 30*b^4*c*d
*e^4*x^4 - 245*a*b^2*c^2*d*e^4*x^4 + 140*a^2*c^3*d*e^4*x^4 + 21*b^3*c^2*d^6
*e*x - 168*a*b*c^3*d^6*e*x + 60*b^4*c*d^2*e^3*x^3 - 490*a*b^2*c^2*d^2*e^3*x
^3 + 280*a^2*c^3*d^2*e^3*x^3 + 3*b^3*c^2*d^7 - 24*a*b*c^3*d^7 + 60*b^4*c*d^
3*e^2*x^2 - 490*a*b^2*c^2*d^3*e^2*x^2 + 280*a^2*c^3*d^3*e^2*x^2 + 30*b^4*c*
d^4*e*x - 245*a*b^2*c^2*d^4*e*x + 140*a^2*c^3*d^4*e*x + 3*b^5*e^3*x^3 - 20*
a*b^3*c*e^3*x^3 - 4*a^2*b*c^2*e^3*x^3 + 6*b^4*c*d^5 - 49*a*b^2*c^2*d^5 + 28
*a^2*c^3*d^5 + 9*b^5*d*e^2*x^2 - 60*a*b^3*c*d*e^2*x^2 - 12*a^2*b*c^2*d*e^2*
x^2 + 9*b^5*d^2*e*x - 60*a*b^3*c*d^2*e*x - 12*a^2*b*c^2*d^2*e*x + 3*b^5*d^3
- 20*a*b^3*c*d^3 - 4*a^2*b*c^2*d^3 + 5*a*b^4*e*x - 37*a^2*b^2*c*e*x + 44*a
^3*c^2*e*x + 5*a*b^4*d - 37*a^2*b^2*c*d + 44*a^3*c^2*d)/((c*e^4*x^4 + 4*c*d
*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x +
b*d^2 + a)^2*(a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e))

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 16086, normalized size of antiderivative = 36.81

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
[Out] ((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7 +
 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b^3
*c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(3
*b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 105*b
^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*d^2*e
^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*a^2*c^
3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4))/(8*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d^3*e + 6
3*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*d^5*e - 4
90*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^3 - 49*a*b^
2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*c + 30*b^4*c*d
^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 245*a*b^2*c^2*d^
4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 +
 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^
2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c
^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d
*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 +
 b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^
2*d*e^7*x^7) - atan((((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*
a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 +
1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4
096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^
8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b
^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^
5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^
6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5
120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 3
27680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6
*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) -
```

1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c
^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^
2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 5376
0*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1
966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e
^2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^
12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360
*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/
(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7
*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
 - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^
3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936
*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*
(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^
15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069
824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^
7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^
10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949
120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*1i + ((3612672*a^6
*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*
c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 33914
88*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240
*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (
((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c
^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 3670
0160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*
c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^
2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a
^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b
^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*
b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9
(b^19 + b^4(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c
^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*
a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a

*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^
16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b
^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*
a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^
12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*
e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b
^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 2
4*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144
*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*
c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^
5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8
 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^1
8*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4
*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^
6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) +
(x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4
*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6
*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^1
5)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440
*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*
c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^1
7*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a
^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3
*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*
c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^1
4*b^2*c^9*e^2)))^(1/2)*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 1693
44*a^3*b*c^8*e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6
 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6
144*a^9*b^2*c^5)) + ((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a
*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1
410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 40
96*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8
*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^
15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5
*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6
 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 51
20*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 32
7680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*
b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^
4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 301056

0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2
 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760
*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 19
66080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^
2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^1
2*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*
a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(
512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*
b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3
 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*
a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) -
41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1
048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*
b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a
^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 262
1440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a
^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(
-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^1
5*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 10698
24*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)
))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7
*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^1
0*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 29491
20*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) - ((3612672*a^6*c^9
*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*
d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a
^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6
*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67
108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d
*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 36700160
*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*
d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 -
1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b
*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c
^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8
+ 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^
19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 -
 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*
b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c -
 b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512
*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c

^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*
c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13
*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (22020096*a^9*c^9*e^12 -
768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12
+ 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c
^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^
5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9
*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9
+ 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^
9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 4
9*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*
e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2
- 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^
7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(
14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5
*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c +
96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(
1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4
*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7
+ 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c
- 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*
c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2
+ 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*
e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^
2*c^9*e^2)))^(1/2)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a
^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 3168
64*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^
3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(
4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^
6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^1
2*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^
12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/
2)*2i - atan((((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9
- 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b
^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 +
49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c -
b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c
*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^
2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c
^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22
020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282
624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12
+ 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 +
4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a

^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b
^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^
5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^
6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5
120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 3
27680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6
*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1
720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^
4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 301056
0*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b
^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2
 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760
*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 19
66080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^
2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 -
 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9
*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49
*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^
2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e
^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2
- 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7
*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612
672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a
^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11
- 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*
c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^
5)) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a
^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a
^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*1i + ((9*(b^4*(-(4*a*c - b^2
)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c
^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 234393
6*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^
8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160
*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2
621440*a^14*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*
e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c
^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(
a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c
^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*
a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 20
27520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^

12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^1
0*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*
c^5)) + ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^
5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^1
3 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a
^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^
6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(
262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 409
60*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2
*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1
069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(
1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720
*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048
*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2
949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6
 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b
^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2
- 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2
 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^
8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*
c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^
12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b
^2*c^3)))*1i)/((567*b^7*c^5*e^10 - 10368*a*b^5*c^6*e^10 - 169344*a^3*b*c^8*
e^10 + 67824*a^2*b^3*c^7*e^10)/(256*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10
*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c
^5)) + ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*
a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5
- 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15
)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 +
720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258
048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2
+ 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*(((22020096*
a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4
*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 2339
6352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^
10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*
c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2
*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 917504

0*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2) + (3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - ((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 + 1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360*a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((

67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b^15*c^2*d*e^13 + 114688*a^5*b^13*c^3
*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 + 9175040*a^7*b^9*c^5*d*e^13 - 367001
60*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^5*c^7*d*e^13 - 117440512*a^10*b^3*c^
8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2
- 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9
*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7
*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 327680*a^8*b^3*c^6*e^14))/(32*(a^4*b^
8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*((9*(b
^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2
+ 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6
*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(51
2*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*
c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10
*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^1
3*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2))*((9*(b^4*(-(4*a*c - b^2)
^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^
3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936
*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 +
1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8
*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*
a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26
21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 -
180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))))
)*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b
^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 106
9824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/
2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a
^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a
^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 294
9120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*2i

3.635 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 255

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\ &\quad+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad+\frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}e}\\ &\quad+\frac{\log(d+ex)}{a^3e}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}\end{aligned}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^
2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*
c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*ar
ctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e+ln(e*x
+d)/a^3/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}$$
$$+\frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$+\frac{b(30a^2c^2-10ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2(b^2-4ac)-3bcx}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{4a\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{2\left(b^2-4ac\right)^2+2bc(b^2-7ac)x}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{4a^2\left(b^2-4ac\right)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+4ac\right)^2}{ax} + \frac{2\left(-b(b^4-9ab^2c+23a^2c^2)-c\left(b^2-4ac\right)^2x\right)}{a(a+bx+cx^2)}\right)dx, x, (d+ex)^2\right)}{4a^2\left(b^2-4ac\right)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^3e} + \frac{\text{Subst}\left(\int \frac{-b(b^4-9ab^2c+23a^2c^2)-c\left(b^2-4ac\right)^2x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^3\left(b^2-4ac\right)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)(d+ex)^2}{4a^2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^3e} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^3e} - \frac{(b(b^4-10ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^3(b^2-4ac)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)(d+ex)^2}{4a^2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^3e} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3e} + \frac{(b(b^4-10ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2a^3(b^2-4ac)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)(d+ex)^2}{4a^2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(b^4-10ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{5/2}e} + \frac{\log(d+ex)}{a^3e} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3e}$$

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{\frac{a^2(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{a(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))} + 4\log(d+ex) - \frac{(b^5-10ab^3c+30a^2b}{}}{}$$

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
```

```
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.43 (sec) , antiderivative size = 966, normalized size of antiderivative = 3.79

method	result
default	$-\frac{\frac{c^{2} e^{5} \left(7 a c-b^{2}\right) a b \,x^{6}}{32 a^{2} c^{2}-16 a \,b^{2} c+2 b^{4}}+\frac{3 \left(7 a c-b^{2}\right) a b \,c^{2} d \,e^{4} x^{5}}{16 a^{2} c^{2}-8 a \,b^{2} c+b^{4}}-\frac{e^{3} a c \left(-210 b \,c^{2} d^{2} a+30 b^{3} c \,d^{2}+16 a^{2} c^{2}-29 a \,b^{2} c+4 b^{4}\right) x^{4}}{4 \left(16 a^{2} c^{2}-8 a \,b^{2} c+b^{4}\right)}-\frac{c d \,e^{2} a \left(-70 b \,c^{2} d^{2} a+10 b^{3} c \,d^{2}+16 a^{2} c^{2}\right.}{16 a^{2} c^{2}-8 a \,b^{2} c+b^{4}}}{}$
risch	Expression too large to display

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*(7*a*
c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-210*a*b*c
^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2*d^4-48*a^
2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c^3*d^2+29*a*
b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a*b^2*c^2*d^4+4
*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c^2-21*a^2*b^2*c
+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*
x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(16*a^2*c^2-8*a*b^
2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2
-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*
b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*b^2*a*c^2*d^3+b^4*c*d^3+23*a^2*b*c
^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_
R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)
*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))+ln(e*x+d)/a^3/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4889 vs. 2(243) = 486.

Time = 0.95 (sec) , antiderivative size = 9908, normalized size of antiderivative = 38.85

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d*e^5*x^5 +
 (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7*a*b*c^3)*d^2)*e^4*x
^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3*c^2 - 7*a*b*c^3)*d^3 + (4*b^4
*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^
3*c^2 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c - a^
2*b*c^2 + 15*(b^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 + 16*a^2
*c^3)*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3*c^2 -
7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 6*a*b^3
*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8
 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c -
8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4
*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a
```

```
^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*
b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 -
 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)
*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a
^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^
6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*
d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^
5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*d^2)*e) - integrate(((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^3*x^3 + 3
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^2*x^2 + (b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 9*a*b^3*c + 23*a^2*b*c^2 + 3*(b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*d^2)*e*x + (b^5 - 9*a*b^3*c + 23*a^2*b*c^2)*d)/(c*e^4*x^4 + 4*c*d
*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
a), x)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2) + log(e*x + d)/(a^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(243) = 486$.

Time = 0.45 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.10

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$

$$-\frac{(a^3b^7ce^3 - 14\,a^4b^5c^2e^3 + 70\,a^5b^3c^3e^3 - 120\,a^6bc^4e^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2 + \sqrt{b^2-4\,ac}e^2x^2 + 2\,bdex + 2\right.\right.}{}$$

$$-\frac{\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{4\,a^3e}$$

$$+\frac{\log\left(|ex+d|\right)}{a^3e}$$

$$+\frac{2\,ab^3c^2d^6 - 14\,a^2bc^3d^6 + 4\,ab^4cd^4 - 29\,a^2b^2c^2d^4 + 16\,a^3c^3d^4 + 2\,ab^5d^2 - 12\,a^2b^3cd^2 - 2\,a^3bc^2d^2 + 2\,(ab^3c^2}{}$$

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*
c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 +
2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a
)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*c
^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 -
2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a^9*b^2*
c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d
```

```
^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^3
*e) + log(abs(e*x + d))/(a^3*e) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 +
 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2
*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*
a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e
^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 2
9*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*
b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*
x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2
- 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2
 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4
*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*
c*d*e - a^3*b*c^2*d*e)*x)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4
*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a
^3*e)
```

Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 19440, normalized size of antiderivative = 76.24

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 -
21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d
^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2
+ 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(
4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(
4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*
d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*
c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4
+ 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + log(d + e*x)/(a^3*e) - (log((((a^3*e*(-(b^
```

2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*
(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5
))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3
*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)^2) + (b*c^2*e^16*(
a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^
(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10
*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2
*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*
a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) + (b*c^3*e^15*(7*a*c - b^2)*(4*
b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c
^2*d^2))/(a^4*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409
*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^
6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4
*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c
^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c
^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*
(4*a*c - b^2)^6))*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e
^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^
2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^
2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*
(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c
)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2
 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^
3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3
*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e)
 - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c
^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*(4*a*c - b^2)^4) + (b*c^4*e^1
7*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b
^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^
4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3
)/(a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 +
 b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5
*d*e^15*x*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10*e - 2048*a^5*c^5
*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8
*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b
^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*atan((x*(((
(((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*
d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^
7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3
*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*
b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 2496
0*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^
18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*

a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c
^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*
a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 -
10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^
2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e +
2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*
c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9
*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 -
294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 -
4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c
^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)
))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 25
60*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 8
0*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2
*c^4*e^2)) + (b*((2*(8960*a^7*b*c^9*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^
3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 1102
4*a^6*b^3*c^8*d*e^16))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*
b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*
(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 +
276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17
- 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240
*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) -
((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560
*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*
d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9
*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 2949
12*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c
*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(
a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c
^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e +
320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)
)/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^
2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 -
10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*
b^2*c)^3*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*
c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^
10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18
))/(32*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10
*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2
c^5)))(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6
*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c
^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^
5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d
*e^15))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*

a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(8960*a^7*b*c^9 *d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6 *d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 11024*a^6*b^3*c^8*d*e^16))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a ^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^1 3*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b ^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^ 12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3 840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a ^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163 840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d* e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^ 10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a ^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b ^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11* b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^ 3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5 *e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120 *a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a ^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 409 6*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e ^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^ 13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7* b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b ^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320* a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(16 3840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d *e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b ^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560* a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7* b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11 *b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b ^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840* a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840 *a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/(4*a^3*e*(4*a* c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640* a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a ^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3* e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e -

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2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4
*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328
*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18
- 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*
c^8*d*e^18))/(16*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2
- 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7
*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 -
 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*
c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*b^10 - 64
00*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*
b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*(4*a*c - b^2)^(
15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*(4*a*c - b
^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*(
4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e
^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900
*a^4*b^2*c^6*e^14) + (x^2*((((((b*((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e
^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 +
 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 -
 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6
144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*
a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19
- 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 2
4960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19
- 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*
e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*
a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c
)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 256
0*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e
^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^
19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c
^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 -
 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b
^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1
280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048
*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e -
40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 6
40*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b*((8
960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*
b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 +
4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a
^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3
*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18
 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6
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- 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(64*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^16 - 21*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3*c^8*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((8960*a^7*b*c^9*e^17 - 6*a^2*b^11*c^4*e^17 + 137*a^3*b^9*c^5*e^17 - 1217*a^4*b^7*c^6*e^17 + 5256*a^5*b^5*c^7*e^17 - 11024*a^6*b^3*c^8*e^17)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((5120*a^10*b*c^9*e^18 + 2*a^4*b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((5120*a^10*b*c^9*e^18 + 2*a^4*

b^13*c^3*e^18 - 36*a^5*b^11*c^4*e^18 + 276*a^6*b^9*c^5*e^18 - 1216*a^7*b^7*
c^6*e^18 + 3456*a^8*b^5*c^7*e^18 - 6144*a^9*b^3*c^8*e^18)/(a^6*b^12 + 4096*
a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b
^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2
*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b
*c^9*e^19 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^
4*e^19 + 24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5
*c^7*e^19 - 294912*a^12*b^3*c^8*e^19))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^
2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^
7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
- 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2
*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 -
10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c
^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^19 - 12*a^6*
b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 + 24960*a^9*
b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^19 - 294912*
a^12*b^3*c^8*e^19))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8
*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 +
5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b
^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 +
30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a
^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 128
0*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*e^1
9 - 12*a^6*b^15*c^2*e^19 + 328*a^7*b^13*c^3*e^19 - 3840*a^8*b^11*c^4*e^19 +
24960*a^9*b^9*c^5*e^19 - 97280*a^10*b^7*c^6*e^19 + 227328*a^11*b^5*c^7*e^1
9 - 294912*a^12*b^3*c^8*e^19))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2
- 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*
c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c +
240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5
)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b
^2)^6*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^
4*b^2*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c
^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*
b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 61
440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(1
5/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*
b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b^12*c^3*e^15 - 93*a^3*
b^10*c^4*e^15 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4
*c^7*e^15 - 7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c
^5*d^2*e^15 + 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024
*a^6*b^3*c^8*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6
- 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 -
6144*a^11*b^2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*
a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*
a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a

^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) - (((b*((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a

^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^
4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e
 - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*
e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*
b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)
^3*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 51
20*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 +
16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e
^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*
b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e
^17 - 163840*a^13*b*c^9*d^2*e^17))/(64*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*
c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 655
36*a^15*c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3
840*a^11*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(1
5/2) + 61440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c
 - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c
^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^1
4)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b
^2*c^4 - 120*a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15 - 93*a^3*b^10*c^4*e^1
5 + 854*a^4*b^8*c^5*e^15 - 3889*a^5*b^6*c^6*e^15 + 8808*a^6*b^4*c^7*e^15 -
7952*a^7*b^2*c^8*e^15 + 6*a^2*b^11*c^4*d^2*e^15 - 137*a^3*b^9*c^5*d^2*e^15
+ 1217*a^4*b^7*c^6*d^2*e^15 - 5256*a^5*b^5*c^7*d^2*e^15 + 11024*a^6*b^3*c^8
*d^2*e^15 - 8960*a^7*b*c^9*d^2*e^15)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5) - (((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4
*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*
e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*
d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*
b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)/(a
^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^
3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e +
320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)
*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120
*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16
384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^1
7 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^
7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^1
7 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80
*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*
c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280
*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^
5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*
a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*

a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^10*e
- 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^
4*e - 40*a*b^8*c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e
^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) -
(b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^14 - 616*a^3*b^4*c^7
*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^2*b^5*c^7*d^2*e^14
- 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6*b^12 + 4096*a^12*c
^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
 - 6144*a^11*b^2*c^5) + (b*((b*((4*a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^1
6 + 1052*a^6*b^10*c^4*e^16 - 5952*a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16
 - 32768*a^9*b^4*c^7*e^16 + 23552*a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^
16 - 36*a^5*b^11*c^4*d^2*e^16 + 276*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6
*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^
10*b*c^9*d^2*e^16)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*
c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*
e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*
c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9
*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a
^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328
*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2
*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912
*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17))/(2*(4*a^3*b^10*e^2 -
4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^
3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 2
40*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5))
)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b*(b^4
+ 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e -
 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^
17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17
+ 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^
17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c
^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 22
7328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c
^9*d^2*e^17))/(8*a^3*e*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e
^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a
^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2
 - 10*a*b^2*c)^2*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*
b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^
8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^
11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^
6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^1
7 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*
b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^1

7))/(32*a^6*e^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4
*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*
e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9
*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(b^6 - 45*a^3*c^3 + 40*
a^2*b^2*c^2 - 11*a*b^4*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*c^
6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11*b
^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 614
40*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(15
/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a
^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 64
00*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*
b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*(4*a*c - b^2)^(5/2))

3.636 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 484

$$\begin{aligned}&\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx\\
&=-\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\
&+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}\,e}\\
&-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}\,e}\end{aligned}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)+1/4*(b^2-2*a
*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1/
8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(-4*a*c+b
^2)^2/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+b*
(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-124*a^2*b*
c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1156, 1135, 1291, 1295, 1180, 211}

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} dx$$
$$= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3e(b^2-4ac)^2(d+ex)} + \frac{36a^2c^2+bc(5b^2-32ac)(d+ex)^2-35ab^2c+5b^4}{8a^2e(b^2-4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}$$
$$- \frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3e(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3e(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+ \frac{-2ac+b^2+bc(d+ex)^2}{4ae(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (
b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 -
 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4
 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5
*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
```

```
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-7bcx^2}{x^2(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{4a\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$+\frac{\text{Subst}\left(\int\frac{3(5b^2-12ac)\,(b^2-5ac)+3bc(5b^2-32ac)x^2}{x^2(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{8a^2\left(b^2-4ac\right)^2e}$$

$$= -\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\left(b^2-4ac\right)^2e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$-\frac{\text{Subst}\left(\int\frac{3b(5b^4-42ab^2c+92a^2c^2)+3c(5b^2-12ac)\,(b^2-5ac)x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{8a^3\left(b^2-4ac\right)^2e}$$

$$= -\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\left(b^2-4ac\right)^2e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$+\frac{\left(3c\left(5b^5-47ab^3c+124a^2bc^2-\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\right)\text{Subst}\left(\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right.}{16a^3\left(b^2-4ac\right)^{5/2}e}$$
$$-\frac{\left(3c\left((5b^2-12ac)\,(b^2-5ac)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{16a^3\left(b^2-4ac\right)^2e}$$

$$= -\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\left(b^2-4ac\right)^2e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2}{8a^2\left(b^2-4ac\right)^2e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$-\frac{3\sqrt{c}\left((5b^2-12ac)\,(b^2-5ac)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}\,e}$$
$$+\frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2-\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{a^3e(d+ex)} + \frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+ \frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ex)}{8a^3\left(-b^2+4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$- \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+e}{\sqrt{b-\sqrt{b^2-}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e}$$
$$- \frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d}{\sqrt{b+\sqrt{b^2}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*
x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(
d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d +
 e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (
3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 3
7*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 1197, normalized size of antiderivative = 2.47

method	result	size
default	Expression too large to display	1197
risch	Expression too large to display	2458

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^2+136*a^2*b*c^2-
99*a*b^3*c+14*b^5)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(364*a^
2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a*b^3*c+14*b^5)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-1645*a*b^2*c^3*d^4+
245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+68*a^3*c
^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*d*e*
(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*
a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129*a*b^4*c+21*b^6)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a*b^2*c^3*d^6+49*b^
4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d^4+204*a^3*c^3*d^2+
75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b*c^2-66*a^2*b^3*c+9*
a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^4*d^6-47*a*b^2*c^3*d^
6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14*b^5*c*d^4+68*a^3*c^3*
d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*a^3*b*c^2-66*a^2*b^3*c+
9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x
^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*a^2*c^2-8*a*b^
2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R^2+2*d*c*e*(60*a^2*c^
2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^2*c^2*d^2+5*b^4*c*d^2+92*a^2*b
*c^2-42*a*b^3*c+5*b^5)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R
*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/a^3/e/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10260 vs. $2(438) = 876$.

Time = 1.11 (sec) , antiderivative size = 10260, normalized size of antiderivative = 21.20

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(ex+d)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b^4*c^2 - 3
7*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b
*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 6*(28*(5*b
^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a
^2*b*c^3)*d)*e^5*x^5 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + (15*
b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^2
*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^2)
*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 4*(42*(5*b^4*c^
2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*
b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^
3 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^6 - 91*a*b^4*c + 25*a^2*
b^2*c^2 + 324*a^3*c^3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^
6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - 227*a*b^3*c^2
 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c
^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d^2 + 2*(12*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)
*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*
a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4
*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
```

```
d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^
3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*
(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*
b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 +
3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*
a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 -
 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*
e) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2
*c^2 + 60*a^2*c^3)*e^2*x^2 + 2*(5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d*e*x
+ (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c
*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. $2(438) = 876$.

Time = 0.40 (sec) , antiderivative size = 1458, normalized size of antiderivative = 3.01

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8*(7*b^4*c^2/((e*x + d)*e) - 47*a*b^2*c^3/((e*x + d)*e) + 52*a^2*c^4/((e
*x + d)*e) + 14*b^5*c/((e*x + d)^3*e) - 99*a*b^3*c^2/((e*x + d)^3*e) + 136*
a^2*b*c^3/((e*x + d)^3*e) + 7*b^6/((e*x + d)^5*e) - 43*a*b^4*c/((e*x + d)^5
*e) + 25*a^2*b^2*c^2/((e*x + d)^5*e) + 68*a^3*c^3/((e*x + d)^5*e) + 9*a*b^5
/((e*x + d)^7*e) - 66*a^2*b^3*c/((e*x + d)^7*e) + 108*a^3*b*c^2/((e*x + d)^
7*e))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(e*x + d)^2 + a/(e*x + d
)^4)^2) + 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*
b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2
*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4 + 2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a
^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*
a*c)*e^2*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2) - (a^3*b^4*e^2
 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*
sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*x + d)*e*sqrt((
a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + sqrt((a^3*b^5*e^2 - 8*a^
```

```
4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a
^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4 - 8*a^5
*b^2*c*e^4 + 16*a^6*c^2*e^4))))/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c
^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*e^3*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 +
 16*a^5*c^2*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^
9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a
^12*b*c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4 - 2*(5*a^4*b^6*c - 57*a^
5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)
*a)*sqrt(b^2 - 4*a*c)*e^2*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^
2) - (a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c
 + 92*a^2*b*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((
e*x + d)*e*sqrt((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 - sqrt((a
^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5
*b^2*c*e^4 + 16*a^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^
4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4))))/((a^7*b^6*c - 12*a^8*b^4*c
^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*e^3*abs(a^3*b^4*e^2 -
8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*abs(a)) - 1/((e*x + d)*a^3*e)
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 18112, normalized size of antiderivative = 37.42

$$\int \frac{1}{(d+ex)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*
e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*
a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*
e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108
*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^
6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 +
 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 +
50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/
(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 +
 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3
*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^
3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 36
4*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e
+ 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*
a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*
```

d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4 - 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b^2*d^2*e^3 + 84*c^2*d^6*e^3 + 2*a*b*e^3 + 20*a*c*d^2*e^3 + 70*b*c*d^4*e^3) + x^7*(36*c^2*d^2*e^7 + 2*b*c*e^7) + x*(a^2*e + 5*b^2*d^4*e + 9*c^2*d^8*e + 6*a*b*d^2*e + 10*a*c*d^4*e + 14*b*c*d^6*e) + x^4*(5*b^2*d*e^4 + 126*c^2*d^5*e^4 + 10*a*c*d*e^4 + 70*b*c*d^3*e^4) + a^2*d + x^2*(10*b^2*d^3*e^2 + 36*c^2*d^7*e^2 + 6*a*b*d*e^2 + 20*a*c*d^3*e^2 + 42*b*c*d^5*e^2) + x^6*(84*c^2*d^3*e^6 + 14*b*c*d*e^6) + x^5*(b^2*e^5 + 126*c^2*d^4*e^5 + 2*a*c*e^5 + 42*b*c*d^2*e^5) + b^2*d^5 + c^2*d^9 + c^2*e^9*x^9 + 2*a*b*d^3 + 2*a*c*d^5 + 2*b*c*d^7 + 9*c^2*d*e^8*x^8) - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4

*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 12
4017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 141733920
7680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a
^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*
b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13
 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145
925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722
688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 283467841536
0*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384
*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*
c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 267
3082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*
a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b
^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^1
1*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12
 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b
^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^
12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869
815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 174731349
1968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*
a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^1
1 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 1687
8108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967
040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 154384780492
8*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648
*a^19*b^2*c^13*d*e^11)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998
60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2
) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 12998

60*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^
7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*
a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(
1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576
*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*
c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*
b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440
*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15
*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14
 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 1240171
80672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*
a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b
^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^1
3*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 23
0686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 2214592512
0*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a
^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^2
3*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25
*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e
^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 26730823
68*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*
b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^
9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^1
2 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 23
0400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c
^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 +
89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 86981581
2096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*
a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*
c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 1
91038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 168781086
72*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a
^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^1
7*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19
*b^2*c^13*d*e^11)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
 b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^

4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 1
8923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^
4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7
*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c -
b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
+ 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17
*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e
^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c
^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16
*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23
*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 27
68240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672
*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*
b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^
11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e
^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 2306867
20*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^1
9*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b
^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7
*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*
c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 +
10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^
15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*
c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^1
2 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3
208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*
a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^
12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374
851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*
a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*
b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*
d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038
464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^
13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b
^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6
*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*
c^13*d*e^11) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c

^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
e^2)))^(1/2)((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
e^2)))^(1/2)((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*
a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c
^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6
2684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15
)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*
b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^
2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 537
60*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 -
 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9
e^2)))^(1/2)(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^1
4 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640
*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^
13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*
e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14
 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 26
2144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*
b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c
^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*
d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*
e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e
^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 1091174
4*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*
c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12
 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918
430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 320834057
0112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22
*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 220
7803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a
^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8

*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12 *e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11 *b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14* c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9* d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d* e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e ^11) + 191102976000*a^17*c^14*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a ^10*b^14*c^7*e^10 + 1312630272*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c ^9*e^10 + 53445353472*a^13*b^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10 + 342580396032*a^15*b^4*c^12*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9* (25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794* a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^1 1*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19 *c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^ 2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^ 7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i - a tan(((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^ 12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a ^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1 /2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^ 12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a ^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1 /2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^1 0 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61266 40*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^ 8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4 *a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8 *b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^

12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 220397568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^

4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11)*1i)/((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2

- 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 + 11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^20*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) + 1185410973696*a^23*b*c^13*e^12 - 245760*a^12*b^23*c^2*e^12 + 10911744*a^13*b^21*c^3*e^12 - 220397568*a^14*b^19*c^4*e^12 + 2673082368*a^15*b^17*c^5*e^12 - 21630025728*a^16*b^15*c^6*e^12 + 122607894528*a^17*b^13*c^7*e^12 - 496773365760*a^18*b^11*c^8*e^12 + 1438679826432*a^19*b^9*c^9*e^12 - 2918430277632*a^20*b^7*c^10*e^12 + 3949222428672*a^21*b^5*c^11*e^12 - 3208340570112*a^22*b^3*c^12*e^12) + x*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^20*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 - 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 333226967040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11 + 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 2207803392*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^14*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 86981581209

6*a^16*b^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968
*a^18*b^4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) - (-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21
 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17
*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 +
19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396
80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694
*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1
/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*
a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 25804
8*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 +
2949120*a^15*b^4*c^8*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627
776*a^26*b*c^13*e^14 - 262144*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e
^14 - 230686720*a^17*b^19*c^4*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 221459
25120*a^19*b^15*c^6*e^14 + 124017180672*a^20*b^13*c^7*e^14 - 496068722688*a
^21*b^11*c^8*e^14 + 1417339207680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^
7*c^10*e^14 + 3779571220480*a^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^1
2*e^14) + 1099511627776*a^26*b*c^13*d*e^13 - 262144*a^15*b^23*c^2*d*e^13 +
11534336*a^16*b^21*c^3*d*e^13 - 230686720*a^17*b^19*c^4*d*e^13 + 2768240640
*a^18*b^17*c^5*d*e^13 - 22145925120*a^19*b^15*c^6*d*e^13 + 124017180672*a^2
0*b^13*c^7*d*e^13 - 496068722688*a^21*b^11*c^8*d*e^13 + 1417339207680*a^22*
b^9*c^9*d*e^13 - 2834678415360*a^23*b^7*c^10*d*e^13 + 3779571220480*a^24*b^
5*c^11*d*e^13 - 3023656976384*a^25*b^3*c^12*d*e^13) - 1185410973696*a^23*b*
c^13*e^12 + 245760*a^12*b^23*c^2*e^12 - 10911744*a^13*b^21*c^3*e^12 + 22039
7568*a^14*b^19*c^4*e^12 - 2673082368*a^15*b^17*c^5*e^12 + 21630025728*a^16*
b^15*c^6*e^12 - 122607894528*a^17*b^13*c^7*e^12 + 496773365760*a^18*b^11*c^
8*e^12 - 1438679826432*a^19*b^9*c^9*e^12 + 2918430277632*a^20*b^7*c^10*e^12
 - 3949222428672*a^21*b^5*c^11*e^12 + 3208340570112*a^22*b^3*c^12*e^12) + x

*(271790899200*a^20*c^14*e^12 - 230400*a^9*b^22*c^3*e^12 + 9861120*a^10*b^2
0*c^4*e^12 - 191038464*a^11*b^18*c^5*e^12 + 2207803392*a^12*b^16*c^6*e^12 -
 16878108672*a^13*b^14*c^7*e^12 + 89374851072*a^14*b^12*c^8*e^12 - 33322696
7040*a^15*b^10*c^9*e^12 + 869815812096*a^16*b^8*c^10*e^12 - 1543847804928*a
^17*b^6*c^11*e^12 + 1747313491968*a^18*b^4*c^12*e^12 - 1101055131648*a^19*b
^2*c^13*e^12) + 271790899200*a^20*c^14*d*e^11 - 230400*a^9*b^22*c^3*d*e^11
+ 9861120*a^10*b^20*c^4*d*e^11 - 191038464*a^11*b^18*c^5*d*e^11 + 220780339
2*a^12*b^16*c^6*d*e^11 - 16878108672*a^13*b^14*c^7*d*e^11 + 89374851072*a^1
4*b^12*c^8*d*e^11 - 333226967040*a^15*b^10*c^9*d*e^11 + 869815812096*a^16*b
^8*c^10*d*e^11 - 1543847804928*a^17*b^6*c^11*d*e^11 + 1747313491968*a^18*b^
4*c^12*d*e^11 - 1101055131648*a^19*b^2*c^13*d*e^11) + 191102976000*a^17*c^1
4*e^10 + 2851200*a^9*b^16*c^6*e^10 - 92568960*a^10*b^14*c^7*e^10 + 13126302
72*a^11*b^12*c^8*e^10 - 10611136512*a^12*b^10*c^9*e^10 + 53445353472*a^13*b
^8*c^10*e^10 - 171591892992*a^14*b^6*c^11*e^10 + 342580396032*a^15*b^4*c^12
*e^10 - 388363714560*a^16*b^2*c^13*e^10))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7
680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2
+ 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8
*e^2 - 2621440*a^16*b^2*c^9*e^2)))^(1/2)*2i

3.637 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 325

$$\begin{aligned}
&\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3\,(b^2-4ac)^2\,e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,e(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&\quad+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,e(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)} \\
&\quad-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4\,(b^2-4ac)^{5/2}\,e} \\
&\quad-\frac{3b\log(d+ex)}{a^4e}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}
\end{aligned}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)^2+1/4*(b^2-2*a*
c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1
/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*c+b
^2)^2/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+30*a^2*b^2*c
^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*
c+b^2)^(5/2)/e-3*b*ln(e*x+d)/a^4/e+3/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^4/
e
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3e\left(b^2-4ac\right)^2(d+ex)^2}$$
$$+\frac{20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2-20ab^2c+3b^4}{4a^2e\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{4a\left(b^2-4ac\right)e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2\left(b^2-4ac\right)^2e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{6(b^2-5ac)(b^2-2ac)+6bc(b^2-6ac)x}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{4a^2\left(b^2-4ac\right)^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2\left(b^2-4ac\right)^2e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \left(\frac{6(b^2-5ac)(b^2-2ac)}{ax^2} - \frac{6b(-b^2+4ac)^2}{a^2x} + \frac{6\left(b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc\left(b^2-4ac\right)^2x\right)}{a^2(a+bx+cx^2)}\right)dx, x, (d+e$$

$$4a^2\left(b^2-4ac\right)^2e$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4e} + \frac{3\text{Subst}\left(\int \frac{b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc(b^2-4ac)^2x}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{2a^4(b^2-4ac)^2 e}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4e} + \frac{(3b)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{4a^4e} + \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\text{Subst}\left(\int \frac{1}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{4a^4(b^2-4ac)^2 e}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4e} + \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e} - \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b+2c(d+ex)^2\right)}{2a^4(b^2-4ac)^2 e}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2} e} - \frac{3b\log(d+ex)}{a^4e} + \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}$$

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3} dx$$
$$= -\frac{1}{2a^3e(d+ex)^2} + \frac{b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2}{4a^2(-b^2+4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2}$$
$$+ \frac{-4b^5+29ab^3c-46a^2bc^2-4b^4c(d+ex)^2+26ab^2c^2(d+ex)^2-28a^2c^3(d+ex)^2}{4a^3(-b^2+4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)}$$
$$- \frac{3b\log(d+ex)}{a^4e}$$
$$+ \frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2\right.}{4a^4(b^2-4ac)^{5/2}e}$$
$$+ \frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{\right.}{4a^4(b^2-4ac)^{5/2}e}$$

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2)
+ (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*
(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4
*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b
^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2
*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30
*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*
a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d +
 e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	1141
risch	Expression too large to display	2190

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*d^2+74*a^2
*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*a*(140*a^2*
c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c^3*d^4+30*b^4
*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^3*c^2*d^2+24*b^5*c*d^2+18*a^3*c^3+7*a^2*
b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(42*a^2*c^4*d^
4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74*a^2*b*c^3*d^2-55*a*b^3*c^2*d^2+8*b^5*c*
d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c^3*d^6+4*b^4*c^2*d^6+74*a^2*b*c^3*d^4-55
*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c^3*d^2+14*a^2*b^2*c^2*d^2-24*a*b^4*c*d^2
+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*c+5*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*b*c*(-16*a^2*c^2+8*
a*b^2*c-b^4)*_R^3+3*d*e^2*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^2+e*(-48*a^2*b
*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d^2+10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b
^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2*d^3-b^5*c*d^3+10*a^3*c^3*d-23*a^2*b^2*c
^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_
R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)
*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a^3/e/(e*x+d)^2-3*b*ln(e*
x+d)/a^4/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7517 vs. $2(311) = 622$.

Time = 1.61 (sec) , antiderivative size = 15165, normalized size of antiderivative = 46.66

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3} \, dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$
$$=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(ex+d)^3}\,dx$$

[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3 + 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*e^6*x^6 + 6*(56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d)*e^5*x^5 + 6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^4*c + 14*a^2*b^2*c^2 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^4 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^3 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^3*x^3 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (168*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 + 4*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^11*x^10 + 10*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^10*x^9 + (2*a^3*b^5*c - 16*a^4*b^3*c^2 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^9*x^8 + 8*(15*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^8*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^7*x^6 + 2*(126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^6*x^5 + (2*a^4*b^5 - 16*a^5*b^3*c + 32*a^6*b*c^2 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 140*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 15*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^5*x^4 + 4*(30*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 28*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^4*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 15*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 12*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^3*x^2 + 2*(5*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c

```
^4)*d^9 + 8*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + 3*(a^3*b^6 - 6
*a^4*b^4*c + 32*a^6*c^3)*d^5 + 4*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^
2*c^3 + 16*a^5*c^4)*d^10 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^8
 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^6 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16
*a^6*b*c^2)*d^4 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d^2)*e) + 3*integrat
e(((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^3*x^3 + 3*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^2*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 -
 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*d^2)*e*x + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3)*d)/(c*e^4
*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 +
 b*d)*e*x + a), x)/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2) - 3*b*log(e*x + d)/
(a^4*e)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(-\frac{b+\frac{2\,a}{(ex+d)^2}}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2\right)\sqrt{-b^2+4\,ac}e} + \frac{3\,b\log\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)}{4\,a^4e} - \frac{1}{2\,(ex+d)^2a^3e} + \frac{5\,b^5c^2-36\,ab^3c^3+58\,a^2bc^4+\frac{2\,(5\,b^6ce-38\,ab^4c^2e+71\,a^2b^2c^3e-14\,a^3c^4e)}{(ex+d)^2e}+\frac{5\,b^7e^2-34\,ab^5ce^2+41\,a^2b^3c^2e^2+42\,a^3bc^3e^2}{(ex+d)^4e^2}+\frac{6\,(ab^6 \cdots}{\cdots}}{4\,(b^2-4\,ac)^2a^4\left(c+\frac{b}{(ex+d)^2}+\frac{a}{(ex+d)^4}\right)^2e}$$

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(e*x
+ d)^2)/sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2
 + 4*a*c)*e) + 3/4*b*log(c + b/(e*x + d)^2 + a/(e*x + d)^4)/(a^4*e) - 1/2/(
(e*x + d)^2*a^3*e) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*c^4 + 2*(5*b^
6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)/((e*x + d)^2*e) +
 (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*b*c^3*e^2)/((e*x
 + d)^4*e^2) + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2*c^2*e^3 - 6*a^4*
c^3*e^3)/((e*x + d)^6*e^3))/((b^2 - 4*a*c)^2*a^4*(c + b/(e*x + d)^2 + a/(e*
x + d)^4)^2*e)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 21465, normalized size of antiderivative = 66.05

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3} dx = \text{Too large to display}$$

[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)

[Out] (log(((27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b
^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2
)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2
c)(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*
b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*(-
(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5
))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 +
 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3
*(4*a*c - b^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2
*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2
 - 20*a*c*d*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^
2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^
8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4
*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c
^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c
^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6))*((27*c
^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 +
10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2)^6) - ((3*
b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(
4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6
- 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2
 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
(6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^
4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*(4*a*c - b
^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30
*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2
*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d
*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4
c))/(a^3(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4
*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)

^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - ((x^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + 4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 12*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e + 168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 216*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 - 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 32*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3*c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^2*c^2*d^4 - 36*a*b^4*c*d^4)/(4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^4*(15*b^2*d^2*e^4 + 210*c^2*d^6*e^4 + 2*a*b*e^4 + 30*a*c*d^2*e^4 + 140*b*c*d^4*e^4) + x^8*(45*c^2*d^2*e^8 + 2*b*c*e^8) + x^5*(6*b^2*d*e^5 + 252*c^2*d^5*e^5 + 12*a*c*d*e^5 + 112*b*c*d^3*e^5) + x^3*(20*b^2*d^3*e^3 + 120*c^2*d^7*e^3 + 8*a*b*d*e^3 + 40*a*c*d^3*e^3 + 112*b*c*d^5*e^3) + x^7*(120*c^2*d^3*e^7 + 16*b*c*d*e^7) + x*(6*b^2*d^5*e + 10*c^2*d^9*e + 2*a^2*d*e + 8*a*b*d^3*e + 12*a*c*d^5*e + 16*b*c*d^7*e) + x^6*(b^2*e^6 + 210*c^2*d^4*e^6 + 2*a*c*e^6 + 56*b*c*d^2*e^6) + x^2*(a^2*e^2 + 15*b^2*d^4*e^2 + 45*c^2*d^8*e^2 + 12*a*b*d^2*e^2 + 30*a*c*d^4*e^2 + 56*b*c*d^6*e^2) + a^2*d^2 + b^2*d^6 + c^2*d^10 + c^2*e^10*x^10 + 2*a*b*d^4 + 2*a*c*d^6 + 2*b*c*d^8 + 10*c^2*d*e^9*x^9) - (3*b*log(d + e*x))/(a^4*e) - (3*atan((x^2*((((27000*a^6*c^11*e^16 + 27*b^12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*b^8*c^7*e^16 - 20601*a^3*b^6*c^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*b^2*c^10*e^16)/(a^9*b^12 + 409

6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a
^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10*e^17 + 54*a^3*b^13*
c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17 - 57204*a^6*b^7*
c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^17)/(a^9*b^12 + 4
096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840
*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^18 + 6*a^6*b^14*
c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*
e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2
*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 -
1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 9
60*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e
- 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^
10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 9728
0*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))
/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2
*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c
^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*
c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3
*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10
*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7
*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840
*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2
*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^
2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((3*((153600*a^13*c^
10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^
18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e
^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c
+ 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*
c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c
^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b
^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^1
2*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 29491
2*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*
e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a
^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6
*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*
b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^
5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^
16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b
^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a
^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c - b^2)^(5/2)*
(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2
 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 -
 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4

- 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4
*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*
b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*
c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*
b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^
12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 2949
12*a^15*b^3*c^8*e^19))/(32*a^8*e^2*(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a
^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2
+ 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^
11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(
3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^
3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^
6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((1
53600*a^13*c^10*e^18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^
8*b^10*c^5*e^18 + 792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a
^11*b^4*c^8*e^18 - 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24
*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6
144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7
680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^
19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^1
9 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7
*e^19 - 294912*a^15*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 -
80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^
2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 -
1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*
c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6
 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e
- 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*
e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19
- 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^
19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19))/(8*a^4*e*(4*a*c
- b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^
6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 40
96*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*
a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^
3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4
*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 -
 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((129600*a^9*b*c^10*e^1
7 + 54*a^3*b^13*c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17
- 57204*a^6*b^7*c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^1
7)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^1
2*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^
18 + 6*a^6*b^14*c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 +
792*a^9*b^8*c^6*e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 -
 199680*a^12*b^2*c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240

*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)
- ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e
- 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c
^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9
*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^1
5*b^3*c^8*e^19))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 +
640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^
12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
+ 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e -
3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)
)/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^
2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 +
30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*e^19 - 12*a^9*
b^15*c^2*e^19 + 328*a^10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^
12*b^9*c^5*e^19 - 97280*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 2949
12*a^15*b^3*c^8*e^19))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 33
5*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 -
6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*
b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c
^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*
b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61
440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(1
5/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*
a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*
a^5*b^2*c^7*e^14) + (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^12*c^5*d*e^15 -
567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3*b^6*c^8*d*e^15
+ 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a^9*b^12 + 4096*a
^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13
*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(129600*a^9*b*c^10*d*e^16 + 54*a^3*b^1
3*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 + 11583*a^5*b^9*c^6*d*e^16 - 57204*
a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d*e^16 - 223200*a^8*b^3*c^9*d*e^16)
)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12
*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(153600*a^13*c^10*
d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5
*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*
b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 2
4*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 -
6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e +
7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d
*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*
c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328
*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096

*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^
2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))
*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 -
80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^
2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
5120*a^8*b^2*c^4*e^2)) - (3*((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*
c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^
8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199
680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*
a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) -
((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e -
120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*
c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a
^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18
- 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^
5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4
e^2)(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*
a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 +
30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20
*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 384
0*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(1
63840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18
- 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c
^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(4*a^
4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e
^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^
9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*
c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b
^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3
+ 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b
^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a
^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840
*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^
18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(16*a^8*e^2*
(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640
*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 +
4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38
40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b^4*c^
2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6*(100*a^6*c^6
- 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5
*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b

^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(4*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (3*((2*(129600*a^9*b*c^10*d*e^16 + 54*a^3*b^13*c^4*d*e^16 - 1233*a^4*b^11*c^5*d*e^16 + 11583*a^5*b^9*c^6*d*e^16 - 57204*a^6*b^7*c^7*d*e^16 + 156276*a^7*b^5*c^8*d*e^16 - 223200*a^8*b^3*c^9*d*e^16))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((2*(153600*a^13*c^10*d*e^17 + 6*a^6*b^14*c^3*d*e^17 - 108*a^7*b^12*c^4*d*e^17 + 588*a^8*b^10*c^5*d*e^17 + 792*a^9*b^8*c^6*d*e^17 - 22272*a^10*b^6*c^7*d*e^17 + 100608*a^11*b^4*c^8*d*e^17 - 199680*a^12*b^2*c^9*d*e^17))/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/((4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b

^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*d*e^18 - 12*a^9*b^15*c^2*d*e^18 + 328*a^10*b^13*c^3*d*e^18 - 3840*a^11*b^11*c^4*d*e^18 + 24960*a^12*b^9*c^5*d*e^18 - 97280*a^13*b^7*c^6*d*e^18 + 227328*a^14*b^5*c^7*d*e^18 - 294912*a^15*b^3*c^8*d*e^18))/(32*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) - (((((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^15 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 - 223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 22272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e

^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b
^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e^
14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^14
 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^1
4 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d^
2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a*b
^10*c^6*d^2*e^14)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8
*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + (3*((3*
((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 31
56*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 1
09824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e
^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^
5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 22272*a^10*b^6*c^7*d^2*e^16 + 10060
8*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^
15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*
b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^
5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^
14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^
8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16
*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 384
0*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6
*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 1
63840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*
b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e
^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^
12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 3
0*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*
a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a
^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a
^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 1638
4*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17
 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b
^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^
17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*
e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*
b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^1
0*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*
b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*(4*a*
c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*
b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*
a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17
 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^1
7 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2
*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^

12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2
*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(32*a^8
e^2(4*a*c - b^2)^5*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b
^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3
 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(3*b^8 + 10*a^4*c^4 + 120*a^2*b
^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) +
 65536*a^18*c^6*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2)
 + 3840*a^14*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2
)^(15/2) + 61440*a^16*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*
a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^1
2*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^
5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*
b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2
*c^5 + 120*a*b^10*c)) - (b*((3*((36*a^3*b^14*c^3*e^15 - 14400*a^10*c^10*e^1
5 - 837*a^4*b^12*c^4*e^15 + 8046*a^5*b^10*c^5*e^15 - 40941*a^6*b^8*c^6*e^15
 + 116532*a^7*b^6*c^7*e^15 - 177588*a^8*b^4*c^8*e^15 + 119520*a^9*b^2*c^9*e
^15 + 54*a^3*b^13*c^4*d^2*e^15 - 1233*a^4*b^11*c^5*d^2*e^15 + 11583*a^5*b^9
*c^6*d^2*e^15 - 57204*a^6*b^7*c^7*d^2*e^15 + 156276*a^7*b^5*c^8*d^2*e^15 -
223200*a^8*b^3*c^9*d^2*e^15 + 129600*a^9*b*c^10*d^2*e^15)/(a^9*b^12 + 4096*
a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^1
3*b^4*c^4 - 6144*a^14*b^2*c^5) - (((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*c^9
*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c^5*
e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*b^3
*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^7*b
^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16 - 2
2272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*b^2
*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^
2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^11*e
 + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*
c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 + 960
*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 - 24
576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^17
- 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*b^9
*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^17
+ 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*b^1
0*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^
7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*
b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^
9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 +
 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/
(4*a^4*e*(4*a*c - b^2)^(5/2)) - (((3*((12*a^6*b^15*c^2*e^16 - 30720*a^13*b*
c^9*e^16 - 300*a^7*b^13*c^3*e^16 + 3156*a^8*b^11*c^4*e^16 - 17976*a^9*b^9*c

^5*e^16 + 59136*a^10*b^7*c^6*e^16 - 109824*a^11*b^5*c^7*e^16 + 101376*a^12*
b^3*c^8*e^16 + 153600*a^13*c^10*d^2*e^16 + 6*a^6*b^14*c^3*d^2*e^16 - 108*a^
7*b^12*c^4*d^2*e^16 + 588*a^8*b^10*c^5*d^2*e^16 + 792*a^9*b^8*c^6*d^2*e^16
- 22272*a^10*b^6*c^7*d^2*e^16 + 100608*a^11*b^4*c^8*d^2*e^16 - 199680*a^12*
b^2*c^9*d^2*e^16)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8
*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) + ((6*b^1
1*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b
^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e^17 +
960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*e^17 -
24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*d^2*e^
17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960*a^12*
b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*d^2*e^
17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(2*(4*a^4*
b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560
*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^1
0*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*
a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*
(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*
(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 1
20*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^12*c^3*e
^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*b^6*c^6*
e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b^15*c^2*
d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17 - 24960
*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*b^5*c^7*
d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^17))/(8*a
^4*e*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*
e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a
^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6
*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2
*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c
^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b
^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) + (27*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17 - 96*a^11*b^
12*c^3*e^17 + 960*a^12*b^10*c^4*e^17 - 5120*a^13*b^8*c^5*e^17 + 15360*a^14*
b^6*c^6*e^17 - 24576*a^15*b^4*c^7*e^17 + 16384*a^16*b^2*c^8*e^17 + 12*a^9*b
^15*c^2*d^2*e^17 - 328*a^10*b^13*c^3*d^2*e^17 + 3840*a^11*b^11*c^4*d^2*e^17
- 24960*a^12*b^9*c^5*d^2*e^17 + 97280*a^13*b^7*c^6*d^2*e^17 - 227328*a^14*
b^5*c^7*d^2*e^17 + 294912*a^15*b^3*c^8*d^2*e^17 - 163840*a^16*b*c^9*d^2*e^1
7))/(64*a^12*e^3*(4*a*c - b^2)^(15/2)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b
^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^1
4*b^2*c^5)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*
a*b^6*c)*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2)^
(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c -
b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*
(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2

```
*(4*a*c - b^2)^(13/2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c
^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^
6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 +
3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b
^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*(4*a*c - b^2)^(5/2
))
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx = \frac{f^4x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) f^4 \arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) f^4 \arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] f^4*x/c-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1156, 1136, 1180, 211}

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx = -\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}} - \frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}} + \frac{f^4x}{c}$$

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f^4 \text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{f^4 x}{c} - \frac{f^4 \text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce}\\
&= \frac{f^4 x}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce}\\
&\quad - \frac{\left(\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2ce}
\end{aligned}
$$

$$= \frac{f^4x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.10

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{f^4\left(2\sqrt{c}(d+ex) - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2c^{3/2}e}$$

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
default	$f^4\left(\frac{x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(-b\,e^2\_R^2-2bde\_R-b\,d^2-a\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}}{2ce}\right.$
risch	$\frac{f^4x}{c} + \frac{f^4\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(-b\,e^2\_R^2-2bde\_R-b\,d^2-a\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}\right.}{2ce}$

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] f^4*(x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(166) = 332$.

Time = 0.29 (sec) , antiderivative size = 1346, normalized size of antiderivative = 6.66

$$\int \frac{(df + efx)^4}{a + b(d + ex)^2 + c(d + ex)^4} dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*
d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*
sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^
6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + s
qrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^
16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/
2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*
f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*
a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4
))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqrt(1/2)*c*sqrt(
-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4
*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a
*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*
a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c
)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b
^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*
f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*
(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^
2*c^3 - 4*a*c^4)*e^2))))/c
```

Sympy [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\cdot\left(48a^2bc^2e^2f^8-28ab^3ce^2f^8+4b^5e^2f^8\right)+a^3f^{16}\right.$$
$$+\frac{f^4x}{c}$$

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**2*
f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3
*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*
e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)
))) + f**4*x/c
```

Maxima [F]

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{(efx+df)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] f^4*x/c - f^4*integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 + 4*
c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x
 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. 2(166) = 332.

Time = 0.30 (sec) , antiderivative size = 1366, normalized size of antiderivative = 6.76

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f^4*x/c + 1/2*((b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
```

```
c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x + sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e
+ (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) + d/e)) - (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) - d/e)^2 + 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*
e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) - d/e)) + (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x + sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)) - (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x - sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e +
b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e)))/(c*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 4605, normalized size of antiderivative = 22.80

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c +
 ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*
a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8
+ b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c
*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c
^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e
^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^
```

2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i + ((2*b^
4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^
3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^
13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1
/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a
*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3
*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 -
 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*e^11*f^8
 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 -
 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x
*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*
f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
 a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e
^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
 b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*
e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^
12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^1
4 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(
1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-
(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*
f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 + b^2*f^8*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*2i + atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*
f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^
13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(
b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f
^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8
*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a

^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i + ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + (f^4*x)/c

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 87

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ace}} + \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*f^3*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*f^3*arctanh((b+2*c*(e*x+d)^
2)/(-4*a*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1156, 1128, 648, 632, 212, 642}

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}} + \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f^3 \text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{f^3 \text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{f^3 \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{(bf^3)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce}
\end{aligned}$$

$$= \frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{(bf^3)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2ce}$$

$$= \frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}e} + \frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^3\left(-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+b(d+ex)^2+c(d+ex)^4\right)\right)}{4ce}$$

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.77

method	result
default	$\frac{f^3\left(\sum_{\_R=\mathrm{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+\left(4d^3 ec+2bde\right)\_Z+d^4c+b d^2+a\right)} \frac{\left(\_R^3 e^3+3\_R^2 d e^2+3\_R d^2 e+d^3\right)\ln\left(x\right.}{2e^3c\_R^3+6cd e^2\_R^2+6c d^2 e\_R+2d^3c+b}\right.}{2e}$
risch	Expression too large to display

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d
*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.13

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left[\frac{\sqrt{b^2-4\,ac}bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\,(b^2c-4\,ac}\right.$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*
d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x +
 b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*
d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c -
 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 + 4*c*d*
e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*f^3*lo
g(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*
c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(75) = 150.

Time = 1.01 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.82

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right) + \left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
 + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqr
t(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

Maxima [F]

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{(efx+df)^3}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{2\,\sqrt{-b^2+4\,ac}ce} + \frac{f^3\log\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+\left(efx^2+2\,dfx\right)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)}{4\,ce}$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
[Out] -1/2*b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))/(sqrt(-b^2 + 4*a*c)*c*e) + 1/4*f^3*log(c*d^4*f^2 + 2*(e*f*x^2
+ 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 +
 2*d*f*x)*b*e*f + a*f^2)/(c*e)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.30

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\frac{4\,a\,c\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$
$$-\frac{b\,f^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$
$$-\frac{b^2\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*
c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2)
- (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*
e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*
a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^
4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e
^2 - 4*b^2*c*e^2)
```

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 170

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

```
[Out] -1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1156, 1144, 211}

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}} - \frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f^2 \text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e}\\
&\quad+\frac{\left(\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e}\\
&= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^2\left(\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}e}$$

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

method	result
default	$\frac{f^2\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b}\right.}{2e}$
risch	$\frac{f^2\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b}\right.}{2e}$

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d
^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*
d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.70

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6+\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4+(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6+\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\log\left(ef^6x+df^6-\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^3\sqrt{-\frac{bf^4-(b^2c-4\,ac^2)\sqrt{\frac{f^8}{(b^2c^2-4\,ac^3)e^4}}e^2}{(b^2c-4\,ac^2)e^2}}\right)$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3
)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*
```

```
c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2*c
- 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2)
)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*
a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b
^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)
*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(
1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4
 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a
*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^
2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 -
sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(
b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c
- 4*a*c^2)*e^2)))
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2f^4+4b^3e^2f^4\right)+af^8,\left(t\mapsto t\log\left(x+\right.\right.\right.$$

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*lo
g(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6
)/(e*f**6))))
```

Maxima [F]

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{(efx+df)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(135) = 270$.

Time = 0.33 (sec) , antiderivative size = 1443, normalized size of antiderivative = 8.49

$$\int \frac{(df + efx)^2}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*f^2*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*f^2*(sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2*f^2)*log(x - sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 6*c*d^2
*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 2*c
*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e) + b*d*e) - 1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*f^2*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*f
^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2*f
^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^
3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e) + 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e) + b*d*e)
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.02

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)+\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d}{8\,(16\,a^2}\right.}{a\,c\,e^{10}\,f^6}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)-\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}}{8\,(16\,a^2\,c^3}\right.}{a\,c\,e^{10}\,f^6}\right.$$

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4
*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 - 2
*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^1
3 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^
4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11*f^4
 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)
^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(
1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*
(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4
 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d
*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11
*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b^2)^3)^(1/2)
- b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2))
)^(1/2)
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 44

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} \, dx = -\frac{f \operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}$$

```
[Out] -f*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1156, 1121, 632, 212}

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} \, dx = -\frac{f \operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\ &= \frac{f\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\ &= -\frac{f\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{e}\\ &= -\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}e}$$

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.95

method	result
default	$\frac{f\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)} \frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+be\_$
risch	$-\frac{f\ln\left(\left(e^2\sqrt{-4ac+b^2}-b\,e^2\right)x^2+\left(2ed\sqrt{-4ac+b^2}-2bde\right)x+d^2\sqrt{-4ac+b^2}-b\,d^2-2a\right)}{2\sqrt{-4ac+b^2}\,e}+\frac{f\ln\left(\left(e^2\sqrt{-4ac+b^2}+b\,e^2\right)x^2+\left(2ed\sqrt{-4ac$

[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 6.23

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left[\frac{f\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac-(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right)}{2\,\sqrt{b^2-4\,ac}e}, -\frac{\sqrt{-b^2+4\,ac}f\arctan\left(-\frac{(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{(b^2-4\,ac)e}\right]$$

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(41) = 82.

Time = 0.59 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = -\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e} + \frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(
2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(4*a
*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*
f))/(2*e)
```

Maxima [F]

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \int \frac{efx+df}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{\sqrt{-b^2+4\,ac}e}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*
f))/(sqrt(-b^2 + 4*a*c)*e)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 477, normalized size of antiderivative = 10.84

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4}\, dx = \frac{f\,\mathrm{atan}\left(\frac{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)\,1{}\mathrm{i}}{2\,e\,\sqrt{b^2-4\,a\,c}}+\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e^8+\cdots\right.}{\cdots}\right.}{2\cdots}}{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}-\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e^8+\cdots\right.}{\cdots}\right.}{2\cdots}}\right.}{e\,\sqrt{b^2-4\,a\,c}}$$

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2
*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2))
+ 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f + 4*
c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*
b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2
- 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8
+ 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c
)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*d^2*e^7*
f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2
 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b
^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

3.642 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef}+\frac{\log(d+ex)}{aef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}$$

```
[Out] ln(e*x+d)/a/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e/f+1/2*b*arctanh((b+2*
c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/f/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1156, 1128, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{ef}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx,x,(d+ex)^2\right)}{2ef}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,(d+ex)^2\right)}{2aef}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2aef}\\
&= \frac{\log(d+ex)}{aef}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4aef}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4aef}\\
&= \frac{\log(d+ex)}{aef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{2aef}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef}+\frac{\log(d+ex)}{aef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$
\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\right.}{4a\sqrt{b^2-4ac}ef}
$$

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\ln(ex+d)}{aef}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2f^2e^2c-a\,b^2f^2e^2\right)\_Z^2+\left(4acef-b^2ef\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac\,e^3f-3b^2e^3f\right)\_R+5c\,e^2\right)x^2+\left(\left(20acd\,e^2\right.\right.\right.\right.}{2}$
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(-e^3c\_R^3-3cd\,e^2\_R^2+e\left(-3c\,d^2-b\right)\_R-d^3c-\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2d^3c+b}}{2ae}}{f}$

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/a/e/f+1/2*sum(_R*ln(((10*a*c*e^3*f-3*b^2*e^3*f)*_R+5*c*e^2)*x^2+(
(20*a*c*d*e^2*f-6*b^2*d*e^2*f)*_R+10*d*c*e)*x+(10*a*c*d^2*e*f-3*b^2*d^2*e*f
-a*b*e*f)*_R+5*c*d^2+2*b),_R=RootOf((4*a^2*c*e^2*f^2-a*b^2*e^2*f^2)*_Z^2+(4
*a*c*e*f-b^2*e*f)*_Z+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-4}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4}\right.$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
+ 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2
 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^
4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6
*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*
log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) -
 (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^
2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x + d))/((a*
b^2 - 4*a^2*c)*e*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(83) = 166.

Time = 17.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.38

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+\cdots}{bce^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4aef\left(4ac-b^2\right)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bc\cdots}{bce^2}\right) + \frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e
 + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) -
 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2
)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c +
 b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**
2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*
a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) -
 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

Maxima [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x + b*d)/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d
^3 + b*d)*e*x + a), x)/(a*f) + log(e*x + d)/(a*e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(95) = 190$.

Time = 0.40 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$$
$$= -\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,aef}$$
$$+\frac{\log\left(\left|ex+d\right|\right)}{aef}$$
$$-\frac{\frac{abce^3f\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}}-\frac{abce^3f\log\left(\left|-be^2x^2+\sqrt{b^2-4\,ac}e^2x^2-2\,bdex+2\,\sqrt{b^2-\right.\right.}{\sqrt{b^2-4\,ac}}}{4\,a^2ce^4f^2}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*
d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a*e*f) + log(abs(e*x + d))/(a*e*
f) - 1/4*(a*b*c*e^3*f*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d
*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sq
rt(b^2 - 4*a*c) - a*b*c*e^3*f*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^
2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 -
 2*a))/sqrt(b^2 - 4*a*c))/(a^2*c*e^4*f^2)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 2520, normalized size of antiderivative = 24.47

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b
*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*c*
e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(4*a
*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^
3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f
^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2*e*f -
 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*
d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e^17)/f))
/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*
(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b^2*e^2*f^2
 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - (((b*(
```

```
2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20
*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*
e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)
) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3*e^3*f^4*(4*a*c
 - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*e^18*f - 20*a*b*
c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b
^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/
2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 - 8*a*b*c)*(4*a*c - b
^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f + ((2*b^2*e*f -
8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17
*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c - b^
2)) - ((2*b^2*e*f - 8*a*c*e*f)^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f
 + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20
*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(4*(4*a*b^
2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^
2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a^2*e^2*f^3*(4
*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(b^2*c^4*e^14*(25*a*c -
 6*b^2)) + (16*a^3*f^3*x^2*(4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((
10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*
e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c
- b^2)) - (((10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f
- 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)))*(2*b^2*e*f
- 8*a*c*e*f)^2)/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a^2*e^2*f^3*(4*a*
b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*((10*b*c^3*e^18)/f + ((2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2
- 16*a^2*c*e^2*f^2)))*(2*b^2*e*f - 8*a*c*e*f))/(4*a*e*f*(4*a*b^2*e^2*f^2 -
16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*e^19*f - 20*a*b*c^
3*e^19*f))/(32*a^3*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f
)^2*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a*e*f^2*(4*a*b^2*e^2*f^2 - 1
6*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(2
5*a*c - 6*b^2))))/(b^2*c^2*e^14) - (2*f^3*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2
 - 14*a*b^2*c)*((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^
2*e^16))/f + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e
^17*f - 20*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/
(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(
2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(32*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(2*a*b^2*c^2*e
^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a*e*f^2*(4*a*b^2*
e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(b^2*c^4*e^14*(25*a*c
- 6*b^2))))/(2*a*e*f*(4*a*c - b^2)^(1/2))
```

3.643 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\begin{aligned}
&\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)} dx \\
&= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2} \\
&\quad - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}ef^2}
\end{aligned}$$

```
[Out] -1/a/e/f^2/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used

$= \{1156, 1137, 1180, 211\}$

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}aef^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}aef^2\sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{aef^2(d+ex)}$$

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^2}\\
&= -\frac{1}{aef^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{aef^2}\\
&= -\frac{1}{aef^2(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2aef^2}\\
&\quad - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2aef^2}\\
&= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2}\\
&\quad - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}ef^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$
\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{\frac{2}{d+ex} + \frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2aef^2}
$$

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4c d\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4d^{3}ec+2bde\right)\_Z+d^{4}c+b\,d^{2}+a\right)}\frac{\left(-\_R^{2}c\,e^{2}-2\_Rcde-c\,d^{2}-b\right)\ln\left(x-\_R\right)}{2e^{3}c\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_R+2d^{3}c+be\_R}}{2ae}}{f^{2}}$
risch	$-\frac{1}{ae\,f^{2}(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16f^{8}e^{4}c^{2}a^{5}-8b^{2}f^{8}e^{4}c\,a^{4}+b^{4}f^{8}e^{4}a^{3}\right)\_Z^{4}+\left(12a^{2}b\,c^{2}e^{2}f^{4}-7a\,b^{3}c\,e^{2}f^{4}+b^{5}e^{2}f^{4}\right)\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(\left(\right.\right.\right.$

[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] 1/f^2*(1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/a/e/(e*x+d))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(167) = 334.

Time = 0.31 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.24

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*

```
c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8))
+ b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x +
 a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^
2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((
a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sq
rt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
- 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) + sqr
t(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((
a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*
c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^
4*c)*e^2*f^4))) - 2)/(a*e^2*f^2*x + a*d*e*f^2)
```

Sympy [A] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2e^4f^8-128a^4b^2ce^4f^8+16a^3b^4e^4f^8\right)+t^2\cdot\left(48a^2bc^2e^2f^4-28ab^3ce^2f^4+4b^5e^2f^4\right)+\right.$$
$$-\frac{1}{adef^2+ae^2f^2x}$$

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**
3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4
 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*
_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d
 - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x
)
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^2}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] -1/(a*e^2*f^2*x + a*d*e*f^2) - integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 + b
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a), x)/(a*f^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad A
rgument ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 4339, normalized size of antiderivative = 21.27

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6)
 - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*
e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e
^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
```

*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16* a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) - (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - 1/(a*e*(d*f^2 + e*f^2*x))

3.644 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 133

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{1}{2aef^3(d+ex)^2}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}ef^3}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}$$

```
[Out] -1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+
d)^4)/a^2/e/f^3-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/
2))/a^2/e/f^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1156, 1128, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = -\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*1/(a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^
2*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^3} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2aef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, (d+ex)^2\right)}{2aef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^2ef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2ef^3} \\
&\quad + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2ef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3} \\
&\quad - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2a^2ef^3} \\
&= -\frac{1}{2aef^3(d+ex)^2} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,ef^3} \\
&\quad - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{-\frac{2a}{(d+ex)^2}-4b\log(d+ex)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}}{4a^2ef^3}$$

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.63

method	result
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+b\,d^2+a\right)}\frac{\left(bc\,e^3\_R^3+3bcd\,e^2\_R^2+e\left(3bc\,d^2-ac+b^2\right)\_R\right.}{2e^3c\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R}}{2a^2e}}{f^3}$
risch	$-\frac{1}{2ae\,f^3(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e\,f^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2f^6-a^2b^2e^2f^6\right)\_Z^2+\left(-4abce\,f^3+b^3e\,f^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4f^6-\right.\right.\right.\right.}{}$

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*_
R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R
*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a
^2/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(123) = 246$.

Time = 0.38 (sec) , antiderivative size = 828, normalized size of antiderivative = 6.23

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$

$$= \left[-\frac{2\,ab^2-8\,a^2c+((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\,ab^2-8\,a^2c+2\,((b^2-2\,ac)e^2x^2+2\,(b^2-2\,ac)dex+(b^2-2\,ac)d^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,ce^2x^2+4\,cdex}{b}\right.}{\cdots}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3
+ 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*
d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)
*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*
d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*
e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^
2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2
 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2
+ 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x
 + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*
(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^3}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] -1/2/(a*e^3*f^3*x^2 + 2*a*d*e^2*f^3*x + a*d^2*e*f^3) + integrate((b*c*e^3*x
^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2 - a*c)*
d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^3) - b*log(e*x + d)/(a^2*e*f^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(123) = 246$.

Time = 0.41 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.68

$$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{b\log\left(|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a|\right)}{4\,a^2ef^3} - \frac{b\log\left(|ex+d|\right)}{a^2ef^3} - \frac{1}{2\,(ex+d)^2aef^3} + \frac{\frac{(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3)\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3)\log\left(\left|-\right.\right.}{}}{4\,a^4ce^4f^6}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c
*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f^3) - b*log(abs(e*x + d)
)/(a^2*e*f^3) - 1/2/((e*x + d)^2*a*e*f^3) + 1/4*((a^2*b^2*c*e^3*f^3 - 2*a^3
*c^2*e^3*f^3)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2
*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(b^2 -
 4*a*c) - (a^2*b^2*c*e^3*f^3 - 2*a^3*c^2*e^3*f^3)*log(abs(-b*e^2*x^2 + sqrt
(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqr
t(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))/(a^4*c*e^4*f^6)
```

Mupad [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 5947, normalized size of antiderivative = 44.71

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx=\text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a*
b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f^9
) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8
*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^
3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^3*e*f
^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*c^5*d*
e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6)
)/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3
*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*
(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^
9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))
/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18*f
^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)
^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*
a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 -
 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c -
 b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a
^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*
e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + ((((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^1
7*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*
(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f
^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*
```

f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e
^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*a*c - b^2)^3)/(32*a^9*e^3*f^18*(4*a
*c - b^2)^(3/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b
^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e
^14) + (16*a^6*f^9*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b
*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (c^5*e^16)/(a^3*f^9
) - ((2*a*c - b^2)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f
^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^
3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b
^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12
*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12
*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*e*f^3
*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19
*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2)^2)/(32*a^7*e^2*f^15*(4*a*c - b^
2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3*e*f^3 - 8*a*b*
c*e*f^3)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9
))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c
^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12*(4*a*c -
b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 -
4*a^2*b^2*e^2*f^6)) + ((12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(
2*a*c - b^2)^3)/(64*a^9*e^3*f^18*(4*a*c - b^2)^(3/2)) + (((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4
*a*b^2*c^3*e^14) + (2*a^3*f^9*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^
2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/(a^3*f^9) + (((((4*a^2*b^3*c^2*e^16*f^6 +
20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^
6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*
a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3 - a^2*c^4*e^15*f^3 +
6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a
^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16*
f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^
16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9

- 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)^2*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(32*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)) + (2*a^3*f^9*(4*a*c - b^2)*(((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3 - a^2*c^4*e^15*f^3 + 6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^3*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(64*a^9*e^3*f^18*(4*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)))*(2*a*c - b^2))/(2*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - 1/(2*a*e*(d^2*f^3 + e^2*f^3*x^2 + 2*d*e*f^3*x)) - (b*log(d + e*x))/(a^2*e*f^3) - (log(((c^5*e^16*x^2)/(a^3*f^9) - ((b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/(a^2*f^6) - ((b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^2))/(a*f^3) + (b*c^2*e^16*(b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c + b^2))/(a*f^3)))/(4*a^2*e*f^3) + (6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2*c^5*d*e^15*x)/(a^3*f^9))*((c^5*e^16*x^2)/(a^3*f^9) - ((b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((c^3*e^15*(4*b^2 - a*c + 6*b*c*d^2))/(a^2*f^6) - ((b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*((2*c^2*e^16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c))/(a*f^3) + (2*c^3*e^18*x^2*(10*a*c + b^2))/(a*f^3) + (b*c^2*e^16*(b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*d

```
^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e
*x))/(a^2*f^3) + (4*c^3*d*e^17*x*(10*a*c + b^2))/(a*f^3)))/(4*a^2*e*f^3) +
(6*b*c^4*e^17*x^2)/(a^2*f^6) + (12*b*c^4*d*e^16*x)/(a^2*f^6)))/(4*a^2*e*f^3
) + (c^4*e^14*(b + c*d^2))/(a^3*f^9) + (2*c^5*d*e^15*x)/(a^3*f^9)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))
```

3.645 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 236

$$\begin{aligned}
&\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx \\
&= -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}ef^4} \\
&\quad+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}ef^4}
\end{aligned}$$

```
[Out] -1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)
)/a^2/e/f^4*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))/a^2/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used

$= \{1156, 1137, 1295, 1180, 211\}$

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}} + \frac{b}{a^2ef^4(d+ex)} - \frac{1}{3aef^4(d+ex)^3}$$

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/3*1/(a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[
c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f
^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^4}\\
&= -\frac{1}{3aef^4(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{3aef^4}\\
&= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{3a^2ef^4}\\
&= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)}\\
&\quad + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2a^2ef^4}\\
&\quad + \frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2a^2ef^4}\\
&= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}ef^4}\\
&\quad + \frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}ef^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$

$$= \frac{-\frac{2a}{(d+ex)^3} + \frac{6b}{d+ex} + \frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2ef^4}$$

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e*f^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{\sum_{\_R=\text{RootOf}\left(c e^4 \_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_Z+d^4c+bd^2+a\right)} \frac{\left(\_R^2 bce^2+2\_R bcde+bcd^2-ac+b^2\right)\ln\left(x-\_ \ldots\right.}{2e^3c\_R^3+6cde^2\_R^2+6cd^2e\_R+2d^3c+be\_ \ldots}}{2a^2e}}{f^4}$
risch	$\frac{\frac{bex^2}{a^2}+\frac{2bdx}{a^2}-\frac{-3bd^2+a}{3ea^2}}{f^4(ex+d)^3}+\left(\sum_{\_R=\text{RootOf}\left(\left(16f^{16}e^4c^2a^7-8a^6b^2ce^4f^{16}+a^5b^4e^4f^{16}\right)\_Z^4+\left(-20bf^8e^2c^3a^3+25b^3f^8e^2c^2a^2-9b^5f^8e^2ca+b^7 \ldots\right.\right.}\right.$

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^
4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c
+b*d^2+a))-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2212 vs. $2(200) = 400$.

Time = 0.30 (sec) , antiderivative size = 2212, normalized size of antiderivative = 9.37

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*
a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 -
4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/
```

((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)

Sympy [A] (verification not implemented)

Time = 105.92 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.74

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}$$
$$+\text{RootSum}\left(t^4\cdot\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-3\right.\right.$$

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d*
*2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**
4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*
f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(
x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8
*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*
e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*
f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2
*c**4*e + b**4*c**3*e))))
```

Maxima [F]

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^4}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] 1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*
f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4) + integrate((b*c*e^2*x^2 + 2
*b*c*d*e*x + b*c*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(200) = 400$.

Time = 0.29 (sec) , antiderivative size = 1353, normalized size of antiderivative = 5.73

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*c
*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2
*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) +
 b*c*d^2 + b^2 - a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*c*e^2*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*c*d*e*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*c*d^2 +
 b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*c*e^2*(sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*c*d*e*(sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*c*d^2 + b^2 - a*c
)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(
2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3
+ 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(a^2*f^4) + 1/3*(3*b*e^2*x^2 + 6*b*
d*e*x + 3*b*d^2 - a)/((e*x + d)^3*a^2*e*f^4)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 5771, normalized size of antiderivative = 24.45

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f^
4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 +
16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8
+ 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^
3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^
14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3
*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^
9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8
*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12
 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 2
0*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2
*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e
^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2
*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a
^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^
20) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12
*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e
^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c
^4*d*e^11*f^12)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b
^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
- 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^
```

3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2
c(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a
^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14
*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*
c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*
a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*
a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12
) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))
^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^
2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)
))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*
b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) + 16*a^10*c^4*e^12*f^16 + 4
*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^
12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e
^10*f^8))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^
8)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
+ 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*
a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^
14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 +
 b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a
^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x
*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12)
+ 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*
f^12)*1i + (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^
3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*
f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e
^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) -

32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^
2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^1
2*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5
*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5
*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i)/(
(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2
c^2(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2
)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*((x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c
^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*
a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) - 16*a^10*c^4*e^12*f^16 - 4*
a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
+ 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^1
2 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) - (-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b
^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^
10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20
 + 8*a^10*b^3*c^2*d*e^13*f^20)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^
8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^
12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^
3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*
c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e^10*f^8))*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*2i

3.646 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\,e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] 1/2*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used

$= \{1156, 1134, 1180, 211\}$

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{f^4\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f^4\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)^{3/2}e} + \frac{\left(\left(b^2+4ac+b\sqrt{b^2-4ac}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)^{3/2}e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4e}$$

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.17

method	result
default	$f^4\left(\frac{-\frac{b\,e^2x^3}{2(4ac-b^2)}-\frac{3bde\,x^2}{2(4ac-b^2)}-\frac{(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(\cdots\right.}}{\cdots}\right.$
risch	$\frac{-\frac{b\,e^2f^4x^3}{2(4ac-b^2)}-\frac{3dbe\,f^4x^2}{2(4ac-b^2)}-\frac{f^4(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d\,f^4(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{f^4\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(\cdots\right.}\right.}{\cdots}$

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^4*((-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*a
)/(4*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^
2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*
_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. 2(235) = 470.

Time = 0.32 (sec) , antiderivative size = 2578, normalized size of antiderivative = 9.24

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(b
*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt
(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 +
```

4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*

```
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(252) = 504.

Time = 11.98 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.30

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{-2adf^4-bd^3f^4-3bde^2f^4x^2-be^3}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde}$$
$$+\operatorname{RootSum}\left(t^4\cdot(1048576a^6c^7e^4-1572864a^5b^2c^6e^4+983040a^4b^4c^5e^4-327680a^3b^6c^4e^4+61440a^2b^8c^3e\right.$$

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(-
2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +
8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2
*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e
**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*
c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Roo
tSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a
**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4
 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e
**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 +
16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f
**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**
2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64
*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f**8 + 4*a*c*d*
f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

Maxima [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^2
)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
```

```
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b
*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 + 2*a*d
)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (
b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(235) = 470$.

Time = 0.30 (sec) , antiderivative size = 1474, normalized size of antiderivative = 5.28

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)^2 - 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e) + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 +
b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) -
(b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^2 + 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
- d/e) + b*d^2*f^4 - 2*a*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)
*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^2
*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2
*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
 + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b*e^2*f^4*(
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e
*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d
^2*f^4 - 2*a*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c
*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
```

```
)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(b^2 - 4*a*c) + 1/2
*(b*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + 3*b*d^2*e*f^4*x + b*d^3*f^4 + 2*a*e*f
^4*x + 2*a*d*f^4)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e
*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 8025, normalized size of antiderivative = 28.76

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^
4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c
^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5
*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4
*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6
*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 +
 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^
4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a
^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i - ((128*a^3*c^4*
d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4
*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a
*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*
a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
(x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^
3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*
f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c
```

^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2
)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 -
96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^
2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a
^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2
*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*
c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((2048*a^4*c^5*e^12*f^4 + 384*a^2
*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 10
24*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16
384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^
2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*
c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b
^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6
e^2)))^(1/2))(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^
8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^
7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 38
40*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^
8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c
^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
c)))(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^
2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 2
4*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^
4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)
^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(
32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2
 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1
/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(

b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (3*a*b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i - ((x*(2*a*f^4 + 3*b*d^2*f^4))/(2*(4*a*c - b^2)) + (b*d^3*f^4 + 2*a*d*f^4)/(2*e*(4*a*c - b^2)) + (b*e^2*f^4*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*f^4*x^2)/(2*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i - ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*

(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)
 - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768
*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2
+ 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6
*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
c)))((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-(4*a*c - b^2)^9)^(1/2) -
b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32
*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 -
 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2
) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/
(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 +
 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
c)))((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2
*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24
*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4
*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6
*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 15
36*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 +
16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^1
4 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 7
68*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*((f^8*(-
(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 -
 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2
)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3
*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*

b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (3*a*b^3*c*e^10*f^12 + 4*a^2*b*c^2*e^10*f^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))))*((f^8*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*2i

3.647 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3(2a+b(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*f^3*
arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1128, 652, 632, 212}

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3(2a+b(d+ex)^2)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{bf^3\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f^3\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{f^3\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2\left(b^2-4ac\right)e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\left(bf^3\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}\right)}{2e}$$

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x
)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2)))/(2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.72

method	result
default	$f^3\left(\frac{-\frac{x^2eb}{2\left(4ac-b^2\right)}-\frac{xbd}{4ac-b^2}-\frac{b\,d^2+2a}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\right.}\right.}{}\right.$
risch	$\frac{-\frac{be\,f^3x^2}{2\left(4ac-b^2\right)}-\frac{bd\,f^3x}{4ac-b^2}-\frac{f^3\left(b\,d^2+2a\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{b\,f^3\ln\left(\left(\left(-\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4ab\,e^2c-b^3e^2\right)x^2+\left(-2\left(-4ac+b\right.\right.\right.\right.}{2e}$

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*((-1/2/(4*a*c-b^2)*x^2*e*b-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*c
-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(97) = 194$.

Time = 0.32 (sec) , antiderivative size = 1077, normalized size of antiderivative = 10.46

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx$$
$$= \left[\frac{(b^3 - 4\,abc)e^2f^3x^2 + 2\,(b^3 - 4\,abc)def^3x + (2\,ab^2 - 8\,a^2c + (b^3 - 4\,abc)d^2)f^3 - (bce^4f^3x^4 + 4\,bcde^3f^3x^3}{2\,((b^4c - 8\,ab^2c^2 + 16\,a^2c^3)e^5x^4 + 4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)de^4x^3 + (b^5 - 8\,ab^3c + 16\,a^2bc^2 + 6\,(b^4c - 8}\right.$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
- 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x
^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d
^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 +
 b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^
2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*
x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^
2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*
f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c
)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e
^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(88) = 176$.

Time = 2.77 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.40

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} - \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e} + \frac{-2af^3-bd^2f^3-2b}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cd}$$

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**
3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3) -
 b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c
*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b
*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 - 2*b*d*e
*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a
*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b*
*2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3
 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d
*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Maxima [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*
```

d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x + (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.96

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ace}} + \frac{bd^2f^5+(efx^2+2\,dfx)bef^4+2\,af^5}{2\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+(efx^2+2\,dfx)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)(b^2e-4\,ac}$$

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*(b*d^2*f^5 + (e*f*x^2 + 2*d*f*x)*b*e*f^4 + 2*a*f^5)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)*(b^2*e - 4*a*c*e))

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.47

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b\,f^3\,\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)^4\left(x\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)+x^2\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,\left(4\,a\,c-b^2\right)^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,\left(4\,a\,c-b^2\right)^{7/2}}\right)-\frac{b^3\,f^6\,(}{}}{2\,b^2\,c^2\,e^6\,f^6}\right.}{e\,\left(4\,a\,c-b^2\right)^{3/2}} - \frac{\frac{f^3\,\left(b\,d^2+2\,a\right)}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{b\,d\,f^3\,x}{4\,a\,c-b^2}+\frac{b\,e\,f^3\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out] (b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)^(

```
7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c -
 b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(16*a
^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a
*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(7/2)))
)/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2))/(2*e
*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a*c - b^2
)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e
) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.648 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] -1/2*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used

$= \{1156, 1133, 1180, 211\}$

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \frac{\sqrt{c}f^2\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}f^2\left(\sqrt{b^2-4ac}+2b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*
f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{f^2\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^2\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)^{3/2}e} - \frac{\left(c\left(2b+\sqrt{b^2-4ac}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2\left(b^2-4ac\right)^{3/2}e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2e}$$

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.23

method	result
default	$f^2\left(\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{(6c d^2+b)x}{8ac-2b^2}+\frac{d(2c d^2+b)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{\sum_{\_R=\text{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+(4c}}{}\right.$
risch	$\frac{\frac{c e^2 f^2 x^3}{4ac-b^2}+\frac{3dce f^2 x^2}{4ac-b^2}+\frac{f^2(6c d^2+b)x}{8ac-2b^2}+\frac{d f^2(2c d^2+b)}{2e(4ac-b^2)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{f^2\left(\sum_{\_R=\text{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6c d^2 e^2+b e^2\right)\_Z^2+(4d^3\right.}\right.}{}$

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*((c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c-
b^2)*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum(
(2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2
*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^
2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. 2(222) = 444.

Time = 0.30 (sec) , antiderivative size = 2600, normalized size of antiderivative = 9.89

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(2
*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c
^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2
```

```
*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4
 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*
a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/
((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c -
4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*
c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e
^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((
3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^
3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4
*c^3)*e^2))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 -
12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^
3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c +
4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a
*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c
)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c
^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4
*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*
x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt
(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c
^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2
*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 2
56*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
)*e^4))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*e^2))))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
```

```
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(
2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*
d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 -
 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 +
(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(222) = 444$.

Time = 0.30 (sec) , antiderivative size = 1482, normalized size of antiderivative = 5.63

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c
```

```
*e^4)) + d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 +
 b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) -
 (2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)^2 + 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) - d/e) + 2*c*d^2*f^2 - b*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e
^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (2*
c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*
(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (2*c*e^
2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 +
4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
) + 2*c*d^2*f^2 - b*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(b^2 - 4*a*c
) - 1/2*(2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + 6*c*d^2*e*f^2*x + 2*c*d^3*f^
2 + b*e*f^2*x + b*d*f^2)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*
c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 7835, normalized size of antiderivative = 29.79

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*e
*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*c
- b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d
^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1/2))
/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
```

^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4
 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 -
 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*
b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*
c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*
c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^
2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^
4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*
e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4
*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7
*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b
*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*
c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e
^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4
*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*
b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*
b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^
7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6 + (3*b
^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (((f
^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b
^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^
10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^
2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8
*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^

5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))*((f^4*(-(4*a*c - b^2)^9)^(1/2) - b^9*f^4 + 768*a^4*b*c^4*f^4 + 96*a^2*b^5*c^2*f^4 - 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i + atan(((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^

(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (-((b^9*f^4)/32 + (f^4*(
-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3
*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*
b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^
5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4
*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7
*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 +
 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 -
128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 204
8*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x
*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4
*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b
^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^
4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2
 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/
2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2
- 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)
/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 -
5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6
 + (3*b^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
 + (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 -
 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 2
4*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^
4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a*c -
 b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 7
68*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b
*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*
f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*
c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*
c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 -
 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f
^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4

```
 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) - (-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f
^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2
 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^
5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((-((b^9*f^4)/32 + (f^4*(-(4*a
*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13
 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a
^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((
b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - 24*a^4*b*c^4*f^4 - 3*a^2*
b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b
^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e
^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^
12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11
*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))))*(-(b^9*f^4 + f^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^4 -
96*a^2*b^5*c^2*f^4 + 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^
2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a
^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*2i
```

3.649 $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 98

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=-\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2cf\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] -1/2*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1156, 1121, 628, 632, 212}

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{2cf\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f(b+2c(d+ex)^2)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
 e*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2
- 4*a*c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}\\
&= \frac{f\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= -\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{(cf)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{\left(b^2-4ac\right)e}\\
&= -\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad + \frac{(2cf)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{\left(b^2-4ac\right)e}
\end{aligned}$$

$$= -\frac{f(b+2c(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}e}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = -\frac{f\left(\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{2(b^2-4ac)e}$$

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*A
rcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b^2
 - 4*a*c)*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.78

method	result
default	$f\left(\frac{\frac{c x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c d^2+b}{2e\left(4ac-b^2\right)}}{c x^4e^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2bdex+b d^2+a}+\frac{c\left(\sum_{\_R=\text{RootOf}\left(c e^4\_Z^4+4cd e^3\_Z^3+\left(6c d^2e^2+b e^2\right)\_Z^2+(\right.}\right.}{}\right.$
risch	$\frac{\frac{cef x^2}{4ac-b^2}+\frac{2cdfx}{4ac-b^2}+\frac{f\left(2c d^2+b\right)}{2e\left(4ac-b^2\right)}}{c x^4e^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2bdex+b d^2+a}+\frac{cf\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}e^2+4ab e^2c-b^3e^2\right)x^2+\left(2(-4ac+b^2)^{\frac{3}{2}}d\right.\right.\right.}{e(-4}$

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*((c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R
*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(
6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 1066, normalized size of antiderivative = 10.88

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx$$
$$= \left[-\frac{2\,(b^2c - 4\,ac^2)e^2fx^2 + 4\,(b^2c - 4\,ac^2)defx + 2\,(c^2e^4fx^4 + 4\,c^2de^3fx^3 + (6\,c^2d^2 + bc)e^2fx^2 + 2\,(2\,c^2d^3}{2\,((b^4c - 8\,ab^2c^2 + 16\,a^2c^3)e^5x^4 + 4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)de^4x^3 + (b^5 - 8\,ab^3c + 16\,a^2bc^2 + 6\,(b^4c -} \right.$$
$$-\frac{2\,(b^2c - 4\,ac^2)e^2fx^2 + 4\,(b^2c - 4\,ac^2)defx - 4\,(c^2e^4fx^4 + 4\,c^2de^3fx^3 + (6}{2\,((b^4c - 8\,ab^2c^2 + 16\,a^2c^3)e^5x^4 + 4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)de^4x^3 + (b^5 - 8\,ab^3c + 16\,a^2bc^2 + 6\,(b^4c -}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^2
*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3
 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2
*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*
c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x
+ 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c
*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*
x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 +
(6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*
d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(87) = 174$.

Time = 2.61 (sec) , antiderivative size = 525, normalized size of antiderivative = 5.36

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx = -\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e} + x^2 + \frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}} + 8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}} - b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}} + bcf + 2c^2d^2f}{2c^2e^2f}\right)}{e} + \frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e} + x^2 + \frac{16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}} - 8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}} + bcf + 2c^2d^2f}{2c^2e^2f}\right)}{e} + \frac{bf + 2cd^2f + 4cde}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\cdot(8ac^2e^5 - 2b^2ce^5) + x^3\cdot(32ac^2de^4 - 8b^2cd}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt(
-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4*c
*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
 c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-
1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*
f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
(b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e
 + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**
4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**
4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**
2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2
*c*d**3*e**2))
```

Maxima [F]

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx = \int \frac{efx + df}{\left((ex + d)^4c + (ex + d)^2b + a\right)^2}\,dx$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)
*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d
^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2
```

```
)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2
*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
 (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.06

$$\int \frac{df + efx}{\left(a + b(d + ex)^2 + c(d + ex)^4\right)^2}\,dx = -\frac{2\,cf\arctan\left(\frac{2\,cd^2f + 2\,(efx^2 + 2\,dfx)ce + bf}{\sqrt{-b^2 + 4\,acf}}\right)}{(b^2 - 4\,ac)\sqrt{-b^2 + 4\,ac}e} - \frac{2\,cd^2f^3 + 2\,(efx^2 + 2\,dfx)cef^2 + bf^3}{2\left(cd^4f^2 + 2\,(efx^2 + 2\,dfx)cd^2ef + (efx^2 + 2\,dfx)^2ce^2 + bd^2f^2 + (efx^2 + 2\,dfx)bef + af^2\right)(b^2e - 4\,ac}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) - 1/2*(2*c*d^2*f^3 + 2*(e*f*x
^2 + 2*d*f*x)*c*e*f^2 + b*f^3)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*
f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a
*f^2)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.51

$$\int \frac{df + efx}{\left(a + b(d + ex)^2 + c(d + ex)^4\right)^2}\,dx = \frac{\frac{f\,(2\,c\,d^2+b)}{2\,e\,(4\,a\,c-b^2)} + \frac{2\,c\,d\,f\,x}{4\,a\,c-b^2} + \frac{c\,e\,f\,x^2}{4\,a\,c-b^2}}{a + x^2\,\left(6\,c\,d^2\,e^2 + b\,e^2\right) + b\,d^2 + c\,d^4 + x\,\left(4\,c\,e\,d^3 + 2\,b\,e\,d\right) + c\,e^4\,x^4 + 4\,c\,d\,e^3\,x^3} + \frac{2\,c\,f\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{8\,b\,c^2\,f^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{4\,b\,c^2\,f^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+}{8\,c^4\,e^6\,f^2}\right.}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*f
*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*
```

```
b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c - b^2
)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c^2*d
*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8*f^
2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/
(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)^(7/2))
 + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*
c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*(4*a*c -
b^2)^(3/2))
```

3.650 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 174

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}ef}+\frac{\log(d+ex)}{a^2ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(3/2)/e/f+ln(e*x+d)/a^2/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/
e/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
```

```
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2a\,(b^2-4ac)\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx, x, (d+ex)^2\right)}{2a\,(b^2-4ac)\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2ef} - \frac{\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^2\,(b^2-4ac)\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2ef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2ef} - \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^2\,(b^2-4ac)\,ef}$$

$$= \frac{b^2 - 2ac + bc(d+ex)^2}{2a(b^2-4ac)ef(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2ef} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^2ef} + \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2a^2(b^2-4ac)ef}$$

$$= \frac{b^2 - 2ac + bc(d+ex)^2}{2a(b^2-4ac)ef(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}ef} + \frac{\log(d+ex)}{a^2ef} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^2ef}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

$$\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + 4\log(d+ex) - \frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} + \frac{(b^3-6a}{}}{4a^2ef}$$

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.32

method	result
default	$-\frac{-\frac{\frac{aebc\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_}{}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{f(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a)}+\frac{\ln(ex+d)}{a^2ef}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2f^2-48a^4b^2c^2e^2f^2+\right.\right.\right.}{}$

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*((1/2*a/(4*a*c-b^2)*e*b*c*x^2+b*c*d*a/(4*a*c-b^2)*x-1/2/e*a*(-b
*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c
*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum((e^3*c*(4
*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b
*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^
3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))+ln(
e*x+d)/a^2/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(164) = 328$.

Time = 0.54 (sec) , antiderivative size = 2486, normalized size of antiderivative = 14.29

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b
*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 -
 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 +
 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d
)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
```

2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c
^2)*e^5*f*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*f*x^3 + (a*b^3 - 4*a^2*b*c +
6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*f*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (
a*b^3 - 4*a^2*b*c)*d)*e^2*f*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e*f) - integrate(((b^2*c - 4*a*c^2)*e^3*x^3
+ 3*(b^2*c - 4*a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*b*c +
3*(b^2*c - 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)*e^4*x^
4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c
+ 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2
+ 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f) + log(e*
x + d)/(a^2*e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(164) = 328$.

Time = 0.40 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.81

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx =$$
$$-\frac{\left(a^2b^3ce^3f-6\,a^3bc^2e^3f\right)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2\right.\right.}{4}$$
$$-\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,a^2ef}$$
$$+\frac{\log\left(|ex+d|\right)}{a^2ef}$$
$$+\frac{abce^2x^2+2\,abcdex+abcd^2+ab^2-2\,a^2c}{2\left(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right)(b^2-4\,ac)a^2ef}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2
*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x +
b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e^3*
f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d
*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(
a^4*b^4*c*e^4*f^2 - 8*a^5*b^2*c^2*e^4*f^2 + 16*a^6*c^3*e^4*f^2) - 1/4*log(a
bs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^
2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f) + log(abs(e*x + d))/(a^2*e*f) + 1
/2*(a*b*c*e^2*x^2 + 2*a*b*c*d*e*x + a*b*c*d^2 + a*b^2 - 2*a^2*c)/((c*e^4*x^
4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b
*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*a^2*e*f)
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 13434, normalized size of antiderivative = 77.21

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^2*
f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c*d*
e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c -
b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c
- b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10
*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)
/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2
```

- 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f*(4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) + (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + log(d + e*x)/(a^2*e*f) + (b*atan((x^2*((((b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a

^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^3*c^5*e^16)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3

- 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e
^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a
c - b^2)^3(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*
b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^
7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))
/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (x*((((((b*(6*a
c - b^2)((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3
*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^
3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*
e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*
a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^
18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*
a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^
2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c -
b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^
2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*
f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b
^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^
3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5
*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f
 - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^
2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^
2)) + (b*((2*(6*a*b^5*c^4*d*e^16*f - 44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6
*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c
^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^
3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c
^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3
*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12
*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e
^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12
*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f
^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3
*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 25
6*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*
c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*
b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 +
 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(32*a^6*e^3*f^
3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48
*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*
a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b
^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((2*(6*a*b^5*c^4*d*e^16*f -
44*a^2*b^3*c^5*d*e^16*f + 80*a^3*b*c^6*d*e^16*f))/(a^3*b^6*f^3 - 64*a^6*c^3
*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (((2*(320*a^5*b*c^6*d*e^17*
f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^
5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^

2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (2*b^3*c^5*d*e^15)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (b*(6*a*c - b^2)*((b*(6*a*c - b^2)*((2*(320*a^5*b*c^6*d*e^17*f^2 - 2*a^2*b^7*c^3*d*e^17*f^2 + 36*a^3*b^5*c^4*d*e^17*f^2 - 192*a^4*b^3*c^5*d*e^17*f^2))/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(2560*a^7*b*c^6*d*e^18*f^3 + 12*a^3*b^9*c^2*d*e^18*f^3 - 184*a^4*b^7*c^3*d*e^18*f^3 + 1056*a^5*b^5*c^4*d*e^18*f^3 - 2688*a^6*b^3*c^5*d*e^18*f^3))/(16*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2)))/(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14) + (((b*((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f + 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^

3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b
^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*
a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 2
56*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b
^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*
b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*
e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*(6*a*c - b
^2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^
16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*
c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f
^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)
+ ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a
^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 -
256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2
*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 -
2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b
^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 1
92*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^
(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
 - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*
a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f
^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*
b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c -
b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^
2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 -
 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f
- 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^
2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(4*a^4*b^8*c^
2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b
^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3
 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*
b^3*c^5*d^2*e^17*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 6
4*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(16*a^6*b^6*f^3*(4
*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*f^
3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9/2))*(3*b^6 - 4
0*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b
^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3
 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16
*a^6*b^6*f^3*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*f^3*(4*a*c - b^2)^(9/2) - 1
92*a^7*b^4*c*f^3*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^(9
/2))*((((4*a*b^6*c^3*e^15*f - 33*a^2*b^4*c^4*e^15*f + 68*a^3*b^2*c^5*e^15*f
 + 6*a*b^5*c^4*d^2*e^15*f + 80*a^3*b*c^6*d^2*e^15*f - 44*a^2*b^3*c^5*d^2*e^
15*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3
) - (((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e
^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7

*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^4*c^4*e^14 - 4*a*b^2*c^5*e^14 + b^3*c^5*d^2*e^14)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*(6*a*c - b^2)*((4*a^2*b^8*c^2*e^16*f^2 - 52*a^3*b^6*c^3*e^16*f^2 + 224*a^4*b^4*c^4*e^16*f^2 - 320*a^5*b^2*c^5*e^16*f^2 - 320*a^5*b*c^6*d^2*e^16*f^2 + 2*a^2*b^7*c^3*d^2*e^16*f^2 - 36*a^3*b^5*c^4*d^2*e^16*f^2 + 192*a^4*b^3*c^5*d^2*e^16*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^2*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^17*f^3 - 48*a^5*b^6*c^3*e^17*f^3 + 192*a^6*b^4*c^4*e^17*f^3 - 256*a^7*b^2*c^5*e^17*f^3 + 2560*a^7*b*c^6*d^2*e^17*f^3 + 12*a^3*b^9*c^2*d^2*e^17*f^3 - 184*a^4*b^7*c^3*d^2*e^17*f^3 + 1056*a^5*b^5*c^4*d^2*e^17*f^3 - 2688*a^6*b^3*c^5*d^2*e^17*f^3))/(32*a^4*e^2*f^2*(4*a*c - b^2)^3*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2*e^14 - 12*a*b^4*c^3*e^14 + 36*a^2*b^2*c^4*e^14)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(6*a*c - b^2))/(2*a^2*e*f*(4*a*c - b^2)^(3/2))

3.651 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{3b^2-10ac}{2a^2(b^2-4ac)\,ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}ef^2}\\ &\quad+\frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}\end{aligned}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/f^2/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d
)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e
*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c
+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1156, 1135, 1295, 1180, 211}

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= -\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^2\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c +
b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c
]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
```

```
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-3bcx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{2a\,(b^2-4ac)\,ef^2}$$

$$= -\frac{3b^2-10ac}{2a^2\,(b^2-4ac)\,ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\text{Subst}\left(\int \frac{-b(3b^2-13ac)-c(3b^2-10ac)x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{2a^2\,(b^2-4ac)\,ef^2}$$

$$= -\frac{3b^2-10ac}{2a^2\,(b^2-4ac)\,ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4a^2\,(b^2-4ac)\,ef^2} - \frac{\left(c\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{4a^2\,(b^2-4ac)\,ef^2}$$

$$= -\frac{3b^2-10ac}{2a^2(b^2-4ac)ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)ef^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}ef^2} - \frac{\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \frac{-\frac{4}{d+ex}+\frac{2(d+ex)(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\cdots}{4a^2ef^2}$$

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\frac{\frac{c e^2 \left(2ac-b^2\right) x^3}{8ac-2b^2}+\frac{3dce\left(2ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(6a c^2 d^2-3b^2 c d^2+3abc-b^3\right)x}{8ac-2b^2}+\frac{d\left(2a c^2 d^2-b^2 c d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c x^4 e^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2bdex+b d^2+a}+\frac{\_R=\text{RootOf}\left(c e^4 \_Z^4+4cd e^3 \_Z^3+\left(6\right.\right.}{}}{}$
risch	Expression too large to display

[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)

[Out] 1/f^2*(-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2*a*c-b^2)/(4*a*c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2)*x+1/2*d/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*d*c*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/a^2/e/(e*x+d))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4520 vs. $2(312) = 624$.

Time = 0.39 (sec) , antiderivative size = 4520, normalized size of antiderivative = 12.56

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3

- 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 19
71*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b
^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4
- 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1054
9*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^
2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3
*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*
a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sq
rt(-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c
+ 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5
625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8
*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e
^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 +
14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(-((a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2
*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*e^2*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c
 - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*
c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
+ 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 1
2*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6
*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^1
1*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c -
385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c
^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -

2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*sqrt(((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - 9*b^7 + 105*a*b^5*c - 385*a^2*b^3*c^2 + 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4))))/((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^2}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + (3*
b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e^2
*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2 + 2*(2*(3*b^2*c - 10*a*c^
2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 +
5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^
2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^
2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a
^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^2) - 1/2*
integrate(((3*b^2*c - 10*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)*d*e*x + 3*
b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(
b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^
2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*
(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(312) = 624$.

Time = 0.34 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.86

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= -\frac{\frac{b^2c}{(efx+df)ef}-\frac{2\,ac^2}{(efx+df)ef}+\frac{b^3f}{(efx+df)^3e}-\frac{3\,abcf}{(efx+df)^3e}}{2\,(a^2b^2-4\,a^3c)\left(c+\frac{bf^2}{(efx+df)^2}+\frac{af^4}{(efx+df)^4}\right)}-\frac{1}{(efx+df)a^2ef}$$
$$+\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}a}e^4f^8+2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab+2}\right.}{}$$
$$-\frac{\left((3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}a}e^4f^8-2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab-2}\right.}{}$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/2*(b^2*c/((e*f*x + d*f)*e*f) - 2*a*c^2/((e*f*x + d*f)*e*f) + b^3*f/((e*f
*x + d*f)^3*e) - 3*a*b*c*f/((e*f*x + d*f)^3*e))/((a^2*b^2 - 4*a^3*c)*(c + b
*f^2/(e*f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)) - 1/((e*f*x + d*f)*a^2*e*f)
+ 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a
*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 + 2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a
*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*e^2*f^4 -
 4*a^3*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3 - 13*a*b*c
)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*x + d*f)*e*
f*sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 + sqrt((a^2*b^3*e^2*f^4 - 4*a^3
*b*c*e^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b^2*c - 4*a^3*
c^2)))/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt
(b^2 - 4*a*c)*e^3*f^6*abs(a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)*abs(a)) - 1/16
*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2
*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 - 2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2
*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*e^2*f^4 - 4*a^3
*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3 - 13*a*b*c)*sqrt
(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*x + d*f)*e*f*sqrt
((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 - sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e
^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b^2*c - 4*a^3*c^2)))
/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 -
 4*a*c)*e^3*f^6*abs(a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 12008, normalized size of antiderivative = 33.36

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^
10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
- 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c
^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 +
 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
```

+ 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
f^4)))^(1/2)((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
+ 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
- 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c
^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^1
0*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^
2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*
e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^1
0*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
c(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
+ 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2
*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a
^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*

e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) - (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6) +

128000*a^10*c^9*e^10*f^4 + 504*a^6*b^8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10
*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 129280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13
 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 128
0*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^
4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4
+ 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^
4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^
2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^
6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^
3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*
a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*
b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a
^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8
*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 2
4*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b
^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14
*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 10
48576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^1
3*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e
^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^1
0 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^
9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^1
2*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 8519
68*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^
12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a
^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^
12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a
^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*
d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^
6)*1i + (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^
10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4

*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c
^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e
^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3
*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^
10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a
^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800
*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^
2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4
 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^
2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*
a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4
*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10
- 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*
a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3
*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*
f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10)
+ 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c
^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 -
 1261568*a^13*b^3*c^7*e^12*f^8 + 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^
12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 +
30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*
c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 +
 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8
*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^1
1*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i)/((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 +
 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4
 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2
f^4)))^(1/2)((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6
 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3
*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a
^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 -
6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*
b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24
*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104

8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
) - (-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2
*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a
^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*
e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b
^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f
^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2
*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c
*e^2*f^4)))^(1/2)*((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 2
40*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^
4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10
*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^1
4*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 15
72864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16
*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e
^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10
 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 19
2*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e
^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 126
1568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c
^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 3011
2*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*
e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144
*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5
*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^
6 - 458752*a^11*b^2*c^8*d*e^11*f^6) + 128000*a^10*c^9*e^10*f^4 + 504*a^6*b^
8*c^5*e^10*f^4 - 8112*a^7*b^6*c^6*e^10*f^4 + 48704*a^8*b^4*c^7*e^10*f^4 - 1

```
29280*a^9*b^2*c^8*e^10*f^4))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 5
1*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e
^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*
c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*2i
 - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*a*b*c*d))/(a*(a*b^2 - 4*a
^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3))/(2*a*(a*b^2 - 4*a^2*c)) - (2*x^
3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2))/(a*(a*b^2 - 4*a^2*c)) + (2*a*b^2 - 8*a^
2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*a*b*c*d^2)/(2*a*e*(a*b^2
- 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*b^2*c*d^2*e - 11*a*b*c*e)
)/(2*a*(a*b^2 - 4*a^2*c)))/(x^2*(10*c*d^3*e^2*f^2 + 3*b*d*e^2*f^2) + x*(a*e
*f^2 + 3*b*d^2*e*f^2 + 5*c*d^4*e*f^2) + x^3*(b*e^3*f^2 + 10*c*d^2*e^3*f^2)
+ b*d^3*f^2 + c*d^5*f^2 + a*d*f^2 + c*e^5*f^2*x^5 + 5*c*d*e^4*f^2*x^4)
```

3.652 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 228

$$\begin{aligned}&\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx\\
&=-\frac{b^2-3ac}{a^2(b^2-4ac)ef^3(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad-\frac{(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}ef^3}\\
&\quad-\frac{2b\log(d+ex)}{a^3ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}\end{aligned}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/f^3/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/
a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b
^2*c+b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3
/2)/e/f^3-2*b*ln(e*x+d)/a^3/e/f^3+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
/f^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used

$= \{1156, 1128, 754, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3} - \frac{2b\log(d+ex)}{a^3ef^3} - \frac{b^2-3ac}{a^2ef^3\,(b^2-4ac)\,(d+ex)^2} - \frac{(6a^2c^2-6ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\,(b^2-4ac)^{3/2}} + \frac{-2ac+b^2+bc(d+ex)^2}{2aef^3\,(b^2-4ac)\,(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}$$

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef^3}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2ef^3}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2a\left(b^2-4ac\right)ef^3}
\end{aligned}
$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{2(-b^2+3ac)}{ax^2}-\frac{2b(-b^2+4ac)}{a^2x}+\frac{2(-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x)}{a^2(a+bx+cx^2)}\right)dx,x,(d+ex)^2\right)}{2a\left(b^2-4ac\right)ef^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2b\log(d+ex)}{a^3ef^3} - \frac{\text{Subst}\left(\int\frac{-b^4+5ab^2c-3a^2c^2-bc(b^2-4ac)x}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{a^3\left(b^2-4ac\right)ef^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2b\log(d+ex)}{a^3ef^3} + \frac{b\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{2a^3ef^3} + \frac{\left(b^4-6ab^2c+6a^2c^2\right)\text{Subst}\left(\int\frac{1}{a+bx+cx^2}dx,x,(d+ex)^2\right)}{2a^3\left(b^2-4ac\right)ef^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{2b\log(d+ex)}{a^3ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}dx,x,b+2c(d+ex)^2\right)}{a^3\left(b^2-4ac\right)ef^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}ef^3} - \frac{2b\log(d+ex)}{a^3ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{-\frac{a}{(d+ex)^2}+\frac{a(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2\right.}}{(b^2-4ac)^{3/2}}}{2a^3ef^3}$$

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e*f^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\frac{\frac{eac\left(2ac-b^2\right)x^2}{8ac-2b^2}+\frac{cda\left(2ac-b^2\right)x}{4ac-b^2}+\frac{a\left(2a\,c^2d^2-b^2c\,d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4d^3ec+2bde\right)\_}}{}}{}$
risch	Expression too large to display

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(-1/a^3*((1/2*e*a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4*
a*c-b^2)*x+1/2/e*a*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d
^2+a)+1/(4*a*c-b^2)/e*sum((e^3*b*c*(-4*a*c+b^2)*_R^3+3*d*e^2*b*c*(-4*a*c+b^
2)*_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a*b*c^
2*d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+d*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))-1/2/a^
2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(220) = 440.

Time = 1.19 (sec) , antiderivative size = 4604, normalized size of antiderivative = 20.19

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] [-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^
2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*
x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*
e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^
3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^
4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 -
8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*
x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c
+ 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*
```

b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3
x^6 + 6(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (
a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^
2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^
3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b
^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)
d^2 + 2(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*
b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6
 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b
^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5
 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^
4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
d^4 + 6(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 -
6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 1
6*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2
*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
 (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(
5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*

c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^3}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3 + 2*(b^2*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4*(b^2*c - 3*a*c^2)*d^3 + (2*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^7*f^3*x^6 + 6*(a^2*b^2*c - 4*a^3*c^2)*d*e^6*f^3*x^5 + (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^3*c^2
```

```
)*d^2)*e^5*f^3*x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*a^3*b*
c)*d)*e^4*f^3*x^3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 6
*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^3*f^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c^2)*d^5
+ 2*(a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*f^3*x + ((a^2*b^
2*c - 4*a^3*c^2)*d^6 + (a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*a^4*c)*d^2)
*e*f^3) + 2*integrate(((b^3*c - 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c - 4*a*b*c^2)*
d*e^2*x^2 + (b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2 + 3*(b^3
*c - 4*a*b*c^2)*d^2)*e*x + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^3*f
^3) - 2*b*log(e*x + d)/(a^3*e*f^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(220) = 440$.

Time = 0.41 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.09

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{(a^3b^4ce^3f^3 - 6\,a^4b^2c^2e^3f^3 + 6\,a^5c^3e^3f^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2 + \sqrt{b^2-4\,ac}e^2x^2 + 2\,bdex + 2\,\sqrt{b^2-4\,ac}d\right.\right.}{}$$
$$+ \frac{b\log\left(\left|ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4 + be^2x^2 + 2\,bdex + bd^2 + a\right|\right)}{2\,a^3ef^3}$$
$$- \frac{2\,b\log\left(\left|ex+d\right|\right)}{a^3ef^3}$$
$$- \frac{2\,ab^2cd^4 - 6\,a^2c^2d^4 + 2\,ab^3d^2 - 7\,a^2bcd^2 + 2\,(ab^2ce^4 - 3\,a^2c^2e^4)x^4 + a^2b^2 - 4\,a^3c + 8\,(ab^2cde^3 - 3\,a^2c^2d}{2\,(ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] 1/2*((a^3*b^4*c*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(b
^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*s
qrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^4*c
*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(b^2 - 4*a*c)*log
(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*
c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*e^4*f^6 - 8*a^
7*b^2*c^2*e^4*f^6 + 16*a^8*c^3*e^4*f^6) + 1/2*b*log(abs(c*e^4*x^4 + 4*c*d*e
^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*
d^2 + a))/(a^3*e*f^3) - 2*b*log(abs(e*x + d))/(a^3*e*f^3) - 1/2*(2*a*b^2*c*
d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^4 - 3*a^2*
c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*d*e^3)*x^3
+ (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^2*b*c*e^2)*x
```

```
^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a^2*b*c*d*e)*x
)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e
^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*(e*x + d)^2*a^3*e*f^3)
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 14830, normalized size of antiderivative = 65.04

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^3*e*f^
3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2*f^3 +
6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6*f^3 + c
*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*f^3*(-(
b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^
2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a
*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2
+ 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*f^3*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*
b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*
c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*
f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a
^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)
^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c
- b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(
4*a*c - b^2)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*
d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^
3)/(a^6*f^9*(4*a*c - b^2)^3))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2
*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2
*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c
))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*f^3*(-(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3
*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(
```

a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c
- b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*
b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b*
c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) +
(8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2
)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b
^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b
*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^3)/(a^6*f^9
*(4*a*c - b^2)^3)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48
*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*
c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (2*b*log(d + e*x))/(a^3*e*f^3) - (at
an(((2*a^9*b^6*f^9*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*f^9*(4*a*c - b^2)^(9/
2) - 24*a^10*b^4*c*f^9*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*f^9*(4*a*c - b
^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*
e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c
*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(276*a^5*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4
*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 - 233*a^4*b^3*c^6*d*e^16*f^3))/(a^6
*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(
480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6
+ 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 6
4*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12
*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6
*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^
8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*
c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*
c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 -
12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6
*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f
^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e
f^3))/(2(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 -
12*a^4*b^4*c*e^2*f^6)) - (((((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b
^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3
- 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 +
3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^
18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b
^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e
^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6

)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^
4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e
*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e
^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*
b^3*c^5*d*e^18*f^9))/(a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*
c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 -
 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((8*(480
*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 3
0*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a
^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*
b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*
e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b
^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3
*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3
*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6
*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c
^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^
7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)
)/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4
*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5
*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 -
64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^
3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((8*(276*a^5
*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 -
233*a^4*b^3*c^6*d*e^16*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f
^9 + 48*a^8*b^2*c^2*f^9) - (((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 4
6*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e
^18*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^
2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*
a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 4
8*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2
*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^
3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(640*a^10*b
*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 26
4*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/(a^9*e^3*f^9*(4*a*c
 - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2
*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*
(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3

+ 72*a*b^6*c))) + x^2*((((4*(54*a^3*c^8*e^16 - 2*b^6*c^5*e^16 + 18*a*b^4*c^6*e^16 - 54*a^2*b^2*c^7*e^16))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*b^5*c^5*e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - ((((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e

^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) + (((4*(6*a^2*b^7*c^4*e^17*f^3 - 65*a^3*b^5*c^5*e^17*f^3 + 233*a^4*b^3*c^6*e^17*f^3 - 276*a^5*b*c^7*e^17*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((4*(480*a^8*c^7*e^18*f^6 - a^4*b^8*c^3*e^18*f^6 + 6*a^5*b^6*c^4*e^18*f^6 + 30*a^6*b^4*c^5*e^18*f^6 - 272*a^7*b^2*c^6*e^18*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(3*a^6*b^9*c^2*e^19*f^9 - 46*a^7*b^7*c^3*e^19*f^9 + 264*a^8*b^5*c^4*e^19*f^9 - 672*a^9*b^3*c^5*e^19*f^9 + 640*a^10*b*c^6*e^19*f^9))/(2*a^9*e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))) + (((((4*(36*a^6*c^7*e^15*f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7*c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e

^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f ^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12 *a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^ 2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5* e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^ 6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6 *f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((((4*(2* a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^ 6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5* d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^ 3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a ^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d ^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9)) /((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a ^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c *e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b *c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e ^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c ^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3* (4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a ^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2* f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^ 5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^ 9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^1 7*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b ^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^ 17*f^9))/(2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12* a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a ^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 2 88*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((4*(36*a^6*c^7*e^15 *f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e ^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7 *c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15* f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^ 4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b *c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (((((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3*(4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(2*a^9*e^3*f^9*(4*a*c - b^2)^(9/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c))))/(36*a^4*c^6*e^14 + b^8*c^2*e^14 - 12*a*b^6*c^3*e^14 + 48*a^2*b^4*c^4*e^14 - 72*a^3*b^2*c^5*e^14))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(a^3*e*f^3*(4*a*c - b^2)^(3/2))

3.653 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 423

$$
\begin{aligned}
&\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx \\
&= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)} \\
&\quad+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}ef^4} \\
&\quad-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}ef^4}
\end{aligned}
$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/f^4/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a
^3/(-4*a*c+b^2)/e/f^4/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/
e/f^4/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-
19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a
*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1135, 1295, 1180, 211}

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$= \frac{b(5b^2-19ac)}{2a^3ef^4(b^2-4ac)(d+ex)} - \frac{5b^2-14ac}{6a^2ef^4(b^2-4ac)(d+ex)^3} + \frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3ef^4(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{-2ac+b^2+bc(d+ex)^2}{2aef^4(b^2-4ac)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 1
9*a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x
)^2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) -
(Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef^4}\\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^4(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-5b^2+14ac-5bcx^2}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{2a\,(b^2-4ac)\,ef^4}\\
&= -\frac{5b^2-14ac}{6a^2\,(b^2-4ac)\,ef^4(d+ex)^3}\\
&\quad + \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^4(d+ex)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-3b(5b^2-19ac)-3c(5b^2-14ac)x^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{6a^2\,(b^2-4ac)\,ef^4}
\end{aligned}
$$

$$= -\frac{5b^2-14ac}{6a^2(b^2-4ac)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)ef^4(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)ef^4(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{-3(5b^4-24ab^2c+14a^2c^2)-3bc(5b^2-19ac)x^2}{a+bx^2+cx^4}dx, x, d+ex\right)}{6a^3(b^2-4ac)ef^4}$$

$$= -\frac{5b^2-14ac}{6a^2(b^2-4ac)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)ef^4(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)ef^4(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\left(c\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx, x, d+ex\right)}{4a^3(b^2-4ac)^{3/2}ef^4} + \frac{\left(c\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx, x, d+ex\right)}{4a^3(b^2-4ac)^{3/2}ef^4}$$

$$= -\frac{5b^2-14ac}{6a^2(b^2-4ac)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)ef^4(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)ef^4(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}ef^4} - \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}ef^4}$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.91

$$\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2}dx$$

$$= \frac{-\frac{4a}{(d+ex)^3} + \frac{24b}{d+ex} + \frac{6(d+ex)(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + \frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3ef^4}$$

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
```

```
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\frac{-\frac{bc e^2\left(3ac-b^2\right)x^3}{2\left(4ac-b^2\right)}-\frac{3dbce\left(3ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(-9bc^2d^2a+3b^3cd^2+2a^2c^2-4ab^2c+b^4\right)x}{8ac-2b^2}+\frac{d\left(-3bc^2d^2a+b^3cd^2+2a^2c^2-4ab^2c+b^4\right)}{2e\left(4ac-b^2\right)}}{cx^4e^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2bdex+bd^2+a}+\frac{\_R=\mathrm{RootC}\ldots}{}}{}$
risch	Expression too large to display

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(-1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*c
-b^2)/(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b
^4)/(4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4
)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x
^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*b^2)*
_R^2+2*b*c*d*e*(-19*a*c+5*b^2)*_R-19*b*c^2*d^2*a+5*b^3*c*d^2+14*a^2*c^2-24*
a*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d
)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*
c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5954 vs. 2(373) = 746.

Time = 0.49 (sec) , antiderivative size = 5954, normalized size of antiderivative = 14.08

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$=\int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^4}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^5
 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4*x
^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (15*
b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c
^2)*d^4 + (45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^
4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3
 - 4*a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a*b^2*
c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b^2*c - 4*a^4*c^
2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4
*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^
4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35
*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 +
(21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2
 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 -
4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4
*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4) + 1
/2*integrate(((5*b^3*c - 19*a*b*c^2)*e^2*x^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*
c^2 + 2*(5*b^3*c - 19*a*b*c^2)*d*e*x + (5*b^3*c - 19*a*b*c^2)*d^2)/((b^2*c
- 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4
+ (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)
/(a^3*f^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(373) = 746$.

Time = 0.32 (sec) , antiderivative size = 2137, normalized size of antiderivative = 5.05

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/4*((5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24
*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (5*b^3*c*e^2*
(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 - 19*a*b
*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2
 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
 d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c
^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)
/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^
3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d
/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*
d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^
3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e -
 b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)) - (5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^
```

```
2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e
^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)))/(a^3*b^2*f^4 - 4*a^4*c*f^4) + 1/2*(b^3*c*e^3*x^3 - 3*a*b*c^2*e^3*x^3 +
3*b^3*c*d*e^2*x^2 - 9*a*b*c^2*d*e^2*x^2 + 3*b^3*c*d^2*e*x - 9*a*b*c^2*d^2*e
*x + b^3*c*d^3 - 3*a*b*c^2*d^3 + b^4*e*x - 4*a*b^2*c*e*x + 2*a^2*c^2*e*x +
b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d)/((a^3*b^2*e*f^4 - 4*a^4*c*e*f^4)*(c*e^4*
x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2
*b*d*e*x + b*d^2 + a)) + 1/3*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 - a)/((e*x
 + d)^3*a^3*e*f^4)
```

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 13781, normalized size of antiderivative = 32.58

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
+ 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
- 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
```

d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816
*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13
*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*
c^9*d*e^11*f^12)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*
a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21
9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
 - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240
*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^
8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15
 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^1
2*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2

401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^
16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e
^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^1
2 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1
871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9
)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*
a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c
 - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
+ 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c
^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^1
1*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1
/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c -
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680
*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*
b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f
^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480
*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4
*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12
*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a
^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c
^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f
^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a
^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^
8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8
*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 - 25*b^6*(-(4
*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*
c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3
c^3(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8

+ 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f
^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*
e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a
c - b^2)^9)^(1/2))/(32(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9
*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 -
6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25
b^6(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6
 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*
c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^
8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*
c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^2
0 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 104857
6*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^
2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*
f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 -
1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^
14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*
f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 286
7200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a
^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f
^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 14586
88*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b
^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*
f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 4
88096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280
*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^
9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8
 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15
- 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225
*b^3*c*d^2*e^3 - 62*a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2
*b^2)) + (3*x^5*(5*b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2))
 + (2*x^3*(15*b^4*d*e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*
d*e^2 - 285*a*b*c^2*d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 +

45*b^3*c*d^5 + 28*a^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3
 - 171*a*b*c^2*d^5))/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b
*c^2*e^5))/(2*a*(4*a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84
*a^2*c^2*d^2*e - 40*a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a
*b*c^2*d^4*e))/(6*a*(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^
4 + 10*a*b^3*d^2 + 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^
2*c*d^4 - 57*a*b*c^2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x*(3*a*d^2*e*f^4 +
5*b*d^4*e*f^4 + 7*c*d^6*e*f^4) + x^4*(35*c*d^3*e^4*f^4 + 5*b*d*e^4*f^4) + x
^2*(10*b*d^3*e^2*f^4 + 21*c*d^5*e^2*f^4 + 3*a*d*e^2*f^4) + x^5*(b*e^5*f^4 +
 21*c*d^2*e^5*f^4) + x^3*(a*e^3*f^4 + 10*b*d^2*e^3*f^4 + 35*c*d^4*e^3*f^4)
 + a*d^3*f^4 + b*d^5*f^4 + c*d^7*f^4 + c*e^7*f^4*x^7 + 7*c*d*e^6*f^4*x^6) +
atan(((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
f^8)))^(1/2)((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816

*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12)*1i + (-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12)*1i)/((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1

/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*
a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*
f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2
*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c
 - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
+ 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 40
96*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 +
 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*
f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20
 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^
19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*
e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20
 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680
*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*
b^3*c^7*d*e^13*f^20) - 917504*a^19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f
^16 - 7936*a^13*b^12*c^3*e^12*f^16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480
*a^15*b^8*c^5*e^12*f^16 + 1536000*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4
*c^7*e^12*f^16 + 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12
*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a
^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c
^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f
^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a
^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^
8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8
*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) - (-(25*b^15 + 25*b^6*(-(4
*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*
c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3
c^3(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8
+ 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f
^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*
e^2*f^8)))^(1/2)*((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*
b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744
*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 6
15*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a
c - b^2)^9)^(1/2))/(32(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9
*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 -
6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 + 25
b^6(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767
*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6

- 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 - 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*c^9*d*e^11*f^12) + 1800*a^9*b^9*c^6*e^10*f^8 - 29080*a^10*b^7*c^7*e^10*f^8 + 176032*a^11*b^5*c^8*e^10*f^8 - 473216*a^12*b^3*c^9*e^10*f^8 + 476672*a^13*b*c^10*e^10*f^8))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*2i

3.654 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 353

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

```
[Out] 1/4*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2-1/8*f^4*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e
*x+d)^2+c*(e*x+d)^4)+3/8*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5
/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*f^4*arctan((e*x+d)*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1156, 1134, 1192, 1180, 211}

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}f^4\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\,(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\,(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\,(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{f^4\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx,x,d+ex\right)}{e}\\
&= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{f^4\text{Subst}\left(\int \frac{2a-5bx^2}{(a+bx^2+cx^4)^2}\,dx,x,d+ex\right)}{4\left(b^2-4ac\right)e}\\
&= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\
&\quad-\frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad+\frac{f^4\text{Subst}\left(\int \frac{3a\left(b^2+4ac\right)-12abcx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{8a\left(b^2-4ac\right)^2e}\\
&= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad+\frac{\left(3c\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{8\left(b^2-4ac\right)^{5/2}e}\\
&\quad-\frac{\left(3c\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{8\left(b^2-4ac\right)^{5/2}e}
\end{aligned}
$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{(d+ex)\left(-7b^2+4ac-12bc(d+ex)^2\right)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$8e$$

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/
((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.01

method	result
default	$f^4\left(\frac{-\frac{3c^2e^6b\,x^7}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{21c^2d\,e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(-252bc\,d^2+4ac-19b^2\right)c\,e^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3\left(-84bc\,d^2+4ac-19b^2\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{e^2\left(420b\,c^2d^4-40a\,c^2\right.}{8\left(16a^2\right.}}{}\right.$
risch	$-\frac{3c^2e^6b\,f^4x^7}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{21c^2d\,e^5b\,f^4x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(-252bc\,d^2+4ac-19b^2\right)c\,e^4f^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3f^4\left(-84bc\,d^2+4ac-19b^2\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{e^2f^4\left(420b\,c^2d^4-40a\,c\right.}{8\left(16a^2\right.}$

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^4*((-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*
b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190
*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^
6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a
*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*
b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6770 vs. $2(305) = 610$.

Time = 0.46 (sec) , antiderivative size = 6770, normalized size of antiderivative = 19.18

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx=\text{Timed out}$$

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx=\int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] -3/8*f^4*integrate((4*b*c*e^2*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2 - 4*a*c)/
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*e^7
*f^4*x^7 + 84*b*c^2*d*e^6*f^4*x^6 + (252*b*c^2*d^2 + 19*b^2*c - 4*a*c^2)*e^
5*f^4*x^5 + 5*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d)*e^4*f^4*x^4 + (420*b*
c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*f^4*x^3 + (25
2*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*f^4
*x^2 + (84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3*
(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + (12*b*c^2*d^7 + (19*b^2*c - 4*a*c^2)*d^5
+ (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*c)*d)*f^4)/((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^
7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3
*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^
5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2
*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2
*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. 2(305) = 610.

Time = 0.33 (sec) , antiderivative size = 1958, normalized size of antiderivative = 5.55

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e)^2 - 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b
*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)) - (4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)^2 + 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*l
og(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c
*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6
*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2
 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 8*b*c*d*e*f^4*(sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 4*b*c*d^2*f^4 - b^2*f^
4 - 4*a*c*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (4*b*c*e^2*f^4*(sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 8*b*c*d*e*f^
4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 4*b*c*
d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2
)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)))/(b^4 -
 8*a*b^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*e^7*f^4*x^7 + 84*b*c^2*d*e^6*f^4*x
^6 + 252*b*c^2*d^2*e^5*f^4*x^5 + 420*b*c^2*d^3*e^4*f^4*x^4 + 420*b*c^2*d^4*
e^3*f^4*x^3 + 19*b^2*c*e^5*f^4*x^5 - 4*a*c^2*e^5*f^4*x^5 + 252*b*c^2*d^5*e^
2*f^4*x^2 + 95*b^2*c*d*e^4*f^4*x^4 - 20*a*c^2*d*e^4*f^4*x^4 + 84*b*c^2*d^6*
e*f^4*x + 190*b^2*c*d^2*e^3*f^4*x^3 - 40*a*c^2*d^2*e^3*f^4*x^3 + 12*b*c^2*d
^7*f^4 + 190*b^2*c*d^3*e^2*f^4*x^2 - 40*a*c^2*d^3*e^2*f^4*x^2 + 95*b^2*c*d^
```

```
4*e*f^4*x - 20*a*c^2*d^4*e*f^4*x + 5*b^3*e^3*f^4*x^3 + 16*a*b*c*e^3*f^4*x^3
 + 19*b^2*c*d^5*f^4 - 4*a*c^2*d^5*f^4 + 15*b^3*d*e^2*f^4*x^2 + 48*a*b*c*d*e
^2*f^4*x^2 + 15*b^3*d^2*e*f^4*x + 48*a*b*c*d^2*e*f^4*x + 5*b^3*d^3*f^4 + 16
*a*b*c*d^3*f^4 + 3*a*b^2*e*f^4*x + 12*a^2*c*e*f^4*x + 3*a*b^2*d*f^4 + 12*a^
2*c*d*f^4)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*
d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^
2*e))
```

Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 13840, normalized size of antiderivative = 39.21

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
- 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 102
4*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*(((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c
^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 229376
0*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*
e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8
*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c
)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 +
20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))
/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*
(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*
a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*
b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4
*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a
^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144
0*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*
f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5
*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*
c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f
^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*
(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*
```

e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*
e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^
8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936
*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*
f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*
a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 +
 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96
*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + (-(9*(b^15*f^8 + f^8*(-
(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160
*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6
*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 -
 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a
^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196608
0*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(
1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^
9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 917504
0*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d
e^13)/(128(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 -
2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 8192
0*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*
a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c
 - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b
^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c
^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^1
2*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*
b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) +
 (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*
f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^
10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6
*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8
+ f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^
8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6
1440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^
10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 +
 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2
- 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*
e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*
b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^
5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*
b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^
3 - 16*a*b^6*c)))*1i)/((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*

b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^
8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*
b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*
b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a
^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13
 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^
4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 220200
96*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*
c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*
c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*
a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a
^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7
*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^
4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c
^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c
^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4
*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 1
92*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12
*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 +
 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 614
4*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/
2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11
520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b
^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 +
720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258
048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2
949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7
*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^
2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^
3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a
^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 +
 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) - (-(9*(b^1
5*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*
c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f
^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a
^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3
*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^
6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^
2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 167
77216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d
*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128
*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^

11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a*b^3*c^4*e^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i - ((x^3*(5*b^3*e^2*f^4 + 16*a*b*c*e^2*f^4 - 40*a*c^2*d^2*e^2*f^4 + 190*b^2*c*d^2*e^2*f^4 + 420*b*c^2*d^4*e^2*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(19*b^2*c*d*e^3*f^4 - 4*a*c^2*d*e^3*f^4 + 84*b*c^2*d^3*e^3*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(19*b^2*c*e^4*f^4 - 4*a*c^2*e^4*f^4 + 252*b*c^2*d^2*e^4*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2*f^4 + 12*a^2*c*f^4 + 15*b^3*d^2*f^4 - 20*a*c^2*d^4*f^4 + 95*b^2*c*d^4*f^4 + 84*b*c^2*d^6*f^4 + 48*a*b*c*d^2*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(15*b^3*d*e*f^4 - 40*a*c^2*d^3*e*f^4 + 190*b^2*c*d^3*e*f^4 + 252*b*c^2*d^5*e*f^4 + 48*a*b*c*d*e*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3*f^4 - 4*a*c^2*d^5*f^4 + 19*b^2*c*d^5*f^4 + 12*b*c^2*d^7*f^4 + 3*a*b^2*d*f^4 + 12*a^2*c*d*f^4 + 16*a*b*c*d^3*f^4)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*f^4*x^7)/(2*(b^4

+ 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*f^4*x^6)/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + atan((((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 +
 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*
a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*
c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*
a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*
a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491
20*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*
e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^1
1*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22
020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*
a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*
b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131
072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 1638
40*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2
*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*
a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7
*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))
/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^1
6*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^1
0*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*
b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4
- 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e
^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^1
2 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15
*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 +
11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a
*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2
+ 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 2
58048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 +
 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c
^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*
a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144
*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8
 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + ((9*
(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b
^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c

^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 10485
76*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14
*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^
8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^1
0*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 -
16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c
^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 2936
0128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280
*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(12
8*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b
^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-
(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2
*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8
- 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11
*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^
2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e
^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c
^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*
a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^1
2*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
)*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560
*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5
*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 +
1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^
4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*
a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26214
40*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^
11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^
12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 -
6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*
e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2
- 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i)/(((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) -
b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f
^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 -
20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c
*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^
2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*
e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*
b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344
064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*
d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b
^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4
- 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3

*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) - ((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4

- 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) + (135*b^5*c^3*e^10*f^12 + 1080*a*b^3*c^4*e^10*f^12 + 432*a^2*b*c^5*e^10*f^12)/(64*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^11*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*2i

3.655 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3(b+2c(d+ex)^2)}{4\,(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3bcf^3\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}\,e}$$

```
[Out] 1/4*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*
b*f^3*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*
f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1156, 1128, 652, 628, 632, 212}

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{3bcf^3\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\,(b^2-4ac)^{5/2}} - \frac{3bf^3(b+2c(d+ex)^2)}{4e\,(b^2-4ac)^2\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^3(2a+b(d+ex)^2)}{4e\,(b^2-4ac)\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f^3\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx,x,d+ex\right)}{e}\\
&= \frac{f^3\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3}\,dx,x,(d+ex)^2\right)}{2e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{(3bf^3)\,\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx,x,(d+ex)^2\right)}{4\,(b^2-4ac)\,e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2}\\
&\quad-\frac{3bf^3(b+2c(d+ex)^2)}{4\,(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad-\frac{(3bcf^3)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2\,(b^2-4ac)^2\,e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2}\\
&\quad-\frac{3bf^3(b+2c(d+ex)^2)}{4\,(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)}\\
&\quad+\frac{(3bcf^3)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{(b^2-4ac)^2\,e}\\
&= \frac{f^3(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2}\\
&\quad-\frac{3bf^3(b+2c(d+ex)^2)}{4\,(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}\,e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$
$$=\frac{f^3\left(-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2}-\frac{12bc\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4\,(b^2-4ac)^2\,e}$$

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.45

method	result
default	$f^3\left(\frac{-\frac{3c^2e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3cd\,e^2b\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{db\left(9c^2\cdots\right.}{16\cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+\cdots\right.}\right.$
risch	$\frac{-\frac{3c^2e^5b\,f^3x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9f^3e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3f^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3cd\,e^2b\,f^3\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\,f^3\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{db\,f^3\left(9\cdots\right.}{1\cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+\cdots\right.}$

[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)

[Out] f^3*((-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. 2(151) = 302.

Time = 0.39 (sec) , antiderivative size = 3843, normalized size of antiderivative = 24.17

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5
*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3*x
^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f^3*
x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(
b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*
(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x + (6*
(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4
*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*e^8*
f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f^3*x^6 +
4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2
 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3
*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c
 + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5
+ a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6
 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c
)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^
2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2
+ 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 +
c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4
+ 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x +
 ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2
 - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*
c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b
^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*

```
a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 -
4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^
2)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3
*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*
f^3 - 12*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2
*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*
d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 1
0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b
^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*
d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3
*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f
^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1
4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(
14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 1
28*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*
b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2
*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(144) = 288$.

Time = 7.32 (sec) , antiderivative size = 1794, normalized size of antiderivative = 11.28

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**
4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*c
- b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*f**
3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b*c**
2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x
**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3
*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**
2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**
2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**2*f**3 -
10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*c**2*d**6*
f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x**4*(-9*b**2
*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e**3*f**3 - 1
20*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3*e**2*f**3 -
54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20*a*b*c*d*e*f**
3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5*e*f**3))/(64*a
**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4
*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a*
*2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6
*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*
d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9)
+ x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8
) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*
e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7)
 + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**
2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d*
*3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**
2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b*
*2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4
*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**
2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a
*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**
2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a*
*2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*
a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b*
*2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**
2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a*
*2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16
*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*
b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**
2*d**7*e**2))
```

Maxima [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] -3*b*c*f^3*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^
2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c +
16*a^2*c^2) - 1/4*(6*b*c^2*e^6*f^3*x^6 + 36*b*c^2*d*e^5*f^3*x^5 + 9*(10*b*c
^2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*f^3*x^3 + 2
*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*f^3*x^2 + 4*(9*b*c^2*d^5
 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e*f^3*x + (6*b*c^2*d^6 + 9*b^2*c*d^4 +
a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70
*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^
6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^
3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(151) = 302.

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.71

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = -\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}} - \frac{6\,bc^2d^6f^7+18\,(efx^2+2\,dfx)bc^2d^4ef^6+18\,(efx^2+2\,dfx)^2bc^2d^2e^2f^5+9\,b^2cd^4f^7+6\,(efx^2+2\,dfx)^3bc^2}{4\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef\right.}$$

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) - 1/4*(6
*b*c^2*d^6*f^7 + 18*(e*f*x^2 + 2*d*f*x)*b*c^2*d^4*e*f^6 + 18*(e*f*x^2 + 2*d
*f*x)^2*b*c^2*d^2*e^2*f^5 + 9*b^2*c*d^4*f^7 + 6*(e*f*x^2 + 2*d*f*x)^3*b*c^2
*e^3*f^4 + 18*(e*f*x^2 + 2*d*f*x)*b^2*c*d^2*e*f^6 + 9*(e*f*x^2 + 2*d*f*x)^2
*b^2*c*e^2*f^5 + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 2*(e*f*x^2 + 2*d*f*x)*b
^3*e*f^6 + 10*(e*f*x^2 + 2*d*f*x)*a*b*c*e*f^6 + a*b^2*f^7 + 8*a^2*c*f^7)/((
c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 +
 b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 16
*a^2*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 1267, normalized size of antiderivative = 7.97

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx =$$

$$-\frac{\frac{8a^2cf^3+ab^2f^3+10abcd^2f^3+2b^3d^2f^3+9b^2cd^4f\cdots}{4e(16a^2c^2-8ab^2c+b^4)}}{x^2\left(6b^2d^2e^2+30bcd^4e^2+2abe^2+28c^2d^6e^2+12acd^2e^2\right)+x^6\left(28c^2d^2e^6+2bce^6\right)+x\left(4eb^2d^3\cdots\right.}$$

$$-\frac{3bcf^3\operatorname{atan}\left(\frac{\left(b^4\left(4ac-b^2\right)^5+16a^2c^2\left(4ac-b^2\right)^5-8ab^2c\left(4ac-b^2\right)^5\right)\left(x^2\left(\frac{9b^2c^4e^8f^6}{a\left(4ac-b^2\right)^{9/2}\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{9b^3c^2f^6\left(32a^2bc\cdots\right.}{2ae^2\left(4ac-b^2\cdots\right.}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^6
*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e
*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3))/(
4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c^2*d^
2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*d^2*f^
3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*b*c
^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*f^3*x^5
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*
a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^
6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e)
 + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*
(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4
 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*
a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b^4*(4*a*c - b^2
)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*
c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^
```

```
3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^
2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((9*b^3*c^2*f^6
*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (18*b^2*c^4*d*e^7*f^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^2*c^4*d^2*e^6*f
^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6
*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8 - 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8
 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(18*b^2*c^4*e^6*f^6)))/(e*(4*a*c - b^
2)^(5/2))
```

3.656 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 375

$$\begin{aligned}
&\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)\,(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad+\frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e} \\
&\quad+\frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}
\end{aligned}$$

```
[Out] -1/4*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2+1/8*f^2*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2
)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*f^2*
arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+2
0*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1156, 1133, 1192, 1180, 211}

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{\sqrt{c}f^2\left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}f^2\left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}} + \frac{f^2(d+ex)\left(c(20ac+b^2)(d+ex)^2+b(8ac+b^2)\right)}{8ae\,(b^2-4ac)^2\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\,(b^2-4ac)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c
)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4))
 + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b
^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*
(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2
 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f^2\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}\\
&= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{f^2\text{Subst}\left(\int \frac{b-10cx^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{4\left(b^2-4ac\right)e}\\
&= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\
&\quad+\frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)\,(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\
&\quad-\frac{f^2\text{Subst}\left(\int \frac{-b(b^2-16ac)-c(b^2+20ac)x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{8a\left(b^2-4ac\right)^2e}
\end{aligned}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{16a\left(b^2-4ac\right)^2e} + \frac{\left(c\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{16a\left(b^2-4ac\right)^2e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e} + \frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.03

$$\int\frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{f^2\left(-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2(d+ex)\left(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2\right)}{a(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4\right.}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b}}}\right.}{16e}$$

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 2
0*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]
*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.37

method	result
default	$f^2\left(\frac{\frac{c^2e^6\left(20ac+b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{7c^2d\,e^5\left(20ac+b^2\right)x^6}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{\left(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3\right)c\,e^4x^5}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{5cd\,e^3\left(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}\right.$
risch	$\frac{\frac{c^2e^6f^2\left(20ac+b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{7c^2d\,e^5f^2\left(20ac+b^2\right)x^6}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{\left(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3\right)c\,e^4f^2x^5}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{5cd\,e^3f^2\left(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}}{}$

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*((1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e
^5*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*
c*d^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(1
40*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1
/8*e^2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^
2+5*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21
*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*
d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)
/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^
2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(1
6*a^2*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*d*c*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c
*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7838 vs. $2(331) = 662$.

Time = 0.53 (sec) , antiderivative size = 7838, normalized size of antiderivative = 20.90

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/8*f^2*integrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c^2)*d*e*x
+ b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d
^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*((b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 7*(b^
2*c^2 + 20*a*c^3)*d*e^6*f^2*x^6 + (2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*
a*c^3)*d^2)*e^5*f^2*x^5 + 5*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b
*c^2)*d)*e^4*f^2*x^4 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*
a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + (21*(b^2*c^2 + 20*a*c^
3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*
e^2*f^2*x^2 + (7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a
*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*x + ((b^2*c
^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2
```

```
*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*
(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^
3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(
a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8
*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15
*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32
*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7
+ 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*
c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
+ 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 3
2*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2679 vs. $2(331) = 662$.

Time = 0.33 (sec) , antiderivative size = 2679, normalized size of antiderivative = 7.14

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*((b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)/(c*e^4)) + d/e)^2 - 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^
2 + b^3*f^2 - 16*a*b*c*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^2*c*e
^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 +
 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)^2 + 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)/(c*e^4)) - d/e) + 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 1
6*a*b*c*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
```

```
)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b^2*c*e^2*f^2*(sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 + 20*a*c^2*e^2*f
^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b
^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/
e) - 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*lo
g(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*
e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*
c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2
 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*f^2*(s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d
*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b
^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b
*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*e^7*
f^2*x^7 + 20*a*c^3*e^7*f^2*x^7 + 7*b^2*c^2*d*e^6*f^2*x^6 + 140*a*c^3*d*e^6*
f^2*x^6 + 21*b^2*c^2*d^2*e^5*f^2*x^5 + 420*a*c^3*d^2*e^5*f^2*x^5 + 35*b^2*c
^2*d^3*e^4*f^2*x^4 + 700*a*c^3*d^3*e^4*f^2*x^4 + 35*b^2*c^2*d^4*e^3*f^2*x^3
 + 700*a*c^3*d^4*e^3*f^2*x^3 + 2*b^3*c*e^5*f^2*x^5 + 28*a*b*c^2*e^5*f^2*x^5
 + 21*b^2*c^2*d^5*e^2*f^2*x^2 + 420*a*c^3*d^5*e^2*f^2*x^2 + 10*b^3*c*d*e^4*
f^2*x^4 + 140*a*b*c^2*d*e^4*f^2*x^4 + 7*b^2*c^2*d^6*e*f^2*x + 140*a*c^3*d^6
*e*f^2*x + 20*b^3*c*d^2*e^3*f^2*x^3 + 280*a*b*c^2*d^2*e^3*f^2*x^3 + b^2*c^2
*d^7*f^2 + 20*a*c^3*d^7*f^2 + 20*b^3*c*d^3*e^2*f^2*x^2 + 280*a*b*c^2*d^3*e^
2*f^2*x^2 + 10*b^3*c*d^4*e*f^2*x + 140*a*b*c^2*d^4*e*f^2*x + b^4*e^3*f^2*x^
3 + 5*a*b^2*c*e^3*f^2*x^3 + 36*a^2*c^2*e^3*f^2*x^3 + 2*b^3*c*d^5*f^2 + 28*a
*b*c^2*d^5*f^2 + 3*b^4*d*e^2*f^2*x^2 + 15*a*b^2*c*d*e^2*f^2*x^2 + 108*a^2*c
^2*d*e^2*f^2*x^2 + 3*b^4*d^2*e*f^2*x + 15*a*b^2*c*d^2*e*f^2*x + 108*a^2*c^2
*d^2*e*f^2*x + b^4*d^3*f^2 + 5*a*b^2*c*d^3*f^2 + 36*a^2*c^2*d^3*f^2 - a*b^3
*e*f^2*x + 16*a^2*b*c*e*f^2*x - a*b^3*d*f^2 + 16*a^2*b*c*d*f^2)/((c*e^4*x^4
 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*
d*e*x + b*d^2 + a)^2*(a*b^4*e - 8*a^2*b^2*c*e + 16*a^3*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 16025, normalized size of antiderivative = 42.73

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out] atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a

^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(
4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4
*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12
*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*
b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*
1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*
b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
- 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8
*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^
8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(2621
44*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a
^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(
a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*
(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1
140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437
76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*
a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1
048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*
b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^
9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214
40*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11
*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2
- 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^
8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c
^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5
*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^
10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
- 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^
9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*
a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*
e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2
*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3
*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a
^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^
6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4
*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*
b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*
c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b

^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1
i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b
^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 -
36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*
b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8
*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(26214
4*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^
4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a
^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(
-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 11
40*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 4377
6*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a
*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 10
48576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b
^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9
*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 262144
0*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*
c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 -
5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8
*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2
- 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b
^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^
2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*
f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 -
25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^1
0*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 +
53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 -
1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9
*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a
*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e
^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*
c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*
b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c
- b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^
3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6
*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*
a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b
^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c
^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^
6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) -
((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*
c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 367
00160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*

c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (8000*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 12720*a^2*b^2*c^6*e^10*f^6 - 84*a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104857

6*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*2i + atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2* b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i)/(((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6

144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 512
0*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327
680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b
^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 +
 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1
863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(
1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*
a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*
a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949
120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*
c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2
949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2
*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*
a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a
^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8
*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9
*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3
*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^
3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e
^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e
^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c
^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 1
6*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11
*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*
(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c
^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6
*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 114
0*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776
*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*
b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104
8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^
14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*
b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440
*a^12*b^2*c^9*e^2)))^(1/2) - ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*
c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 917
5040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^
7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24
*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*
a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^
3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*
a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c
^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1

720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 348
80*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 18636
80*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*
b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*
b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*
a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*
e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 29491
20*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^1
2*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*
b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b
^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c
^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4
*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7
*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^
20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 -
 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 +
 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e
^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^
10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4
 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2
*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 +
 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3
*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^
8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^1
7*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^
2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5
*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15
*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576
*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c
^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*
c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^1
2*b^2*c^9*e^2)))^(1/2) + (8000*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 127
20*a^2*b^2*c^6*e^10*f^6 - 84*a*b^4*c^5*e^10*f^6)/(256*(a^2*b^12 + 4096*a^8*
c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4
 - 6144*a^7*b^2*c^5))))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1
720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 348
80*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 18636
80*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*
b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*
b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*
a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*2i + ((x^5*(2*b^3*c*e^
4*f^2 + 21*b^2*c^2*d^2*e^4*f^2 + 28*a*b*c^2*e^4*f^2 + 420*a*c^3*d^2*e^4*f^2
))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4*e^2*f^2 + 36*a^2*c^2*e^

```
2*f^2 + 35*b^2*c^2*d^4*e^2*f^2 + 5*a*b^2*c*e^2*f^2 + 700*a*c^3*d^4*e^2*f^2
+ 20*b^3*c*d^2*e^2*f^2 + 280*a*b*c^2*d^2*e^2*f^2))/(8*a*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) + (x*(3*b^4*d^2*f^2 - a*b^3*f^2 + 140*a*c^3*d^6*f^2 + 10*b^3*c
*d^4*f^2 + 108*a^2*c^2*d^2*f^2 + 7*b^2*c^2*d^6*f^2 + 16*a^2*b*c*f^2 + 15*a*
b^2*c*d^2*f^2 + 140*a*b*c^2*d^4*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (x^2*(3*b^4*d*e*f^2 + 108*a^2*c^2*d*e*f^2 + 420*a*c^3*d^5*e*f^2 + 20*b^3*
c*d^3*e*f^2 + 21*b^2*c^2*d^5*e*f^2 + 15*a*b^2*c*d*e*f^2 + 280*a*b*c^2*d^3*e
*f^2))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3*f^2 + 20*a*c^3*d^7*f
^2 + 2*b^3*c*d^5*f^2 + 36*a^2*c^2*d^3*f^2 + b^2*c^2*d^7*f^2 - a*b^3*d*f^2 +
 16*a^2*b*c*d*f^2 + 5*a*b^2*c*d^3*f^2 + 28*a*b*c^2*d^5*f^2)/(8*a*e*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(20*a*c^3*d*e^5*f^2 + b^2*c^2*d*e^5*f^2))/
(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(7*b^2*c^2*d^3*e^3*f^2 + 2*b^
3*c*d*e^3*f^2 + 140*a*c^3*d^3*e^3*f^2 + 28*a*b*c^2*d*e^3*f^2))/(8*a*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (f^2*x^7*(20*a*c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
```

3.657 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)^2 e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{6c^2f\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}e}$$

```
[Out] -1/4*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c
*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1156, 1121, 628, 632, 212}

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{6c^2f\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}} + \frac{3cf(b+2c(d+ex)^2)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f(b+2c(d+ex)^2)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(f*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
 e*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{f\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}\\ &= \frac{f\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}\end{aligned}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(3cf)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}dx, x, (d+ex)^2\right)}{2(b^2-4ac)e}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3cf(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(3c^2f)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}dx, x, (d+ex)^2\right)}{(b^2-4ac)^2e}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3cf(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(6c^2f)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b+2c(d+ex)^2\right)}{(b^2-4ac)^2e}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3cf(b+2c(d+ex)^2)}{2(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}e}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}dx = \frac{f\left(\frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{24c^2\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4(b^2-4ac)^2e}$$

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)
^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.55

method	result
default	$f\left(\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4-\cdots\right.}{16a^2c\cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+c\cdots\right.}\right.$
risch	$\frac{\frac{3c^3e^5f\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18f\,e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3f\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2f\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{cef\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cdf\left(9c^2d^4+9\cdots\right.}{16a^2c^2-\cdots}}{\left(c\,x^4e^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2bdex+b\,d^2+a\right)\cdots}$

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*((3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2
*d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c
*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5
*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*
c^2*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^
3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. 2(145) = 290.

Time = 0.56 (sec) , antiderivative size = 3748, normalized size of antiderivative = 24.50

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5
 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10
*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^
3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^8 + 8*
c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^
```

3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x - 24*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2

```
*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e
*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^
4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4
*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^
7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^
2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(
b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*
x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^
6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4
+ 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(139) = 278$.

Time = 7.21 (sec) , antiderivative size = 1707, normalized size of antiderivative = 11.16

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3}\, dx = \text{Too large to display}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*f
*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**
5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1/(4
*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c**2*
f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqrt(-1/
(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5) + 36*a
*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a*c - b*
```

```
*2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*c*f + 20
*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*c**3*d**6
*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**2*e**4*f +
 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**3*e**3*f) +
 x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**2*f + 180*c*
*3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c**2*d**3*e*f
+ 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d
**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*
a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d*
*4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b*
*5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3
*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*
e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d
**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7
+ 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*
d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c
*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b
*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*
c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2
*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b
*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b*
*3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*
c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a
**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 179
2*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3
*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*
c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a*
*3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 51
2*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b*
*3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5
*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Maxima [F]

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{efx+df}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*f*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
 b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*
a^2*c^2) + 1/4*(12*c^3*e^6*f*x^6 + 72*c^3*d*e^5*f*x^5 + 18*(10*c^3*d^2 + b*
c^2)*e^4*f*x^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*f*x^3 + 4*(45*c^3*d^4 + 27
*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*f*x^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 + (b^2
```

```
*c + 5*a*c^2)*d)*e*f*x + (12*c^3*d^6 + 18*b*c^2*d^4 - b^3 + 10*a*b*c + 4*(b
^2*c + 5*a*c^2)*d^2)*f)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
+ 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(145) = 290$.

Time = 0.33 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.80

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{6\,c^2f\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}e} + \frac{12\,c^3d^6f^5+36\,(efx^2+2\,dfx)c^3d^4ef^4+36\,(efx^2+2\,dfx)^2c^3d^2e^2f^3+18\,bc^2d^4f^5+12\,(efx^2+2\,dfx)^3c}{4\,(cd^4f^2+2\,(efx^2+2\,dfx)cd^2}$$

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) + 1/4*(12*c
^3*d^6*f^5 + 36*(e*f*x^2 + 2*d*f*x)*c^3*d^4*e*f^4 + 36*(e*f*x^2 + 2*d*f*x)^
2*c^3*d^2*e^2*f^3 + 18*b*c^2*d^4*f^5 + 12*(e*f*x^2 + 2*d*f*x)^3*c^3*e^3*f^2
 + 36*(e*f*x^2 + 2*d*f*x)*b*c^2*d^2*e*f^4 + 18*(e*f*x^2 + 2*d*f*x)^2*b*c^2*
e^2*f^3 + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 4*(e*f*x^2 + 2*d*f*x)*b^2*c*
e*f^4 + 20*(e*f*x^2 + 2*d*f*x)*a*c^2*e*f^4 - b^3*f^5 + 10*a*b*c*f^5)/((c*d^
4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d
^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2
*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.84

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{x^2\left(e\,f\,b^2\,c+27\,e\,f\,b\,c^2\,d^2+45\,e\,f\,c^3\,d^4+5\,a\,e\,f\,c^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\left(6\,b^2\,d^2\,e^2+30\,b\,c\,d^4\,e^2+2\,a\,b\,e^2+28\,c^2\,d^6\,e^2+12\,a\,c\,d^2\,e^2\right)+x^6\left(28\,c^2\,d^2\,e^6+2\,b\,c\,e^6\right)+x\left(4\,e\,b^2\,d^3+1\right.}$$

$$+\frac{6\,c^2\,f\,\mathrm{atan}\left(\frac{\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\left(4\,a\,c-b^2\right)^5\right)\left(x^2\left(\frac{36\,c^6\,e^8\,f^2}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,b\,c^4\,f^2\left(16\,a^2\,b\,c^4\,e^{10}\right.}{a\,e^2\left(4\,a\,c-b^2\right)^{15/2}}\right.\right.}{}\right.}{}$$

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2*c
*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(b^4 +
16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
+ (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2
+ 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c
^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b
*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 4
0*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^
2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8
*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*f*atan
(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^
2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*c^4*e^
10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((72*
c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72
*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2
*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (36*c^6*d^2*e^6*f^
2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(
32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^2*e^8 - 8
*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6*f^2)))/(e*(4*a*c - b^2)^(5/2))
```

3.658 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)\,ef\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\\ &\quad+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\left(b^2-4ac\right)^2 ef\,\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\ &\quad+\frac{b(b^4-10ab^2c+30a^2c^2)\,\mathrm{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2} ef}\\ &\quad+\frac{\log(d+ex)}{a^3ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}\end{aligned}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*
a*c+b^2)^2/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e/f+
ln(e*x+d)/a^3/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$=-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}$$
$$+\frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$+\frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}$$
$$+\frac{-2ac+b^2+bc(d+ex)^2}{4aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{-2(b^2-4ac)-3bcx}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{4a\,(b^2-4ac)\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\text{Subst}\left(\int \frac{2(b^2-4ac)^2+2bc(b^2-7ac)x}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{4a^2\,(b^2-4ac)^2\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\text{Subst}\left(\int \left(\frac{2(-b^2+4ac)^2}{ax} + \frac{2\left(-b(b^4-9ab^2c+23a^2c^2)-c(b^2-4ac)^2x\right)}{a(a+bx+cx^2)}\right) dx, x, (d+ex)^2\right)}{4a^2\,(b^2-4ac)^2\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^3ef} + \frac{\text{Subst}\left(\int \frac{-b(b^4-9ab^2c+23a^2c^2)-c(b^2-4ac)^2x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^3\,(b^2-4ac)^2\,ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^3ef} - \frac{\text{Subst}\left(\int\frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4a^3ef} - \frac{(b(b^4-10ab^2c+30a^2c^2))\,\text{Subst}\left(\int\frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4a^3\left(b^2-4ac\right)^2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex)}{a^3ef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef} + \frac{(b(b^4-10ab^2c+30a^2c^2))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{2a^3\left(b^2-4ac\right)^2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b(b^4-10ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}ef} + \frac{\log(d+ex)}{a^3ef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}$$

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.46

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{\frac{a^2\left(-b^2+2ac-bc(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{a\left(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2\right)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))} + 4\log(d+ex) - \frac{\left(b^5-10ab^3c+30a^2b\right.}{}}{}$$

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
```

```
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.59

method	result
default	$-\frac{\frac{c^2 e^5 \left(7ac-b^2\right) a b\, x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{3\left(7ac-b^2\right) a b\, c^2 d\, e^4 x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{e^3 a c\left(-210b\,c^2d^2a+30b^3c\,d^2+16a^2c^2-29a\,b^2c+4b^4\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{c d\, e^2 a\left(-70b\,c^2d^2a+10b^3c\,d^2+16a^2c^2 \cdots\right.}{16a^2c^2-8a\,b^2c+b^4 \cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*
(7*a*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-210*
a*b*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2*d^4-
48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c^3*d^2+
29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8*a*b^2*c
+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a*b^2*c^2*
d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c^2-21*a^2*
b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2
*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(16*a^2*c^2-8
*a*b^2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^
2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23
*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*b^2*a*c^2*d^3+b^4*c*d^3+23*a^
2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*
d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b
*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))+ln(e*x+d)/a^3/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4898 vs. 2(258) = 516.

Time = 1.58 (sec) , antiderivative size = 9926, normalized size of antiderivative = 36.76

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d*e^5*x^5 +
 (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7*a*b*c^3)*d^2)*e^4*x
^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3*c^2 - 7*a*b*c^3)*d^3 + (4*b^4
*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^
3*c^2 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c - a^
2*b*c^2 + 15*(b^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 + 16*a^2
*c^3)*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3*c^2 -
7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 6*a*b^3
*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*f*x
```

```
^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*f*x^7 + 2*(a^2*b^5*
c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4
*c^4)*d^2)*e^7*f*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3
 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*f*x^5 + (a^2*b^6 - 6
*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4
 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*f*x^4 + 4*(14*(a^
2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*f*x^3 +
2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a
^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*f*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)
*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c +
16*a^5*b*c^2)*d)*e^2*f*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8
+ a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^
4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d^2)*e*f) - integrate(((b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*e^3*x^3 + 3*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^2*x^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 9*a*b^3*c + 23*a^2*b*c^2 + 3*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e*x + (b^5 - 9*a*b^3*c + 23*a^2*b*c^2)*d)/
(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f) + log(e*x
 + d)/(a^3*e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(258) = 516$.

Time = 0.43 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.99

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx =$$

$$-\frac{\left(a^3b^7ce^3f-14\,a^4b^5c^2e^3f+70\,a^5b^3c^3e^3f-120\,a^6bc^4e^3f\right)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bde\right.\right.}{}$$

$$-\frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,a^3ef}$$

$$+\frac{\log\left(\left|ex+d\right|\right)}{a^3ef}$$

$$+\frac{2\,ab^3c^2d^6-14\,a^2bc^3d^6+4\,ab^4cd^4-29\,a^2b^2c^2d^4+16\,a^3c^3d^4+2\,ab^5d^2-12\,a^2b^3cd^2-2\,a^3bc^2d^2+2\,(ab^3c^2}{}$$

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e^3*f - 120*
a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^
```

```
2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d
^2 + 2*a)) - (a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e^3*f
 - 120*a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4
*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 -
4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*e^4*f^2 - 16*a^7*b^6*c^2*e^4*f^2 + 96*a^8*b^
4*c^3*e^4*f^2 - 256*a^9*b^2*c^4*e^4*f^2 + 256*a^10*c^5*e^4*f^2) - 1/4*log(a
bs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^
2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^3*e*f) + log(abs(e*x + d))/(a^3*e*f) + 1
/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4
 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a
*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 2
10*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)
*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 2
9*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105
*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c
^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*
c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 1
6*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)/((c*e^4*x
^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*
b*d*e*x + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*e*f)
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 22621, normalized size of antiderivative = 83.78

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 -
21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d
^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^3*f + 40*b*c*d
```

^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^6*e^2*f + 2*a*b
*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^3*e*f + 8*c^2*d^
7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^4*(b^2*e^4*f + 70
*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(56*c^2*d^3*e^5*f +
12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*e^6*f) + b^2*d^4*f
+ c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^4*f + 2*b*c*d^6*f + 8*c
^2*d*e^7*f*x^7) - (log((((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)
/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*
c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^
16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^
2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b +
3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*
a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^
2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a
^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71
*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))
/(a^4*f^2*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2
*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^
6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4))
)/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6
) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c
 - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c -
 b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6))*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 1
0*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*f*(-(b^2*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)
*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b
^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*f*(-(
b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2)
 - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*
e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*
a*b^2*c))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2
*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) - (b*c^3*e^15*(7*a*c - b^2
)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*
b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) + (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*
c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) + (2*b*c^4*d
*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a
*c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4
*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^
2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^
15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b
^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)) + log(d + e*x)/(a^3*e*f) - (b*atan((x*((((((b*((2*(5120*

a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((2*(6*a^2*b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f - 5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^

6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(32*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d*e^15))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(6*a^2*b^11*c^4*d*e^16*f - 137*a^3*b^9*c^5*d*e^16*f + 1217*a^4*b^7*c^6*d*e^16*f - 5256*a^5*b^5*c^7*d*e^16*f + 11024*a^6*b^3*c^8*d*e^16*f - 8960*a^7*b*c^9*d*e^16*f))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456

*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 409
6*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3
*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048
*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^
4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e
^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 2496
0*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*
c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3*b^10*e^2*f^2 - 40
96*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5
120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c
^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3
840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^
2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*
(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f
 + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 1
2*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4
*d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3
+ 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(4*a^3*
e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^
5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 8
0*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3
+ 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144
*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b
^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5*
c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f
 - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f
^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9
*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d
*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(16*a^6*e^2*f^2*(4*a*c - b^2)^
5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 25
60*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(
a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3
- 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(
b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6
*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2
*c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^
6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 384
0*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b
^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c
^5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*
b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) + (x^2*((((((b*
((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18
*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3
*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3
 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^

10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + (b*((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f + 137*a^3*b^9*c^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f - 11024*a^6*b^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^

2)^(5/2)) - (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(64*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^16 - 21*a*b^7*c^6*e^16 + 147*a^2*b^5*c^7*e^16 - 343*a^3*b^3*c^8*e^16)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (((8960*a^7*b*c^9*e^17*f - 6*a^2*b^11*c^4*e^17*f + 137*a^3*b^9*c^5*e^17*f - 1217*a^4*b^7*c^6*e^17*f + 5256*a^5*b^5*c^7*e^17*f - 11024*a^6*b^3*c^8*e^17*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) - (b*((b*((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18*f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3*c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*

b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^
3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f
^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2
*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3
)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(
b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6
*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(1
2*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^1
9*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^3 - 227328*a
^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^13*b*c^9*e^1
9*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e
^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*
c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*
a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4
*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3
*e*f*(4*a*c - b^2)^(5/2)) - (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*
e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*
a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*
c^3*e^19*f^3 + 3840*a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97
280*a^10*b^7*c^6*e^19*f^3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*
c^8*e^19*f^3 - 163840*a^13*b*c^9*e^19*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5
*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 256
0*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a
^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 -
 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b
^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6*
(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*
c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6
f^3(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840
*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^
2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^
5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b
^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) - (((b*((4*a^2*b
^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a^4*b^8*c^5*e^15*f - 3889*a^5
*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 7952*a^7*b^2*c^8*e^15*f - 8960*
a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*f - 137*a^3*b^9*c^5*d^2*e^15
*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5*c^7*d^2*e^15*f + 11024*a^6*
b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 +
 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*
a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2
+ 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^
6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 512
0*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d
^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2

+ 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*
f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^
9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e
*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a
^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3
*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*
a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^
17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 32
8*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^
9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*
d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 -
4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 +
5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12
*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 +
3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^
5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f -
40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b
^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a
^4*b^8*c*e^2*f^2)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^
2)^(5/2)) - (((b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 10
52*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^
16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^
10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e
^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 34
56*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3
+ 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^
6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f -
2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b
^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^1
7*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11
*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f
^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^
7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^
5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*
e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096
*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 512
0*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6
*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 384
0*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*
c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2
*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f +
2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b
^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 +
15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2
*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f

^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960
*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b
^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c
- b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e
^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c
*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b
^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c
^5*f^3)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b
^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2
 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^
2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + (b^3*(b^4 + 30*a^2*
c^2 - 10*a*b^2*c)^3*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 9
60*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*
e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 1638
40*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^
3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^1
7*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3
+ 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(64*a^9*e^3*f^3*(4*a*c - b^2)^(15/2)*(
a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3
- 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(
3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a
^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4*a*c - b^2)^(15/2)
- 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*f^3*(4*a*c -
 b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4
*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(4*a*c - b^2)^(15/2)
))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160
*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 -
6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*
a*b^8*c)) - (3*b*((((4*a^2*b^12*c^3*e^15*f - 93*a^3*b^10*c^4*e^15*f + 854*a
^4*b^8*c^5*e^15*f - 3889*a^5*b^6*c^6*e^15*f + 8808*a^6*b^4*c^7*e^15*f - 795
2*a^7*b^2*c^8*e^15*f - 8960*a^7*b*c^9*d^2*e^15*f + 6*a^2*b^11*c^4*d^2*e^15*
f - 137*a^3*b^9*c^5*d^2*e^15*f + 1217*a^4*b^7*c^6*d^2*e^15*f - 5256*a^5*b^5
*c^7*d^2*e^15*f + 11024*a^6*b^3*c^8*d^2*e^15*f)/(a^6*b^12*f^3 + 4096*a^12*c
^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3
840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - (((4*a^4*b^14*c^2*e^16*f^2
- 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^4*e^16*f^2 - 5952*a^7*b^8*c^5
*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768*a^9*b^4*c^7*e^16*f^2 + 23552
*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e^16*f^2 + 2*a^4*b^13*c^3*d^2*
e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*a^6*b^9*c^5*d^2*e^16*f^2 - 12
16*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7*d^2*e^16*f^2 - 6144*a^9*b^3*
c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 2
40*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^
11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1
280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^
2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^

10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17
*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^
6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4
*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^1
7*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3
))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2
- 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^
2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*
f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)
))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*
e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096
*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 512
0*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6
*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4
*b^8*c*e^2*f^2)) - (b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^1
4 - 616*a^3*b^4*c^7*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^
2*b^5*c^7*d^2*e^14 - 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6
*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1
280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (b*(
(b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^
4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768
*a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e
^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276*
a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7
*d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^
6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 38
40*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*
e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4
0*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^
9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*
f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^
13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2
*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3
 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294
912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f
^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*
e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*
b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4
*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f
*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 20
48*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*
c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f
^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^

6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(32*a^6*e^2*f^2*(4*a*c - b^2)^5*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c)*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 3840*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*f^3*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^6*(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2*b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*a^3*e*f*(4*a*c - b^2)^(5/2))

3.659 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 499

$$\begin{aligned}
&\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2(b^2-4ac)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}ef^2} \\
&\quad-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}ef^2}
\end{aligned}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^2/(e*x+d)+1/4*(b^2
-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)
^4)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5
*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^2/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b
^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^
2/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1156, 1135, 1291, 1295, 1180, 211}

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$=-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3ef^2\,(b^2-4ac)^2\,(d+ex)}+\frac{36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\,(b^2-4ac)^2\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)\,(b^2-5ac)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3ef^2\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)\,(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3ef^2\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^2\,(b^2-4ac)\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
```

```
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-7bcx^2}{x^2(a+bx^2+cx^4)^2}\,dx,x,d+ex\right)}{4a(b^2-4ac)ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2(b^2-4ac)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{3(5b^2-12ac)(b^2-5ac)+3bc(5b^2-32ac)x^2}{x^2(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{8a^2(b^2-4ac)^2ef^2}$$

$$= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2(b^2-4ac)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{3b(5b^4-42ab^2c+92a^2c^2)+3c(5b^2-12ac)(b^2-5ac)x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{8a^3(b^2-4ac)^2ef^2}$$

$$= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2(b^2-4ac)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(3c\left(5b^5-47ab^3c+124a^2bc^2-\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,\right.}{16a^3(b^2-4ac)^{5/2}ef^2} - \frac{\left(3c\left((5b^2-12ac)(b^2-5ac)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{16a^3(b^2-4ac)^2ef^2}$$

$$= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2 ef^2(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2(b^2-4ac)^2 ef^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}ef^2} + \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2-\sqrt{b^2-4ac}(5b^4-37ab^2c+60a^2c^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

Mathematica [A] (verified)

Time = 6.14 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.15

$$\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$= -\frac{1}{a^3ef^2(d+ex)} + \frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2(-b^2+4ac)ef^2(a+b(d+ex)^2+c(d+ex)^4)^2}$$

$$+ \frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+e}{8a^3(-b^2+4ac)^2ef^2(a+b(d+ex)^2+c(d+ex)^4)}$$

$$- \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d}{\sqrt{b-\sqrt{b^2}}}\right.}{8\sqrt{2}a^3(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}ef^2}$$

$$- \frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{}}}\right.}{8\sqrt{2}a^3(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
+ e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2
*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*
c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*(-5*b
^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt
[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	1201
risch	Expression too large to display	2710

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2*(-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^2+136*a^2
*b*c^2-99*a*b^3*c+14*b^5)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*
(364*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a*b^3*c+14
*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-1645*a*b^2*c
^3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+6
8*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1
/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^2*b*c^3*d
^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129*a*b^4*c+2
1*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a*b^2*c^3*d^
6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d^4+204*a^3*c
^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b*c^2-66*a^2*b
^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^4*d^6-47*a*b^2
*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14*b^5*c*d^4+68*a
^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*a^3*b*c^2-66*a^2
*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^
2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*a^2*c^2
-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R^2+2*d*c*e*(60
*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^2*c^2*d^2+5*b^4*c*d^2+9
2*a^2*b*c^2-42*a*b^3*c+5*b^5)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c
*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+
b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/a^3/e/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10518 vs. 2(453) = 906.

Time = 1.14 (sec) , antiderivative size = 10518, normalized size of antiderivative = 21.08

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^2}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b^4*c^2 - 3
7*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b
*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 6*(28*(5*b
^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a
^2*b*c^3)*d)*e^5*x^5 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + (15*
b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^2
*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^2)
*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 4*(42*(5*b^4*c^
2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*
b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^
3 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^6 - 91*a*b^4*c + 25*a^2*
b^2*c^2 + 324*a^3*c^3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^
6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - 227*a*b^3*c^2
 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c
^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d^2 + 2*(12*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c - 227*a*b^3*c^2 +
392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)
*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 1
```

```
8*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*
b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3
*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a
^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^
6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^
6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*
d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^
4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3
*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2
*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6
+ 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^
4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2) - 3/8*integrate((5*b^5 - 42*a*b^
3*c + 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*e^2*x^2 + 2*(5*b
^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d*e*x + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*
c^3)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f^
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1704 vs. $2(453) = 906$.

Time = 0.40 (sec) , antiderivative size = 1704, normalized size of antiderivative = 3.41

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] -1/8*(7*b^4*c^2/((e*f*x + d*f)*e*f) - 47*a*b^2*c^3/((e*f*x + d*f)*e*f) + 52
*a^2*c^4/((e*f*x + d*f)*e*f) + 14*b^5*c*f/((e*f*x + d*f)^3*e) - 99*a*b^3*c^
2*f/((e*f*x + d*f)^3*e) + 136*a^2*b*c^3*f/((e*f*x + d*f)^3*e) + 7*b^6*f^3/(
(e*f*x + d*f)^5*e) - 43*a*b^4*c*f^3/((e*f*x + d*f)^5*e) + 25*a^2*b^2*c^2*f^
3/((e*f*x + d*f)^5*e) + 68*a^3*c^3*f^3/((e*f*x + d*f)^5*e) + 9*a*b^5*f^5/((
e*f*x + d*f)^7*e) - 66*a^2*b^3*c*f^5/((e*f*x + d*f)^7*e) + 108*a^3*b*c^2*f^
5/((e*f*x + d*f)^7*e))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*f^2/(e*
f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)^2) - 1/((e*f*x + d*f)*a^3*e*f) + 3/64
```

```
*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 1273
6*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2*sqrt(
b^2 - 4*a*c)*a)*e^4*f^8 + 2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3
 - 240*a^7*c^4)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f
^4*abs(a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4) - (a^3*b
^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4)^2*(5*b^5 - 42*a*b^3*
c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/(
(e*f*x + d*f)*e*f*sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f^4 + 16*a^5*b*c^
2*e^2*f^4 + sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f^4 + 16*a^5*b*c^2*e^2*
f^4)^2 - 4*(a^4*b^4*e^4*f^8 - 8*a^5*b^2*c*e^4*f^8 + 16*a^6*c^2*e^4*f^8)*(a^
3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4*f^8 - 8*a^5*b^2*c*e^4*
f^8 + 16*a^6*c^2*e^4*f^8))))/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3
- 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*e^3*f^6*abs(a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*
e^2*f^4 + 16*a^5*c^2*e^2*f^4)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c
+ 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3
*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 - 2*(5*
a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^3*b^4*e^2*f^4 - 8*a^4*
b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4) - (a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^
4 + 16*a^5*c^2*e^2*f^4)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b -
2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*x + d*f)*e*f*sqrt((a^3*b^5
*e^2*f^4 - 8*a^4*b^3*c*e^2*f^4 + 16*a^5*b*c^2*e^2*f^4 - sqrt((a^3*b^5*e^2*f
^4 - 8*a^4*b^3*c*e^2*f^4 + 16*a^5*b*c^2*e^2*f^4)^2 - 4*(a^4*b^4*e^4*f^8 - 8
*a^5*b^2*c*e^4*f^8 + 16*a^6*c^2*e^4*f^8)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^
5*c^3)))/(a^4*b^4*e^4*f^8 - 8*a^5*b^2*c*e^4*f^8 + 16*a^6*c^2*e^4*f^8))))/((
a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c
)*e^3*f^6*abs(a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4)*a
bs(a))
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 20580, normalized size of antiderivative = 41.24

$$\int \frac{1}{(df+efx)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*
c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61
26640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160
*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f
^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^1
```

2*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^1

2*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^
12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^
6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 +
 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6
 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11
*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*
d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^
2*c^13*d*e^11*f^6)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) +
18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a
^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^
7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c
- b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2)
 + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576
*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4
+ 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*
b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f
^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2717
90899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b
^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*
c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c
^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c
^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^
4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 2
5*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2
 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 1990
5600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a
^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2
*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))
)/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2
*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^
12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^
2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^
8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2)
 + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 129986
0*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7
*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048
576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f
^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^
13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^
2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2
62144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 23068672
0*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 2214592512
0*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 49606872

2688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 28346
78415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 +
3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10
) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^1
0 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^1
0 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13
*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8
*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*
b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976
384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a
^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*
b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*
b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^1
9*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a
^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 118541097369
6*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^
22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*
c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^1
4*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15
*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 154384780492
8*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 11010
55131648*a^19*b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b
^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15
*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 -
43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a
^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c
- b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e
^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b
^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^
4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^
15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4))
)^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6
+ 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 22078
03392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 8937485
1072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 8698158
12096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 17473
13491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (
-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17
794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5
*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*
c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*
b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c -
b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*
a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2
*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080

*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9
*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c
- b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b
^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^
6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 22
5*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a
*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^2
0*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^1
0*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2
*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120
*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^
4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14
*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f
^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14
*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e
^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c
^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*
b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^2
3*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^1
9*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19
*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 4960687226
88*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 283
4678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f
^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*
f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 -
2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 12
2607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1
438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 -
3949222428672*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^
8 - 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6
- 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 1910
38464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 1687
8108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 3
33226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^
6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*
e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 - 25*b^6
*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 18
8095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*
a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^
3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*
c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(51
2*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4
- 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^
10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4
+ 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^1

8*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^

11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976000*a^17*c^14*e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14*c^7*e^10*f^4 + 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c^9*e^10*f^4 + 53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^11*e^10*f^4 + 342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c^13*e^10*f^4))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - atan(((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 1779

4*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b
^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^
8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^
19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b
^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^
9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f
^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a
^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e
^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10
- 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 27
68240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 12
4017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 -
1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^
10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^
14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*
d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e
^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d
*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13
*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a
^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571
220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10)
- 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397
568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 2163002572
8*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 4967733657
60*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 291843027
7632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 320834
0570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 2717
90899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^
10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^
12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072
*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 8698158
12096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1
747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*
f^6)*1i + (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*
b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 626841
60*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c
*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2
*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b
^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^
4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a
^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*
c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f
^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 -

16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^

11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6)*1i)/((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f

^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) + 245760*a^12*b^23*c^2*e^12*f^8 - 10911744*a^13*b^21*c^3*e^12*f^8 + 220397568*a^14*b^19*c^4*e^12*f^8 - 2673082368*a^15*b^17*c^5*e^12*f^8 + 21630025728*a^16*b^15*c^6*e^12*f^8 - 122607894528*a^17*b^13*c^7*e^12*f^8 + 496773365760*a^18*b^11*c^8*e^12*f^8 - 1438679826432*a^19*b^9*c^9*e^12*f^8 + 2918430277632*a^20*b^7*c^10*e^12*f^8 - 3949222428672*a^21*b^5*c^11*e^12*f^8 + 3208340570112*a^22*b^3*c^12*e^12*f^8 - 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*((-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225 *a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6) + 191102976000*a^17*c^14*e^10*f^4 + 2851200*a^9*b^16*c^6*e^10*f^4 - 92568960*a^10*b^14*c^7*e^10*f^4 + 1312630272*a^11*b^12*c^8*e^10*f^4 - 10611136512*a^12*b^10*c^9*e^10*f^4 + 53445353472*a^13*b^8*c^10*e^10*f^4 - 171591892992*a^14*b^6*c^11*e^10*f^4 + 342580396032*a^15*b^4*c^12*e^10*f^4 - 388363714560*a^16*b^2*c^13*e^10*f^4))*(-(9*(25*b^21 + 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*2i - ((x^4*(15*b^6*e^3 + 324*a^

3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4 - 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b^2*d^2*e^3*f^2 + 84*c^2*d^6*e^3*f^2 + 2*a*b*e^3*f^2 + 20*a*c*d^2*e^3*f^2 + 70*b*c*d^4*e^3*f^2) + x^6*(84*c^2*d^3*e^6*f^2 + 14*b*c*d*e^6*f^2) + x^2*(10*b^2*d^3*e^2*f^2 + 36*c^2*d^7*e^2*f^2 + 6*a*b*d*e^2*f^2 + 20*a*c*d^3*e^2*f^2 + 42*b*c*d^5*e^2*f^2) + x^4*(5*b^2*d*e^4*f^2 + 126*c^2*d^5*e^4*f^2 + 10*a*c*d*e^4*f^2 + 70*b*c*d^3*e^4*f^2) + x^7*(36*c^2*d^2*e^7*f^2 + 2*b*c*e^7*f^2) + x^5*(b^2*e^5*f^2 + 126*c^2*d^4*e^5*f^2 + 2*a*c*e^5*f^2 + 42*b*c*d^2*e^5*f^2) + x*(a^2*e*f^2 + 5*b^2*d^4*e*f^2 + 9*c^2*d^8*e*f^2 + 6*a*b*d^2*e*f^2 + 10*a*c*d^4*e*f^2 + 14*b*c*d^6*e*f^2) + a^2*d*f^2 + b^2*d^5*f^2 + c^2*d^9*f^2 + c^2*e^9*f^2*x^9 + 2*a*b*d^3*f^2 + 2*a*c*d^5*f^2 + 2*b*c*d^7*f^2 + 9*c^2*d*e^8*f^2*x^8)

3.660 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 343

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3\,(b^2-4ac)^2\,ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef^3(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef^3(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4\,(b^2-4ac)^{5/2}\,ef^3} - \frac{3b\log(d+ex)}{a^4ef^3} + \frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2+1/4*(b^2-
2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d
)^4)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+
30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2)
)/a^4/(-4*a*c+b^2)^(5/2)/e/f^3-3*b*ln(e*x+d)/a^4/e/f^3+3/4*b*ln(a+b*(e*x+d)
^2+c*(e*x+d)^4)/a^4/e/f^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\begin{aligned}&\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx\\&=\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3ef^3\left(b^2-4ac\right)^2(d+ex)^2}\\&+\frac{20a^2c^2+3bc(b^2-6ac)\left(d+ex\right)^2-20ab^2c+3b^4}{4a^2ef^3\left(b^2-4ac\right)^2(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\\&-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\left(b^2-4ac\right)^{5/2}}\\&+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\end{aligned}$$

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{4a(b^2-4ac)\,ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{6(b^2-5ac)(b^2-2ac)+6bc(b^2-6ac)x}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{4a^2(b^2-4ac)^2\,ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \left(\frac{6(b^2-5ac)(b^2-2ac)}{ax^2} - \frac{6b(-b^2+4ac)^2}{a^2x} + \frac{6\left(b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc(b^2-4ac)^2x\right)}{a^2(a+bx+cx^2)}\right)dx, x, (d+e

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4ef^3} + \frac{3\text{Subst}\left(\int \frac{b^6-9ab^4c+23a^2b^2c^2-10a^3c^3+bc(b^2-4ac)^2x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2a^4(b^2-4ac)^2 ef^3}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4ef^3} + \frac{(3b)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^4ef^3} + \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4a^4(b^2-4ac)^2 ef^3}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3b\log(d+ex)}{a^4ef^3} + \frac{3b\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^4ef^3} - \frac{(3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d+ex)^2\right)}{2a^4(b^2-4ac)^2 ef^3}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 ef^3(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)(d+ex)^2}{4a^2(b^2-4ac)^2 ef^3(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2} ef^3} - \frac{3b\log(d+ex)}{a^4ef^3} + \frac{3b\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^4ef^3}$$

Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.34

$$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$= \frac{-\frac{2a}{(d+ex)^2} + \frac{a^2(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4c(d+ex)^2-26ab^2c^2(d+ex)^2+28a^2c^3(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))} - 12}{}$$

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d
+ e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) - (a*(4*b
^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*(d + e*x)^2 - 26*a*b^2*c^2*(d + e*
x)^2 + 28*a^2*c^3*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) - 12*b*Log[d + e*x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b
*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2
- 4*a*c)^(5/2) + (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*
Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2))/(
4*a^4*e*f^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.00 (sec) , antiderivative size = 1145, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1145
risch	Expression too large to display	2364

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(-1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*d^2
+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*a*(1
40*a^2*c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c^3*d^4
+30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^3*c^2*d^2+24*b^5*c*d^2+18*a^3*c^3
+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(42*a^2
*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74*a^2*b*c^3*d^2-55*a*b^3*c^2*d^2+8
*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c
+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c^3*d^6+4*b^4*c^2*d^6+74*a^2*b*c^3
*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c^3*d^2+14*a^2*b^2*c^2*d^2-24*a*b^
4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*c+5*a*b^5)/(16*a^2*c^2-8*a*b^2*c+
b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+
2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*b*c*(-16*a^2
*c^2+8*a*b^2*c-b^4)*_R^3+3*d*e^2*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^2+e*(-4
8*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d^2+10*a^3*c^3-23*a^2*b^2*c^2+9*a*
b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2*d^3-b^5*c*d^3+10*a^3*c^3*d-23*a^
2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2
+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+b*d^2+a)))-1/2/a^3/e/(e*x+d)^2-3*
b*ln(e*x+d)/a^4/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7550 vs. 2(329) = 658.

Time = 4.42 (sec) , antiderivative size = 15231, normalized size of antiderivative = 44.41

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^3}\,dx$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^2 - 7*a*b^
2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3 +
56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*e^6*x^6 + 6*(56*(b^4*c^2 - 7*a
*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d)*e
^5*x^5 + 6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^4*c +
 14*a^2*b^2*c^2 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^
4 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b^5*c -
29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^
4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^3 + 2*(3*b^6 - 18*a*b
^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^3*x^3 + 2*a^2*b^4 - 16*a^3*b^2*c +
32*a^4*c^2 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (168
*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^3*c + 122*a^
3*b*c^2 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*(3*b^6 - 18*a
*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5 - 68*a^2*b^3*c
 + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^7 + 9*
(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 + 4*(3*b^6 - 18*a*b^4*c + 7*a^2
*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b^5 - 68*a^2*b^3*c + 122*a^3*b*c^2)*d)*e*
x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^11*f^3*x^10 + 10*(a^3*b^4*
c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^10*f^3*x^9 + (2*a^3*b^5*c - 16*a^4*b^
3*c^2 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e
^9*f^3*x^8 + 8*(15*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + 2*(a^3*
b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^8*f^3*x^7 + (a^3*b^6 - 6*a^4*b^4
*c + 32*a^6*c^3 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 56*(
```

```
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^7*f^3*x^6 + 2*(126*(a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 56*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1
6*a^5*b*c^3)*d^3 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^6*f^3*x^5 +
(2*a^4*b^5 - 16*a^5*b^3*c + 32*a^6*b*c^2 + 210*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
 + 16*a^5*c^4)*d^6 + 140*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 1
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^5*f^3*x^4 + 4*(30*(a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 28*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^
5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 2*(a^4*b^5 - 8*
a^5*b^3*c + 16*a^6*b*c^2)*d)*e^4*f^3*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 56*(a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 15*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*
d^4 + 12*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(5*(a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 8*(a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*d^7 + 3*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 4*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)
*d)*e^2*f^3*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^10 + 2*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^8 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*
c^3)*d^6 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^4 + (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*d^2)*e*f^3) + 3*integrate(((b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*e^3*x^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^2*x^2 + (b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 1
0*a^3*c^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e*x + (b^6 - 9*a*b^
4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 +
(6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/((a^4*b^4 -
8*a^5*b^2*c + 16*a^6*c^2)*f^3) - 3*b*log(e*x + d)/(a^4*e*f^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. $2(329) = 658$.

Time = 0.43 (sec) , antiderivative size = 1791, normalized size of antiderivative = 5.22

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] 3/4*((a^4*b^8*c*e^3*f^3 - 14*a^5*b^6*c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3 -
 140*a^7*b^2*c^4*e^3*f^3 + 80*a^8*c^5*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(b*
e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x
 + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*e^3*f^3 - 14*a^5*b^6*
c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3 - 140*a^7*b^2*c^4*e^3*f^3 + 80*a^8*c^5
*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2
- 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
```

```
*a)))/(a^8*b^8*c*e^4*f^6 - 16*a^9*b^6*c^2*e^4*f^6 + 96*a^10*b^4*c^3*e^4*f^6
 - 256*a^11*b^2*c^4*e^4*f^6 + 256*a^12*c^5*e^4*f^6) - 1/4*(6*b^4*c^2*e^8*x^
8 - 42*a*b^2*c^3*e^8*x^8 + 60*a^2*c^4*e^8*x^8 + 48*b^4*c^2*d*e^7*x^7 - 336*
a*b^2*c^3*d*e^7*x^7 + 480*a^2*c^4*d*e^7*x^7 + 168*b^4*c^2*d^2*e^6*x^6 - 117
6*a*b^2*c^3*d^2*e^6*x^6 + 1680*a^2*c^4*d^2*e^6*x^6 + 336*b^4*c^2*d^3*e^5*x^
5 - 2352*a*b^2*c^3*d^3*e^5*x^5 + 3360*a^2*c^4*d^3*e^5*x^5 + 420*b^4*c^2*d^4
*e^4*x^4 - 2940*a*b^2*c^3*d^4*e^4*x^4 + 4200*a^2*c^4*d^4*e^4*x^4 + 12*b^5*c
*e^6*x^6 - 87*a*b^3*c^2*e^6*x^6 + 138*a^2*b*c^3*e^6*x^6 + 336*b^4*c^2*d^5*e
^3*x^3 - 2352*a*b^2*c^3*d^5*e^3*x^3 + 3360*a^2*c^4*d^5*e^3*x^3 + 72*b^5*c*d
*e^5*x^5 - 522*a*b^3*c^2*d*e^5*x^5 + 828*a^2*b*c^3*d*e^5*x^5 + 168*b^4*c^2*
d^6*e^2*x^2 - 1176*a*b^2*c^3*d^6*e^2*x^2 + 1680*a^2*c^4*d^6*e^2*x^2 + 180*b
^5*c*d^2*e^4*x^4 - 1305*a*b^3*c^2*d^2*e^4*x^4 + 2070*a^2*b*c^3*d^2*e^4*x^4
+ 48*b^4*c^2*d^7*e*x - 336*a*b^2*c^3*d^7*e*x + 480*a^2*c^4*d^7*e*x + 240*b^
5*c*d^3*e^3*x^3 - 1740*a*b^3*c^2*d^3*e^3*x^3 + 2760*a^2*b*c^3*d^3*e^3*x^3 +
 6*b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d^8 + 180*b^5*c*d^4*e^2*x^2
- 1305*a*b^3*c^2*d^4*e^2*x^2 + 2070*a^2*b*c^3*d^4*e^2*x^2 + 6*b^6*e^4*x^4 -
 36*a*b^4*c*e^4*x^4 + 14*a^2*b^2*c^2*e^4*x^4 + 100*a^3*c^3*e^4*x^4 + 72*b^5
*c*d^5*e*x - 522*a*b^3*c^2*d^5*e*x + 828*a^2*b*c^3*d^5*e*x + 24*b^6*d*e^3*x
^3 - 144*a*b^4*c*d*e^3*x^3 + 56*a^2*b^2*c^2*d*e^3*x^3 + 400*a^3*c^3*d*e^3*x
^3 + 12*b^5*c*d^6 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 + 36*b^6*d^2*e^2*x
^2 - 216*a*b^4*c*d^2*e^2*x^2 + 84*a^2*b^2*c^2*d^2*e^2*x^2 + 600*a^3*c^3*d^2
*e^2*x^2 + 24*b^6*d^3*e*x - 144*a*b^4*c*d^3*e*x + 56*a^2*b^2*c^2*d^3*e*x +
400*a^3*c^3*d^3*e*x + 6*b^6*d^4 - 36*a*b^4*c*d^4 + 14*a^2*b^2*c^2*d^4 + 100
*a^3*c^3*d^4 + 9*a*b^5*e^2*x^2 - 68*a^2*b^3*c*e^2*x^2 + 122*a^3*b*c^2*e^2*x
^2 + 18*a*b^5*d*e*x - 136*a^2*b^3*c*d*e*x + 244*a^3*b*c^2*d*e*x + 9*a*b^5*d
^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a
^4*c^2)/((a^3*b^4*e*f^3 - 8*a^4*b^2*c*e*f^3 + 16*a^5*c^2*e*f^3)*(c*e^5*x^5
+ 5*c*d*e^4*x^4 + 10*c*d^2*e^3*x^3 + 10*c*d^3*e^2*x^2 + 5*c*d^4*e*x + b*e^3
*x^3 + c*d^5 + 3*b*d*e^2*x^2 + 3*b*d^2*e*x + b*d^3 + a*e*x + a*d)^2) + 3/4*
b*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4
 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^4*e*f^3) - 3*b*log(abs(e*x + d))/
(a^4*e*f^3)
```

Mupad [B] (verification not implemented)

Time = 24.18 (sec) , antiderivative size = 25334, normalized size of antiderivative = 73.86

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] (log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^
```

2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^
4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3
*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*
e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c
*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30
*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*f
^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c
 - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*
e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 10
0*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^
3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*
c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*
a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (
18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3
 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b
^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3
*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*
e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6))*((27*c^
5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6) - ((
3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8
*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*
c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b
^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e
f^3(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a
c - b^2)^5))^(1/2))((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^
2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3
*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*f^3*(-(b^6 -
20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^
(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c
*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 -
30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^17*x*(
b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2))
)/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e
^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*
c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b^4 + 10*a^2*
c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b
^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4
+ 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6)))*(6*b^11*e*f^3 - 12
0*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b
^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c
^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*
b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - ((x^4*(6*b^6*e^3 + 100*a^3*c^3*e
^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + 4200*a^2*c^4*d^4*e^3 + 420*b^
4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^2*d^2*e^3 + 2070*a^2*b*c^3*d^

2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 12*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e + 168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 216*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 - 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 32*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3*c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^2*c^2*d^4 - 36*a*b^4*c*d^4)/(4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^4*(15*b^2*d^2*e^4*f^3 + 210*c^2*d^6*e^4*f^3 + 2*a*b*e^4*f^3 + 30*a*c*d^2*e^4*f^3 + 140*b*c*d^4*e^4*f^3) + x^7*(120*c^2*d^3*e^7*f^3 + 16*b*c*d*e^7*f^3) + x*(6*b^2*d^5*e*f^3 + 10*c^2*d^9*e*f^3 + 2*a^2*d*e*f^3 + 8*a*b*d^3*e*f^3 + 12*a*c*d^5*e*f^3 + 16*b*c*d^7*e*f^3) + x^3*(20*b^2*d^3*e^3*f^3 + 120*c^2*d^7*e^3*f^3 + 8*a*b*d*e^3*f^3 + 40*a*c*d^3*e^3*f^3 + 112*b*c*d^5*e^3*f^3) + x^2*(a^2*e^2*f^3 + 15*b^2*d^4*e^2*f^3 + 45*c^2*d^8*e^2*f^3 + 12*a*b*d^2*e^2*f^3 + 30*a*c*d^4*e^2*f^3 + 56*b*c*d^6*e^2*f^3) + x^5*(6*b^2*d*e^5*f^3 + 252*c^2*d^5*e^5*f^3 + 12*a*c*d*e^5*f^3 + 112*b*c*d^3*e^5*f^3) + x^8*(45*c^2*d^2*e^8*f^3 + 2*b*c*e^8*f^3) + x^6*(b^2*e^6*f^3 + 210*c^2*d^4*e^6*f^3 + 2*a*c*e^6*f^3 + 56*b*c*d^2*e^6*f^3) + a^2*d^2*f^3 + b^2*d^6*f^3 + c^2*d^10*f^3 + c^2*e^10*f^3*x^10 + 2*a*b*d^4*f^3 + 2*a*c*d^6*f^3 + 2*b*c*d^8*f^3 + 10*c^2*d*e^9*f^3*x^9) - (3*b*log(d + e*x))/(a^4*e*f^3) + (3*atan((x^2*((((((54*a^3*b^13*c^4*e^17*f^3 - 1233*a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57204*a^6*b^7*c^7*e^17*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9*e^17*f^3 + 129600*a^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4

*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a
^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*
b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 384
0*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6
*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 1
63840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*
f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5
*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^
3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 +
960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6))
- (27000*a^6*c^11*e^16 + 27*b^12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*
b^8*c^7*e^16 - 20601*a^3*b^6*c^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*
b^2*c^10*e^16)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240
*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^
14*b^2*c^5*f^9) + (3*((3*((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*
f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c
^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 -
199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b
^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^
4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*
a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4
*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 38
40*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^
6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 -
163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6
+ 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2
*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^
10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4
*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^
3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3
)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c
^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 2
27328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*
c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*
a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120
*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*

f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3
840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*
b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6
144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680
*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9
+ 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^
7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^
9 - 163840*a^16*b*c^9*e^19*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^1
0*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c
^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9
 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^1
2*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 1
0*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*
a*c - b^2)^6*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7
675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((3*((54*a^3*b^13*
c^4*e^17*f^3 - 1233*a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57
204*a^6*b^7*c^7*e^17*f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9
*e^17*f^3 + 129600*a^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 -
 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a
^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6
*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*
f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*
b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*
c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*
b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c
^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^1
3*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9
+ 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*
b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 40
96*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5
120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2
*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^
4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10
*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (((3*((153600*a^13*c^10*e^18
*f^6 + 6*a^6*b^14*c^3*e^18*f^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c
^5*e^18*f^6 + 792*a^9*b^8*c^6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 1006
08*a^11*b^4*c^8*e^18*f^6 - 199680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3

- 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(64*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) - (x*((((2*(27000*a^6*c^11*d*e^15 + 27*b^12*c^5*d*e^15 - 567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3*b^6*c^8*d*e^15 + 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((2*(129600*a^9*b*c^10*d*e^16*f^3 + 54*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4

*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e
^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)
- (((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b
^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f
^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 19968
0*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1
0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*
f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^
5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b
^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 +
328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b
^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e
^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*
c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8
*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9
- 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*
a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f
^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3
+ 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 +
640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f
^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b
*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*
c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2
*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8
*c*e^2*f^6)) - (3*((3*((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e
^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a
^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8
*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f^9 + 4096*a^15*c^
6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1
5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/((4*a^4*b^10*
e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 +
4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*
b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^
3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - (
3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^
9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3
*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^1

5*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18
*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 2273
28*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9))/(4*a^4*e*f^3*
(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^
6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^
5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2
40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*
a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^
4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a
*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^
2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*
a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^
18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97
280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*
b^3*c^8*d*e^18*f^9))/(16*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10*e^2*f^6 -
4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 +
5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15
*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^
9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10*a^4*c^4 +
120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^6
*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*
c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*((((3*((2*(153600*a^13*c^10*d*
e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^
8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e
^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))
/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*
f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9
) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c
^8*d*e^18*f^9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c
^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b
^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*
a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^1
4*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e
f^3(4*a*c - b^2)^(5/2)) - (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^
4*c)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*
b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^
9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a
^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c
^8*d*e^18*f^9))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096
*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 512

0*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6
*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 +
3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*
c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e
*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f
^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*
e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (3*((2*(129600*a^9*b*c^10*d*e^16*f^3 + 5
4*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4*b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^
6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3
 - 223200*a^8*b^3*c^9*d*e^16*f^3))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a
^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b
^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - (((2*(153600*a^13*c^10*d*e^17*f^6 + 6
*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d
*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 10
0608*a^11*b^4*c^8*d*e^17*f^6 - 199680*a^12*b^2*c^9*d*e^17*f^6))/(a^9*b^12*f
^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a
^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11
*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 -
 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18
*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^1
1*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*
d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^
9))/((4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 -
 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6
)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2
*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^
9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7
*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^
10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*
c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) +
 (27*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(163840*a^16*b*c^9*
d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + 328*a^10*b^13*c^3*d*e^18*f^9 - 38
40*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b^9*c^5*d*e^18*f^9 - 97280*a^13*b^
7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e^18*f^9 - 294912*a^15*b^3*c^8*d*e
^18*f^9))/(32*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 1
80*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(1
3/2)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*
b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2
)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4
*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^1
5*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(
15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 +

27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3
*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14) + (((((36*
a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^4*e^15*f^
3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 116532*a^7*b^
6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*e^15*f^3
+ 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 - 1233*a^4*
b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6*b^7*c^7*
d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^9*d^2*e^1
5*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^
8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c
^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^
8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*
f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*
a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^1
6*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792
*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b
^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 409
6*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*
c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 -
120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^
3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^
11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*
f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^1
6*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e
^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9
- 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 22732
8*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*
b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^
4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*
f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*
a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11
*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 -
3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 -
4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6
+ 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*
b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c
^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e
^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*
c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - (27*b^13*c^4*e^14 - 594*a*b^11*c^5*e
^14 + 43200*a^6*b*c^10*e^14 + 5319*a^2*b^9*c^6*e^14 - 24732*a^3*b^7*c^7*e^1
4 + 62748*a^4*b^5*c^8*e^14 - 82080*a^5*b^3*c^9*e^14 + 27000*a^6*c^11*d^2*e^
14 + 27*b^12*c^5*d^2*e^14 + 4779*a^2*b^8*c^7*d^2*e^14 - 20601*a^3*b^6*c^8*d
^2*e^14 + 47790*a^4*b^4*c^9*d^2*e^14 - 56700*a^5*b^2*c^10*d^2*e^14 - 567*a*
b^10*c^6*d^2*e^14)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 +
240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 614

4*a^14*b^2*c^5*f^9) + (3*((3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*
e^16*f^6 + 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*
a^10*b^7*c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*
e^16*f^6 + 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^
6*b^14*c^3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*
d^2*e^16*f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f
^6 + 100608*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)
+ ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c
^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^
2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*
a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^
17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*
a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11
*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^
2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^1
7*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2
*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e
^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b
^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*
c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(
4*a*c - b^2)^(5/2)) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(
6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e
*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^
17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13
*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f
^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*
b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4
*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^
17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^
9))/(8*a^4*e*f^3*(4*a*c - b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2
*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^
4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^
10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^
4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 1
0*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (9*(b^6 - 20*a^3*c^3 + 30*a
^2*b^2*c^2 - 10*a*b^4*c)^2*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c
^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^
4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b
^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9
 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*
b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e
^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9
 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294

912*a^15*b^3*c^8*d^2*e^17*f^9))/(32*a^8*e^2*f^6*(4*a*c - b^2)^5*(4*a^4*b^10
*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^
3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9
+ 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12
*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 10
*a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c)*(16*a^12*b^12*f^
9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13
*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/
2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(
4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*
c^2*(4*a*c - b^2)^6*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3
*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*
e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 38
40*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)) + (b*
((3*((36*a^3*b^14*c^3*e^15*f^3 - 14400*a^10*c^10*e^15*f^3 - 837*a^4*b^12*c^
4*e^15*f^3 + 8046*a^5*b^10*c^5*e^15*f^3 - 40941*a^6*b^8*c^6*e^15*f^3 + 1165
32*a^7*b^6*c^7*e^15*f^3 - 177588*a^8*b^4*c^8*e^15*f^3 + 119520*a^9*b^2*c^9*
e^15*f^3 + 129600*a^9*b*c^10*d^2*e^15*f^3 + 54*a^3*b^13*c^4*d^2*e^15*f^3 -
1233*a^4*b^11*c^5*d^2*e^15*f^3 + 11583*a^5*b^9*c^6*d^2*e^15*f^3 - 57204*a^6
*b^7*c^7*d^2*e^15*f^3 + 156276*a^7*b^5*c^8*d^2*e^15*f^3 - 223200*a^8*b^3*c^
9*d^2*e^15*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 24
0*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a
^14*b^2*c^5*f^9) - (((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6
+ 3156*a^8*b^11*c^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*
c^6*e^16*f^6 - 109824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6
+ 153600*a^13*c^10*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^
3*d^2*e^16*f^6 - 108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*
f^6 + 792*a^9*b^8*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 1006
08*a^11*b^4*c^8*d^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*
f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*
a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^1
1*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3
- 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^
9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*
c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +
16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*
c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*
e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^
9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(
2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25
60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(
a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^
9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9))
)*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^
2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*

e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3
*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)))*(b^6 - 20*a^3
*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) - ((
(3*((12*a^6*b^15*c^2*e^16*f^6 - 300*a^7*b^13*c^3*e^16*f^6 + 3156*a^8*b^11*c
^4*e^16*f^6 - 17976*a^9*b^9*c^5*e^16*f^6 + 59136*a^10*b^7*c^6*e^16*f^6 - 10
9824*a^11*b^5*c^7*e^16*f^6 + 101376*a^12*b^3*c^8*e^16*f^6 + 153600*a^13*c^1
0*d^2*e^16*f^6 - 30720*a^13*b*c^9*e^16*f^6 + 6*a^6*b^14*c^3*d^2*e^16*f^6 -
108*a^7*b^12*c^4*d^2*e^16*f^6 + 588*a^8*b^10*c^5*d^2*e^16*f^6 + 792*a^9*b^8
*c^6*d^2*e^16*f^6 - 22272*a^10*b^6*c^7*d^2*e^16*f^6 + 100608*a^11*b^4*c^8*d
^2*e^16*f^6 - 199680*a^12*b^2*c^9*d^2*e^16*f^6)/(a^9*b^12*f^9 + 4096*a^15*c
^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9
+ 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b
^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^
3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*
c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15
360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^
8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9
- 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*
a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b
^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(2*(4*a^4*b^10*e^2
*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^
2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 40
96*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6
*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(b^6 - 20*a^3*c
^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e*f^3*(4*a*c - b^2)^(5/2)) + (3*(
b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)*(6*b^11*e*f^3 - 120*a*b^9*c
*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*
f^3 + 7680*a^4*b^3*c^4*e*f^3)*(4*a^10*b^14*c^2*e^17*f^9 - 96*a^11*b^12*c^3*
e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8*c^5*e^17*f^9 + 15360*
a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 + 16384*a^16*b^2*c^8*e^
17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 32
8*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12
*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c
^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(8*a^4*e*f^3*(4*a*c -
b^2)^(5/2)*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2
*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e
^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b
^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*
c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a
^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*
a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^
7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) + (27
*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(4*a^10*b^14*c^2*e^17*f
^9 - 96*a^11*b^12*c^3*e^17*f^9 + 960*a^12*b^10*c^4*e^17*f^9 - 5120*a^13*b^8
*c^5*e^17*f^9 + 15360*a^14*b^6*c^6*e^17*f^9 - 24576*a^15*b^4*c^7*e^17*f^9 +

16384*a^16*b^2*c^8*e^17*f^9 - 163840*a^16*b*c^9*d^2*e^17*f^9 + 12*a^9*b^15*c^2*d^2*e^17*f^9 - 328*a^10*b^13*c^3*d^2*e^17*f^9 + 3840*a^11*b^11*c^4*d^2*e^17*f^9 - 24960*a^12*b^9*c^5*d^2*e^17*f^9 + 97280*a^13*b^7*c^6*d^2*e^17*f^9 - 227328*a^14*b^5*c^7*d^2*e^17*f^9 + 294912*a^15*b^3*c^8*d^2*e^17*f^9))/(64*a^12*e^3*f^9*(4*a*c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9*(4*a*c - b^2)^(15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10*c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c^6*e^14 - 32400*a^5*b^2*c^7*e^14)*(100*a^6*c^6 - 6*b^12 - 960*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^10*c)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*a^4*e*f^3*(4*a*c - b^2)^(5/2))

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 340

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

```
[Out] -d*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^2/(a+b*(e*x+d)^3
+c*(e*x+d)^6)^(1/2)+1/2*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*(e*x+d)^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)
^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/e^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

= {1403, 1804, 1362, 440, 1399, 524}

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$= \frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

```
[In] Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*e
^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1403

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-d+x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{d}{\sqrt{a+bx^3+cx^6}}+\frac{x}{\sqrt{a+bx^3+cx^6}}\right)\,dx, x, d+ex\right)}{e^2} \\
&= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2}-\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2} \\
&= \frac{\left(\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx, x, d+ex\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} \\
&\quad -\frac{\left(d\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx, x, d+ex\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}
\end{aligned}$$

$$= -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Mathematica [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

```
[In] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\left(ex+d\right)^{3}+c\left(ex+d\right)^{6}}}dx$$

```
[In] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
[Out] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{\left(ex+d\right)^{6}c+\left(ex+d\right)^{3}b+a}}\,dx$$

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (20*
c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d^2
)*e*x + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$=\int \frac{x}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x^5}}$$

```
[In] integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
[Out] Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 +
 c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d
**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{a+b\,(d+e\,x)^3+c\,(d+e\,x)^6}}\,dx$$

```
[In] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
[Out] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 398

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\text{arctanh}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

```
[Out] 1/3*arctanh(1/2*(b+2*c*(e*x+d)^3)/c^(1/2)/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
)/e^3/c^(1/2)+d^2*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*
c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^3
/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)-d*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*
c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(
1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1403, 1804, 1362, 440, 1399, 524, 1366, 635, 212}

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$= \frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$
$$-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

```
[In] Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*
Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^
3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*c*(d + e*x)^3)/
(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1403

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
```

```
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-d+x)^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2}{\sqrt{a+bx^3+cx^6}} - \frac{2dx}{\sqrt{a+bx^3+cx^6}} + \frac{x^2}{\sqrt{a+bx^3+cx^6}}\right)\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} - \frac{(2d)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, (d+ex)^3\right)}{3e^3} - \frac{\left(2d\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx, x, d+ex\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\left(d^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\,dx, x, d+ex\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2c(d+ex)^3}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3e^3}$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$
$$- \frac{d(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$
$$+ \frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

```
[In] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\left(ex+d\right)^3+c\left(ex+d\right)^6}}dx$$

```
[In] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
[Out] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{\left(ex+d\right)^6c+\left(ex+d\right)^3b+a}}\,dx$$

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (2
0*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d
^2)*e*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$= \int \frac{x^2}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x}}$$

```
[In] integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**
3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*
c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{a+b\,(d+e\,x)^3+c\,(d+e\,x)^6}}\,dx$$

```
[In] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

```
[Out] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

```
[Out] 1/21*(2+3*x)^7+1/42*(2+3*x)^14+1/63*(2+3*x)^21
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1404, 14}

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1404

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
```

```
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^6\left(1+x^7+x^{14}\right)\,dx,x,2+3x\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(x^6+x^{13}+x^{20}\right)\,dx,x,2+3x\right)\\ &= \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx=\frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(28) = 56.

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

method	result
gosper	$\frac{x(2324522934x^{20}+32543321076x^{19}+216955473840x^{18}+916034222880x^{17}+2748102668640x^{16}+6229032715584x^{15}+110738}{}$
default	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
norman	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
risch	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$
parallelrisch	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x$

```
[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14),x,method=_RETURNVERBOSE)
[Out] 1/14*x*(2324522934*x^20+32543321076*x^19+216955473840*x^18+916034222880*x^1
7+2748102668640*x^16+6229032715584*x^15+11073835938816*x^14+15819767221203*
x^13+18456408111708*x^12+17772887593188*x^11+14218430440032*x^10+9479154235
824*x^9+5266441986624*x^8+2430891860544*x^7+926214166962*x^6+288242703252*x
^5+72097012008*x^4+14148077328*x^3+2098628448*x^2+221323200*x+14795648)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.15

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x**
18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 1581976
7221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217428
8*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 + 6615
8154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 + 14990
2032*x**3 + 15808800*x**2 + 1056832*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(28) = 56$.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{1}{63} (3x+2)^{21} + \frac{1}{42} (3x+2)^{14} + \frac{1}{21} (3x+2)^7$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{(3x+2)^7 \left(3 (3x+2)^7 + 2 (3x+2)^{14} + 6\right)}{126}$$

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
[Out] ((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21} + \frac{1}{42}(2+3x)^{28}+\frac{1}{105}(2+3x)^{35}$$

```
[Out] 1/21*(2+3*x)^7+1/21*(2+3*x)^14+1/21*(2+3*x)^21+1/42*(2+3*x)^28+1/105*(2+3*x
)^35
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1404, 1366, 625\}$

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21} + \frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1404

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^6\left(1+x^7+x^{14}\right)^2\,dx,x,2+3x\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+x+x^2\right)^2\,dx,x,(2+3x)^7\right)\\
&= \frac{1}{21}\text{Subst}\left(\int \left(1+2x+3x^2+2x^3+x^4\right)\,dx,x,(2+3x)^7\right)\\
&= \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+\frac{1}{105}(2+3x)^{35}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(56) = 112.

Time = 0.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.36

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)^2\,dx$$
$$= 17451466816x + 443569828128x^2 + 7299544818384x^3 + 87406679578680x^4 + \frac{4057390785756924x^5}{5} + 6077684727888102x^6 + 37727143432895007x^7 + 197897276851452864x^8 + 889942562270387136x^9 + \frac{17344958593049772048x^{10}}{5} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263918929616324x^{13} + 221699757548270194389x^{14} + 465517091041681015296x^{15} + 872775774067455498528x^{16} + 1463104032160519033200x^{17} + 2194577166014752240080x^{18} + 2945285062308448290360x^{19} + 3534290697929473864098x^{20} + \frac{26506949038858918036881x^{21}}{7} + 3614565944605222108800x^{22} + 3064515076512846852480x^{23} + 2298383223254096766840x^{24} + \frac{7584660010542711771792x^{25}}{5} + 875152864622814086340x^{26} + 437576396725285446564x^{27} + \frac{2625458326972530284475x^{28}}{14} + 67899784121041365504x^{29} + \frac{10184967618156204825 6x^{30}}{5} + 49282101378175184 64x^{31} + 924039400840784712x^{32} + 126005372841925188x^{33} + 11118121133111046x^{34} + \frac{16677181699666569x^{35}}{35}$$

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 +
 (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 +
197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^1
0)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 940692639189
29616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 8
72775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660147522
40080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (2
6506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 306451507651
2846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^2
5)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (262545832
6972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256
*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 12600537284
1925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(46) = 92$.

Time = 0.59 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

method	result
gosper	$\frac{x(33354363399333138x^{34}+778268479317773220x^{33}+8820376098934763160x^{32}+64682758058854929840x^{31}+3449747096472}{}$
default	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263$
norman	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263$
risch	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263$
parallelrisch	$\frac{17344958593049772048}{5}x^{10} + 11821487501620716192x^{11} + 35454069480572048124x^{12} + 94069263$

```
[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*x*(33354363399333138*x^34+778268479317773220*x^33+8820376098934763160*
x^32+64682758058854929840*x^31+344974709647226292480*x^30+14258954665418686
75584*x^29+4752984888472895585280*x^28+13127291634862651422375*x^27+3063034
7770769981259480*x^26+61260700523596986043800*x^25+106185240147597964805088
*x^24+160886825627786773678800*x^23+214516055355899279673600*x^22+253019616
122365547616000*x^21+265069490388589180368810*x^20+247400348855063170486860
*x^19+206169954361591380325200*x^18+153620401621032656805600*x^17+102417282
251236332324000*x^16+61094304184721884896960*x^15+32586196372917671070720*x
^14+15518983028378913607230*x^13+6584848474325073142680*x^12+24817848636400
43368680*x^11+827504125113450133440*x^10+242829420302696808672*x^9+62295979
358927099520*x^8+13852809379601700480*x^7+2640900040302650490*x^6+425437930
952167140*x^5+56803471000596936*x^4+6118467570507600*x^3+510968137286880*x^
2+31049887968960*x+1221602677120)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{16677181699666569}{35}x^{35} + 11118121133111046\,x^{34} + 126005372841925188\,x^{33} + 924039400840784712\,x^{32} + 4928210137817518464\,x^{31} + \frac{101849676181562048256}{5}x^{30} + 67899784121041365504\,x^{29} + \frac{2625458326972530284475}{14}x^{28} + 437576396725285446564\,x^{27} + 875152864622814086340\,x^{26} + \frac{7584660010542711771792}{5}x^{25} + 2298383223254096766840\,x^{24} + 3064515076512846852480\,x^{23} + 3614565944605222108800\,x^{22} + \frac{26506949038858918036881}{7}x^{21} + 3534290697929473864098\,x^{20} + 2945285062308448290360\,x^{19} + 2194577166014752240080\,x^{18} + 1463104032160519033200\,x^{17} + 872775774067455498528\,x^{16} + 465517091041681015296\,x^{15} + 221699757548270194389\,x^{14} + 94069263918929616324\,x^{13} + 35454069480572048124\,x^{12} + 11821487501620716192\,x^{11} + \frac{17344958593049772048}{5}x^{10} + 889942562270387136\,x^{9} + 197897276851452864\,x^{8} + 37727143432895007\,x^{7} + 6077684727888102\,x^{6} + \frac{4057390785756924}{5}x^{5} + 87406679578680\,x^{4} + 7299544818384\,x^{3} + 443569828128\,x^{2} + 17451466816\,x$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(41) = 82.

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.34

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 dx$$
$$= \frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} + 67899784121041365504x^{29} + \frac{2625458326972530284475x^{28}}{14} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792x^{25}}{5} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881x^{21}}{7} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048x^{10}}{5} + 889942562270387136x^{9} + 197897276851452864x^{8} + 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924x^{5}}{5} + 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x$$

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x
**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815
62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**2
8/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 75846600
10542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852
480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7
 + 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 21945771660
14752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x*
*16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 940692639
18929616324*x**13 + 35454069480572048124*x**12 + 11821487501620716192*x**11
 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452
864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405739078575692
4*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 dx$$
$$= \frac{16677181699666569}{35}x^{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256}{5}x^{30} + 67899784121041365504x^{29} + \frac{262545832697253028447 5}{14}x^{28} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792}{5}x^{25} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881}{7}x^{21} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048}{5}x^{10} + 889942562270387136x^9 + 197897276851452864x^8 + 37727143432895007x^7 + 6077684727888102x^6 + \frac{4057390785756924}{5}x^5 + 87406679578680x^4 + 7299544818384x^3 + 443569828128x^2 + 17451466816x$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 262545832697253028447 5/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{105} (3x+2)^{35} + \frac{1}{42} (3x+2)^{28} + \frac{1}{21} (3x+2)^{21} + \frac{1}{21} (3x+2)^{14} + \frac{1}{21} (3x+2)^7$$

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)
^14 + 1/21*(3*x + 2)^7
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{(3x+2)^7}{21} + \frac{(3x+2)^{14}}{21} + \frac{(3x+2)^{21}}{21} + \frac{(3x+2)^{28}}{42} + \frac{(3x+2)^{35}}{105}$$

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

```
[Out] (3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*x
 + 2)^35/105
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-ty
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```